Computer Algebra
Independent Integration Tests

Summer 2024

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/334-7.2.4

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**417**]. This is test number [334].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.52 (415)	0.48 (2)
Rubi	99.28 (414)	0.72 (3)
Maple	84.41 (352)	15.59 (65)
Fricas	42.45 (177)	57.55 (240)
Maxima	40.77 (170)	59.23 (247)
Reduce	28.78 (120)	71.22 (297)
Mupad	18.94 (79)	81.06 (338)
Giac	12.23 (51)	87.77 (366)
Sympy	12.23 (51)	87.77 (366)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

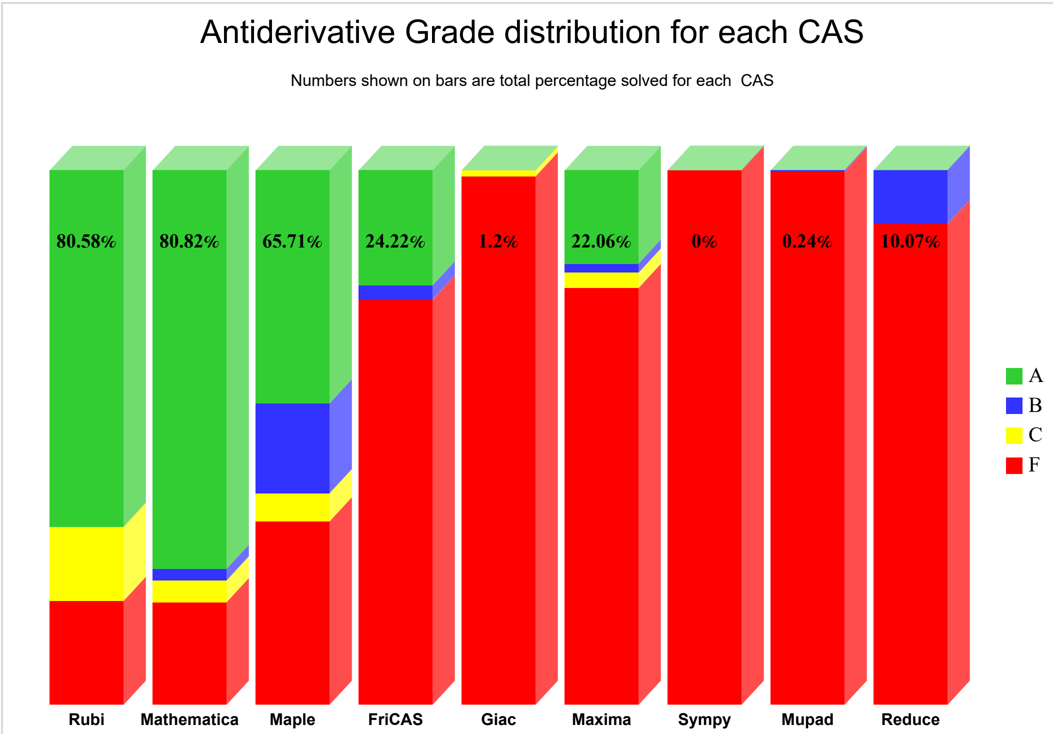
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

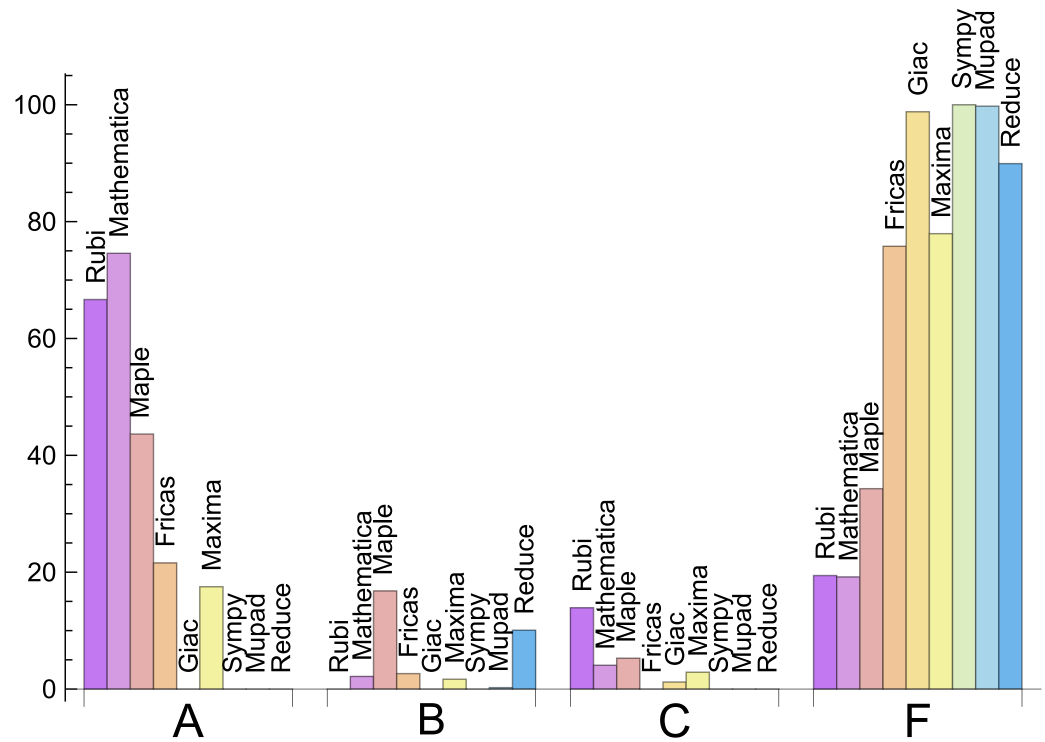
System	% A grade	% B grade	% C grade	% F grade
Mathematica	74.580	2.158	4.077	19.185
Rubi	66.667	0.000	13.909	19.424
Maple	43.645	16.787	5.276	34.293
Fricas	21.583	2.638	0.000	75.779
Maxima	17.506	1.679	2.878	77.938
Giac	0.000	0.000	1.199	98.801
Mupad	0.000	0.240	0.000	99.760
Reduce	0.000	10.072	0.000	89.928
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	2	100.00	0.00	0.00
Rubi	3	100.00	0.00	0.00
Maple	65	100.00	0.00	0.00
Fricas	240	95.42	0.00	4.58
Maxima	247	92.31	0.00	7.69
Reduce	297	100.00	0.00	0.00
Giac	366	50.55	0.82	48.63
Mupad	338	0.00	100.00	0.00
Sympy	366	69.13	30.87	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Maxima	0.23
Reduce	0.26
Maple	0.81
Rubi	1.12
Mathematica	2.17
Giac	2.81
Mupad	3.58
Sympy	25.87

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	28.68	1.02	28.00	1.00
Sympy	32.06	1.16	29.00	1.03
Giac	34.84	1.04	28.00	1.00
Reduce	121.81	2.19	89.00	1.40
Maxima	179.23	2.89	135.50	1.00
Fricas	188.55	1.78	114.00	1.51
Rubi	215.01	0.93	174.00	1.00
Mathematica	282.95	1.09	159.00	1.00
Maple	454.56	1.85	240.00	1.21

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

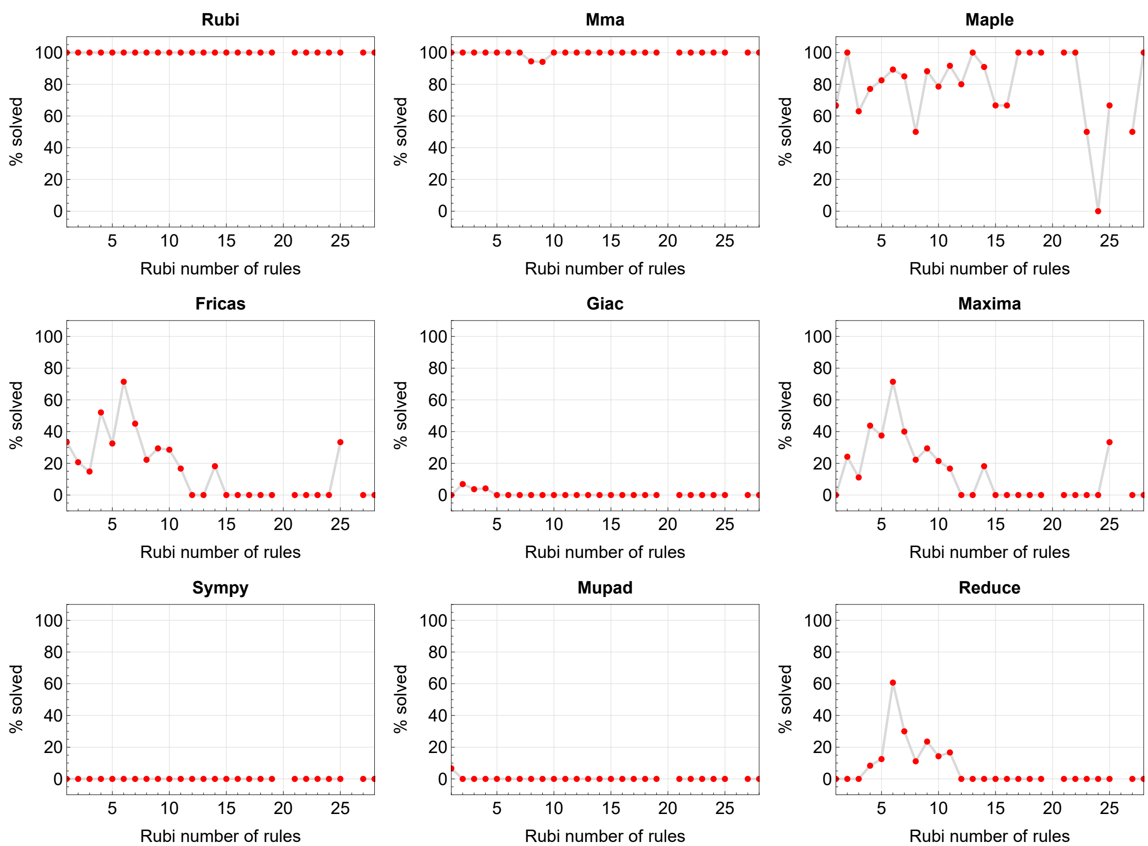


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

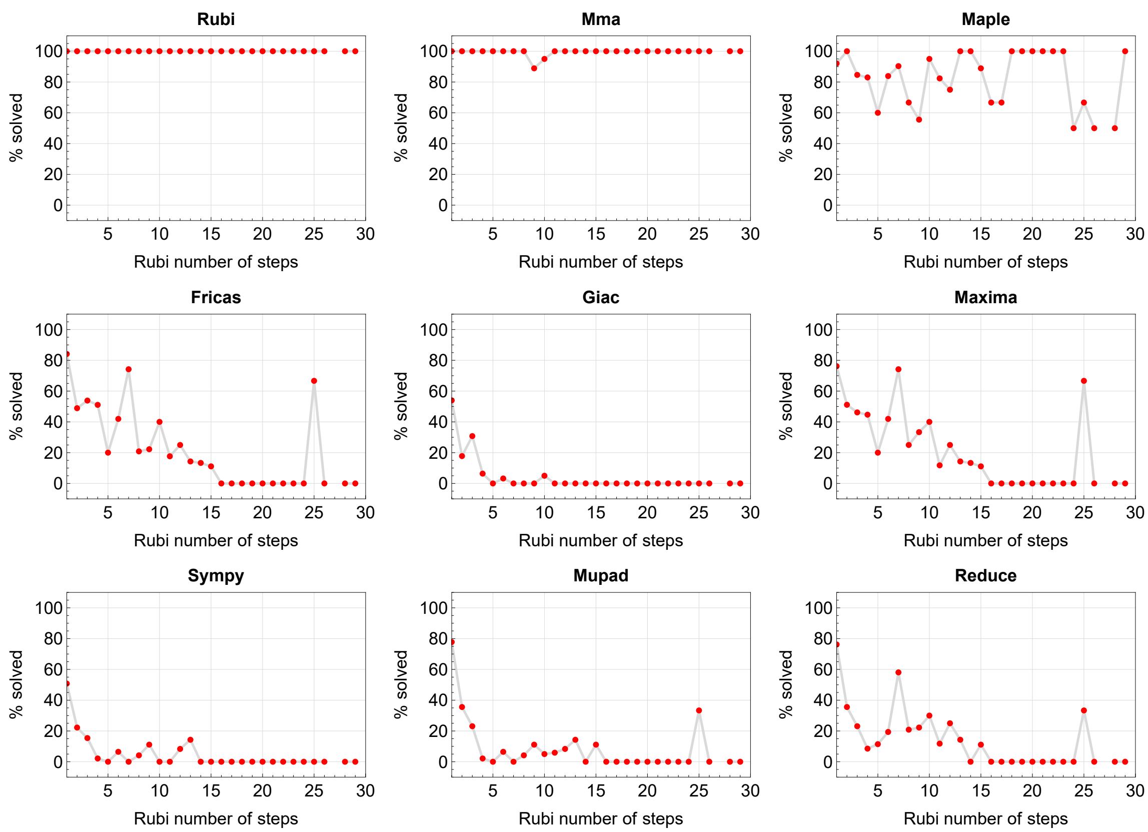


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

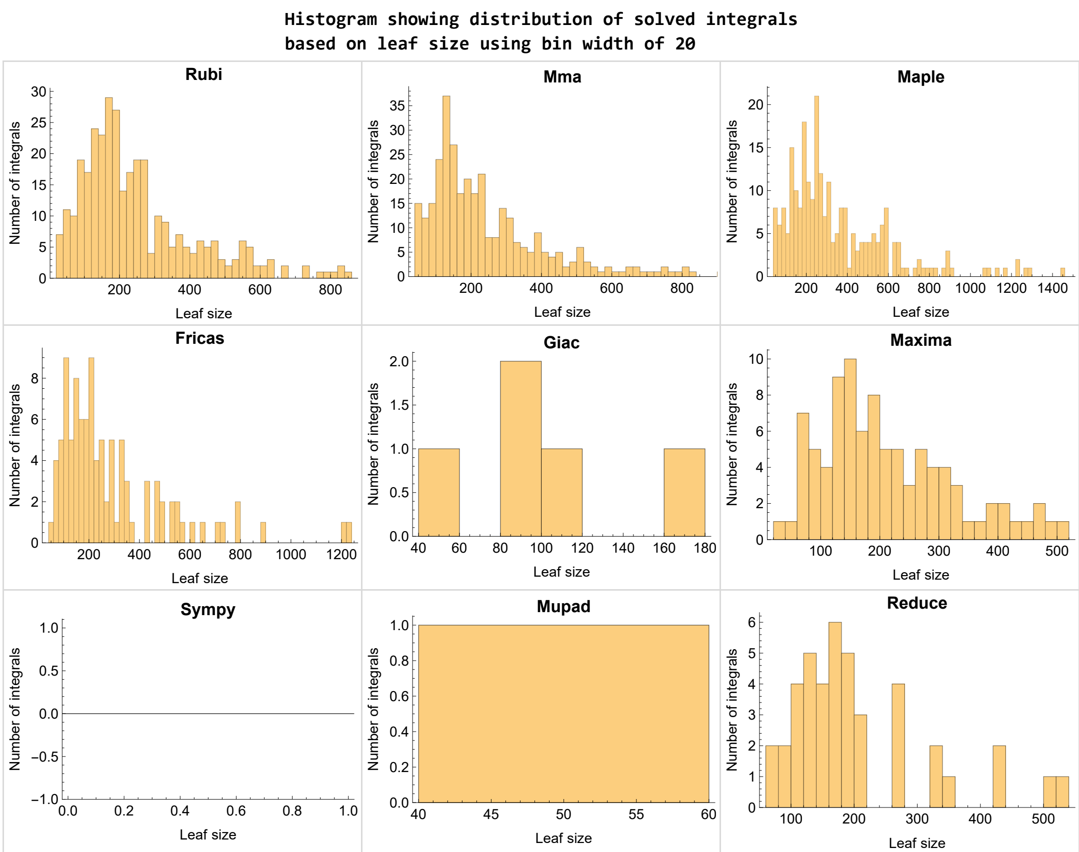


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

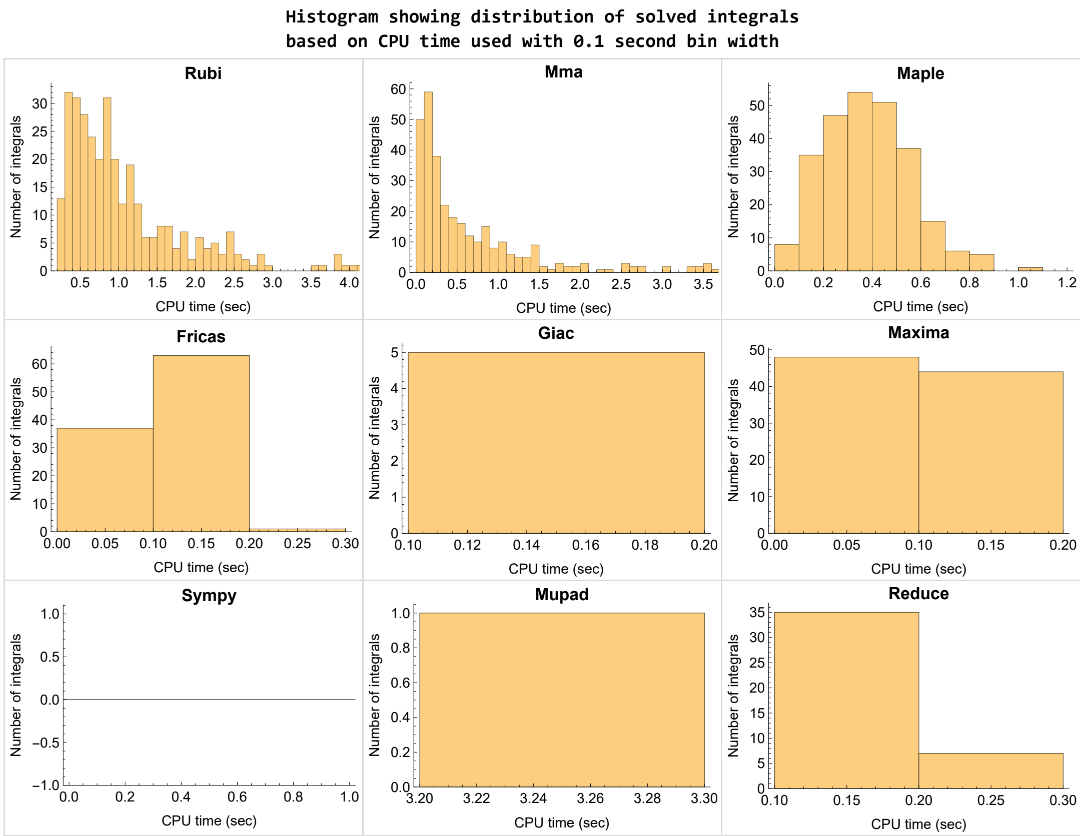


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

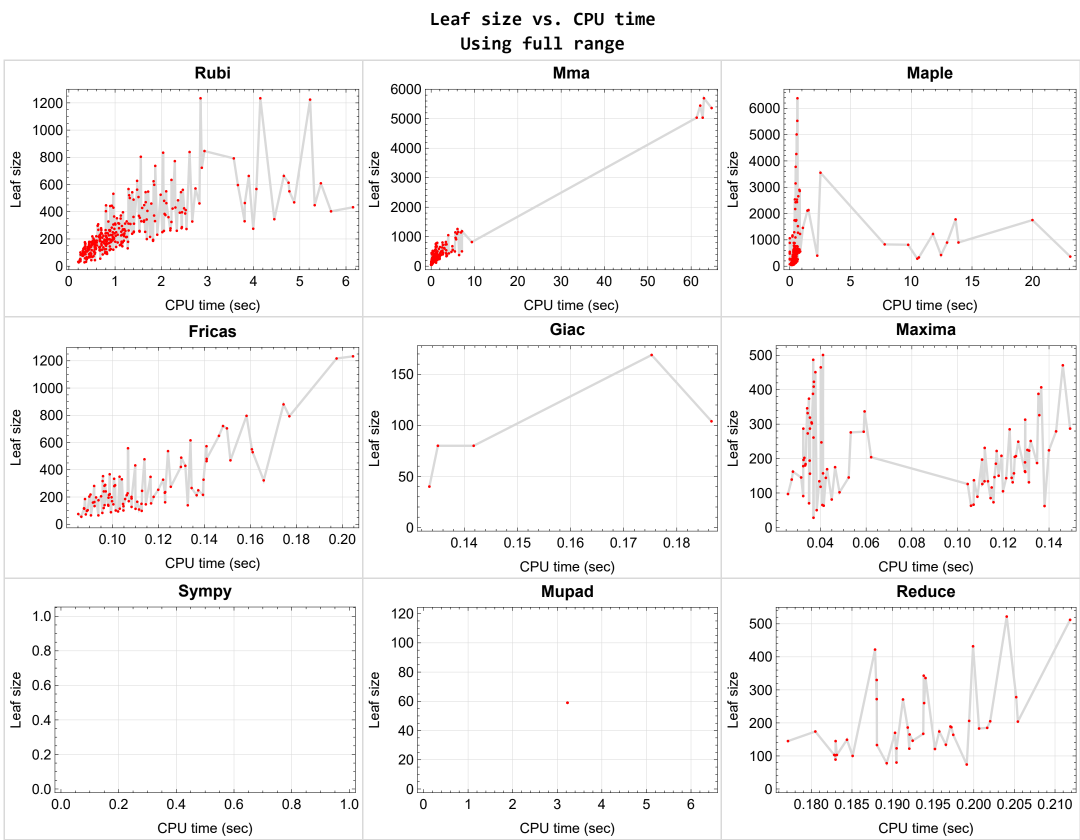


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{143, 144, 145, 216, 217, 218, 219, 220, 221, 230, 236, 237, 242, 243, 248, 249, 255, 256, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 277, 278, 282, 283, 287, 288, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 316, 321, 322, 326, 327, 331, 332, 337, 338, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 415, 416, 417}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {6, 8, 15, 16, 17, 18, 24, 26, 27, 146, 147, 148, 152, 153, 154, 164, 166, 172, 174, 180, 181, 190, 192, 209, 375, 377, 386}

Mathematica {4, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 30, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 57, 58, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 97, 98, 99, 107, 110, 112, 115, 117, 120, 125, 127, 136, 160, 162, 163, 164, 165, 166, 168, 170, 171, 172, 173, 174, 176, 178, 179, 180, 181, 182, 184, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 215, 229, 231, 232, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 307, 308, 310, 312, 313, 315, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 339, 340, 361, 363, 370, 372, 374, 376, 379, 381, 383, 385, 387, 388, 389, 395, 396, 398, 399, 400, 401, 403, 404, 407, 408, 409, 410, 411}

Maple {388, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 407, 408, 409, 410, 411}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

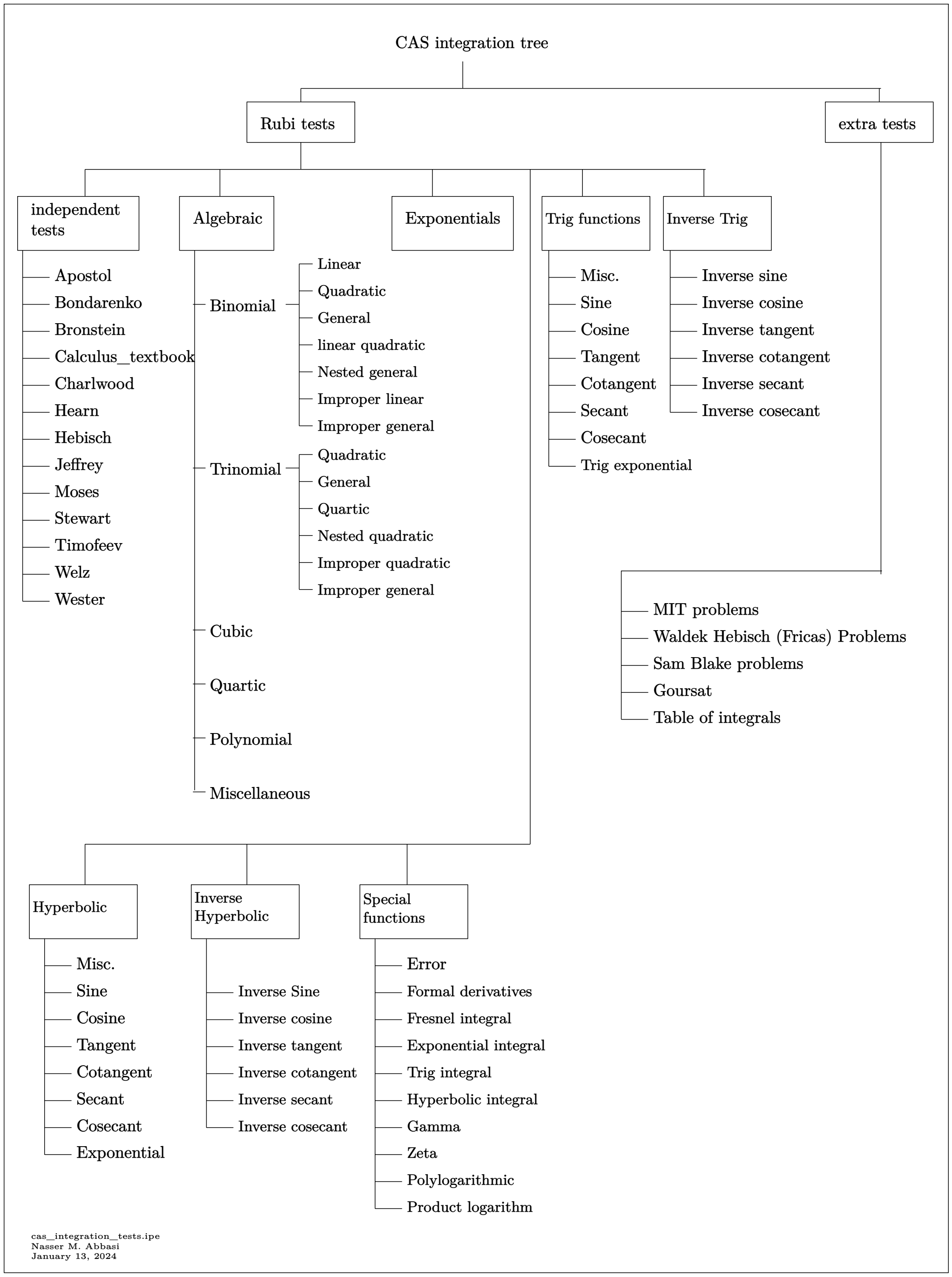
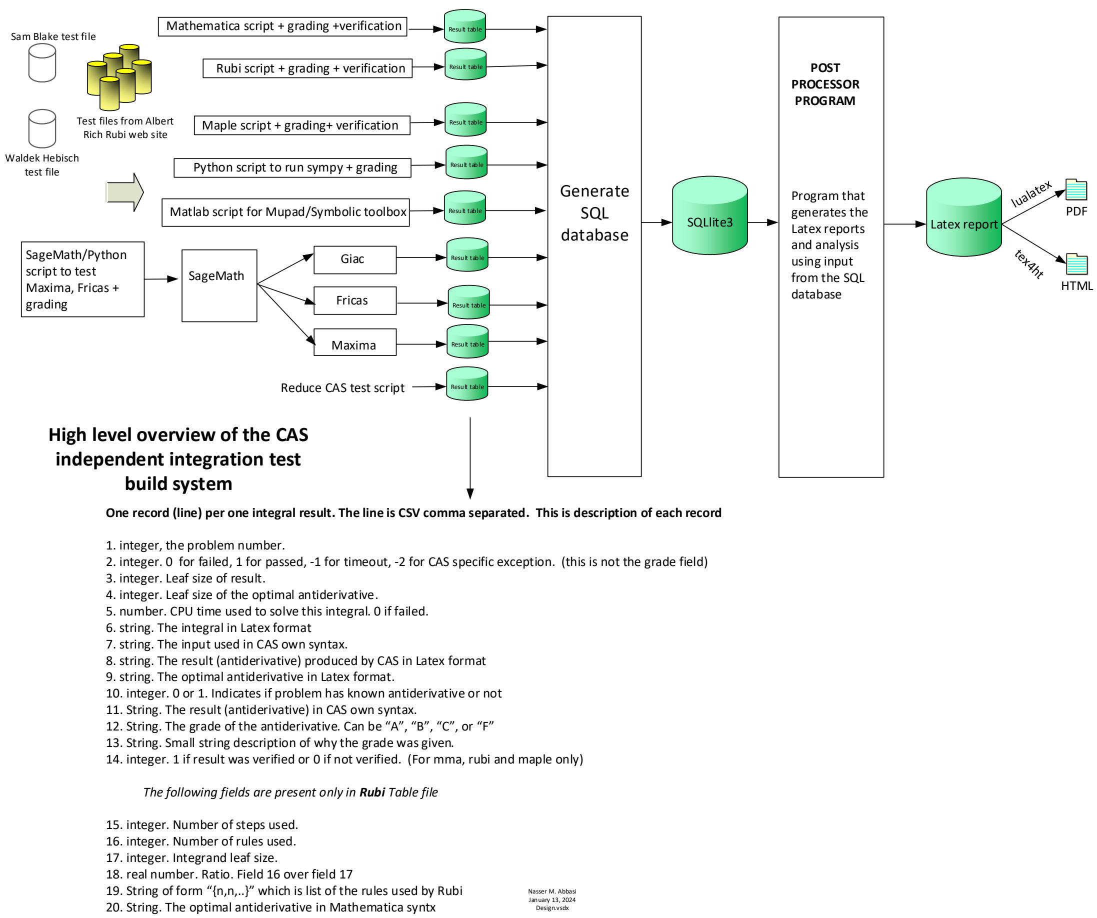


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 167, 168, 169, 170, 171, 173, 177, 178, 179, 181, 183, 184, 185, 186, 187, 188, 189, 191, 198, 200, 206, 208, 209, 210, 211, 212, 213, 215, 222, 223, 224, 225, 226, 227, 229, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 251, 252, 253, 254, 257, 258, 259, 260, 273, 279, 281, 284, 286, 289, 290, 291, 294, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 335, 336, 339, 340, 341, 342, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414 }

B grade { }

C grade { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 164, 166, 172, 174, 180, 190, 192, 193, 194, 195, 196, 197, 199, 201, 202, 203, 204, 205, 207, 214, 228, 274, 275, 276, 280, 285, 292, 293 }

F normal fail { 175, 176, 182 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 222, 223, 224, 225, 226, 227, 228, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 251, 252, 253, 254, 257, 258, 259, 260, 273, 274, 275, 276, 279, 280, 281, 284, 285, 286, 289, 290, 291, 292, 293, 294, 307, 308, 310, 312, 313, 315, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 335, 336, 339, 340, 341, 342, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 390, 391, 393, 395, 397, 405, 406, 412, 413, 414 }

B grade { 33, 42, 44, 165, 173, 181, 191, 200, 229 }

C grade { 387, 389, 392, 394, 396, 398, 399, 400, 401, 402, 403, 404, 407, 408, 409, 410, 411 }

F normal fail { 309, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 59, 62, 63, 64, 65, 66, 67, 71, 72, 77, 78, 79, 80, 81, 82, 83, 87, 88, 93, 94, 95, 96, 97, 98, 99, 101, 104, 105, 107, 108, 109, 110, 111, 113, 116, 117, 118, 119, 120, 121, 125, 127, 128, 130, 132, 133, 136, 137, 159, 167, 172, 174, 175, 177, 180, 182, 183, 193, 194, 196, 202, 210, 211, 212, 214, 223, 224, 225, 226, 228, 231, 232, 233, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 251, 252, 254, 257, 258, 260, 273, 279, 280, 284, 285, 289, 290, 291, 292, 294, 336, 342, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 390 }

B grade { 55, 56, 57, 58, 60, 61, 68, 69, 70, 73, 74, 75, 76, 84, 85, 86, 89, 90, 91, 92, 100, 102, 103, 106, 112, 114, 115, 122, 123, 124, 126, 129, 131, 134, 135, 160, 161, 162, 164, 166, 168, 169, 170, 176, 178, 184, 185, 186, 187, 188, 190, 192, 195, 197, 199, 201, 203, 204, 205, 207, 208, 209, 222, 274, 276, 281, 286, 397, 405, 406 }

C grade { 32, 387, 388, 389, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 407, 408, 409, 410, 411 }

F normal fail { 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 163, 165, 171, 173, 179, 181, 189, 191, 198, 200, 206, 213, 215, 227, 229, 234, 253, 259, 275, 293, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 335, 339, 340, 341, 412, 413, 414 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 60, 61, 62, 63, 64, 73, 74, 75, 76, 77, 78, 79, 80, 90, 91, 92, 93, 94, 95, 96, 101, 103, 106, 108, 109, 111, 113, 119, 121, 123, 130, 132, 137, 159, 161, 167, 169, 175, 177, 183, 185, 209, 211, 223, 225, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 377, 378, 379, 380, 381, 382, 384, 386 }

B grade { 59, 135, 187, 254, 260, 294, 336, 342, 397, 405, 406 }

C grade { }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 97, 98, 99, 100, 102, 104, 105, 107, 110, 112, 114, 115, 116, 117, 118, 120, 122, 124, 125, 126, 127, 128, 129, 131, 133, 134, 136, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 165, 166, 168, 170, 171, 172, 173, 174, 176, 178, 179, 180, 181, 182, 184, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 215, 222, 224, 226, 227, 228, 229, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 251, 252, 253, 257, 258, 259, 273, 274, 275, 276, 279, 280, 281, 284, 285, 286, 289, 290, 291, 292, 293, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 335, 339, 340, 341, 365, 367, 374, 376, 383, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 407, 408, 409, 410, 411, 412, 413, 414 }

F(-1) timedout fail { }

F(-2) exception fail { 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317 }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 9, 10, 12, 14, 16, 18, 23, 25, 27, 60, 61, 62, 63, 64, 74, 75, 76, 77, 78, 79, 80, 91, 92, 93, 94, 95, 96, 101, 103, 108, 111, 114, 116, 121, 122, 124, 128, 159, 161, 167, 169, 175, 177, 183, 185, 187, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 377, 378, 379, 380, 381, 382, 384, 386 }

B grade { 11, 13, 19, 20, 21, 22, 40 }

C grade { 59, 73, 90, 106, 130, 132, 135, 137, 209, 211, 223, 225 }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 97, 98, 99, 100, 102, 104, 105, 107, 109, 110, 112, 113, 115, 117, 118, 119, 120, 123, 125, 126, 127, 133, 134, 136, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 165, 166, 168, 170, 171, 172, 173, 174, 176, 178, 179, 180, 181, 182, 184, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 212, 213, 214, 215, 226, 227, 228, 229, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 251, 252, 253, 254, 257, 258, 259, 260, 273, 274, 275, 276, 279, 280, 281, 284, 285, 286, 289, 290, 291, 292, 293, 294, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 335, 336, 339, 340, 341, 342, 365, 367, 374, 376, 383, 385, 388, 390, 392, 394, 396, 398, 399, 404, 405, 406, 407, 408, 412, 413, 414 }

F(-1) timedout fail { }

F(-2) exception fail { 129, 131, 208, 210, 222, 224, 387, 389, 391, 393, 395, 397, 400, 401, 402, 403, 409, 410, 411 }

Giac

A grade { }

B grade { }

C grade { 132, 135, 137, 211, 225 }

F normal fail { 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 68, 69, 84, 85, 100, 102, 103, 104, 105, 106, 107, 108, 112, 113, 114, 115, 116, 117, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 134, 136, 138, 139, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 168, 176, 184, 186, 187, 188, 189, 190, 191, 192, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 215, 222, 224, 226, 227, 228, 229, 231, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 252, 253, 254, 258, 259, 260, 274, 275, 276, 279, 280, 281, 284, 285, 286, 290, 292, 293, 294, 308, 310, 313, 315, 317, 318, 323, 328, 334, 335, 336, 340, 341, 342, 365, 366, 367, 368, 374, 375, 376, 377, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413, 414 }

F(-1) timedout fail { 353, 356, 357 }

F(-2) exception fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 37, 38, 47, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 109, 110, 111, 119, 121, 130, 140, 141, 142, 146, 147, 148, 159, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 185, 193, 194, 202, 209, 216, 217, 218, 223, 232, 236, 242, 248, 251, 257, 261, 264, 266, 268, 269, 273, 277, 282, 287, 289, 291, 295, 298, 300, 302, 303, 307, 309, 311, 312, 314, 316, 319, 320, 321, 322, 324, 325, 326, 327, 329, 330, 331, 332, 333, 339, 351, 352, 360, 361, 362, 363, 364, 369, 370, 371, 372, 373, 378, 379, 380, 381, 382, 388, 396, 404, 405 }

Mupad

A grade { }

B grade { 294 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 251, 252, 253, 254, 257, 258, 259, 260, 273, 274, 275, 276, 279, 280, 281, 284, 285, 286, 289, 290, 291, 292, 293, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 335, 336, 339, 340, 341, 342, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 70, 71, 72, 80, 81, 82, 83, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 142, 148, 149, 150, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 238, 239, 240, 241, 250, 251, 252, 253, 254, 257, 258, 259, 260, 273, 274, 275, 276, 279, 280, 281, 289, 290, 291, 292, 293, 294, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 320, 333, 334, 335, 336, 339, 340, 341, 342, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 414 }

F(-1) timedout fail { 19, 20, 52, 53, 54, 61, 68, 69, 73, 74, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 118, 119, 126, 127, 128, 138, 139, 140, 141, 145, 146, 147, 151, 152, 153, 154, 156, 157, 167, 168, 175, 176, 177, 178, 179, 180, 181, 182, 207, 216, 217, 244, 245, 246, 247, 248, 249, 268, 272, 284, 285, 286, 287, 288, 301, 302, 305, 306, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 348, 349, 351, 352, 355, 356, 357, 359, 378, 399, 400, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 416, 417 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 377, 378, 379, 380, 381, 382, 384, 386 }

C grade { }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 250, 251, 252, 253, 254, 257, 258, 259, 260, 273, 274, 275, 276, 279, 280, 281, 284, 285, 286, 289, 290, 291, 292, 293, 294, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 333, 334, 335, 336, 339, 340, 341, 342, 365, 367, 374, 376, 383, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	163	91	94	184	113	0	0	121	0
N.S.	1	1.08	0.60	0.62	1.22	0.75	0.00	0.00	0.80	0.00
time (sec)	N/A	0.356	0.111	0.199	0.034	0.110	0.000	0.000	0.195	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	147	166	151	202	108	0	0	122	0
N.S.	1	1.09	1.23	1.12	1.50	0.80	0.00	0.00	0.90	0.00
time (sec)	N/A	0.355	0.069	0.181	0.033	0.112	0.000	0.000	0.192	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	127	89	86	145	103	0	0	102	0
N.S.	1	1.05	0.74	0.71	1.20	0.85	0.00	0.00	0.84	0.00
time (sec)	N/A	0.350	0.075	0.151	0.032	0.089	0.000	0.000	0.183	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	101	100	136	162	98	0	0	103	0
N.S.	1	1.03	1.02	1.39	1.65	1.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.250	0.106	0.210	0.028	0.089	0.000	0.000	0.183	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	89	71	71	97	83	0	0	89	0
N.S.	1	1.03	0.83	0.83	1.13	0.97	0.00	0.00	1.03	0.00
time (sec)	N/A	0.272	0.052	0.000	0.026	0.096	0.000	0.000	0.183	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	140	130	131	0	0	0	0	75	0
N.S.	1	1.20	1.11	1.12	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.672	0.095	0.322	0.000	0.000	0.000	0.000	0.194	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	110	92	66	127	0	0	74	0
N.S.	1	0.97	1.45	1.21	0.87	1.67	0.00	0.00	0.97	0.00
time (sec)	N/A	0.534	0.117	0.100	0.107	0.108	0.000	0.000	0.199	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	141	107	128	0	0	0	0	71	0
N.S.	1	1.04	0.79	0.95	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.023	0.066	0.230	0.000	0.000	0.000	0.000	0.205	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	93	127	117	89	146	0	0	78	0
N.S.	1	1.03	1.41	1.30	0.99	1.62	0.00	0.00	0.87	0.00
time (sec)	N/A	0.337	0.170	0.122	0.109	0.115	0.000	0.000	0.189	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	200	124	124	319	165	0	0	164	0
N.S.	1	0.97	0.60	0.60	1.55	0.80	0.00	0.00	0.80	0.00
time (sec)	N/A	0.829	0.128	0.203	0.035	0.111	0.000	0.000	0.197	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	231	194	192	346	161	0	0	165	0
N.S.	1	1.16	0.97	0.96	1.73	0.80	0.00	0.00	0.82	0.00
time (sec)	N/A	0.674	0.130	0.187	0.034	0.123	0.000	0.000	0.192	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	180	116	116	261	153	0	0	145	0
N.S.	1	1.02	0.66	0.66	1.47	0.86	0.00	0.00	0.82	0.00
time (sec)	N/A	0.809	0.115	0.192	0.037	0.117	0.000	0.000	0.183	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	133	126	170	287	149	0	0	146	0
N.S.	1	0.98	0.93	1.25	2.11	1.10	0.00	0.00	1.07	0.00
time (sec)	N/A	0.328	0.137	0.194	0.035	0.105	0.000	0.000	0.192	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	155	99	99	194	133	0	0	133	0
N.S.	1	1.08	0.69	0.69	1.36	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.440	0.072	0.145	0.035	0.092	0.000	0.000	0.188	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	241	208	192	0	0	0	0	120	0
N.S.	1	1.31	1.13	1.04	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.988	0.428	0.333	0.000	0.000	0.000	0.000	0.214	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	145	131	139	143	201	0	0	134	0
N.S.	1	1.04	0.94	0.99	1.02	1.44	0.00	0.00	0.96	0.00
time (sec)	N/A	0.886	0.123	0.168	0.121	0.118	0.000	0.000	0.197	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	249	182	206	0	0	0	0	138	0
N.S.	1	1.24	0.91	1.03	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	1.024	0.221	0.333	0.000	0.000	0.000	0.000	0.195	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	163	135	150	137	213	0	0	123	0
N.S.	1	1.15	0.95	1.06	0.96	1.50	0.00	0.00	0.87	0.00
time (sec)	N/A	0.512	0.127	0.164	0.107	0.137	0.000	0.000	0.190	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	241	147	154	465	201	0	0	205	0
N.S.	1	0.94	0.57	0.60	1.82	0.79	0.00	0.00	0.80	0.00
time (sec)	N/A	0.794	0.166	0.210	0.040	0.108	0.000	0.000	0.202	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	219	162	233	501	197	0	0	206	0
N.S.	1	0.97	0.72	1.03	2.22	0.87	0.00	0.00	0.91	0.00
time (sec)	N/A	0.508	0.209	0.197	0.041	0.093	0.000	0.000	0.199	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	221	139	146	388	189	0	0	186	0
N.S.	1	0.97	0.61	0.64	1.71	0.83	0.00	0.00	0.82	0.00
time (sec)	N/A	1.194	0.181	0.203	0.037	0.108	0.000	0.000	0.192	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	161	150	202	423	185	0	0	187	0
N.S.	1	0.97	0.90	1.22	2.55	1.11	0.00	0.00	1.13	0.00
time (sec)	N/A	0.437	0.235	0.194	0.037	0.088	0.000	0.000	0.197	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	194	123	130	302	169	0	0	174	0
N.S.	1	1.02	0.64	0.68	1.58	0.88	0.00	0.00	0.91	0.00
time (sec)	N/A	0.908	0.116	0.141	0.036	0.100	0.000	0.000	0.180	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	362	305	245	0	0	0	0	161	0
N.S.	1	1.51	1.28	1.03	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	1.399	0.436	0.374	0.000	0.000	0.000	0.000	0.215	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	177	136	178	231	249	0	0	174	0
N.S.	1	0.93	0.72	0.94	1.22	1.31	0.00	0.00	0.92	0.00
time (sec)	N/A	0.714	0.244	0.167	0.112	0.137	0.000	0.000	0.196	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	370	303	258	0	0	0	0	179	0
N.S.	1	1.39	1.13	0.97	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	1.516	0.461	0.352	0.000	0.000	0.000	0.000	0.201	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	187	142	195	208	253	0	0	183	0
N.S.	1	0.94	0.72	0.98	1.05	1.28	0.00	0.00	0.92	0.00
time (sec)	N/A	0.815	0.251	0.164	0.119	0.120	0.000	0.000	0.201	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	187	227	232	0	0	0	0	74	0
N.S.	1	1.18	1.44	1.47	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.192	0.370	0.350	0.000	0.000	0.000	0.000	0.222	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	146	151	217	0	0	0	0	69	0
N.S.	1	1.04	1.08	1.55	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.158	0.193	0.294	0.000	0.000	0.000	0.000	0.206	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	155	181	0	0	0	0	64	0
N.S.	1	1.00	1.52	1.77	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.821	0.199	0.283	0.000	0.000	0.000	0.000	0.207	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	85	137	0	0	0	0	58	0
N.S.	1	1.01	1.15	1.85	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.475	0.064	0.207	0.000	0.000	0.000	0.000	0.202	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	58	64	180	0	0	0	0	54	0
N.S.	1	0.98	1.08	3.05	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.350	0.013	0.368	0.000	0.000	0.000	0.000	0.196	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	65	129	191	0	0	0	0	58	0
N.S.	1	1.07	2.11	3.13	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.450	0.259	0.269	0.000	0.000	0.000	0.000	0.202	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	132	143	0	0	0	0	65	0
N.S.	1	1.04	1.39	1.51	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.568	0.193	0.351	0.000	0.000	0.000	0.000	0.221	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	117	212	243	0	0	0	0	87	0
N.S.	1	0.99	1.80	2.06	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.664	0.297	0.323	0.000	0.000	0.000	0.000	0.220	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	178	223	190	0	0	0	0	85	0
N.S.	1	1.13	1.42	1.21	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.913	0.224	0.388	0.000	0.000	0.000	0.000	0.221	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	198	244	254	0	0	0	0	167	0
N.S.	1	1.12	1.38	1.44	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.882	0.647	0.365	0.000	0.000	0.000	0.000	0.217	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	180	209	202	0	0	0	0	160	0
N.S.	1	1.19	1.38	1.34	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	1.003	0.465	0.267	0.000	0.000	0.000	0.000	0.201	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	121	206	192	0	0	0	0	156	0
N.S.	1	0.98	1.66	1.55	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	0.961	0.596	0.279	0.000	0.000	0.000	0.000	0.214	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	64	134	65	0	0	89	0
N.S.	1	1.00	0.87	1.05	2.20	1.07	0.00	0.00	1.46	0.00
time (sec)	N/A	0.407	0.152	0.176	0.040	0.094	0.000	0.000	0.232	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	118	189	192	0	0	0	0	148	0
N.S.	1	0.98	1.58	1.60	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.885	0.813	0.217	0.000	0.000	0.000	0.000	0.212	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	244	254	0	0	0	0	164	0
N.S.	1	1.04	2.10	2.19	0.00	0.00	0.00	0.00	1.41	0.00
time (sec)	N/A	0.717	0.524	0.331	0.000	0.000	0.000	0.000	0.200	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	196	283	207	0	0	0	0	165	0
N.S.	1	1.24	1.79	1.31	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.871	0.499	0.376	0.000	0.000	0.000	0.000	0.207	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	207	289	280	0	0	0	0	195	0
N.S.	1	1.45	2.02	1.96	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.977	1.078	0.373	0.000	0.000	0.000	0.000	0.216	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	313	377	251	0	0	0	0	184	0
N.S.	1	1.41	1.70	1.13	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.289	1.180	0.465	0.000	0.000	0.000	0.000	0.225	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	249	287	256	0	0	0	0	273	0
N.S.	1	1.32	1.53	1.36	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.951	1.083	0.397	0.000	0.000	0.000	0.000	0.224	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	133	83	136	0	101	0	0	162	0
N.S.	1	1.23	0.77	1.26	0.00	0.94	0.00	0.00	1.50	0.00
time (sec)	N/A	0.438	0.230	0.193	0.000	0.105	0.000	0.000	0.217	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	180	287	256	0	0	0	0	271	0
N.S.	1	0.97	1.54	1.38	0.00	0.00	0.00	0.00	1.46	0.00
time (sec)	N/A	1.141	1.084	0.376	0.000	0.000	0.000	0.000	0.207	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	64	86	0	98	0	0	158	0
N.S.	1	0.98	0.70	0.95	0.00	1.08	0.00	0.00	1.74	0.00
time (sec)	N/A	0.436	0.210	0.183	0.000	0.113	0.000	0.000	0.204	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	174	316	256	0	0	0	0	262	0
N.S.	1	0.97	1.76	1.42	0.00	0.00	0.00	0.00	1.46	0.00
time (sec)	N/A	1.057	0.608	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	200	328	324	0	0	0	0	289	0
N.S.	1	1.17	1.92	1.89	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.976	0.878	0.407	0.000	0.000	0.000	0.000	0.216	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	270	362	269	0	0	0	0	285	0
N.S.	1	1.24	1.66	1.23	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	1.169	1.110	0.421	0.000	0.000	0.000	0.000	0.223	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	312	402	391	0	0	0	0	328	0
N.S.	1	1.13	1.45	1.41	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	1.409	1.010	0.499	0.000	0.000	0.000	0.000	0.216	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	406	471	314	0	0	0	0	304	0
N.S.	1	1.43	1.66	1.11	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	1.700	1.267	0.530	0.000	0.000	0.000	0.000	0.227	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	231	198	878	0	0	0	0	98	0
N.S.	1	0.83	0.71	3.16	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.913	0.886	0.398	0.000	0.000	0.000	0.000	0.233	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	176	151	367	0	0	0	0	77	0
N.S.	1	0.88	0.75	1.83	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.167	0.795	0.247	0.000	0.000	0.000	0.000	0.223	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	144	278	0	0	0	0	51	0
N.S.	1	1.00	1.16	2.24	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.438	0.393	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	137	286	0	0	0	0	54	0
N.S.	1	1.00	1.16	2.42	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.494	0.389	0.335	0.000	0.000	0.000	0.000	0.222	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	88	88	153	150	463	0	0	68	0
N.S.	1	0.74	0.74	1.29	1.26	3.89	0.00	0.00	0.57	0.00
time (sec)	N/A	0.334	0.089	0.354	0.118	0.141	0.000	0.000	0.215	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	134	128	1742	146	550	0	0	88	0
N.S.	1	0.67	0.64	8.75	0.73	2.76	0.00	0.00	0.44	0.00
time (sec)	N/A	0.411	0.148	0.389	0.116	0.161	0.000	0.000	0.229	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	178	146	2537	207	616	0	0	109	0
N.S.	1	0.64	0.52	9.09	0.74	2.21	0.00	0.00	0.39	0.00
time (sec)	N/A	0.478	0.166	0.421	0.126	0.134	0.000	0.000	0.236	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	175	169	197	205	203	0	0	109	0
N.S.	1	0.64	0.62	0.72	0.75	0.75	0.00	0.00	0.40	0.00
time (sec)	N/A	0.461	0.169	0.502	0.125	0.090	0.000	0.000	0.270	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	132	126	181	144	176	0	0	89	0
N.S.	1	0.68	0.65	0.93	0.74	0.90	0.00	0.00	0.46	0.00
time (sec)	N/A	0.402	0.117	0.612	0.124	0.094	0.000	0.000	0.235	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	87	98	155	81	142	0	0	66	0
N.S.	1	0.74	0.83	1.31	0.69	1.20	0.00	0.00	0.56	0.00
time (sec)	N/A	0.306	0.119	0.322	0.045	0.098	0.000	0.000	0.229	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	150	233	394	0	0	0	0	53	0
N.S.	1	0.70	1.09	1.85	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.806	0.626	0.319	0.000	0.000	0.000	0.000	0.210	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	165	307	436	0	0	0	0	66	0
N.S.	1	0.70	1.31	1.86	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.830	0.833	0.346	0.000	0.000	0.000	0.000	0.214	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	215	290	541	0	0	0	0	85	0
N.S.	1	0.68	0.92	1.72	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.758	0.804	0.418	0.000	0.000	0.000	0.000	0.219	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	325	337	783	0	0	0	0	146	0
N.S.	1	0.90	0.94	2.18	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	2.221	3.521	0.352	0.000	0.000	0.000	0.000	0.276	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	270	270	883	0	0	0	0	126	0
N.S.	1	0.96	0.96	3.14	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	1.180	1.474	0.330	0.000	0.000	0.000	0.000	0.270	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	216	235	546	0	0	0	0	101	0
N.S.	1	1.10	1.19	2.77	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.648	0.831	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	209	223	244	0	0	0	0	102	0
N.S.	1	1.06	1.13	1.24	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.723	0.905	0.388	0.000	0.000	0.000	0.000	0.233	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	210	259	270	0	0	0	0	113	0
N.S.	1	1.03	1.28	1.33	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.818	0.799	0.395	0.000	0.000	0.000	0.000	0.238	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	98	94	2171	189	573	0	0	120	0
N.S.	1	0.59	0.57	13.08	1.14	3.45	0.00	0.00	0.72	0.00
time (sec)	N/A	0.354	0.064	0.446	0.129	0.141	0.000	0.000	0.268	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	143	136	3145	163	649	0	0	140	0
N.S.	1	0.58	0.55	12.73	0.66	2.63	0.00	0.00	0.57	0.00
time (sec)	N/A	0.421	0.162	0.477	0.129	0.146	0.000	0.000	0.259	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	191	154	4262	225	721	0	0	160	0
N.S.	1	0.58	0.47	12.99	0.69	2.20	0.00	0.00	0.49	0.00
time (sec)	N/A	0.796	0.208	0.540	0.131	0.148	0.000	0.000	0.284	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	231	170	5523	287	793	0	0	180	0
N.S.	1	0.56	0.42	13.50	0.70	1.94	0.00	0.00	0.44	0.00
time (sec)	N/A	1.194	0.269	0.620	0.149	0.177	0.000	0.000	0.291	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	229	209	257	285	275	0	0	177	0
N.S.	1	0.57	0.52	0.64	0.71	0.69	0.00	0.00	0.44	0.00
time (sec)	N/A	0.906	0.173	0.648	0.123	0.125	0.000	0.000	0.311	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	186	153	241	223	245	0	0	157	0
N.S.	1	0.58	0.48	0.75	0.69	0.76	0.00	0.00	0.49	0.00
time (sec)	N/A	0.522	0.201	0.502	0.131	0.113	0.000	0.000	0.281	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	143	135	225	161	215	0	0	137	0
N.S.	1	0.59	0.56	0.93	0.66	0.88	0.00	0.00	0.56	0.00
time (sec)	N/A	0.432	0.098	0.441	0.130	0.098	0.000	0.000	0.255	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	98	107	200	102	185	0	0	115	0
N.S.	1	0.59	0.65	1.21	0.62	1.12	0.00	0.00	0.70	0.00
time (sec)	N/A	0.323	0.177	0.494	0.048	0.106	0.000	0.000	0.238	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	232	336	499	0	0	0	0	103	0
N.S.	1	0.79	1.15	1.71	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.157	0.912	0.490	0.000	0.000	0.000	0.000	0.253	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	241	500	538	0	0	0	0	126	0
N.S.	1	0.77	1.61	1.73	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.202	1.310	0.356	0.000	0.000	0.000	0.000	0.224	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	259	574	570	0	0	0	0	117	0
N.S.	1	0.81	1.79	1.78	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.224	1.044	0.424	0.000	0.000	0.000	0.000	0.239	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	431	500	1743	0	0	0	0	195	0
N.S.	1	0.95	1.10	3.84	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	2.306	5.474	0.443	0.000	0.000	0.000	0.000	0.305	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	376	415	1289	0	0	0	0	175	0
N.S.	1	1.01	1.12	3.47	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	2.090	3.551	0.371	0.000	0.000	0.000	0.000	0.282	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	306	347	885	0	0	0	0	150	0
N.S.	1	1.10	1.25	3.18	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.963	1.692	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	316	305	303	0	0	0	0	152	0
N.S.	1	0.97	0.94	0.93	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.071	1.472	0.428	0.000	0.000	0.000	0.000	0.266	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	312	319	340	0	0	0	0	162	0
N.S.	1	1.06	1.09	1.16	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	1.087	1.259	0.412	0.000	0.000	0.000	0.000	0.258	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	313	400	2429	0	0	0	0	165	0
N.S.	1	1.07	1.37	8.29	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	1.218	2.693	0.473	0.000	0.000	0.000	0.000	0.253	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	111	105	3775	224	704	0	0	171	0
N.S.	1	0.51	0.48	17.24	1.02	3.21	0.00	0.00	0.78	0.00
time (sec)	N/A	0.375	0.103	0.485	0.140	0.150	0.000	0.000	0.269	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	172	147	5007	187	796	0	0	191	0
N.S.	1	0.57	0.49	16.58	0.62	2.64	0.00	0.00	0.63	0.00
time (sec)	N/A	0.438	0.100	0.564	0.135	0.158	0.000	0.000	0.313	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	203	165	6382	251	880	0	0	212	0
N.S.	1	0.53	0.43	16.58	0.65	2.29	0.00	0.00	0.55	0.00
time (sec)	N/A	0.533	0.130	0.632	0.132	0.174	0.000	0.000	0.354	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	241	179	273	313	353	0	0	226	0
N.S.	1	0.53	0.39	0.60	0.68	0.77	0.00	0.00	0.49	0.00
time (sec)	N/A	0.843	0.161	0.500	0.130	0.096	0.000	0.000	0.373	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	198	163	257	249	317	0	0	206	0
N.S.	1	0.52	0.43	0.68	0.66	0.84	0.00	0.00	0.54	0.00
time (sec)	N/A	0.874	0.114	0.543	0.127	0.096	0.000	0.000	0.390	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	153	145	241	185	281	0	0	186	0
N.S.	1	0.51	0.49	0.81	0.62	0.94	0.00	0.00	0.62	0.00
time (sec)	N/A	0.708	0.087	0.467	0.117	0.092	0.000	0.000	0.338	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	108	117	216	118	241	0	0	163	0
N.S.	1	0.50	0.54	0.99	0.54	1.11	0.00	0.00	0.75	0.00
time (sec)	N/A	0.542	0.194	0.538	0.040	0.096	0.000	0.000	0.306	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	326	471	620	0	0	0	0	152	0
N.S.	1	0.86	1.24	1.64	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	1.624	2.597	0.454	0.000	0.000	0.000	0.000	0.264	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	335	596	390	0	0	0	0	176	0
N.S.	1	0.83	1.48	0.97	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.629	3.069	0.460	0.000	0.000	0.000	0.000	0.268	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	343	660	691	0	0	0	0	178	0
N.S.	1	0.84	1.62	1.70	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.684	1.277	0.418	0.000	0.000	0.000	0.000	0.257	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	220	171	568	0	0	0	0	82	0
N.S.	1	1.00	0.77	2.57	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.678	0.861	0.341	0.000	0.000	0.000	0.000	0.218	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	160	113	162	131	146	0	0	73	0
N.S.	1	0.99	0.70	1.00	0.81	0.90	0.00	0.00	0.45	0.00
time (sec)	N/A	0.540	0.228	0.446	0.131	0.100	0.000	0.000	0.218	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	132	141	300	0	0	0	0	61	0
N.S.	1	0.96	1.02	2.17	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.588	0.587	0.270	0.000	0.000	0.000	0.000	0.203	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	85	134	63	117	0	0	49	0
N.S.	1	0.97	1.15	1.81	0.85	1.58	0.00	0.00	0.66	0.00
time (sec)	N/A	0.441	0.186	0.335	0.041	0.088	0.000	0.000	0.214	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	53	53	89	0	0	0	0	38	0
N.S.	1	0.95	0.95	1.59	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.376	0.013	0.154	0.000	0.000	0.000	0.000	0.211	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	86	153	326	0	0	0	0	41	0
N.S.	1	0.54	0.96	2.04	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.733	0.282	0.382	0.000	0.000	0.000	0.000	0.200	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	71	71	208	116	266	0	0	49	0
N.S.	1	0.97	0.97	2.85	1.59	3.64	0.00	0.00	0.67	0.00
time (sec)	N/A	0.483	0.060	0.398	0.115	0.134	0.000	0.000	0.209	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	168	309	431	0	0	0	0	69	0
N.S.	1	0.67	1.23	1.72	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.766	0.824	0.434	0.000	0.000	0.000	0.000	0.216	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	154	174	192	134	480	0	0	73	0
N.S.	1	0.96	1.08	1.19	0.83	2.98	0.00	0.00	0.45	0.00
time (sec)	N/A	0.494	0.306	0.451	0.112	0.141	0.000	0.000	0.223	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	162	145	325	0	489	0	0	108	0
N.S.	1	0.70	0.62	1.40	0.00	2.11	0.00	0.00	0.47	0.00
time (sec)	N/A	0.501	0.076	0.452	0.000	0.130	0.000	0.000	0.233	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	243	192	301	0	0	0	0	119	0
N.S.	1	1.08	0.85	1.33	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.824	1.175	0.421	0.000	0.000	0.000	0.000	0.241	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	113	97	255	157	429	0	0	98	0
N.S.	1	0.76	0.66	1.72	1.06	2.90	0.00	0.00	0.66	0.00
time (sec)	N/A	0.407	0.060	0.369	0.125	0.132	0.000	0.000	0.257	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	159	279	0	0	0	0	108	0
N.S.	1	1.00	1.11	1.95	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.482	0.597	0.426	0.000	0.000	0.000	0.000	0.266	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	136	135	0	327	0	0	85	0
N.S.	1	1.00	1.79	1.78	0.00	4.30	0.00	0.00	1.12	0.00
time (sec)	N/A	0.295	0.376	0.345	0.000	0.122	0.000	0.000	0.212	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	180	70	0	0	0	80	0
N.S.	1	1.00	0.86	2.14	0.83	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.254	0.020	0.335	0.035	0.000	0.000	0.000	0.218	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	159	337	483	0	0	0	0	114	0
N.S.	1	0.69	1.47	2.11	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.792	2.025	0.538	0.000	0.000	0.000	0.000	0.212	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	122	114	266	144	0	0	0	96	0
N.S.	1	0.78	0.73	1.69	0.92	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.673	0.068	0.474	0.042	0.000	0.000	0.000	0.236	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	254	439	588	0	0	0	0	147	0
N.S.	1	0.77	1.33	1.79	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	1.531	3.427	0.544	0.000	0.000	0.000	0.000	0.232	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	177	161	367	0	0	0	0	108	0
N.S.	1	0.72	0.65	1.49	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.573	0.089	0.532	0.000	0.000	0.000	0.000	0.227	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	175	167	398	0	529	0	0	217	0
N.S.	1	0.75	0.72	1.72	0.00	2.28	0.00	0.00	0.94	0.00
time (sec)	N/A	0.484	0.120	0.471	0.000	0.161	0.000	0.000	0.268	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	264	225	366	0	0	0	0	252	0
N.S.	1	1.18	1.00	1.63	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.894	0.644	0.505	0.000	0.000	0.000	0.000	0.263	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	138	122	257	175	469	0	0	208	0
N.S.	1	0.87	0.77	1.63	1.11	2.97	0.00	0.00	1.32	0.00
time (sec)	N/A	0.405	0.107	0.418	0.046	0.151	0.000	0.000	0.247	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	108	101	458	169	0	0	0	196	0
N.S.	1	0.81	0.76	3.44	1.27	0.00	0.00	0.00	1.47	0.00
time (sec)	N/A	0.373	0.141	0.491	0.043	0.000	0.000	0.000	0.230	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	107	162	220	0	421	0	0	195	0
N.S.	1	0.84	1.28	1.73	0.00	3.31	0.00	0.00	1.54	0.00
time (sec)	N/A	0.308	0.395	0.427	0.000	0.130	0.000	0.000	0.238	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	179	132	477	157	0	0	0	197	0
N.S.	1	1.10	0.81	2.94	0.97	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.440	0.054	0.000	0.041	0.000	0.000	0.000	0.227	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	265	376	560	0	0	0	0	284	0
N.S.	1	0.87	1.23	1.84	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	1.167	6.449	0.535	0.000	0.000	0.000	0.000	0.216	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	173	173	434	0	0	0	0	217	0
N.S.	1	0.73	0.73	1.83	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.506	0.254	0.534	0.000	0.000	0.000	0.000	0.228	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	384	502	655	0	0	0	0	315	0
N.S.	1	0.94	1.22	1.60	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	2.072	7.104	0.581	0.000	0.000	0.000	0.000	0.232	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	226	237	403	276	0	0	0	226	0
N.S.	1	0.70	0.73	1.24	0.85	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	1.131	0.156	0.530	0.053	0.000	0.000	0.000	0.236	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	153	93	456	0	0	0	0	23	0
N.S.	1	1.06	0.64	3.14	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.586	0.323	0.475	0.000	0.000	0.000	0.000	0.197	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	74	144	62	101	0	0	23	0
N.S.	1	1.04	0.67	1.31	0.56	0.92	0.00	0.00	0.21	0.00
time (sec)	N/A	0.387	0.093	0.418	0.138	0.101	0.000	0.000	0.202	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	75	223	0	0	0	0	23	0
N.S.	1	1.00	0.85	2.53	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.360	0.145	0.282	0.000	0.000	0.000	0.000	0.229	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	55	65	28	72	0	40	21	0
N.S.	1	1.02	1.15	1.35	0.58	1.50	0.00	0.83	0.44	0.00
time (sec)	N/A	0.241	0.066	0.349	0.037	0.088	0.000	0.133	0.209	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	51	0	0	0	0	20	0
N.S.	1	1.00	1.41	1.59	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.203	0.015	0.142	0.000	0.000	0.000	0.000	0.192	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	66	113	270	0	0	0	0	23	0
N.S.	1	0.64	1.10	2.62	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.383	0.134	0.418	0.000	0.000	0.000	0.000	0.196	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	48	57	168	73	88	0	80	23	0
N.S.	1	1.02	1.21	3.57	1.55	1.87	0.00	1.70	0.49	0.00
time (sec)	N/A	0.251	0.023	0.398	0.116	0.099	0.000	0.135	0.203	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	125	234	349	0	0	0	0	23	0
N.S.	1	0.74	1.39	2.08	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.583	0.260	0.436	0.000	0.000	0.000	0.000	0.198	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	108	117	137	126	169	0	169	23	0
N.S.	1	0.99	1.07	1.26	1.16	1.55	0.00	1.55	0.21	0.00
time (sec)	N/A	0.396	1.132	0.454	0.111	0.107	0.000	0.175	0.195	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	95	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.309	0.078	0.000	0.000	0.000	0.000	0.000	0.857	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	115	0	0	0	0	0	100	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.344	0.059	0.000	0.000	0.000	0.000	0.000	0.926	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	385	423	387	0	0	0	0	0	616	0
N.S.	1	1.10	1.01	0.00	0.00	0.00	0.00	0.00	1.60	0.00
time (sec)	N/A	2.245	0.681	0.000	0.000	0.000	0.000	0.000	0.399	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	306	290	0	0	0	0	0	349	0
N.S.	1	1.12	1.06	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	1.006	0.278	0.000	0.000	0.000	0.000	0.000	0.296	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	195	191	0	0	0	0	0	158	0
N.S.	1	1.14	1.12	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.771	0.153	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	32	31	41	31	51	29
N.S.	1	1.00	1.07	1.00	1.19	1.15	1.52	1.15	1.89	1.07
time (sec)	N/A	0.324	3.396	0.525	0.140	0.106	3.442	0.136	0.222	3.499

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	30	43	58	30	66	29
N.S.	1	1.00	1.07	1.00	1.11	1.59	2.15	1.11	2.44	1.07
time (sec)	N/A	0.789	8.418	0.499	0.143	0.095	50.763	0.129	0.236	3.429

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	32	57	0	31	83	29
N.S.	1	1.00	1.07	1.00	1.19	2.11	0.00	1.15	3.07	1.07
time (sec)	N/A	0.886	11.172	0.490	0.151	0.094	0.000	0.139	0.272	3.626

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	718	548	350	0	0	0	0	0	162	0
N.S.	1	0.76	0.49	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.716	0.853	0.000	0.000	0.000	0.000	0.000	0.449	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	454	396	274	0	0	0	0	0	106	0
N.S.	1	0.87	0.60	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.171	0.502	0.000	0.000	0.000	0.000	0.000	0.325	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	265	223	0	0	0	0	0	49	0
N.S.	1	0.97	0.81	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.795	0.179	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	147	0	0	0	0	0	55	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.347	0.055	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	297	216	0	0	0	0	0	101	0
N.S.	1	0.99	0.72	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.622	0.164	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	430	319	0	0	0	0	0	134	0
N.S.	1	0.96	0.71	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.964	0.470	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	864	596	387	0	0	0	0	0	179	0
N.S.	1	0.69	0.45	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	3.659	1.496	0.000	0.000	0.000	0.000	0.000	1.512	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	546	428	288	0	0	0	0	0	117	0
N.S.	1	0.78	0.53	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.586	0.635	0.000	0.000	0.000	0.000	0.000	0.498	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	283	229	0	0	0	0	0	55	0
N.S.	1	0.92	0.74	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.928	0.140	0.000	0.000	0.000	0.000	0.000	0.309	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	153	0	0	0	0	0	67	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.602	0.302	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	330	225	0	0	0	0	0	116	0
N.S.	1	0.96	0.66	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.568	0.288	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	526	484	413	0	0	0	0	0	155	0
N.S.	1	0.92	0.79	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	2.363	0.667	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	128	124	0	0	0	0	0	27	0
N.S.	1	0.99	0.96	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.315	0.060	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	425	237	557	326	349	0	0	127	0
N.S.	1	1.29	0.72	1.69	0.99	1.06	0.00	0.00	0.38	0.00
time (sec)	N/A	2.495	0.392	0.860	0.136	0.103	0.000	0.000	0.284	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	328	241	678	0	0	0	0	115	0
N.S.	1	1.03	0.76	2.13	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	2.672	1.943	0.356	0.000	0.000	0.000	0.000	0.266	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	163	181	489	204	280	0	0	100	0
N.S.	1	0.88	0.97	2.63	1.10	1.51	0.00	0.00	0.54	0.00
time (sec)	N/A	0.704	0.517	0.457	0.062	0.101	0.000	0.000	0.254	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	181	235	527	0	0	0	0	82	0
N.S.	1	0.89	1.15	2.58	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.752	0.684	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	402	238	449	0	0	0	0	0	88	0
N.S.	1	0.59	1.12	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.370	1.084	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	186	270	582	0	0	0	0	88	0
N.S.	1	0.79	1.15	2.49	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.822	1.483	0.474	0.000	0.000	0.000	0.000	0.215	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	427	250	5035	0	0	0	0	0	102	0
N.S.	1	0.59	11.79	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	2.024	62.667	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	213	304	1751	0	0	0	0	104	0
N.S.	1	0.65	0.93	5.37	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.238	0.860	0.536	0.000	0.000	0.000	0.000	0.266	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	663	262	582	388	432	0	0	209	0
N.S.	1	1.46	0.58	1.28	0.86	0.95	0.00	0.00	0.46	0.00
time (sec)	N/A	3.895	0.494	0.694	0.135	0.110	0.000	0.000	0.353	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	550	485	1721	0	0	0	0	198	0
N.S.	1	1.25	1.10	3.90	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	4.774	4.678	0.501	0.000	0.000	0.000	0.000	0.307	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	221	208	515	278	367	0	0	183	0
N.S.	1	0.75	0.71	1.75	0.94	1.24	0.00	0.00	0.62	0.00
time (sec)	N/A	0.804	1.441	0.773	0.059	0.099	0.000	0.000	0.286	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	356	374	1061	0	0	0	0	166	0
N.S.	1	1.10	1.15	3.27	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	1.536	1.830	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	573	396	650	0	0	0	0	0	170	0
N.S.	1	0.69	1.13	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	2.450	2.042	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	393	433	464	0	0	0	0	168	0
N.S.	1	0.89	0.98	1.05	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	2.402	4.091	0.515	0.000	0.000	0.000	0.000	0.254	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	630	387	5444	0	0	0	0	0	201	0
N.S.	1	0.61	8.64	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	2.518	62.100	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	403	583	500	0	0	0	0	186	0
N.S.	1	0.97	1.40	1.20	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	5.676	1.771	0.565	0.000	0.000	0.000	0.000	0.266	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	0	288	648	471	558	0	0	292	0
N.S.	1	0.00	0.39	0.89	0.64	0.76	0.00	0.00	0.40	0.00
time (sec)	N/A	0.000	0.573	0.714	0.146	0.107	0.000	0.000	0.395	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	0	910	2528	0	0	0	0	281	0
N.S.	1	0.00	1.39	3.87	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	5.761	0.588	0.000	0.000	0.000	0.000	0.392	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	259	234	581	337	477	0	0	265	0
N.S.	1	0.64	0.58	1.43	0.83	1.18	0.00	0.00	0.65	0.00
time (sec)	N/A	1.528	1.475	0.766	0.059	0.114	0.000	0.000	0.364	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	561	740	1737	0	0	0	0	249	0
N.S.	1	1.21	1.60	3.76	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	2.467	3.555	0.503	0.000	0.000	0.000	0.000	0.320	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	758	613	963	0	0	0	0	0	253	0
N.S.	1	0.81	1.27	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	4.756	5.640	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	663	554	589	0	0	0	0	253	0
N.S.	1	1.13	0.95	1.01	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	4.654	4.988	0.595	0.000	0.000	0.000	0.000	0.318	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	777	609	5694	0	0	0	0	0	286	0
N.S.	1	0.78	7.33	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	5.456	62.956	0.000	0.000	0.000	0.000	0.000	0.300	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	616	0	803	652	0	0	0	0	269	0
N.S.	1	0.00	1.30	1.06	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.000	2.611	0.628	0.000	0.000	0.000	0.000	0.316	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	470	255	561	407	348	0	0	133	0
N.S.	1	1.29	0.70	1.54	1.12	0.95	0.00	0.00	0.36	0.00
time (sec)	N/A	1.664	0.462	0.773	0.137	0.117	0.000	0.000	0.281	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	372	295	1092	0	0	0	0	122	0
N.S.	1	1.06	0.84	3.11	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.845	1.511	0.466	0.000	0.000	0.000	0.000	0.250	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	271	201	533	279	282	0	0	111	0
N.S.	1	1.12	0.83	2.21	1.16	1.17	0.00	0.00	0.46	0.00
time (sec)	N/A	1.203	0.407	0.586	0.143	0.097	0.000	0.000	0.242	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	189	228	563	0	0	0	0	99	0
N.S.	1	0.83	1.00	2.48	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.678	1.237	0.365	0.000	0.000	0.000	0.000	0.221	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	108	149	314	145	218	0	0	83	0
N.S.	1	0.97	1.34	2.83	1.31	1.96	0.00	0.00	0.75	0.00
time (sec)	N/A	0.358	0.708	0.464	0.052	0.090	0.000	0.000	0.229	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	53	53	149	0	0	0	0	69	0
N.S.	1	0.95	0.95	2.66	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.247	0.014	0.203	0.000	0.000	0.000	0.000	0.232	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	138	315	0	0	0	0	0	74	0
N.S.	1	0.48	1.09	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.659	0.655	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	138	237	476	0	0	0	0	83	0
N.S.	1	0.71	1.22	2.44	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.747	0.889	0.575	0.000	0.000	0.000	0.000	0.219	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	452	254	5036	0	0	0	0	0	107	0
N.S.	1	0.56	11.14	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.406	61.281	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	263	370	1520	0	0	0	0	111	0
N.S.	1	0.79	1.11	4.58	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	2.196	0.932	0.668	0.000	0.000	0.000	0.000	0.259	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	412	343	756	0	0	0	0	191	0
N.S.	1	0.94	0.78	1.72	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	2.530	1.578	0.620	0.000	0.000	0.000	0.000	0.254	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	287	319	648	0	0	0	0	168	0
N.S.	1	0.83	0.92	1.87	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	2.447	1.364	0.553	0.000	0.000	0.000	0.000	0.232	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	199	270	738	0	0	0	0	178	0
N.S.	1	0.77	1.05	2.87	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	1.687	1.969	0.641	0.000	0.000	0.000	0.000	0.225	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	125	219	341	0	0	0	0	151	0
N.S.	1	0.64	1.12	1.74	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.804	1.346	0.507	0.000	0.000	0.000	0.000	0.230	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	140	126	578	0	0	0	0	142	0
N.S.	1	0.71	0.64	2.92	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.664	0.262	0.533	0.000	0.000	0.000	0.000	0.218	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	471	260	613	0	0	0	0	0	181	0
N.S.	1	0.55	1.30	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	2.337	3.035	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	279	127	1271	0	0	0	0	164	0
N.S.	1	0.82	0.37	3.73	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	2.440	0.490	0.674	0.000	0.000	0.000	0.000	0.229	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	650	433	5362	0	0	0	0	0	221	0
N.S.	1	0.67	8.25	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	6.152	64.729	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	461	382	2901	0	0	0	0	429	0
N.S.	1	1.05	0.87	6.64	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	2.827	1.854	0.793	0.000	0.000	0.000	0.000	0.282	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	330	356	635	0	0	0	0	381	0
N.S.	1	1.02	1.10	1.96	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	3.805	3.635	0.608	0.000	0.000	0.000	0.000	0.252	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	249	264	2657	0	0	0	0	364	0
N.S.	1	0.72	0.77	7.72	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	1.403	1.777	0.701	0.000	0.000	0.000	0.000	0.246	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	189	388	582	0	0	0	0	362	0
N.S.	1	0.66	1.36	2.03	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.946	2.377	0.622	0.000	0.000	0.000	0.000	0.244	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	278	289	2435	0	0	0	0	361	0
N.S.	1	0.87	0.91	7.63	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	2.201	0.935	0.593	0.000	0.000	0.000	0.000	0.236	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	585	448	818	0	0	0	0	0	460	0
N.S.	1	0.77	1.40	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	5.322	9.391	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	469	457	2857	0	0	0	0	392	0
N.S.	1	1.01	0.98	6.16	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	4.878	2.757	0.821	0.000	0.000	0.000	0.000	0.247	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	295	116	488	0	0	0	0	25	0
N.S.	1	1.21	0.48	2.01	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.149	0.344	0.392	0.000	0.000	0.000	0.000	0.223	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	215	123	343	105	150	0	0	25	0
N.S.	1	1.09	0.62	1.73	0.53	0.76	0.00	0.00	0.13	0.00
time (sec)	N/A	1.255	0.112	0.463	0.120	0.113	0.000	0.000	0.211	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	140	87	239	0	0	0	0	25	0
N.S.	1	0.93	0.58	1.58	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.560	0.162	0.308	0.000	0.000	0.000	0.000	0.227	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	54	82	50	114	0	80	23	0
N.S.	1	1.08	0.73	1.11	0.68	1.54	0.00	1.08	0.31	0.00
time (sec)	N/A	0.315	0.076	0.371	0.038	0.103	0.000	0.142	0.202	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	51	0	0	0	0	22	0
N.S.	1	1.00	1.41	1.59	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.220	0.018	0.191	0.000	0.000	0.000	0.000	0.204	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	106	151	0	0	0	0	0	25	0
N.S.	1	0.58	0.83	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.545	0.171	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	103	111	241	0	0	0	0	25	0
N.S.	1	0.83	0.90	1.94	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.786	0.368	0.451	0.000	0.000	0.000	0.000	0.201	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	194	233	0	0	0	0	0	25	0
N.S.	1	0.65	0.78	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.823	0.758	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	136	0	0	267	31
N.S.	1	1.00	1.06	0.94	1.00	4.39	0.00	0.00	8.61	1.00
time (sec)	N/A	5.431	2.125	1.860	0.254	0.107	0.000	0.000	0.642	3.557

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	87	0	0	174	31
N.S.	1	1.00	1.06	0.94	1.00	2.81	0.00	0.00	5.61	1.00
time (sec)	N/A	2.976	1.073	1.292	0.179	0.103	0.000	0.000	0.464	3.637

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	43	32	0	80	31
N.S.	1	1.00	1.06	0.94	1.00	1.39	1.03	0.00	2.58	1.00
time (sec)	N/A	1.030	0.746	1.273	0.174	0.102	56.704	0.000	0.285	3.593

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	58	32	31	88	31
N.S.	1	1.00	1.06	0.94	1.00	1.87	1.03	1.00	2.84	1.00
time (sec)	N/A	0.334	3.685	0.428	0.178	0.093	17.264	0.189	0.221	3.610

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	70	32	31	156	31
N.S.	1	1.00	1.06	0.94	1.00	2.26	1.03	1.00	5.03	1.00
time (sec)	N/A	0.351	4.405	0.618	0.238	0.088	21.933	0.208	0.229	3.778

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	38	27	26	29	26
N.S.	1	1.00	1.08	0.92	1.00	1.46	1.04	1.00	1.12	1.00
time (sec)	N/A	0.277	0.720	0.411	0.166	0.087	9.583	0.184	0.199	3.545

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	345	136	520	0	0	0	0	25	0
N.S.	1	1.10	0.43	1.65	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	4.448	0.509	0.444	0.000	0.000	0.000	0.000	0.194	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	259	140	375	131	205	0	0	25	0
N.S.	1	1.07	0.58	1.54	0.54	0.84	0.00	0.00	0.10	0.00
time (sec)	N/A	2.057	0.118	0.516	0.124	0.095	0.000	0.000	0.186	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	158	98	255	0	0	0	0	25	0
N.S.	1	0.84	0.52	1.36	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.131	0.198	0.368	0.000	0.000	0.000	0.000	0.185	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	95	101	111	65	159	0	104	23	0
N.S.	1	0.90	0.96	1.06	0.62	1.51	0.00	0.99	0.22	0.00
time (sec)	N/A	0.788	0.079	0.440	0.041	0.092	0.000	0.187	0.194	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	51	0	0	0	0	22	0
N.S.	1	1.00	1.41	1.59	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.372	0.018	0.254	0.000	0.000	0.000	0.000	0.187	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	154	488	0	0	0	0	0	25	0
N.S.	1	0.58	1.84	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.104	0.471	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	125	137	313	0	0	0	0	25	0
N.S.	1	0.75	0.82	1.87	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.908	0.403	0.528	0.000	0.000	0.000	0.000	0.227	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	461	273	1051	0	0	0	0	0	25	0
N.S.	1	0.59	2.28	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.546	3.951	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	68	31	30	117	30
N.S.	1	1.00	1.07	0.93	1.00	2.27	1.03	1.00	3.90	1.00
time (sec)	N/A	0.316	3.219	0.414	0.197	0.092	58.782	0.194	0.265	3.719

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	190	188	297	0	0	0	0	27	0
N.S.	1	0.56	0.55	0.88	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.676	0.372	0.287	0.000	0.000	0.000	0.000	0.231	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	173	171	254	0	0	0	0	27	0
N.S.	1	0.58	0.58	0.86	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.883	0.331	0.648	0.000	0.000	0.000	0.000	0.206	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	90	103	165	0	0	0	0	27	0
N.S.	1	0.65	0.74	1.19	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.827	0.287	0.195	0.000	0.000	0.000	0.000	0.226	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	123	127	0	0	0	0	0	25	0
N.S.	1	0.62	0.64	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.773	0.292	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	90	105	165	0	0	0	0	24	0
N.S.	1	0.65	0.76	1.19	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.559	0.298	0.241	0.000	0.000	0.000	0.000	0.224	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	27	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	0.96	1.00
time (sec)	N/A	0.690	1.412	0.311	0.110	0.082	0.908	0.000	0.213	3.314

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	31	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	1.11	1.00
time (sec)	N/A	0.529	1.707	0.313	0.110	0.091	1.405	0.145	0.245	3.242

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	224	215	318	0	0	0	0	60	0
N.S.	1	0.56	0.54	0.80	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.749	0.656	0.305	0.000	0.000	0.000	0.000	0.284	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	191	188	297	0	0	0	0	60	0
N.S.	1	0.56	0.55	0.88	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.667	0.598	0.224	0.000	0.000	0.000	0.000	0.290	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	174	172	252	0	0	0	0	58	0
N.S.	1	0.59	0.58	0.85	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.574	0.570	0.543	0.000	0.000	0.000	0.000	0.248	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	141	147	231	0	0	0	0	57	0
N.S.	1	0.59	0.62	0.97	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.469	0.474	0.231	0.000	0.000	0.000	0.000	0.255	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	58	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	2.07	1.00
time (sec)	N/A	1.300	1.382	0.221	0.112	0.087	7.768	0.000	0.248	3.239

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	61	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	2.18	1.00
time (sec)	N/A	1.251	2.279	0.285	0.118	0.084	9.627	0.147	0.276	3.418

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	223	216	318	0	0	0	0	91	0
N.S.	1	0.56	0.54	0.80	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.013	0.943	0.312	0.000	0.000	0.000	0.000	0.274	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	240	233	363	0	0	0	0	91	0
N.S.	1	0.55	0.53	0.83	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.714	0.952	0.256	0.000	0.000	0.000	0.000	0.307	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	223	216	318	0	0	0	0	89	0
N.S.	1	0.56	0.54	0.80	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.661	0.900	0.341	0.000	0.000	0.000	0.000	0.269	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	190	191	297	0	0	0	0	88	0
N.S.	1	0.56	0.56	0.88	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.552	0.704	0.256	0.000	0.000	0.000	0.000	0.281	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	45	0	0	89	28
N.S.	1	1.00	1.07	0.93	1.00	1.61	0.00	0.00	3.18	1.00
time (sec)	N/A	1.357	1.394	0.256	0.136	0.082	0.000	0.000	0.269	3.615

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	0	28	92	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.00	1.00	3.29	1.00
time (sec)	N/A	1.621	2.077	0.265	0.140	0.090	0.000	0.154	0.304	3.721

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	55	69	87	0	0	0	0	25	0
N.S.	1	0.56	0.70	0.89	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.679	0.097	0.246	0.000	0.000	0.000	0.000	0.195	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	44	60	78	0	0	0	0	25	0
N.S.	1	0.68	0.92	1.20	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.564	0.080	0.329	0.000	0.000	0.000	0.000	0.200	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	44	60	67	0	0	0	0	25	0
N.S.	1	0.68	0.92	1.03	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.486	0.089	0.178	0.000	0.000	0.000	0.000	0.182	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	50	0	0	0	0	0	23	0
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.338	0.075	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	28	47	48	0	55	0	0	22	0
N.S.	1	0.97	1.62	1.66	0.00	1.90	0.00	0.00	0.76	0.00
time (sec)	N/A	0.213	0.054	0.234	0.000	0.086	0.000	0.000	0.181	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	24	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	1.00	1.00	1.04	1.00
time (sec)	N/A	0.263	0.589	0.414	0.108	0.093	1.112	0.138	0.189	3.223

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	37	26	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.54	1.08	1.00	1.04	1.00
time (sec)	N/A	0.318	0.766	0.309	0.105	0.082	3.701	0.133	0.194	3.170

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	123	130	182	0	0	0	0	39	0
N.S.	1	0.62	0.66	0.92	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.558	0.437	0.351	0.000	0.000	0.000	0.000	0.197	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	90	99	161	0	0	0	0	39	0
N.S.	1	0.65	0.71	1.16	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.484	0.369	0.217	0.000	0.000	0.000	0.000	0.202	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	68	81	0	0	0	0	0	50	0
N.S.	1	0.74	0.88	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.511	0.262	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	35	54	55	0	65	0	0	49	0
N.S.	1	0.97	1.50	1.53	0.00	1.81	0.00	0.00	1.36	0.00
time (sec)	N/A	0.235	0.134	0.238	0.000	0.091	0.000	0.000	0.194	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	0	37	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	0.00	1.32	1.00
time (sec)	N/A	0.303	1.511	0.290	0.109	0.083	1.737	0.000	0.198	3.236

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	53	29	28	41	28
N.S.	1	1.00	1.07	0.93	1.00	1.89	1.04	1.00	1.46	1.00
time (sec)	N/A	0.305	2.230	0.329	0.112	0.090	3.080	0.139	0.212	3.121

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	63	27	28	277	28
N.S.	1	1.00	1.07	0.93	1.00	2.25	0.96	1.00	9.89	1.00
time (sec)	N/A	0.359	4.489	0.300	0.140	0.100	7.171	0.166	0.258	3.196

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	61	26	0	83	26
N.S.	1	1.00	1.08	0.92	1.00	2.35	1.00	0.00	3.19	1.00
time (sec)	N/A	0.343	6.379	0.289	0.131	0.092	6.749	0.000	0.218	3.156

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	60	26	25	81	25
N.S.	1	1.00	1.08	0.92	1.00	2.40	1.04	1.00	3.24	1.00
time (sec)	N/A	0.290	0.172	0.270	0.125	0.092	5.980	0.163	0.207	3.095

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	64	27	0	83	28
N.S.	1	1.00	1.07	0.93	1.00	2.29	0.96	0.00	2.96	1.00
time (sec)	N/A	0.439	8.076	0.393	0.141	0.095	17.872	0.000	0.213	3.188

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	68	29	28	87	28
N.S.	1	1.00	1.07	0.93	1.00	2.43	1.04	1.00	3.11	1.00
time (sec)	N/A	0.480	9.583	0.366	0.148	0.095	38.702	0.167	0.243	3.165

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	38	0	0	63	28
N.S.	1	1.00	1.07	0.93	1.00	1.36	0.00	0.00	2.25	1.00
time (sec)	N/A	0.509	1.340	0.549	0.109	0.084	0.000	0.000	0.293	3.116

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	27	0	27	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.96	0.00	0.96	1.00
time (sec)	N/A	0.471	0.256	0.599	0.106	0.096	2.254	0.000	0.238	3.032

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	50	27	28	39	28
N.S.	1	1.00	1.07	0.93	1.00	1.79	0.96	1.00	1.39	1.00
time (sec)	N/A	0.315	0.923	0.325	0.112	0.104	2.415	0.152	0.197	3.629

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	63	27	28	85	28
N.S.	1	1.00	1.07	0.93	1.00	2.25	0.96	1.00	3.04	1.00
time (sec)	N/A	0.332	1.358	0.510	0.139	0.089	65.616	0.174	0.616	3.722

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	86	0	28	125	28
N.S.	1	1.00	1.07	0.93	1.00	3.07	0.00	1.00	4.46	1.00
time (sec)	N/A	0.306	1.728	0.540	0.144	0.111	0.000	0.184	0.230	3.740

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	352	322	555	0	0	0	0	41	0
N.S.	1	1.01	0.92	1.59	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.040	0.533	0.823	0.000	0.000	0.000	0.000	0.210	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	262	130	279	0	0	0	0	41	0
N.S.	1	1.70	0.84	1.81	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.294	0.413	0.229	0.000	0.000	0.000	0.000	0.214	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	252	217	0	0	0	0	0	39	0
N.S.	1	1.02	0.88	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.652	0.360	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	131	121	269	0	0	0	0	38	0
N.S.	1	0.90	0.83	1.84	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.274	0.203	0.275	0.000	0.000	0.000	0.000	0.207	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	442	43	27	0	42	28
N.S.	1	1.00	1.07	0.93	15.79	1.54	0.96	0.00	1.50	1.00
time (sec)	N/A	1.109	9.142	0.405	0.531	0.114	2.295	0.000	0.202	3.248

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	427	49	29	28	48	28
N.S.	1	1.00	1.07	0.93	15.25	1.75	1.04	1.00	1.71	1.00
time (sec)	N/A	0.467	3.553	0.346	0.402	0.104	5.927	0.150	0.231	3.196

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	310	338	599	0	0	0	0	88	0
N.S.	1	0.88	0.95	1.69	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.332	0.808	0.303	0.000	0.000	0.000	0.000	0.298	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	360	327	583	0	0	0	0	86	0
N.S.	1	1.03	0.94	1.68	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	2.156	0.715	0.626	0.000	0.000	0.000	0.000	0.232	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	179	232	445	0	0	0	0	85	0
N.S.	1	0.73	0.94	1.81	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.281	0.458	0.257	0.000	0.000	0.000	0.000	0.264	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	476	43	27	0	87	28
N.S.	1	1.00	1.07	0.93	17.00	1.54	0.96	0.00	3.11	1.00
time (sec)	N/A	2.196	18.224	0.303	0.700	0.102	16.435	0.000	0.222	3.181

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	484	49	29	28	92	28
N.S.	1	1.00	1.07	0.93	17.29	1.75	1.04	1.00	3.29	1.00
time (sec)	N/A	0.898	20.084	0.515	0.728	0.090	56.981	0.153	0.261	3.131

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	460	446	773	0	0	0	0	133	0
N.S.	1	1.01	0.98	1.70	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.785	1.309	0.349	0.000	0.000	0.000	0.000	0.342	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	464	436	759	0	0	0	0	131	0
N.S.	1	1.04	0.97	1.69	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	3.810	1.071	0.497	0.000	0.000	0.000	0.000	0.264	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	229	343	623	0	0	0	0	130	0
N.S.	1	0.65	0.98	1.77	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.942	0.756	0.312	0.000	0.000	0.000	0.000	0.306	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	524	60	0	0	132	28
N.S.	1	1.00	1.07	0.93	18.71	2.14	0.00	0.00	4.71	1.00
time (sec)	N/A	2.570	16.198	0.383	0.903	0.089	0.000	0.000	0.262	3.253

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	534	66	0	28	137	28
N.S.	1	1.00	1.07	0.93	19.07	2.36	0.00	1.00	4.89	1.00
time (sec)	N/A	1.208	22.182	0.360	0.900	0.106	0.000	0.161	0.308	3.226

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	215	190	519	0	0	0	0	64	0
N.S.	1	0.64	0.56	1.54	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.950	0.488	0.464	0.000	0.000	0.000	0.000	0.231	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	169	149	387	0	0	0	0	64	0
N.S.	1	0.72	0.63	1.64	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.140	0.365	0.253	0.000	0.000	0.000	0.000	0.216	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	165	144	375	0	0	0	0	64	0
N.S.	1	0.70	0.61	1.58	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.818	0.313	0.411	0.000	0.000	0.000	0.000	0.193	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	121	117	243	0	0	0	0	64	0
N.S.	1	0.89	0.86	1.79	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.887	0.270	0.238	0.000	0.000	0.000	0.000	0.207	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	115	107	0	0	0	0	0	62	0
N.S.	1	0.88	0.82	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.675	0.214	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	37	50	57	0	75	0	0	60	59
N.S.	1	1.03	1.39	1.58	0.00	2.08	0.00	0.00	1.67	1.64
time (sec)	N/A	0.250	0.026	0.106	0.000	0.085	0.000	0.000	0.201	3.229

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	481	80	29	0	63	28
N.S.	1	1.00	1.07	0.93	17.18	2.86	1.04	0.00	2.25	1.00
time (sec)	N/A	0.465	3.954	0.306	0.592	0.090	5.964	0.000	0.215	3.355

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	491	86	31	28	69	28
N.S.	1	1.00	1.07	0.93	17.54	3.07	1.11	1.00	2.46	1.00
time (sec)	N/A	0.508	1.307	0.292	0.593	0.079	17.837	0.158	0.207	3.372

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	503	106	29	28	141	28
N.S.	1	1.00	1.07	0.93	17.96	3.79	1.04	1.00	5.04	1.00
time (sec)	N/A	0.927	5.121	0.304	0.720	0.086	56.438	0.183	0.251	3.415

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	511	104	27	0	139	26
N.S.	1	1.00	1.08	0.92	19.65	4.00	1.04	0.00	5.35	1.00
time (sec)	N/A	0.313	20.221	0.280	0.731	0.099	50.437	0.000	0.248	3.408

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	502	103	27	25	137	25
N.S.	1	1.00	1.08	0.92	20.08	4.12	1.08	1.00	5.48	1.00
time (sec)	N/A	0.720	3.605	0.270	0.673	0.092	40.247	0.180	0.239	3.579

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	530	108	29	0	140	28
N.S.	1	1.00	1.07	0.93	18.93	3.86	1.04	0.00	5.00	1.00
time (sec)	N/A	0.316	18.778	0.437	0.975	0.092	108.196	0.000	0.233	3.834

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	538	114	0	28	146	28
N.S.	1	1.00	1.07	0.93	19.21	4.07	0.00	1.00	5.21	1.00
time (sec)	N/A	0.401	15.879	0.412	0.968	0.091	0.000	0.187	0.255	3.728

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	577	54	0	0	95	30
N.S.	1	1.00	1.07	0.93	19.23	1.80	0.00	0.00	3.17	1.00
time (sec)	N/A	0.457	1.427	1.042	1.578	0.097	0.000	0.000	0.273	3.467

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	514	44	31	0	45	30
N.S.	1	1.00	1.07	0.93	17.13	1.47	1.03	0.00	1.50	1.00
time (sec)	N/A	0.445	0.320	0.875	1.101	0.108	5.804	0.000	0.210	3.573

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	544	82	31	30	68	30
N.S.	1	1.00	1.07	0.93	18.13	2.73	1.03	1.00	2.27	1.00
time (sec)	N/A	0.580	1.026	0.383	0.481	0.095	22.544	0.168	0.199	4.657

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	596	108	0	30	144	30
N.S.	1	1.00	1.07	0.93	19.87	3.60	0.00	1.00	4.80	1.00
time (sec)	N/A	0.509	1.493	0.558	0.912	0.100	0.000	0.204	0.232	4.667

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	700	146	0	30	216	30
N.S.	1	1.00	1.07	0.93	23.33	4.87	0.00	1.00	7.20	1.00
time (sec)	N/A	0.541	1.882	0.600	1.060	0.108	0.000	0.224	0.208	4.357

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	463	300	0	0	0	0	0	86	0
N.S.	1	1.79	1.16	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	2.111	1.719	0.000	0.000	0.000	0.000	0.000	0.734	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	525	384	0	0	0	0	0	86	0
N.S.	1	1.54	1.13	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.987	1.147	0.000	0.000	0.000	0.000	0.000	0.629	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	275	0	0	0	0	0	0	84	0
N.S.	1	1.14	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	3.991	0.000	0.000	0.000	0.000	0.000	0.000	0.510	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	231	246	0	0	0	0	0	860	0
N.S.	1	0.99	1.06	0.00	0.00	0.00	0.00	0.00	3.69	0.00
time (sec)	N/A	1.056	1.061	0.000	0.000	0.000	0.000	0.000	1.309	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	30	0	87	0	85	27
N.S.	1	1.00	1.07	0.93	1.11	0.00	3.22	0.00	3.15	1.00
time (sec)	N/A	3.305	1.107	0.251	0.898	0.000	7.900	0.000	0.499	4.418

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	479	571	527	0	0	0	0	0	131	0
N.S.	1	1.19	1.10	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	2.741	2.589	0.000	0.000	0.000	0.000	0.000	0.981	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	462	723	498	0	0	0	0	0	131	0
N.S.	1	1.56	1.08	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	2.878	2.274	0.000	0.000	0.000	0.000	0.000	0.819	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	567	0	0	0	0	0	0	129	0
N.S.	1	1.56	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	4.060	0.000	0.000	0.000	0.000	0.000	0.000	0.706	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	351	331	387	0	0	0	0	0	1045	0
N.S.	1	0.94	1.10	0.00	0.00	0.00	0.00	0.00	2.98	0.00
time (sec)	N/A	1.865	1.452	0.000	0.000	0.000	0.000	0.000	1.693	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	30	0	133	0	130	29
N.S.	1	1.00	1.07	0.93	1.03	0.00	4.59	0.00	4.48	1.00
time (sec)	N/A	4.526	1.009	0.254	0.858	0.000	8.339	0.000	0.739	3.997

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	50	72	0	0	0	0	0	30	0
N.S.	1	0.77	1.11	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.386	0.079	0.000	0.000	0.000	0.000	0.000	0.313	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	184	182	0	0	0	0	0	30	0
N.S.	1	0.73	0.72	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.670	0.721	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	379	279	241	0	0	0	0	0	28	0
N.S.	1	0.74	0.64	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.086	0.919	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	184	214	0	0	0	0	0	27	0
N.S.	1	0.73	0.85	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.822	0.547	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	29	0	30	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.00	0.00	1.03	1.00
time (sec)	N/A	1.025	0.317	0.232	0.357	0.105	4.468	0.000	0.237	3.790

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	31	0	30	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.07	0.00	1.03	1.00
time (sec)	N/A	0.595	0.289	0.256	0.359	0.098	6.333	0.000	0.294	3.773

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	658	444	438	0	0	0	0	0	64	0
N.S.	1	0.67	0.67	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.930	2.069	0.000	0.000	0.000	0.000	0.000	0.320	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	578	404	500	0	0	0	0	0	62	0
N.S.	1	0.70	0.87	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.802	1.444	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	308	384	0	0	0	0	0	61	0
N.S.	1	0.68	0.85	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.635	1.392	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	0	0	62	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.00	0.00	2.14	1.00
time (sec)	N/A	1.669	0.357	0.364	0.350	0.115	0.000	0.000	0.238	3.695

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	0	0	61	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.00	0.00	2.10	1.00
time (sec)	N/A	1.598	0.744	0.227	0.366	0.093	0.000	0.000	0.298	3.749

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	870	567	677	0	0	0	0	0	97	0
N.S.	1	0.65	0.78	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.291	4.894	0.000	0.000	0.000	0.000	0.000	0.340	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	793	532	633	0	0	0	0	0	95	0
N.S.	1	0.67	0.80	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.963	2.528	0.000	0.000	0.000	0.000	0.000	0.314	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	674	447	538	0	0	0	0	0	94	0
N.S.	1	0.66	0.80	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.804	3.483	0.000	0.000	0.000	0.000	0.000	0.320	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	0	0	95	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.00	0.00	3.28	1.00
time (sec)	N/A	2.273	0.366	0.234	0.383	0.097	0.000	0.000	0.283	3.627

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	0	0	94	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.00	0.00	3.24	1.00
time (sec)	N/A	2.211	0.537	0.246	0.396	0.091	0.000	0.000	0.343	3.712

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	323	265	292	0	0	0	0	0	29	0
N.S.	1	0.82	0.90	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.874	0.863	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	170	212	0	0	0	0	0	29	0
N.S.	1	0.81	1.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.549	0.535	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	138	154	0	0	0	0	0	27	0
N.S.	1	0.90	1.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.461	0.196	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	56	53	0	213	0	0	26	0
N.S.	1	0.98	1.27	1.20	0.00	4.84	0.00	0.00	0.59	0.00
time (sec)	N/A	0.237	0.030	0.109	0.000	0.106	0.000	0.000	0.187	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	39	27	28	29	28
N.S.	1	1.00	1.07	0.93	1.00	1.39	0.96	1.00	1.04	1.00
time (sec)	N/A	0.297	1.906	0.301	0.343	0.099	6.211	9.736	0.201	3.744

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	41	29	28	29	28
N.S.	1	1.00	1.07	0.93	1.00	1.46	1.04	1.00	1.04	1.00
time (sec)	N/A	0.295	1.067	0.316	0.347	0.107	28.555	9.739	0.203	3.847

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	379	279	291	0	0	0	0	0	34	0
N.S.	1	0.74	0.77	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.697	0.841	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	184	213	0	0	0	0	0	34	0
N.S.	1	0.73	0.84	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.572	0.592	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	152	153	0	0	0	0	0	32	0
N.S.	1	0.84	0.84	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.608	0.230	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	57	57	54	0	221	0	0	31	0
N.S.	1	0.93	0.93	0.89	0.00	3.62	0.00	0.00	0.51	0.00
time (sec)	N/A	0.392	0.037	0.110	0.000	0.099	0.000	0.000	0.209	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	42	29	29	34	29
N.S.	1	1.00	1.07	0.93	1.00	1.45	1.00	1.00	1.17	1.00
time (sec)	N/A	0.497	0.446	0.296	0.354	0.111	6.290	9.700	0.227	3.853

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	44	31	29	34	29
N.S.	1	1.00	1.07	0.93	1.00	1.52	1.07	1.00	1.17	1.00
time (sec)	N/A	0.510	0.457	0.330	0.353	0.109	28.553	9.833	0.225	3.856

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	56	31	29	59	29
N.S.	1	1.00	1.07	0.93	1.00	1.93	1.07	1.00	2.03	1.00
time (sec)	N/A	0.526	0.804	0.346	0.380	0.106	63.591	9.774	0.252	3.688

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	54	29	27	57	27
N.S.	1	1.00	1.07	0.93	1.00	2.00	1.07	1.00	2.11	1.00
time (sec)	N/A	0.391	0.586	0.280	0.374	0.110	60.512	9.827	0.218	3.597

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	53	27	26	56	26
N.S.	1	1.00	1.08	0.92	1.00	2.04	1.04	1.00	2.15	1.00
time (sec)	N/A	0.298	0.128	0.322	0.366	0.092	40.727	9.736	0.218	3.664

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	55	0	29	57	29
N.S.	1	1.00	1.07	0.93	1.00	1.90	0.00	1.00	1.97	1.00
time (sec)	N/A	0.334	0.606	0.447	0.387	0.108	0.000	9.692	0.216	3.907

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	57	0	29	59	29
N.S.	1	1.00	1.07	0.93	1.00	1.97	0.00	1.00	2.03	1.00
time (sec)	N/A	0.315	0.622	0.363	0.381	0.109	0.000	9.757	0.225	3.859

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	42	31	30	33	30
N.S.	1	1.00	1.07	0.93	1.00	1.40	1.03	1.00	1.10	1.00
time (sec)	N/A	0.282	0.529	0.312	0.368	0.103	78.944	9.600	0.232	4.090

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	32	43	0	0	71	31
N.S.	1	1.00	1.07	1.00	1.10	1.48	0.00	0.00	2.45	1.07
time (sec)	N/A	0.262	0.580	0.164	0.928	0.102	0.000	0.000	0.382	3.581

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	32	31	0	0	46	29
N.S.	1	1.00	1.07	1.00	1.19	1.15	0.00	0.00	1.70	1.07
time (sec)	N/A	0.233	0.392	0.139	0.874	0.098	0.000	0.000	0.320	3.623

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	0	20	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	0.00	1.25	1.12
time (sec)	N/A	0.193	0.029	0.126	0.348	0.109	27.750	0.000	0.242	3.343

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	34	33	27	33	35	31
N.S.	1	1.00	1.07	1.00	1.17	1.14	0.93	1.14	1.21	1.07
time (sec)	N/A	0.282	1.002	0.201	0.367	0.105	69.985	9.810	0.257	3.661

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	32	45	0	32	42	31
N.S.	1	1.00	1.07	1.00	1.10	1.55	0.00	1.10	1.45	1.07
time (sec)	N/A	0.276	1.259	0.214	0.413	0.114	0.000	9.622	0.272	3.695

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	44	0	0	70	31
N.S.	1	1.00	1.06	0.94	1.00	1.42	0.00	0.00	2.26	1.00
time (sec)	N/A	0.322	0.926	0.219	0.408	0.104	0.000	0.000	0.334	3.901

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	31	0	0	33	31
N.S.	1	1.00	1.06	0.94	1.00	1.00	0.00	0.00	1.06	1.00
time (sec)	N/A	0.313	0.241	0.195	0.415	0.119	0.000	0.000	0.248	3.748

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	46	32	31	37	31
N.S.	1	1.00	1.06	0.94	1.00	1.48	1.03	1.00	1.19	1.00
time (sec)	N/A	0.311	0.643	0.310	0.414	0.104	78.968	10.071	0.219	3.812

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	31	58	0	31	62	31
N.S.	1	1.00	1.06	0.94	1.00	1.87	0.00	1.00	2.00	1.00
time (sec)	N/A	0.371	0.916	0.467	0.391	0.117	0.000	10.291	0.205	3.814

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	173	122	123	178	140	0	0	185	0
N.S.	1	0.98	0.69	0.69	1.01	0.79	0.00	0.00	1.05	0.00
time (sec)	N/A	0.602	0.074	0.290	0.032	0.097	0.000	0.000	0.202	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	157	140	215	196	136	0	0	189	0
N.S.	1	0.98	0.87	1.34	1.22	0.84	0.00	0.00	1.17	0.00
time (sec)	N/A	0.570	0.114	0.216	0.033	0.098	0.000	0.000	0.197	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	137	101	105	139	119	0	0	145	0
N.S.	1	0.99	0.73	0.76	1.01	0.86	0.00	0.00	1.05	0.00
time (sec)	N/A	0.426	0.071	0.220	0.027	0.096	0.000	0.000	0.177	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	121	120	197	156	114	0	0	149	0
N.S.	1	0.99	0.98	1.61	1.28	0.93	0.00	0.00	1.22	0.00
time (sec)	N/A	0.433	0.092	0.213	0.035	0.088	0.000	0.000	0.184	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	76	76	91	94	0	0	103	0
N.S.	1	1.05	0.81	0.81	0.97	1.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.466	0.056	0.110	0.032	0.102	0.000	0.000	0.183	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	274	129	128	0	0	0	0	91	0
N.S.	1	1.04	0.49	0.48	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.986	0.136	0.504	0.000	0.000	0.000	0.000	0.186	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	105	102	63	132	0	0	80	0
N.S.	1	1.01	1.40	1.36	0.84	1.76	0.00	0.00	1.07	0.00
time (sec)	N/A	0.304	0.080	0.122	0.106	0.105	0.000	0.000	0.190	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	258	100	135	0	0	0	0	70	0
N.S.	1	1.03	0.40	0.54	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.888	0.078	0.364	0.000	0.000	0.000	0.000	0.191	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	128	141	85	139	0	0	100	0
N.S.	1	1.03	1.36	1.50	0.90	1.48	0.00	0.00	1.06	0.00
time (sec)	N/A	0.317	0.165	0.142	0.114	0.133	0.000	0.000	0.185	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	267	192	214	305	231	0	0	336	0
N.S.	1	1.02	0.74	0.82	1.17	0.89	0.00	0.00	1.29	0.00
time (sec)	N/A	0.638	0.186	0.257	0.036	0.123	0.000	0.000	0.194	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	266	214	364	332	227	0	0	343	0
N.S.	1	0.88	0.71	1.21	1.10	0.75	0.00	0.00	1.14	0.00
time (sec)	N/A	0.703	0.224	0.239	0.034	0.107	0.000	0.000	0.194	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	226	163	182	247	198	0	0	271	0
N.S.	1	1.06	0.76	0.85	1.15	0.93	0.00	0.00	1.27	0.00
time (sec)	N/A	0.866	0.148	0.247	0.040	0.100	0.000	0.000	0.191	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	246	183	304	273	195	0	0	278	0
N.S.	1	1.06	0.79	1.30	1.17	0.84	0.00	0.00	1.19	0.00
time (sec)	N/A	0.700	0.199	0.240	0.034	0.098	0.000	0.000	0.205	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	180	130	138	180	163	0	0	204	0
N.S.	1	1.11	0.80	0.85	1.11	1.01	0.00	0.00	1.26	0.00
time (sec)	N/A	0.596	0.120	0.171	0.033	0.092	0.000	0.000	0.205	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	372	228	223	0	0	0	0	193	0
N.S.	1	1.09	0.67	0.65	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	1.125	0.391	0.436	0.000	0.000	0.000	0.000	0.201	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	144	128	178	134	236	0	0	170	0
N.S.	1	1.02	0.91	1.26	0.95	1.67	0.00	0.00	1.21	0.00
time (sec)	N/A	0.559	0.158	0.195	0.113	0.123	0.000	0.000	0.190	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	329	173	196	0	0	0	0	169	0
N.S.	1	1.02	0.54	0.61	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.251	0.270	0.517	0.000	0.000	0.000	0.000	0.196	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	168	133	191	126	216	0	0	167	0
N.S.	1	1.20	0.95	1.36	0.90	1.54	0.00	0.00	1.19	0.00
time (sec)	N/A	0.863	0.153	0.184	0.104	0.139	0.000	0.000	0.194	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	276	319	451	334	0	0	512	0
N.S.	1	1.00	0.76	0.87	1.24	0.92	0.00	0.00	1.40	0.00
time (sec)	N/A	0.980	0.269	0.263	0.038	0.101	0.000	0.000	0.212	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	442	294	538	487	330	0	0	522	0
N.S.	1	1.11	0.74	1.35	1.22	0.83	0.00	0.00	1.31	0.00
time (sec)	N/A	0.895	0.357	0.289	0.037	0.104	0.000	0.000	0.204	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	313	236	273	374	289	0	0	422	0
N.S.	1	1.02	0.77	0.89	1.22	0.94	0.00	0.00	1.37	0.00
time (sec)	N/A	1.000	0.205	0.260	0.035	0.097	0.000	0.000	0.188	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	328	256	430	409	286	0	0	432	0
N.S.	1	1.05	0.82	1.38	1.32	0.92	0.00	0.00	1.39	0.00
time (sec)	N/A	0.855	0.259	0.257	0.037	0.101	0.000	0.000	0.200	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	253	193	211	287	241	0	0	330	0
N.S.	1	1.05	0.80	0.88	1.19	1.00	0.00	0.00	1.37	0.00
time (sec)	N/A	1.133	0.157	0.177	0.033	0.097	0.000	0.000	0.188	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	508	344	348	0	0	0	0	322	0
N.S.	1	1.00	0.68	0.68	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.507	0.697	0.467	0.000	0.000	0.000	0.000	0.199	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	217	182	273	222	327	0	0	272	0
N.S.	1	1.01	0.85	1.28	1.04	1.53	0.00	0.00	1.27	0.00
time (sec)	N/A	0.805	0.212	0.196	0.117	0.140	0.000	0.000	0.188	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	480	278	293	0	0	0	0	276	0
N.S.	1	1.19	0.69	0.73	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	2.063	0.501	0.556	0.000	0.000	0.000	0.000	0.194	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	198	184	270	197	322	0	0	260	0
N.S.	1	0.95	0.88	1.30	0.95	1.55	0.00	0.00	1.25	0.00
time (sec)	N/A	0.948	0.224	0.208	0.111	0.166	0.000	0.000	0.194	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	524	364	0	0	0	0	65	0
N.S.	1	1.00	0.84	0.58	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.477	0.790	23.088	0.000	0.000	0.000	0.000	0.222	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	512	2111	0	0	0	0	49	0
N.S.	1	1.00	0.98	4.05	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.305	0.390	1.454	0.000	0.000	0.000	0.000	0.232	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	457	284	0	0	0	0	52	0
N.S.	1	1.00	0.84	0.52	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.661	0.370	10.496	0.000	0.000	0.000	0.000	0.221	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	447	386	0	0	0	0	37	0
N.S.	1	1.00	1.00	0.86	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.630	0.097	0.403	0.000	0.000	0.000	0.000	0.215	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	397	232	0	0	0	0	46	0
N.S.	1	1.00	0.79	0.46	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.319	0.216	0.370	0.000	0.000	0.000	0.000	0.220	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	472	489	410	381	0	0	0	0	44	0
N.S.	1	1.04	0.87	0.81	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.400	0.195	0.555	0.000	0.000	0.000	0.000	0.252	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	549	331	0	0	0	0	58	0
N.S.	1	1.00	1.01	0.61	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.372	0.938	10.626	0.000	0.000	0.000	0.000	0.310	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	531	550	518	466	0	0	0	0	65	0
N.S.	1	1.04	0.98	0.88	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	2.107	0.345	0.599	0.000	0.000	0.000	0.000	0.374	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	657	424	0	0	0	0	75	0
N.S.	1	1.00	1.05	0.68	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.834	1.033	12.450	0.000	0.000	0.000	0.000	0.415	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	694	2132	0	0	0	0	123	0
N.S.	1	1.00	1.23	3.79	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.477	1.194	1.540	0.000	0.000	0.000	0.000	0.324	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	123	399	0	537	0	0	90	0
N.S.	1	1.00	1.09	3.53	0.00	4.75	0.00	0.00	0.80	0.00
time (sec)	N/A	0.301	0.246	2.260	0.000	0.124	0.000	0.000	0.321	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	581	598	754	505	0	0	0	0	138	0
N.S.	1	1.03	1.30	0.87	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.848	1.063	0.628	0.000	0.000	0.000	0.000	0.468	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	616	634	792	635	0	0	0	0	168	0
N.S.	1	1.03	1.29	1.03	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	2.221	1.409	0.800	0.000	0.000	0.000	0.000	0.764	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	777	897	0	0	0	0	149	0
N.S.	1	1.00	0.93	1.07	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	2.613	1.203	12.946	0.000	0.000	0.000	0.000	0.338	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	792	792	719	813	0	0	0	0	144	0
N.S.	1	1.00	0.91	1.03	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	3.570	2.710	9.748	0.000	0.000	0.000	0.000	0.316	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	804	804	734	828	0	0	0	0	137	0
N.S.	1	1.00	0.91	1.03	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.558	3.304	7.812	0.000	0.000	0.000	0.000	0.312	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	846	846	821	900	0	0	0	0	155	0
N.S.	1	1.00	0.97	1.06	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	2.936	3.395	13.887	0.000	0.000	0.000	0.000	0.520	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	727	737	1097	3551	0	0	0	0	238	0
N.S.	1	1.01	1.51	4.88	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.870	5.956	2.527	0.000	0.000	0.000	0.000	0.574	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	220	192	1169	0	1217	0	0	180	0
N.S.	1	0.95	0.83	5.06	0.00	5.27	0.00	0.00	0.78	0.00
time (sec)	N/A	0.594	0.577	0.224	0.000	0.197	0.000	0.000	0.573	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	166	183	1126	0	1233	0	0	172	0
N.S.	1	0.99	1.10	6.74	0.00	7.38	0.00	0.00	1.03	0.00
time (sec)	N/A	0.356	0.688	0.218	0.000	0.205	0.000	0.000	0.575	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	745	772	1147	1225	0	0	0	0	265	0
N.S.	1	1.04	1.54	1.64	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	2.294	5.579	0.844	0.000	0.000	0.000	0.000	1.029	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	805	834	1261	1455	0	0	0	0	299	0
N.S.	1	1.04	1.57	1.81	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	2.041	6.074	1.078	0.000	0.000	0.000	0.000	2.463	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1224	1224	1185	1752	0	0	0	0	272	0
N.S.	1	1.00	0.97	1.43	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	5.222	7.116	19.974	0.000	0.000	0.000	0.000	0.580	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1143	1222	0	0	0	0	271	0
N.S.	1	1.00	0.93	0.99	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	4.145	6.874	11.788	0.000	0.000	0.000	0.000	0.597	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1161	1778	0	0	0	0	263	0
N.S.	1	1.00	0.94	1.44	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	2.850	6.241	13.648	0.000	0.000	0.000	0.000	0.616	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	518	544	397	0	0	0	0	0	656	0
N.S.	1	1.05	0.77	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	2.427	0.792	0.000	0.000	0.000	0.000	0.000	0.397	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	350	293	0	0	0	0	0	360	0
N.S.	1	1.09	0.91	0.00	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	1.120	0.292	0.000	0.000	0.000	0.000	0.000	0.304	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	205	186	0	0	0	0	0	149	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.671	0.413	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	43	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.87	1.09
time (sec)	N/A	0.359	2.617	0.622	0.118	0.111	18.600	0.131	0.306	3.560

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	65	25
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	2.83	1.09
time (sec)	N/A	0.409	2.059	0.605	0.119	0.094	0.000	0.127	0.898	3.648

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	47	0	25	87	25
N.S.	1	1.00	1.09	1.00	1.09	2.04	0.00	1.09	3.78	1.09
time (sec)	N/A	0.333	3.227	0.641	0.124	0.096	0.000	0.130	2.365	3.598

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [54] had the largest ratio of [1.08000000000000007]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	8	1.08	23	0.348
2	A	7	7	1.09	23	0.304
3	A	6	6	1.05	23	0.261
4	A	4	4	1.03	21	0.190
5	A	4	4	1.03	20	0.200
6	C	12	11	1.20	23	0.478
7	A	7	6	0.97	23	0.261
8	C	13	12	1.04	23	0.522
9	A	6	5	1.03	23	0.217
10	A	7	6	0.97	25	0.240
11	A	11	10	1.16	25	0.400
12	A	7	6	1.02	25	0.240
13	A	5	5	0.98	23	0.217
14	A	7	6	1.08	22	0.273
15	C	17	16	1.31	25	0.640
16	A	8	7	1.04	25	0.280
17	C	18	17	1.24	25	0.680
18	A	12	11	1.15	25	0.440
19	A	7	6	0.94	25	0.240
20	A	10	10	0.97	25	0.400
21	A	7	6	0.97	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	6	0.97	23	0.261
23	A	7	6	1.02	22	0.273
24	C	22	21	1.51	25	0.840
25	A	7	6	0.93	25	0.240
26	C	23	22	1.39	25	0.880
27	A	10	9	0.94	25	0.360
28	C	14	13	1.18	25	0.520
29	C	13	12	1.04	25	0.480
30	C	10	9	1.00	25	0.360
31	C	9	8	1.01	23	0.348
32	C	7	6	0.98	22	0.273
33	C	8	7	1.07	25	0.280
34	C	11	10	1.04	25	0.400
35	C	11	10	0.99	25	0.400
36	C	16	15	1.13	25	0.600
37	C	14	13	1.12	25	0.520
38	C	15	14	1.19	25	0.560
39	C	10	9	0.98	25	0.360
40	A	2	2	1.00	23	0.087
41	C	10	9	0.98	22	0.409
42	C	11	10	1.04	25	0.400
43	C	15	14	1.24	25	0.560
44	C	15	14	1.45	25	0.560
45	C	22	21	1.41	25	0.840
46	C	15	14	1.32	25	0.560
47	A	7	7	1.23	25	0.280
48	C	12	11	0.97	25	0.440
49	A	3	3	0.98	23	0.130
50	C	12	11	0.97	22	0.500
51	C	14	13	1.17	25	0.520
52	C	19	18	1.24	25	0.720
53	C	19	18	1.13	25	0.720

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	C	28	27	1.43	25	1.080
55	A	7	7	0.83	27	0.259
56	A	5	5	0.88	27	0.185
57	A	3	3	1.00	24	0.125
58	A	3	3	1.00	27	0.111
59	A	5	5	0.74	27	0.185
60	A	4	4	0.67	27	0.148
61	A	4	4	0.64	27	0.148
62	A	3	3	0.64	27	0.111
63	A	3	3	0.68	27	0.111
64	A	4	4	0.74	25	0.160
65	A	8	7	0.70	27	0.259
66	A	8	7	0.70	27	0.259
67	A	10	9	0.68	27	0.333
68	A	12	12	0.90	27	0.444
69	A	10	10	0.96	27	0.370
70	A	8	8	1.10	24	0.333
71	A	8	8	1.06	27	0.296
72	A	8	8	1.03	27	0.296
73	A	6	5	0.59	27	0.185
74	A	6	5	0.58	27	0.185
75	A	6	5	0.58	27	0.185
76	A	6	5	0.56	27	0.185
77	A	4	4	0.57	27	0.148
78	A	4	4	0.58	27	0.148
79	A	4	4	0.59	27	0.148
80	A	4	4	0.59	25	0.160
81	A	12	11	0.79	27	0.407
82	A	13	12	0.77	27	0.444
83	A	13	12	0.81	27	0.444
84	A	18	17	0.95	27	0.630
85	A	16	15	1.01	27	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	11	1.10	24	0.458
87	A	14	13	0.97	27	0.481
88	A	14	13	1.06	27	0.481
89	A	14	13	1.07	27	0.481
90	A	7	6	0.51	27	0.222
91	A	7	6	0.57	27	0.222
92	A	6	5	0.53	27	0.185
93	A	4	4	0.53	27	0.148
94	A	4	4	0.52	27	0.148
95	A	4	4	0.51	27	0.148
96	A	5	5	0.50	25	0.200
97	A	16	15	0.86	27	0.556
98	A	16	15	0.83	27	0.556
99	A	17	16	0.84	27	0.593
100	A	5	5	1.00	27	0.185
101	A	4	4	0.99	27	0.148
102	A	3	3	0.96	27	0.111
103	A	2	2	0.97	25	0.080
104	A	1	1	0.95	24	0.042
105	A	6	5	0.54	27	0.185
106	A	2	2	0.97	27	0.074
107	A	8	7	0.67	27	0.259
108	A	4	4	0.96	27	0.148
109	A	4	4	0.70	27	0.148
110	A	10	9	1.08	27	0.333
111	A	4	4	0.76	27	0.148
112	A	5	5	1.00	27	0.185
113	A	4	4	1.00	25	0.160
114	A	2	2	1.00	24	0.083
115	A	10	9	0.69	27	0.333
116	A	7	6	0.78	27	0.222
117	A	15	14	0.77	27	0.519

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	0.72	27	0.185
119	A	6	6	0.75	27	0.222
120	A	11	10	1.18	27	0.370
121	A	4	4	0.87	27	0.148
122	A	6	5	0.81	27	0.185
123	A	4	4	0.84	25	0.160
124	A	5	5	1.10	24	0.208
125	A	14	13	0.87	27	0.481
126	A	6	5	0.73	27	0.185
127	A	19	18	0.94	27	0.667
128	A	6	5	0.70	27	0.185
129	A	5	5	1.06	22	0.227
130	A	4	4	1.04	22	0.182
131	A	3	3	1.00	22	0.136
132	A	2	2	1.02	20	0.100
133	A	1	1	1.00	19	0.053
134	A	6	5	0.64	22	0.227
135	A	2	2	1.02	22	0.091
136	A	8	7	0.74	22	0.318
137	A	4	4	0.99	22	0.182
138	A	1	1	1.00	30	0.033
139	A	1	1	1.00	31	0.032
140	A	9	9	1.10	27	0.333
141	A	8	8	1.12	27	0.296
142	A	6	6	1.14	25	0.240
143	N/A	1	0	1.00	27	0.000
144	N/A	6	0	1.00	27	0.000
145	N/A	10	0	1.00	27	0.000
146	A	12	12	0.76	29	0.414
147	A	8	8	0.87	29	0.276
148	A	3	3	0.97	29	0.103
149	A	1	1	1.00	29	0.034

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	5	5	0.99	29	0.172
151	A	8	8	0.96	29	0.276
152	A	12	12	0.69	35	0.343
153	A	8	8	0.78	35	0.229
154	A	3	3	0.92	35	0.086
155	A	1	1	1.00	35	0.029
156	A	5	5	0.96	35	0.143
157	A	8	8	0.92	35	0.229
158	A	1	1	0.99	24	0.042
159	A	14	14	1.29	29	0.483
160	A	11	11	1.03	29	0.379
161	A	7	7	0.88	27	0.259
162	A	5	5	0.89	26	0.192
163	A	9	8	0.59	29	0.276
164	C	11	10	0.79	29	0.345
165	A	11	10	0.59	29	0.345
166	C	16	15	0.65	29	0.517
167	A	25	25	1.46	29	0.862
168	A	21	21	1.25	29	0.724
169	A	9	8	0.75	27	0.296
170	A	12	12	1.10	26	0.462
171	A	16	15	0.69	29	0.517
172	C	20	19	0.89	29	0.655
173	A	17	16	0.61	29	0.552
174	C	26	25	0.97	29	0.862
175	F	0	0	N/A	0.000	N/A
176	F	0	0	N/A	0.000	N/A
177	A	10	9	0.64	27	0.333
178	A	14	14	1.21	26	0.538
179	A	24	23	0.81	29	0.793
180	C	29	28	1.13	29	0.966
181	A	25	24	0.78	29	0.828

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	F	0	0	N/A	0.000	N/A
183	A	14	14	1.29	29	0.483
184	A	11	11	1.06	29	0.379
185	A	7	7	1.12	29	0.241
186	A	5	5	0.83	29	0.172
187	A	2	2	0.97	27	0.074
188	A	1	1	0.95	26	0.038
189	A	7	6	0.48	29	0.207
190	C	10	9	0.71	29	0.310
191	A	11	10	0.56	29	0.345
192	C	13	12	0.79	29	0.414
193	C	19	18	0.94	29	0.621
194	C	14	13	0.83	29	0.448
195	C	13	12	0.77	29	0.414
196	C	10	9	0.64	27	0.333
197	C	10	9	0.71	26	0.346
198	A	16	15	0.55	29	0.517
199	C	20	19	0.82	29	0.655
200	A	28	27	0.67	29	0.931
201	C	19	18	1.05	29	0.621
202	C	20	19	1.02	29	0.655
203	C	16	15	0.72	29	0.517
204	C	11	10	0.66	27	0.370
205	C	14	13	0.87	26	0.500
206	A	26	25	0.77	29	0.862
207	C	24	23	1.01	29	0.793
208	A	11	11	1.21	24	0.458
209	A	8	8	1.09	24	0.333
210	A	5	5	0.93	24	0.208
211	A	3	3	1.08	22	0.136
212	A	1	1	1.00	21	0.048
213	A	7	6	0.58	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	C	9	8	0.83	24	0.333
215	A	11	10	0.65	24	0.417
216	N/A	25	0	1.00	31	0.000
217	N/A	15	0	1.00	31	0.000
218	N/A	6	0	1.00	31	0.000
219	N/A	1	0	1.00	31	0.000
220	N/A	1	0	1.00	31	0.000
221	N/A	1	0	1.00	26	0.000
222	A	11	11	1.10	24	0.458
223	A	10	10	1.07	24	0.417
224	A	6	6	0.84	24	0.250
225	A	4	4	0.90	22	0.182
226	A	1	1	1.00	21	0.048
227	A	8	7	0.58	24	0.292
228	C	10	9	0.75	24	0.375
229	A	15	14	0.59	24	0.583
230	N/A	1	0	1.00	30	0.000
231	A	4	3	0.56	28	0.107
232	A	4	3	0.58	28	0.107
233	A	4	3	0.65	28	0.107
234	A	4	3	0.62	26	0.115
235	A	6	5	0.65	25	0.200
236	N/A	2	0	1.00	28	0.000
237	N/A	2	0	1.00	28	0.000
238	A	4	3	0.56	28	0.107
239	A	4	3	0.56	28	0.107
240	A	4	3	0.59	26	0.115
241	A	5	4	0.59	25	0.160
242	N/A	2	0	1.00	28	0.000
243	N/A	2	0	1.00	28	0.000
244	A	4	3	0.56	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	4	3	0.55	28	0.107
246	A	4	3	0.56	26	0.115
247	A	6	5	0.56	25	0.200
248	N/A	2	0	1.00	28	0.000
249	N/A	2	0	1.00	28	0.000
250	A	5	4	0.56	24	0.167
251	A	5	4	0.68	24	0.167
252	A	5	4	0.68	24	0.167
253	A	4	3	1.00	22	0.136
254	A	1	1	0.97	21	0.048
255	N/A	1	0	1.00	24	0.000
256	N/A	1	0	1.00	24	0.000
257	A	5	4	0.62	28	0.143
258	A	5	4	0.65	28	0.143
259	A	9	8	0.74	26	0.308
260	A	1	1	0.97	25	0.040
261	N/A	1	0	1.00	28	0.000
262	N/A	1	0	1.00	28	0.000
263	N/A	1	0	1.00	28	0.000
264	N/A	1	0	1.00	26	0.000
265	N/A	1	0	1.00	25	0.000
266	N/A	1	0	1.00	28	0.000
267	N/A	1	0	1.00	28	0.000
268	N/A	1	0	1.00	28	0.000
269	N/A	1	0	1.00	28	0.000
270	N/A	1	0	1.00	28	0.000
271	N/A	1	0	1.00	28	0.000
272	N/A	1	0	1.00	28	0.000
273	A	6	5	1.01	28	0.179
274	C	15	14	1.70	28	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
275	C	16	15	1.02	26	0.577
276	C	14	13	0.90	25	0.520
277	N/A	13	0	1.00	28	0.000
278	N/A	2	0	1.00	28	0.000
279	A	8	7	0.88	28	0.250
280	C	15	14	1.03	26	0.538
281	A	8	7	0.73	25	0.280
282	N/A	12	0	1.00	28	0.000
283	N/A	4	0	1.00	28	0.000
284	A	7	6	1.01	28	0.214
285	C	14	13	1.04	26	0.500
286	A	7	6	0.65	25	0.240
287	N/A	11	0	1.00	28	0.000
288	N/A	3	0	1.00	28	0.000
289	A	6	5	0.64	28	0.179
290	A	6	5	0.72	28	0.179
291	A	6	5	0.70	28	0.179
292	C	14	13	0.89	28	0.464
293	C	12	11	0.88	26	0.423
294	A	1	1	1.03	25	0.040
295	N/A	2	0	1.00	28	0.000
296	N/A	2	0	1.00	28	0.000
297	N/A	3	0	1.00	28	0.000
298	N/A	1	0	1.00	26	0.000
299	N/A	3	0	1.00	25	0.000
300	N/A	1	0	1.00	28	0.000
301	N/A	1	0	1.00	28	0.000
302	N/A	1	0	1.00	30	0.000
303	N/A	1	0	1.00	30	0.000
304	N/A	2	0	1.00	30	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	N/A	1	0	1.00	30	0.000
306	N/A	1	0	1.00	30	0.000
307	A	5	4	1.79	27	0.148
308	A	5	4	1.54	27	0.148
309	A	10	9	1.14	25	0.360
310	A	5	4	0.99	24	0.167
311	N/A	9	0	1.00	27	0.000
312	A	5	4	1.19	29	0.138
313	A	5	4	1.56	29	0.138
314	A	9	8	1.56	27	0.296
315	A	5	4	0.94	26	0.154
316	N/A	8	0	1.00	29	0.000
317	A	8	7	0.77	19	0.368
318	A	4	3	0.73	29	0.103
319	A	4	3	0.74	27	0.111
320	A	6	5	0.73	26	0.192
321	N/A	2	0	1.00	29	0.000
322	N/A	2	0	1.00	29	0.000
323	A	4	3	0.67	29	0.103
324	A	4	3	0.70	27	0.111
325	A	5	4	0.68	26	0.154
326	N/A	2	0	1.00	29	0.000
327	N/A	2	0	1.00	29	0.000
328	A	4	3	0.65	29	0.103
329	A	4	3	0.67	27	0.111
330	A	6	5	0.66	26	0.192
331	N/A	2	0	1.00	29	0.000
332	N/A	2	0	1.00	29	0.000
333	A	5	4	0.82	28	0.143
334	A	5	4	0.81	28	0.143
335	A	6	5	0.90	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	1	1	0.98	25	0.040
337	N/A	1	0	1.00	28	0.000
338	N/A	1	0	1.00	28	0.000
339	A	5	4	0.74	29	0.138
340	A	5	4	0.73	29	0.138
341	A	6	5	0.84	27	0.185
342	A	1	1	0.93	26	0.038
343	N/A	1	0	1.00	29	0.000
344	N/A	1	0	1.00	29	0.000
345	N/A	1	0	1.00	29	0.000
346	N/A	1	0	1.00	27	0.000
347	N/A	1	0	1.00	26	0.000
348	N/A	1	0	1.00	29	0.000
349	N/A	1	0	1.00	29	0.000
350	N/A	1	0	1.00	30	0.000
351	N/A	1	0	1.00	29	0.000
352	N/A	1	0	1.00	27	0.000
353	N/A	1	0	1.00	16	0.000
354	N/A	1	0	1.00	29	0.000
355	N/A	1	0	1.00	29	0.000
356	N/A	1	0	1.00	31	0.000
357	N/A	1	0	1.00	31	0.000
358	N/A	1	0	1.00	31	0.000
359	N/A	1	0	1.00	31	0.000
360	A	7	7	0.98	19	0.368
361	A	7	7	0.98	19	0.368
362	A	5	5	0.99	19	0.263
363	A	5	5	0.99	17	0.294
364	A	4	4	1.05	16	0.250
365	A	4	4	1.04	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
366	A	5	4	1.01	19	0.211
367	A	4	4	1.03	19	0.211
368	A	6	5	1.03	19	0.263
369	A	7	6	1.02	21	0.286
370	A	10	9	0.88	21	0.429
371	A	7	6	1.06	21	0.286
372	A	8	7	1.06	19	0.368
373	A	7	6	1.11	18	0.333
374	A	4	4	1.09	21	0.190
375	A	8	7	1.02	21	0.333
376	A	4	4	1.02	21	0.190
377	A	10	9	1.20	21	0.429
378	A	7	6	1.00	21	0.286
379	A	12	11	1.11	21	0.524
380	A	7	6	1.02	21	0.286
381	A	9	8	1.05	19	0.421
382	A	7	6	1.05	18	0.333
383	A	4	4	1.00	21	0.190
384	A	7	6	1.01	21	0.286
385	A	4	4	1.19	21	0.190
386	A	10	9	0.95	21	0.429
387	A	2	2	1.00	21	0.095
388	A	2	2	1.00	21	0.095
389	A	2	2	1.00	21	0.095
390	A	2	2	1.00	19	0.105
391	A	2	2	1.00	18	0.111
392	A	2	2	1.04	21	0.095
393	A	2	2	1.00	21	0.095
394	A	2	2	1.04	21	0.095
395	A	2	2	1.00	21	0.095
396	A	2	2	1.00	21	0.095
397	A	5	4	1.00	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
398	A	2	2	1.03	21	0.095
399	A	2	2	1.03	21	0.095
400	A	2	2	1.00	21	0.095
401	A	2	2	1.00	21	0.095
402	A	2	2	1.00	18	0.111
403	A	2	2	1.00	21	0.095
404	A	2	2	1.01	21	0.095
405	A	11	10	0.95	21	0.476
406	A	6	5	0.99	19	0.263
407	A	2	2	1.04	21	0.095
408	A	2	2	1.04	21	0.095
409	A	2	2	1.00	21	0.095
410	A	2	2	1.00	21	0.095
411	A	2	2	1.00	18	0.111
412	A	8	8	1.05	23	0.348
413	A	7	7	1.09	23	0.304
414	A	5	5	1.11	21	0.238
415	N/A	1	0	1.00	23	0.000
416	N/A	1	0	1.00	23	0.000
417	N/A	1	0	1.00	23	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^4(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int x^4(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx = -\frac{152bd\sqrt{-1+cx}\sqrt{1+cx}}{3675c^5} - \frac{76bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^3} - \frac{19bdx^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c} + \frac{1}{49}bcdx^6\sqrt{-1+cx}\sqrt{1+cx} + \frac{1}{5}dx^5(a+b\text{arccosh}(cx)) - \frac{1}{7}c^2dx^7(a+b\text{arccosh}(cx))$$

output
```
-152/3675*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-76/3675*b*d*x^2*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c^3-19/1225*b*d*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/49*b*c
*d*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/5*d*x^5*(a+b*arccosh(c*x))-1/7*c^2*d*
x^7*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

$$\int x^4(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))\,dx$$
$$=\frac{d\left(-105ax^5(-7+5c^2x^2)+\frac{b\sqrt{-1+cx}\sqrt{1+cx}(-152-76c^2x^2-57c^4x^4+75c^6x^6)}{c^5}-105bx^5(-7+5c^2x^2)\operatorname{arccosh}(cx)\right)}{3675}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*(-105*a*x^5*(-7 + 5*c^2*x^2) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-152 -
76*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6))/c^5 - 105*b*x^5*(-7 + 5*c^2*x^2)*Ar
cCosh[c*x]))/3675
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.348$, Rules used = {6336, 27, 960, 111, 27, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))\,dx$$
$$\Big\downarrow \text{6336}$$
$$-bc\int\frac{dx^5(7-5c^2x^2)}{35\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^2dx^7(a+b\operatorname{arccosh}(cx))+\frac{1}{5}dx^5(a+b\operatorname{arccosh}(cx))$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{35}bcd\int\frac{x^5(7-5c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^2dx^7(a+b\operatorname{arccosh}(cx))+\frac{1}{5}dx^5(a+b\operatorname{arccosh}(cx))$$
$$\Big\downarrow \text{960}$$

$$-\frac{1}{35}bcd\left(\frac{19}{7}\int \frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2dx^7(a+b\text{arccosh}(cx))+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 111}$$

$$-\frac{1}{35}bcd\left(\frac{19}{7}\left(\frac{\int \frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2dx^7(a+b\text{arccosh}(cx))+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{35}bcd\left(\frac{19}{7}\left(\frac{4\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2dx^7(a+b\text{arccosh}(cx))+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 111}$$

$$-\frac{1}{35}bcd\left(\frac{19}{7}\left(\frac{4\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2dx^7(a+b\text{arccosh}(cx))+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{35}bcd\left(\frac{19}{7}\left(\frac{4\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2dx^7(a+b\text{arccosh}(cx))+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{1}{7}c^2dx^7(a+b\text{arccosh}(cx))+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))-\frac{1}{35}bcd\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)$$

input
```
Int[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/35*(b*c*d*((-5*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/7 + (19*((x^4*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(5*c^2) + (4*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^
4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2)))/(5*c^2)))/7)) + (d*x^5*(
a + b*ArcCosh[c*x]))/5 - (c^2*d*x^7*(a + b*ArcCosh[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

method	result
parts	$-da\left(\frac{1}{7}c^2x^7-\frac{1}{5}x^5\right)-\frac{db\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(75c^6x^6-57c^4x^4-76c^2x^2-152\right)}{3675}\right)}{c^5}$
derivativedivides	$\frac{-da\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-db\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(75c^6x^6-57c^4x^4-76c^2x^2-152\right)}{3675}\right)}{c^5}$
default	$\frac{-da\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-db\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(75c^6x^6-57c^4x^4-76c^2x^2-152\right)}{3675}\right)}{c^5}$
orering	$\frac{(975c^8x^8-1377c^6x^6-228c^4x^4-608c^2x^2+608)(-c^2d\,x^2+d)(a+b\operatorname{arccosh}(cx))}{3675c^6x(c^2x^2-1)}-\frac{(75c^6x^6-57c^4x^4-76c^2x^2-152)\Big(}{}$

input
```
int(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/7*c^2*x^7-1/5*x^5)-d*b/c^5*(1/7*arccosh(c*x)*c^7*x^7-1/5*arccosh(c
*x)*c^5*x^5-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-57*c^4*x^4-76*c
^2*x^2-152))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^4\left(d-c^2dx^2\right)\left(a+b\operatorname{arccosh}(cx)\right)dx =$$

$$-\frac{525\,ac^7dx^7-735\,ac^5dx^5+105\left(5\,bc^7dx^7-7\,bc^5dx^5\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(75\,bc^6dx^6-57\,bc^4dx^4-\right.}{3675\,c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/3675*(525*a*c^7*d*x^7 - 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 - 7*b*c^5*
d*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d*x^6 - 57*b*c^4*d*x^4 - 7
6*b*c^2*d*x^2 - 152*b*d)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [F]

$$\int x^4(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx = -d\left(\int\left(-ax^4\right)\,dx+\int ac^2x^6\,dx+\int\left(-bx^4\operatorname{acosh}(cx)\right)\,dx+\int bc^2x^6\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate(x**4*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
-d*(Integral(-a*x**4, x) + Integral(a*c**2*x**6, x) + Integral(-b*x**4*aco
sh(c*x), x) + Integral(b*c**2*x**6*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\int x^4(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx = -\frac{1}{7}\,ac^2dx^7+\frac{1}{5}\,adx^5-\frac{1}{245}\left(35\,x^7\operatorname{arcosh}(cx)-\left(\frac{5\,\sqrt{c^2x^2-1}x^6}{c^2}+\frac{6\,\sqrt{c^2x^2-1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2-1}x^2}{c^6}+\frac{16\,\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*
x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c
^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arccosh(c*x) - (3
*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2
- 1)/c^6)*c)*b*d
```

Giac [F(-2)]

Exception generated.

$$\int x^4(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx = \int x^4\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int x^4(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{d\left(-525acosh(cx)\,b\,c^7x^7+735acosh(cx)\,b\,c^5x^5+75\sqrt{c^2x^2-1}\,b\,c^6x^6-57\sqrt{c^2x^2-1}\,b\,c^4x^4-76\sqrt{c^2x^2-}\right.}{3675c^5}$$

input
```
int(x^4*(-c^2*d*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(d*( - 525*acosh(c*x)*b*c**7*x**7 + 735*acosh(c*x)*b*c**5*x**5 + 75*sqrt(c
**2*x**2 - 1)*b*c**6*x**6 - 57*sqrt(c**2*x**2 - 1)*b*c**4*x**4 - 76*sqrt(c
**2*x**2 - 1)*b*c**2*x**2 - 152*sqrt(c**2*x**2 - 1)*b - 525*a*c**7*x**7 +
735*a*c**5*x**5))/(3675*c**5)
```

3.2 $\int x^3(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int x^3(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx = -\frac{bdx\sqrt{-1+cx}\sqrt{1+cx}}{24c^3} - \frac{bdx^3\sqrt{-1+cx}\sqrt{1+cx}}{36c} + \frac{1}{36}bcdx^5\sqrt{-1+cx}\sqrt{1+cx} - \frac{bd\text{arccosh}(cx)}{24c^4} + \frac{1}{4}dx^4(a+b\text{arccosh}(cx)) - \frac{1}{6}c^2dx^6(a+b\text{arccosh}(cx))$$

output
```
-1/24*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*b*d*x^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c+1/36*b*c*d*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/24*b*d*arccosh(c
*x)/c^4+1/4*d*x^4*(a+b*arccosh(c*x))-1/6*c^2*d*x^6*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.23

$$\int x^3\left(d-c^2dx^2\right)(a+b\operatorname{arccosh}(cx))\,dx=\frac{1}{4}adx^4-\frac{1}{6}ac^2dx^6-\frac{bdx\sqrt{-1+cx}\sqrt{1+cx}}{24c^3}-\frac{bdx^3\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{4}bdx^4\operatorname{arccosh}(cx)-\frac{1}{6}bc^2dx^6\operatorname{arccosh}(cx)-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{-1+cx}}{\sqrt{1+cx}}\right)}{12c^4}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(a*d*x^4)/4 - (a*c^2*d*x^6)/6 - (b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c
^3) - (b*d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/36 + (b*d*x^4*ArcCosh[c*x])/4 - (b*c^2*d*x^6*ArcCosh[
c*x])/6 - (b*d*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(12*c^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6336, 27, 960, 111, 27, 101, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d-c^2dx^2\right)(a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int\frac{dx^4\left(3-2c^2x^2\right)}{12\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{6}c^2dx^6(a+b\operatorname{arccosh}(cx))+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{12}bcd\int \frac{x^4(3-2c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{6}c^2dx^6(a+b\operatorname{arccosh}(cx))+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 960$$

$$-\frac{1}{12}bcd\left(\frac{4}{3}\int \frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2dx^6(a+b\operatorname{arccosh}(cx))+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 111$$

$$-\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{\int \frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2dx^6(a+b\operatorname{arccosh}(cx))+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{3\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2dx^6(a+b\operatorname{arccosh}(cx))+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 101$$

$$-\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2dx^6(a+b\operatorname{arccosh}(cx))+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 43$$

$$-\frac{1}{6}c^2dx^6(a+b\operatorname{arccosh}(cx))+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))-\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{3\left(\frac{\operatorname{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)$$

input

```
Int[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*d*x^6*(a + b*ArcCosh[c*x]))/6 - (b*c
*d*(-1/3*(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*((x^3*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])/(4*c^2) + (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCos
h[c*x]/(2*c^3)))/(4*c^2)))/3))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

method	result
parts	$-da\left(\frac{1}{6}c^2x^6-\frac{1}{4}x^4\right)-\frac{db\left(\frac{\operatorname{arccosh}(cx)c^6x^6}{6}-\frac{\operatorname{arccosh}(cx)c^4x^4}{4}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-2c^5x^5\sqrt{c^2x^2-1}+2\sqrt{c^2x^2-1}\,c^3x^3+\right.}{72\sqrt{c^2x^2-1}}\right.}{c^4}$
derivativedivides	$\frac{-da\left(\frac{1}{6}c^6x^6-\frac{1}{4}c^4x^4\right)-db\left(\frac{\operatorname{arccosh}(cx)c^6x^6}{6}-\frac{\operatorname{arccosh}(cx)c^4x^4}{4}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-2c^5x^5\sqrt{c^2x^2-1}+2\sqrt{c^2x^2-1}\,c^3x^3+3cx\sqrt{c^2\right.}{72\sqrt{c^2x^2-1}}\right.}{c^4}$
default	$\frac{-da\left(\frac{1}{6}c^6x^6-\frac{1}{4}c^4x^4\right)-db\left(\frac{\operatorname{arccosh}(cx)c^6x^6}{6}-\frac{\operatorname{arccosh}(cx)c^4x^4}{4}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-2c^5x^5\sqrt{c^2x^2-1}+2\sqrt{c^2x^2-1}\,c^3x^3+3cx\sqrt{c^2\right.}{72\sqrt{c^2x^2-1}}\right.}{c^4}$
orering	$\frac{(22c^6x^6-34c^4x^4-9c^2x^2+12)\left(-c^2d\,x^2+d\right)(a+b\operatorname{arccosh}(cx))}{72c^4(c^2x^2-1)}-\frac{(2c^4x^4-2c^2x^2-3)\left(3x^2\left(-c^2d\,x^2+d\right)(a+b\operatorname{arccosh}\right.}{72}$

input
```
int(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/6*c^2*x^6-1/4*x^4)-d*b/c^4*(1/6*arccosh(c*x)*c^6*x^6-1/4*arccosh(c
*x)*c^4*x^4+1/72*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-2*c^5*x^5*(c^2*x^2-1)^(1/2)
+2*(c^2*x^2-1)^(1/2)*c^3*x^3+3*c*x*(c^2*x^2-1)^(1/2)+3*ln(c*x+(c^2*x^2-1)^
(1/2)))/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80

$$\int x^3(d - c^2dx^2)(a + b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{12\,ac^6dx^6 - 18\,ac^4dx^4 + 3\,(4\,bc^6dx^6 - 6\,bc^4dx^4 + bd)\log\left(cx + \sqrt{c^2x^2 - 1}\right) - (2\,bc^5dx^5 - 2\,bc^3dx^3 - 3}{72\,c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/72*(12*a*c^6*d*x^6 - 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 - 6*b*c^4*d*x^4
+ b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d*x^5 - 2*b*c^3*d*x^3 - 3*b
*c*d*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [F]

$$\int x^3(d - c^2dx^2)(a + b\operatorname{arccosh}(cx))\,dx = -d\left(\int(-ax^3)\,dx + \int ac^2x^5\,dx + \int(-bx^3\operatorname{acosh}(cx))\,dx + \int bc^2x^5\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate(x**3*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
-d*(Integral(-a*x**3, x) + Integral(a*c**2*x**5, x) + Integral(-b*x**3*aco
sh(c*x), x) + Integral(b*c**2*x**5*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.50

$$\int x^3(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx = -\frac{1}{6}\,ac^2dx^6 + \frac{1}{4}\,adx^4 - \frac{1}{288}\left(48\,x^6\,\text{arcosh}\,(cx) - \left(\frac{8\,\sqrt{c^2x^2-1}x^5}{c^2} + \frac{10\,\sqrt{c^2x^2-1}x^3}{c^4} + \frac{15\,\sqrt{c^2x^2-1}x}{c^6} + \frac{15\,\log\left(2\,c^2x+2\,\sqrt{\vphantom{x}}\right.}{c^7}\right.\right. + \frac{1}{32}\left(8\,x^4\,\text{arcosh}\,(cx) - \left(\frac{2\,\sqrt{c^2x^2-1}x^3}{c^2} + \frac{3\,\sqrt{c^2x^2-1}x}{c^4} + \frac{3\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bd$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*
x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c
^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d + 1/32*(8*x^4
*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 +
 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d
```

Giac [F(-2)]

Exception generated.

$$\int x^3(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d-c^2dx^2) (a+b\text{arccosh}(cx)) \, dx = \int x^3 (a+b\,\text{acosh}(c\,x)) \left(d-c^2\,d\,x^2\right) \, dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int x^3 (d-c^2dx^2) (a+b\text{arccosh}(cx)) \, dx = \frac{d\left(-12acosh(cx)\,b\,c^6x^6+18acosh(cx)\,b\,c^4x^4+2\sqrt{c^2x^2-1}\,b\,c^5x^5-2\sqrt{c^2x^2-1}\,b\,c^3x^3-3\sqrt{c^2x^2-1}\,bcx}{72c^4}$$

input
```
int(x^3*(-c^2*d*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(d*( - 12*acosh(c*x)*b*c**6*x**6 + 18*acosh(c*x)*b*c**4*x**4 + 2*sqrt(c**2
*x**2 - 1)*b*c**5*x**5 - 2*sqrt(c**2*x**2 - 1)*b*c**3*x**3 - 3*sqrt(c**2*x
**2 - 1)*b*c*x - 3*log(sqrt(c**2*x**2 - 1) + c*x)*b - 12*a*c**6*x**6 + 18*
a*c**4*x**4))/(72*c**4)
```

3.3 $\int x^2(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int x^2(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx = -\frac{26bd\sqrt{-1+cx}\sqrt{1+cx}}{225c^3} - \frac{13bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{225c} + \frac{1}{25}bcdx^4\sqrt{-1+cx}\sqrt{1+cx} + \frac{1}{3}dx^3(a+b\text{arccosh}(cx)) - \frac{1}{5}c^2dx^5(a+b\text{arccosh}(cx))$$

output
```
-26/225*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-13/225*b*d*x^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c+1/25*b*c*d*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/3*d*x^3*(a+b*a
rccosh(c*x))-1/5*c^2*d*x^5*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int x^2(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx = -\frac{d(15ac^3x^3(-5+3c^2x^2)+b\sqrt{-1+cx}\sqrt{1+cx}(26+13c^2x^2-9c^4x^4)+15bc^3x^3(-5+3c^2x^2)\text{arccosh}(}{225c^3}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/225*(d*(15*a*c^3*x^3*(-5 + 3*c^2*x^2) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(26 + 13*c^2*x^2 - 9*c^4*x^4) + 15*b*c^3*x^3*(-5 + 3*c^2*x^2)*ArcCosh[c*x]
))/c^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6336, 27, 960, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d-c^2dx^2)(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int \frac{dx^3(5-3c^2x^2)}{15\sqrt{cx-1}\sqrt{cx+1}}dx - \frac{1}{5}c^2dx^5(a+b\text{arccosh}(cx)) + \frac{1}{3}dx^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bcd\int \frac{x^3(5-3c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx - \frac{1}{5}c^2dx^5(a+b\text{arccosh}(cx)) + \frac{1}{3}dx^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{960}$$

$$-\frac{1}{15}bcd\left(\frac{13}{5}\int\frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{3}{5}x^4\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{5}c^2dx^5(a+b\text{arccosh}(cx))+\frac{1}{3}dx^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{111}$$

$$-\frac{1}{15}bcd\left(\frac{13}{5}\left(\frac{\int\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)-\frac{3}{5}x^4\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{5}c^2dx^5(a+b\text{arccosh}(cx))+\frac{1}{3}dx^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bcd\left(\frac{13}{5}\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)-\frac{3}{5}x^4\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{5}c^2dx^5(a+b\text{arccosh}(cx))+\frac{1}{3}dx^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{83}$$

$$-\frac{1}{5}c^2dx^5(a+b\text{arccosh}(cx))+\frac{1}{3}dx^3(a+b\text{arccosh}(cx))-\frac{1}{15}bcd\left(\frac{13}{5}\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)-\frac{3}{5}x^4\sqrt{cx-1}\sqrt{cx+1}\right)$$

input
```
Int[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/15*(b*c*d*((-3*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/5 + (13*((2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2)))
/5)) + (d*x^3*(a + b*ArcCosh[c*x]))/3 - (c^2*d*x^5*(a + b*ArcCosh[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

method	result
parts	$-da\left(\frac{1}{5}c^2x^5-\frac{1}{3}x^3\right)-\frac{db\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4x^4-13c^2x^2-26)}{225}\right)}{c^3}$
derivativedivides	$\frac{-da\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-db\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4x^4-13c^2x^2-26)}{225}\right)}{c^3}$
default	$\frac{-da\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-db\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4x^4-13c^2x^2-26)}{225}\right)}{c^3}$
orering	$\frac{(81c^6x^6-145c^4x^4-78c^2x^2+52)(-c^2d\,x^2+d)(a+b\operatorname{arccosh}(cx))}{225c^4x(c^2x^2-1)}-\frac{(9c^4x^4-13c^2x^2-26)\left(2x(-c^2d\,x^2+d)(a+b\operatorname{arcc}\right.}{}$

input
```
int(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/5*c^2*x^5-1/3*x^3)-d*b/c^3*(1/5*arccosh(c*x)*c^5*x^5-1/3*c^3*x^3*a
rccosh(c*x)-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*x^4-13*c^2*x^2-26))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int x^2(d-c^2dx^2)\,(a+b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{45\,ac^5dx^5-75\,ac^3dx^3+15\,(3\,bc^5dx^5-5\,bc^3dx^3)\log\left(cx+\sqrt{c^2x^2-1}\right)-(9\,bc^4dx^4-13\,bc^2dx^2-26\,b}{225\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/225*(45*a*c^5*d*x^5 - 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 - 5*b*c^3*d*x^
3)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^4*d*x^4 - 13*b*c^2*d*x^2 - 26*b*d
)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [F]

$$\int x^2(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))\,dx = -d\left(\int(-ax^2)\,dx + \int ac^2x^4\,dx + \int(-bx^2\operatorname{acosh}(cx))\,dx + \int bc^2x^4\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate(x**2*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
-d*(Integral(-a*x**2, x) + Integral(a*c**2*x**4, x) + Integral(-b*x**2*aco
sh(c*x), x) + Integral(b*c**2*x**4*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

$$\int x^2(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))\,dx = -\frac{1}{5}ac^2dx^5 - \frac{1}{75}\left(15x^5\operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^2d + \frac{1}{3}adx^3 + \frac{1}{9}\left(3x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/5*a*c^2*d*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^
2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^2*d + 1/
3*a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqr
t(c^2*x^2 - 1)/c^4))*b*d
```

Giac [F(-2)]

Exception generated.

$$\int x^2(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx=\int x^2\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int x^2(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{d\big(-45acosh(cx)\,b\,c^5x^5+75acosh(cx)\,b\,c^3x^3+9\sqrt{c^2x^2-1}\,b\,c^4x^4-13\sqrt{c^2x^2-1}\,b\,c^2x^2-26\sqrt{c^2x^2-1}\,b}{225c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(d*( - 45*acosh(c*x)*b*c**5*x**5 + 75*acosh(c*x)*b*c**3*x**3 + 9*sqrt(c**2
*x**2 - 1)*b*c**4*x**4 - 13*sqrt(c**2*x**2 - 1)*b*c**2*x**2 - 26*sqrt(c**2
*x**2 - 1)*b - 45*a*c**5*x**5 + 75*a*c**3*x**3))/(225*c**3)
```

3.4 $\int x(d - c^2dx^2)(a + b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int x(d - c^2dx^2)(a + b\text{arccosh}(cx))\,dx = -\frac{3bdx\sqrt{-1+cx}\sqrt{1+cx}}{32c} + \frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c} + \frac{3bd\text{arccosh}(cx)}{32c^2} - \frac{d(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}$$

output
```
-3/32*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/16*b*d*x*(c*x-1)^(3/2)*(c*x+1)
^(3/2)/c+3/32*b*d*arccosh(c*x)/c^2-1/4*d*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))
/c^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int x(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx =$$

$$-\frac{d\Big(cx\big(b\sqrt{-1+cx}\sqrt{1+cx}(5-2c^2x^2)+8acx(-2+c^2x^2)\big)+8bc^2x^2(-2+c^2x^2)\,\text{arccosh}(cx)+10b\text{arc}}{32c^2}$$

input
```
Integrate[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/32*(d*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5 - 2*c^2*x^2) + 8*a*c*x*(-
2 + c^2*x^2)) + 8*b*c^2*x^2*(-2 + c^2*x^2)*ArcCosh[c*x] + 10*b*ArcTanh[Sqr
t[(-1 + c*x)/(1 + c*x)]]))/c^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6329, 40, 40, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d-c^2dx^2)\,(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6329}$$

$$\frac{bd\int (cx-1)^{3/2}(cx+1)^{3/2}dx}{4c}-\frac{d(1-c^2x^2)^2\,(a+b\text{arccosh}(cx))}{4c^2}$$

$$\Big\downarrow \text{40}$$

$$\frac{bd\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)}{4c}-\frac{d(1-c^2x^2)^2\,(a+b\text{arccosh}(cx))}{4c^2}$$

$$\Big\downarrow \text{40}$$

$$\frac{bd\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)}{4c}-\frac{d(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}$$

$$\Big\downarrow \text{43}$$

$$\frac{bd\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)}{4c}-\frac{d(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}$$

input
```
Int[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/4*(d*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/c^2 + (b*d*((x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/4 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[
c*x]/(2*c)))/4))/(4*c)
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{da\left(c^2x^2-1\right)^2}{4}-db\left(\frac{\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{c^2x^2\operatorname{arccosh}(cx)}{2}+\frac{\operatorname{arccosh}(cx)}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2\sqrt{c^2x^2-1}\,c^3x^3-5cx\sqrt{c^2x^2-1}+3\ln(\right.}{32\sqrt{c^2x^2-1}}\right.}{c^2}$
default	$\frac{-\frac{da\left(c^2x^2-1\right)^2}{4}-db\left(\frac{\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{c^2x^2\operatorname{arccosh}(cx)}{2}+\frac{\operatorname{arccosh}(cx)}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2\sqrt{c^2x^2-1}\,c^3x^3-5cx\sqrt{c^2x^2-1}+3\ln(\right.}{32\sqrt{c^2x^2-1}}\right.}{c^2}$
orering	$\frac{(14c^4x^4-33c^2x^2+10)\left(-c^2d\,x^2+d\right)(a+b\operatorname{arccosh}(cx))}{32c^2(c^2x^2-1)}-\frac{(2c^2x^2-5)\left(\left(-c^2d\,x^2+d\right)(a+b\operatorname{arccosh}(cx))-2x^2c^2d(a+b\right.}{32c^2}$
parts	$-\frac{da(c^2x^2-1)^2}{4c^2}-\frac{db\left(\frac{\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{c^2x^2\operatorname{arccosh}(cx)}{2}+\frac{\operatorname{arccosh}(cx)}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2\sqrt{c^2x^2-1}\,c^3x^3-5cx\sqrt{c^2x^2-1}+\right.}{32\sqrt{c^2x^2-1}}\right.}{c^2}$

input
```
int(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/4*d*a*(c^2*x^2-1)^2-d*b*(1/4*arccosh(c*x)*c^4*x^4-1/2*c^2*x^2*ar
ccosh(c*x)+1/4*arccosh(c*x)-1/32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(2*(c^2*x^2-1
)^(1/2)*c^3*x^3-5*c*x*(c^2*x^2-1)^(1/2)+3*ln(c*x+(c^2*x^2-1)^(1/2)))/(c^2*
x^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int x\left(d-c^2dx^2\right)\left(a+b\operatorname{arccosh}(cx)\right)dx =$$

$$-\frac{8\,ac^4dx^4-16\,ac^2dx^2+\left(8\,bc^4dx^4-16\,bc^2dx^2+5\,bd\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2\,bc^3dx^3-5\,bcdx\right)\sqrt{c^2x}}{32\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/32*(8*a*c^4*d*x^4 - 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 - 16*b*c^2*d*x^2 +
5*b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*d*x^3 - 5*b*c*d*x)*sqrt(c^2
*x^2 - 1))/c^2
```

Sympy [F]

$$\int x(d-c^2dx^2)\,(a+b\operatorname{arccosh}(cx))\,dx = -d\left(\int(-ax)\;dx+\int ac^2x^3\,dx+\int(-bx\operatorname{acosh}(cx))\;dx+\int bc^2x^3\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate(x*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
-d*(Integral(-a*x, x) + Integral(a*c**2*x**3, x) + Integral(-b*x*acosh(c*x
), x) + Integral(b*c**2*x**3*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.65

$$\int x(d-c^2dx^2)\,(a+b\operatorname{arccosh}(cx))\,dx = -\frac{1}{4}\,ac^2dx^4 - \frac{1}{32}\left(8\,x^4\operatorname{arcosh}(cx)-\left(\frac{2\,\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bc^2d + \frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c^3}\right)\right)bd$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/4*a*c^2*d*x^4 - 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2
 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)
*c)*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)
*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)\,(a+b\mathrm{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)\,(a+b\mathrm{arccosh}(cx))\,dx = \int x\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int x(d-c^2dx^2)\,(a+b\mathrm{arccosh}(cx))\,dx$$
$$= \frac{d\left(-8acosh(cx)\,b\,c^4x^4+16acosh(cx)\,b\,c^2x^2+2\sqrt{c^2x^2-1}\,b\,c^3x^3-5\sqrt{c^2x^2-1}\,bcx-5\log\left(\sqrt{c^2x^2-1}+\right.\right.}{32c^2}$$

input
```
int(x*(-c^2*d*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(d*( - 8*acosh(c*x)*b*c**4*x**4 + 16*acosh(c*x)*b*c**2*x**2 + 2*sqrt(c**2*
x**2 - 1)*b*c**3*x**3 - 5*sqrt(c**2*x**2 - 1)*b*c*x - 5*log(sqrt(c**2*x**2
 - 1) + c*x)*b - 8*a*c**4*x**4 + 16*a*c**2*x**2))/(32*c**2)
```

3.5 $\int (d - c^2dx^2)(a + b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int (d - c^2dx^2)(a + b\text{arccosh}(cx))\,dx = -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}}{9c} + \frac{1}{9}bcdx^2\sqrt{-1+cx}\sqrt{1+cx} + dx(a + b\text{arccosh}(cx)) - \frac{1}{3}c^2dx^3(a + b\text{arccosh}(cx))$$

output
```
-7/9*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/9*b*c*d*x^2*(c*x-1)^(1/2)*(c*x+1)
^(1/2)+d*x*(a+b*arccosh(c*x))-1/3*c^2*d*x^3*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int (d - c^2dx^2)(a + b\text{arccosh}(cx))\,dx = \frac{d\left(b\sqrt{-1+cx}\sqrt{1+cx}(-7+c^2x^2) + a(9cx - 3c^3x^3) - 3bcx(-3+c^2x^2)\,\text{arccosh}(cx)\right)}{9c}$$

input
```
Integrate[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-7 + c^2*x^2) + a*(9*c*x - 3*c^3*x^3)
- 3*b*c*x*(-3 + c^2*x^2)*ArcCosh[c*x]))/(9*c)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6309, 27, 960, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6309}$$

$$-bc\int \frac{dx\left(3-c^2x^2\right)}{3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2dx^3(a+b\text{arccosh}(cx))+dx(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bcd\int \frac{x\left(3-c^2x^2\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2dx^3(a+b\text{arccosh}(cx))+dx(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{960}$$

$$-\frac{1}{3}bcd\left(\frac{7}{3}\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{3}c^2dx^3(a+b\text{arccosh}(cx))+dx(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{83}$$

$$-\frac{1}{3}c^2dx^3(a+b\text{arccosh}(cx))+dx(a+b\text{arccosh}(cx))-\frac{1}{3}bcd\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)$$

input
```
Int[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/3*(b*c*d*((7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) - (x^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/3)) + d*x*(a + b*ArcCosh[c*x]) - (c^2*d*x^3*(a + b*ArcCos
h[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
parts	$-da\left(\frac{1}{3}c^2x^3-x\right)-\frac{db\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2x^2-7)}{9}\right)}{c}$	71
derivativedivides	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-db\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2x^2-7)}{9}\right)}{c}$	73
default	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-db\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2x^2-7)}{9}\right)}{c}$	73
orering	$\frac{x(5c^2x^2-23)(-c^2d\,x^2+d)(a+b\operatorname{arccosh}(cx))}{9c^2x^2-9}-\frac{(c^2x^2-7)\left(-2c^2dx(a+b\operatorname{arccosh}(cx))+\frac{(-c^2d\,x^2+d)bc}{\sqrt{cx-1}\,\sqrt{cx+1}}\right)}{9c^2}$	103

input
```
int((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/3*c^2*x^3-x)-d*b/c*(1/3*c^3*x^3*arccosh(c*x)-c*x*arccosh(c*x)-1/9*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*x^2-7))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \left(d-c^2dx^2\right)\left(a+b\operatorname{arccosh}(cx)\right)dx = -\frac{3\,ac^3dx^3-9\,acdx+3\,(bc^3dx^3-3\,bcdx)\log\left(cx+\sqrt{c^2x^2-1}\right)-(bc^2dx^2-7\,bd)\sqrt{c^2x^2-1}}{9\,c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/9*(3*a*c^3*d*x^3 - 9*a*c*d*x + 3*(b*c^3*d*x^3 - 3*b*c*d*x)*log(c*x + sq
rt(c^2*x^2 - 1)) - (b*c^2*d*x^2 - 7*b*d)*sqrt(c^2*x^2 - 1))/c
```

Sympy [F]

$$\int (d - c^2dx^2)(a + b\text{arccosh}(cx))\,dx = -d\left(\int (-a)\,dx + \int (-b\operatorname{acosh}(cx))\,dx + \int ac^2x^2\,dx + \int bc^2x^2\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
-d*(Integral(-a, x) + Integral(-b*acosh(c*x), x) + Integral(a*c**2*x**2, x
) + Integral(b*c**2*x**2*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int (d - c^2dx^2)(a + b\text{arccosh}(cx))\,dx = -\frac{1}{3}ac^2dx^3 - \frac{1}{9}\left(3x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^2d + adx + \frac{\left(cx\operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bd}{c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/3*a*c^2*d*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
+ 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d + a*d*x + (c*x*arccosh(c*x) - sqrt(c^2
*x^2 - 1))*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)(a+b\text{arccosh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)(a+b\text{arccosh}(cx))\,dx=\int \left(a+b\,\mathrm{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \left(d-c^2dx^2\right)(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{d\left(-3acosh(cx)\,b\,c^3x^3+9acosh(cx)\,bcx+\sqrt{c^2x^2-1}\,b\,c^2x^2+2\sqrt{c^2x^2-1}\,b-9\sqrt{cx+1}\,\sqrt{cx-1}\,b-3a\,c\right.}{9c}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(d*( - 3*acosh(c*x)*b*c**3*x**3 + 9*acosh(c*x)*b*c*x + sqrt(c**2*x**2 - 1)
*b*c**2*x**2 + 2*sqrt(c**2*x**2 - 1)*b - 9*sqrt(c*x + 1)*sqrt(c*x - 1)*b -
 3*a*c**3*x**3 + 9*a*c*x))/(9*c)
```

3.6 $\int \frac{(d-c^2dx^2)(a+b\,\mathrm{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\begin{aligned}\int \frac{(d-c^2dx^2)\,(a+b\mathrm{arccosh}(cx))}{x}\,dx &= \frac{1}{4}bcdx\sqrt{-1+cx}\sqrt{1+cx}-\frac{1}{4}bd\mathrm{arccosh}(cx)\\ &+\frac{1}{2}d\big(1-c^2x^2\big)\,(a+b\mathrm{arccosh}(cx))\\ &-\frac{d(a+b\mathrm{arccosh}(cx))^2}{2b}\\ &+d(a+b\mathrm{arccosh}(cx))\log\left(1+e^{2\mathrm{arccosh}(cx)}\right)\\ &+\frac{1}{2}bd\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{arccosh}(cx)}\right)\end{aligned}$$

output

```
1/4*b*c*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/4*b*d*arccosh(c*x)+1/2*d*(-c^2*x
^2+1)*(a+b*arccosh(c*x))-1/2*d*(a+b*arccosh(c*x))^2/b+d*(a+b*arccosh(c*x))
*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d*polylog(2,-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx = -\frac{1}{2}ac^2dx^2+\frac{1}{4}bcdx\sqrt{-1+cx}\sqrt{1+cx} - \frac{1}{2}bc^2dx^2\text{arccosh}(cx) + \frac{1}{2}bd\text{arctanh}\left(\frac{\sqrt{-1+cx}}{\sqrt{1+cx}}\right) + ad\log(x)+\frac{1}{2}bd\left(\text{arccosh}(cx)\left(\text{arccosh}(cx)+2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right) - \text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\right)$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
-1/2*(a*c^2*d*x^2) + (b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*c^2*d*x
^2*ArcCosh[c*x])/2 + (b*d*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/2 + a*d*L
og[x] + (b*d*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])])
 - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {6334, 40, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx$$

$$\Big\downarrow \text{6334}$$

$$d\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bcd\int \sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 40$$

$$d\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 43$$

$$d\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 6297$$

$$\frac{d\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{d\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{d\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{id\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 4201$$

$$\frac{id\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+$$

$$\frac{1}{2}d(1-c^2x^2)\,(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 2620$$

$$\frac{id\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+\right.}{b}$$

$$\frac{1}{2}d(1-c^2x^2)\,(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 2715$$

$$\frac{id\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\right.}{b}$$

$$\frac{1}{2}d(1-c^2x^2)\,(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

$$\Big\downarrow \; 2838$$

$$\frac{id\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx\right.}{b}$$

$$\frac{1}{2}d(1-c^2x^2)\,(a+b\text{arccosh}(cx))+\frac{1}{2}bcd\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(d*(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 + (b*c*d*((x*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/2 + (I*d*((-1/2*I)*(a + b*ArcCosh[c*x])
^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])]) + (
b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6334
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Simp[b*c*((-d
)^p/(2*p))   Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

method	result
parts	$-da\left(\frac{c^2x^2}{2}-\ln(x)\right)-\frac{db\operatorname{arccosh}(cx)^2}{2}-\frac{db\operatorname{arccosh}(cx)c^2x^2}{2}+\frac{bcdx\sqrt{cx-1}\,\sqrt{cx+1}}{4}+\frac{bd\operatorname{arccosh}(cx)}{4}+$
derivativedivides	$-da\left(\frac{c^2x^2}{2}-\ln(cx)\right)-\frac{db\operatorname{arccosh}(cx)^2}{2}+\frac{bcdx\sqrt{cx-1}\,\sqrt{cx+1}}{4}-\frac{db\operatorname{arccosh}(cx)c^2x^2}{2}+\frac{bd\operatorname{arccosh}(cx)}{4}$
default	$-da\left(\frac{c^2x^2}{2}-\ln(cx)\right)-\frac{db\operatorname{arccosh}(cx)^2}{2}+\frac{bcdx\sqrt{cx-1}\,\sqrt{cx+1}}{4}-\frac{db\operatorname{arccosh}(cx)c^2x^2}{2}+\frac{bd\operatorname{arccosh}(cx)}{4}$

input
```
int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/2*c^2*x^2-ln(x))-1/2*d*b*arccosh(c*x)^2-1/2*d*b*arccosh(c*x)*c^2*x
^2+1/4*b*c*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/4*b*d*arccosh(c*x)+d*b*arccos
h(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx=\int -\frac{(c^2dx^2-d)(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx=-d\left(\int \left(-\frac{a}{x}\right)\,dx+\int ac^2x\,dx+\int \left(-\frac{b\,\text{acosh}\,(cx)}{x}\right)\,dx+\int bc^2x\,\text{acosh}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x,x)
```

output
```
-d*(Integral(-a/x, x) + Integral(a*c**2*x, x) + Integral(-b*acosh(c*x)/x,
x) + Integral(b*c**2*x*acosh(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx=\int -\frac{(c^2dx^2-d)(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
-1/2*a*c^2*d*x^2 + a*d*log(x) - integrate(b*c^2*d*x*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)) - b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x}\,dx = \frac{d\left(-2acosh(cx)\,b\,c^2x^2+\sqrt{c^2x^2-1}\,bcx+4\left(\int \frac{acosh(cx)}{x}dx\right)b+\log\left(\sqrt{c^2x^2-1}+cx\right)b+4\log(x)\,a-2a\,c\right.}{4}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acosh(c*x))/x,x)
```

output
```
(d*( - 2*acosh(c*x)*b*c**2*x**2 + sqrt(c**2*x**2 - 1)*b*c*x + 4*int(acosh(
c*x)/x,x)*b + log(sqrt(c**2*x**2 - 1) + c*x)*b + 4*log(x)*a - 2*a*c**2*x**
2))/4
```

3.7 $\int \frac{(d-c^2dx^2)(a+b\,\mathbf{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^2}\,dx = bcd\sqrt{-1+cx}\sqrt{1+cx}-\frac{d(a+b\operatorname{arccosh}(cx))}{x} - c^2dx(a+b\operatorname{arccosh}(cx)) + bcd\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)-d*(a+b*arccosh(c*x))/x-c^2*d*x*(a+b*arcc
osh(c*x))+b*c*d*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.45

$$\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^2}\,dx = -\frac{ad}{x}-ac^2dx+bcd\sqrt{-1+cx}\sqrt{1+cx} - \frac{bd\operatorname{arccosh}(cx)}{x}-bc^2dx\operatorname{arccosh}(cx) + \frac{bcd\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((a*d)/x) - a*c^2*d*x + b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (b*d*ArcCosh
[c*x])/x - b*c^2*d*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[
-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6336, 25, 27, 960, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int -\frac{d(c^2x^2+1)}{x\sqrt{cx-1}\sqrt{cx+1}}dx+c^2(-d)x(a+b\text{arccosh}(cx))-\frac{d(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{25}$$

$$bc\int \frac{d(c^2x^2+1)}{x\sqrt{cx-1}\sqrt{cx+1}}dx+c^2(-d)x(a+b\text{arccosh}(cx))-\frac{d(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$bcd\int \frac{c^2x^2+1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+c^2(-d)x(a+b\text{arccosh}(cx))-\frac{d(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{960}$$

$$bcd\left(\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\sqrt{cx-1}\sqrt{cx+1}\right)+c^2(-d)x(a+b\text{arccosh}(cx))-\frac{d(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{103}$$

$$bcd\left(c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)+c^2(-d)x(a+b\text{arccosh}(cx))-\frac{d(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{ 218}$$

$$c^2(-d)x(a+b\text{arccosh}(cx))-\frac{d(a+b\text{arccosh}(cx))}{x}+bcd\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcCosh[c*x]))/x) - c^2*d*x*(a + b*ArcCosh[c*x]) + b*c*d*(Sqrt
[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
parts	$-da\left(c^2x+\frac{1}{x}\right)-dbc\left(cx\,\operatorname{arccosh}(cx)+\frac{\operatorname{arccosh}(cx)}{cx}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x^2-}}\right.\right.}{\sqrt{c^2x^2-1}}\right.$
derivativedivides	$c\left(-da\left(cx+\frac{1}{cx}\right)-db\left(cx\,\operatorname{arccosh}(cx)+\frac{\operatorname{arccosh}(cx)}{cx}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x}}\right.\right.}{\sqrt{c^2x^2-1}}\right.\right.$
default	$c\left(-da\left(cx+\frac{1}{cx}\right)-db\left(cx\,\operatorname{arccosh}(cx)+\frac{\operatorname{arccosh}(cx)}{cx}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x}}\right.\right.}{\sqrt{c^2x^2-1}}\right.\right.$

input
```
int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-d*a*(c^2*x+1/x)-d*b*c*(c*x*arccosh(c*x)+arccosh(c*x)/c/x-(c*x-1)^(1/2)*(c
*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*((c^2*x^2-1)^(1/2)-arctan(1/(c^2*x^2-1)^(1/2
))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{ac^2dx^2 - 2\,bcdx\arctan\left(-cx+\sqrt{c^2x^2-1}\right) - \sqrt{c^2x^2-1}bcdx - (bc^2+b)dx\log\left(-cx+\sqrt{c^2x^2-1}\right) + }{x}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

output
```
-(a*c^2*d*x^2 - 2*b*c*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2
- 1)*b*c*d*x - (b*c^2 + b)*d*x*log(-c*x + sqrt(c^2*x^2 - 1)) + a*d + (b*c^
2*d*x^2 - (b*c^2 + b)*d*x + b*d)*log(c*x + sqrt(c^2*x^2 - 1)))/x
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x^2}\,dx = -d\left(\int ac^2\,dx + \int \left(-\frac{a}{x^2}\right)dx + \int bc^2\operatorname{acosh}(cx)\,dx + \int \left(-\frac{b\operatorname{acosh}(cx)}{x^2}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

output
```
-d*(Integral(a*c**2, x) + Integral(-a/x**2, x) + Integral(b*c**2*acosh(c*x
), x) + Integral(-b*acosh(c*x)/x**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^2}\,dx = -ac^2dx - \left(cx\operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bcd - \left(c\arcsin\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)bd - \frac{ad}{x}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

output
```
-a*c^2*d*x - (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c*d - (c*arcsin(1/(c
*abs(x))) + arccosh(c*x)/x)*b*d - a*d/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\operatorname{acosh}(c\,x))\,(d-c^2\,d\,x^2)}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \frac{(d - c^2dx^2)(a + b\text{arccosh}(cx))}{x^2} dx$$
$$= \frac{d\left(-acosh(cx)\, b\, c^2x^2 - acosh(cx)\, b - 2atan\left(\sqrt{c^2x^2 - 1} + cx\right) bcx + \sqrt{cx + 1}\,\sqrt{cx - 1}\, bcx - a\, c^2x^2 - a\right)}{x}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acosh(c*x))/x^2,x)
```

output
```
(d*( - acosh(c*x)*b*c**2*x**2 - acosh(c*x)*b - 2*atan(sqrt(c**2*x**2 - 1)
+ c*x)*b*c*x + sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - a*c**2*x**2 - a))/x
```

3.8 $\int \frac{(d-c^2dx^2)(a+b\,\mathbf{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(d-c^2dx^2)\,(a+b\text{arccosh}(cx))}{x^3}\,dx = \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\text{arccosh}(cx) - \frac{d(1-c^2x^2)\,(a+b\text{arccosh}(cx))}{2x^2} + \frac{c^2d(a+b\text{arccosh}(cx))^2}{2b} - c^2d(a+b\text{arccosh}(cx))\log\left(1+e^{2\text{arccosh}(cx)}\right) - \frac{1}{2}bc^2d\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)$$

output
```
1/2*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x-1/2*b*c^2*d*arccosh(c*x)-1/2*d*(-c
^2*x^2+1)*(a+b*arccosh(c*x))/x^2+1/2*c^2*d*(a+b*arccosh(c*x))^2/b-c^2*d*(a
+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*b*c^2*d*pol
ylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{(d-c^2dx^2)\,(a+b\text{arccosh}(cx))}{x^3}\,dx = -\frac{ad}{2x^2}+\frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x}-\frac{bd\text{arccosh}(cx)}{2x^2} - ac^2d\log(x)-\frac{1}{2}bc^2d\big(\text{arccosh}(cx)\,\big(\text{arccosh}(cx) + 2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\big) - \text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\big)$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*d*ArcCosh
[c*x])/(2*x^2) - a*c^2*d*Log[x] - (b*c^2*d*(ArcCosh[c*x]*(ArcCosh[c*x] + 2
*Log[1 + E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6335, 108, 27, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)\,(a+b\text{arccosh}(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{6335}$$

$$c^2(-d)\int \frac{a+b\text{arccosh}(cx)}{x}dx-\frac{1}{2}bcd\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2}dx-\frac{d(1-c^2x^2)\,(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \text{108}$$

$$c^2(-d)\int \frac{a+b\text{arccosh}(cx)}{x}dx-\frac{1}{2}bcd\left(\int \frac{c^2}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)-\frac{d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 27$$

$$c^2(-d)\int \frac{a+b\text{arccosh}(cx)}{x}dx-\frac{1}{2}bcd\left(c^2\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)-\frac{d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 43$$

$$c^2(-d)\int \frac{a+b\text{arccosh}(cx)}{x}dx-\frac{d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$$\Big\downarrow \; 6297$$

$$-\frac{c^2d\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}-\frac{d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{c^2d\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{c^2d\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$$\Big\downarrow \; 26$$

$$-\frac{ic^2d\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{d\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$$\Big\downarrow \; 4201$$

$$-\frac{ic^2d\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}-\frac{d(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$\big\downarrow$ 2620

$$-\frac{ic^2d\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(\right.}{b}-\frac{d(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$\big\downarrow$ 2715

$$-\frac{ic^2d\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx\right.\right.}{b}-\frac{d(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

$\big\downarrow$ 2838

$$-\frac{ic^2d\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}\right.}{b}-\frac{d(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bcd\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(d*(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/x^2 - (b*c*d*(-((Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/x) + c*ArcCosh[c*x]))/2 - (I*c^2*d*((-1/2*I)*(a + b*ArcCo
sh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x
])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6335
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*((-d)^p/(f*(m + 1)))   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(
m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

method	result
parts	$-da\,c^2\ln(x)-\frac{da}{2x^2}-db\,c^2\left(-\frac{\operatorname{arccosh}(cx)^2}{2}+\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^2x^2}+\operatorname{arccosh}(c\right.$
derivativedivides	$c^2\left(-da\left(\ln(cx)+\frac{1}{2c^2x^2}\right)-db\left(-\frac{\operatorname{arccosh}(cx)^2}{2}+\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^2x^2}+\operatorname{arccosl}\right.\right.$
default	$c^2\left(-da\left(\ln(cx)+\frac{1}{2c^2x^2}\right)-db\left(-\frac{\operatorname{arccosh}(cx)^2}{2}+\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^2x^2}+\operatorname{arccosl}\right.\right.$

input
```
int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-d*a*c^2*ln(x)-1/2*d*a/x^2-d*b*c^2*(-1/2*arccosh(c*x)^2+1/2*(-(c*x-1)^(1/2
)*(c*x+1)^(1/2)*c*x+c^2*x^2+arccosh(c*x))/c^2/x^2+arccosh(c*x)*ln(1+(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2))
```

Fricas [F]

$$\int \frac{(d - c^2dx^2)\,(a + b\operatorname{arccosh}(cx))}{x^3}\,dx = \int -\frac{(c^2dx^2 - d)(b\operatorname{arcosh}(cx) + a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d - c^2dx^2)\,(a + b\operatorname{arccosh}(cx))}{x^3}\,dx = -d\left(\int \left(-\frac{a}{x^3}\right)dx + \int \frac{ac^2}{x}\,dx + \int \left(-\frac{b\operatorname{acosh}(cx)}{x^3}\right)dx + \int \frac{bc^2\operatorname{acosh}(cx)}{x}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**3,x)
```

output
```
-d*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*acosh(c*x)/
x**3, x) + Integral(b*c**2*acosh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)(a + b\text{arccosh}(cx))}{x^3} dx = \int -\frac{(c^2dx^2 - d)(b \operatorname{arcosh}(cx) + a)}{x^3} dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

output
```
-b*c^2*d*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) - a*c^2*d*
log(x) + 1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d/x^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)(a + b\text{arccosh}(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)(a + b\text{arccosh}(cx))}{x^3} dx = \int \frac{(a + b \operatorname{acosh}(c\,x))\,(d - c^2\,d\,x^2)}{x^3} dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d - c^2dx^2)(a + b\text{arccosh}(cx))}{x^3} dx$$
$$= \frac{d\left(-acosh(cx)\, b - \sqrt{c^2x^2 - 1}\, bcx - 2\left(\int \frac{acosh(cx)}{x} dx\right) b\, c^2x^2 - 2\log(x)\, a\, c^2x^2 - a - b\, c^2x^2\right)}{2x^2}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acosh(c*x))/x^3,x)
```

output
```
(d*( - acosh(c*x)*b - sqrt(c**2*x**2 - 1)*b*c*x - 2*int(acosh(c*x)/x,x)*b*
c**2*x**2 - 2*log(x)*a*c**2*x**2 - a - b*c**2*x**2))/(2*x**2)
```

3.9 $\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d(a+b\operatorname{arccosh}(cx))}{3x^3} + \frac{c^2d(a+b\operatorname{arccosh}(cx))}{x} - \frac{5}{6}bc^3d\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-1/3*d*(a+b*arccosh(c*x))/x^3+c^2
*d*(a+b*arccosh(c*x))/x-5/6*b*c^3*d*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.41

$$\int \frac{(d-c^2dx^2)(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = -\frac{ad}{3x^3} + \frac{ac^2d}{x} + \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{bd\operatorname{arccosh}(cx)}{3x^3} + \frac{bc^2d\operatorname{arccosh}(cx)}{x} - \frac{5bc^3d\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{6\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(a*d)/x^3 + (a*c^2*d)/x + (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2
) - (b*d*ArcCosh[c*x])/(3*x^3) + (b*c^2*d*ArcCosh[c*x])/x - (5*b*c^3*d*Sqr
t[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6336, 27, 956, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{ 6336}$$

$$-bc\int -\frac{d(1-3c^2x^2)}{3x^3\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{c^2d(a+b\text{arccosh}(cx))}{x}-\frac{d(a+b\text{arccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}bcd\int \frac{1-3c^2x^2}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{c^2d(a+b\text{arccosh}(cx))}{x}-\frac{d(a+b\text{arccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{ 956}$$

$$\frac{1}{3}bcd\left(\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}-\frac{5}{2}c^2\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{c^2d(a+b\text{arccosh}(cx))}{x}-\frac{d(a+b\text{arccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{ 103}$$

$$\frac{1}{3}bcd\left(\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}-\frac{5}{2}c^3\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)+\frac{c^2d(a+b\text{arccosh}(cx))}{x}-\frac{d(a+b\text{arccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{c^2d(a+b\text{arccosh}(cx))}{x}-\frac{d(a+b\text{arccosh}(cx))}{3x^3}+\frac{1}{3}bcd\left(\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}-\frac{5}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcCosh[c*x]))/x^3 + (c^2*d*(a + b*ArcCosh[c*x]))/x + (b*c*
d*((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) - (5*c^2*ArcTan[Sqrt[-1 + c*x]*S
qrt[1 + c*x]])/2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

method	result
parts	$-da\left(-\frac{c^2}{x}+\frac{1}{3x^3}\right)-db\,c^3\left(-\frac{\operatorname{arccosh}(cx)}{cx}+\frac{\operatorname{arccosh}(cx)}{3c^3x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(5\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\right.}{6c^2x^2\sqrt{c^2x^2-1}}\right.$
derivativedivides	$c^3\left(-da\left(-\frac{1}{cx}+\frac{1}{3c^3x^3}\right)-db\left(-\frac{\operatorname{arccosh}(cx)}{cx}+\frac{\operatorname{arccosh}(cx)}{3c^3x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(5\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x\right.}{6c^2x^2\sqrt{c^2x^2-1}}\right.\right.$
default	$c^3\left(-da\left(-\frac{1}{cx}+\frac{1}{3c^3x^3}\right)-db\left(-\frac{\operatorname{arccosh}(cx)}{cx}+\frac{\operatorname{arccosh}(cx)}{3c^3x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(5\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x\right.}{6c^2x^2\sqrt{c^2x^2-1}}\right.\right.$

input
```
int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-d*a*(-c^2/x+1/3/x^3)-d*b*c^3*(-arccosh(c*x)/c/x+1/3*arccosh(c*x)/c^3/x^3-
1/6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(5*arctan(1/(c^2*x^2-1)^(1/2))*c^2*x^2+(c^
2*x^2-1)^(1/2))/c^2/x^2/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{(d-c^2dx^2)\,(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = -\frac{10\,bc^3dx^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-6\,ac^2dx^2+2\,(3\,bc^2-b)dx^3\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-}}{6\,x^3}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(10*b*c^3*d*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 6*a*c^2*d*x^2 + 2*
(3*b*c^2 - b)*d*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*b*c*
d*x + 2*a*d - 2*(3*b*c^2*d*x^2 - (3*b*c^2 - b)*d*x^3 - b*d)*log(c*x + sqrt
(c^2*x^2 - 1)))/x^3
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = -d\left(\int\left(-\frac{a}{x^4}\right)dx+\int\frac{ac^2}{x^2}\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x^4}\right)dx+\int\frac{bc^2\operatorname{acosh}(cx)}{x^2}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

output
```
-d*(Integral(-a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(-b*acosh(c*
x)/x**4, x) + Integral(b*c**2*acosh(c*x)/x**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{(d-c^2dx^2)\,(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = \left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)bc^2d - \frac{1}{6}\left(\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\operatorname{arcosh}(cx)}{x^3}\right)bd+\frac{ac^2d}{x}-\frac{ad}{3x^3}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

output
```
(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*c^2*d - 1/6*((c^2*arcsin(1/(c*
abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d + a*c^2*d/x
- 1/3*a*d/x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)(a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{(d-c^2dx^2)\,(a+b\text{arccosh}(cx))}{x^4}\,dx$$
$$= \frac{d\left(6acosh(cx)\,b\,c^2x^2-2acosh(cx)\,b+10atan\left(\sqrt{c^2x^2-1}+cx\right)b\,c^3x^3-\sqrt{c^2x^2-1}\,bcx+6a\,c^2x^2-2a\right)}{6x^3}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acosh(c*x))/x^4,x)
```

output
```
(d*(6*acosh(c*x)*b*c**2*x**2 - 2*acosh(c*x)*b + 10*atan(sqrt(c**2*x**2 - 1
) + c*x)*b*c**3*x**3 - sqrt(c**2*x**2 - 1)*b*c*x + 6*a*c**2*x**2 - 2*a))/(
6*x**3)
```

3.10 $\int x^4(d-c^2dx^2)^2\,(a+b\,\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\begin{aligned}
&\int x^4\left(d-c^2dx^2\right)^2(a+b\,\text{arccosh}(cx))\,dx \\
&= -\frac{8bd^2\sqrt{-1+cx}\sqrt{1+cx}}{315c^5}+\frac{4bd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{945c^5} \\
&\quad -\frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}}{525c^5} \\
&\quad -\frac{10bd^2(-1+cx)^{7/2}(1+cx)^{7/2}}{441c^5}-\frac{bd^2(-1+cx)^{9/2}(1+cx)^{9/2}}{81c^5} \\
&\quad +\frac{1}{5}d^2x^5(a+b\,\text{arccosh}(cx))-\frac{2}{7}c^2d^2x^7(a+b\,\text{arccosh}(cx))+\frac{1}{9}c^4d^2x^9(a+b\,\text{arccosh}(cx))
\end{aligned}$$

output
```
-8/315*b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5+4/945*b*d^2*(c*x-1)^(3/2)*(c*
x+1)^(3/2)/c^5-1/525*b*d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^5-10/441*b*d^2*(c
*x-1)^(7/2)*(c*x+1)^(7/2)/c^5-1/81*b*d^2*(c*x-1)^(9/2)*(c*x+1)^(9/2)/c^5+1
/5*d^2*x^5*(a+b*arccosh(c*x))-2/7*c^2*d^2*x^7*(a+b*arccosh(c*x))+1/9*c^4*d
^2*x^9*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60

$$\int x^4 (d - c^2 d x^2)^2 (a + b\text{arccosh}(cx))\, dx$$
$$= \frac{d^2 (315ac^5x^5(63 - 90c^2x^2 + 35c^4x^4) - b\sqrt{-1 + cx}\sqrt{1 + cx}(2104 + 1052c^2x^2 + 789c^4x^4 - 2650c^6x^6 + 12}{99225c^5}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*(315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8
) + 315*b*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*ArcCosh[c*x]))/(99225*c^5
)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6336, 27, 1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (d - c^2 d x^2)^2 (a + b\text{arccosh}(cx))\, dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int \frac{d^2x^5(35c^4x^4 - 90c^2x^2 + 63)}{315\sqrt{cx-1}\sqrt{cx+1}}dx + \frac{1}{9}c^4d^2x^9(a + b\text{arccosh}(cx)) - \frac{2}{7}c^2d^2x^7(a + b\text{arccosh}(cx)) + \frac{1}{5}d^2x^5(a + b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{315}bcd^2\int \frac{x^5(35c^4x^4 - 90c^2x^2 + 63)}{\sqrt{cx-1}\sqrt{cx+1}}dx + \frac{1}{9}c^4d^2x^9(a + b\text{arccosh}(cx)) - \frac{2}{7}c^2d^2x^7(a + b\text{arccosh}(cx)) + \frac{1}{5}d^2x^5(a + b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1905}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{x^5(35c^4x^4-90c^2x^2+63)}{\sqrt{c^2x^2-1}}dx}{315\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{9}c^4d^2x^9(a+\text{barccosh}(cx))-\frac{2}{7}c^2d^2x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^2x^5(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{1578}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{x^4(35c^4x^4-90c^2x^2+63)}{\sqrt{c^2x^2-1}}dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{9}c^4d^2x^9(a+\text{barccosh}(cx))-\frac{2}{7}c^2d^2x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^2x^5(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{1195}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \left(\frac{35(c^2x^2-1)^{7/2}}{c^4}+\frac{50(c^2x^2-1)^{5/2}}{c^4}+\frac{3(c^2x^2-1)^{3/2}}{c^4}-\frac{4\sqrt{c^2x^2-1}}{c^4}+\frac{8}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{9}c^4d^2x^9(a+\text{barccosh}(cx))-\frac{2}{7}c^2d^2x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^2x^5(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}c^4d^2x^9(a+\text{barccosh}(cx))-\frac{2}{7}c^2d^2x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^2x^5(a+\text{barccosh}(cx))-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{70(c^2x^2-1)^{9/2}}{9c^6}+\frac{100(c^2x^2-1)^{7/2}}{7c^6}+\frac{6(c^2x^2-1)^{5/2}}{5c^6}-\frac{8(c^2x^2-1)^{3/2}}{3c^6}+\frac{16\sqrt{c^2x^2-1}}{c^6}\right)}{630\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/630*(b*c*d^2*Sqrt[-1 + c^2*x^2]*((16*Sqrt[-1 + c^2*x^2])/c^6 - (8*(-1 +
 c^2*x^2)^(3/2))/(3*c^6) + (6*(-1 + c^2*x^2)^(5/2))/(5*c^6) + (100*(-1 + c
^2*x^2)^(7/2))/(7*c^6) + (70*(-1 + c^2*x^2)^(9/2))/(9*c^6)))/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b*ArcCosh[c*x]))/5 - (2*c^2*d^2*x^7*(a +
 b*ArcCosh[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcCosh[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60

method	result
parts	$d^2a\left(\frac{1}{9}c^4x^9-\frac{2}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^2b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{2\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(122}{}\right.}{c^5}$
derivativedivides	$\frac{d^2a\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{2\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1225c^8x^8-265}{9}\right.}{c^5}$
default	$\frac{d^2a\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{2\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1225c^8x^8-265}{9}\right.}{c^5}$
orering	$\frac{(20825c^{10}x^{10}-54450c^8x^8+36757c^6x^6+5260c^4x^4+12624c^2x^2-8416)(-c^2d\,x^2+d)^2(a+b\operatorname{arccosh}(cx))}{99225c^6(cx-1)(cx+1)x(c^2x^2-1)}-\frac{(1225c^8x^8}{}$

input
```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/9*c^4*x^9-2/7*c^2*x^7+1/5*x^5)+d^2*b/c^5*(1/9*arccosh(c*x)*c^9*x^
9-2/7*arccosh(c*x)*c^7*x^7+1/5*arccosh(c*x)*c^5*x^5-1/99225*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*(1225*c^8*x^8-2650*c^6*x^6+789*c^4*x^4+1052*c^2*x^2+2104))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int x^4\left(d-c^2dx^2\right)^2(a+b\operatorname{arccosh}(cx))\,dx$$
$$=\frac{11025\,ac^9d^2x^9-28350\,ac^7d^2x^7+19845\,ac^5d^2x^5+315\,(35\,bc^9d^2x^9-90\,bc^7d^2x^7+63\,bc^5d^2x^5)\log\left(cx+\right.}{99225\,c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*d^2*x^9 - 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
 315*(35*b*c^9*d^2*x^9 - 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sq
rt(c^2*x^2 - 1)) - (1225*b*c^8*d^2*x^8 - 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^
2*x^4 + 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [F]

$$\int x^4 (d - c^2 d x^2)^2 (a + b\text{arccosh}(cx))\, dx = d^2 \left(\int ax^4\, dx + \int \left(-2ac^2x^6\right)\, dx + \int ac^4x^8\, dx + \int bx^4 \operatorname{acosh}(cx)\, dx + \int \left(-2bc^2x^6 \operatorname{acosh}(cx)\right)\, dx + \int bc^4x^8 \operatorname{acosh}(cx)\, dx \right)$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
d**2*(Integral(a*x**4, x) + Integral(-2*a*c**2*x**6, x) + Integral(a*c**4*
x**8, x) + Integral(b*x**4*acosh(c*x), x) + Integral(-2*b*c**2*x**6*acosh(
c*x), x) + Integral(b*c**4*x**8*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.55

$$\int x^4 (d - c^2 d x^2)^2 (a + b\text{arccosh}(cx))\, dx = \frac{1}{9} ac^4d^2x^9 - \frac{2}{7} ac^2d^2x^7 + \frac{1}{2835} \left(315\, x^9 \operatorname{arcosh}(cx) - \left(\frac{35\sqrt{c^2x^2 - 1}x^8}{c^2} + \frac{40\sqrt{c^2x^2 - 1}x^6}{c^4} + \frac{48\sqrt{c^2x^2 - 1}x^4}{c^6} + \frac{64\sqrt{c^2x^2 - 1}x}{c^8} \right.\right.$$

$$+ \frac{1}{5} ad^2x^5 - \frac{2}{245} \left(35\, x^7 \operatorname{arcosh}(cx) - \left(\frac{5\sqrt{c^2x^2 - 1}x^6}{c^2} + \frac{6\sqrt{c^2x^2 - 1}x^4}{c^4} + \frac{8\sqrt{c^2x^2 - 1}x^2}{c^6} + \frac{16\sqrt{c^2x^2 - 1}}{c^8} \right) c \right) bc$$

$$+ \frac{1}{75} \left(15\, x^5 \operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2 - 1}x^4}{c^2} + \frac{4\sqrt{c^2x^2 - 1}x^2}{c^4} + \frac{8\sqrt{c^2x^2 - 1}}{c^6} \right) c \right) bd^2$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arccosh(c*x) - (35
*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^
2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10
)*c)*b*c^4*d^2 + 1/5*a*d^2*x^5 - 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*
x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c
^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arccosh(c*x) -
(3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 - 1)/c^6)*c)*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \left(d - c^2 d x^2\right)^2 (a + b\text{arccosh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(d - c^2 d x^2\right)^2 (a + b\text{arccosh}(cx))\, dx = \int x^4\, (a + b\,\text{acosh}(c\,x))\, \left(d - c^2\,d\,x^2\right)^2\, dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int x^4(d-c^2dx^2)^2(a+b\operatorname{arccosh}(cx))\,dx$$
$$=\frac{d^2(11025acosh(cx)\,b\,c^9x^9-28350acosh(cx)\,b\,c^7x^7+19845acosh(cx)\,b\,c^5x^5-1225\sqrt{c^2x^2-1}\,b\,c^8x^8+26$$

input
```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(d**2*(11025*acosh(c*x)*b*c**9*x**9 - 28350*acosh(c*x)*b*c**7*x**7 + 19845
*acosh(c*x)*b*c**5*x**5 - 1225*sqrt(c**2*x**2 - 1)*b*c**8*x**8 + 2650*sqrt
(c**2*x**2 - 1)*b*c**6*x**6 - 789*sqrt(c**2*x**2 - 1)*b*c**4*x**4 - 1052*s
qrt(c**2*x**2 - 1)*b*c**2*x**2 - 2104*sqrt(c**2*x**2 - 1)*b + 11025*a*c**9
*x**9 - 28350*a*c**7*x**7 + 19845*a*c**5*x**5))/(99225*c**5)
```

3.11 $\int x^3(d-c^2dx^2)^2(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int x^3\left(d-c^2dx^2\right)^2(a+b\text{arccosh}(cx))\,dx = -\frac{73bd^2x\sqrt{-1+cx}\sqrt{1+cx}}{3072c^3}-\frac{73bd^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{4608c} + \frac{43bcd^2x^5\sqrt{-1+cx}\sqrt{1+cx}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{-1+cx}\sqrt{1+cx}-\frac{73bd^2\text{arccosh}(cx)}{3072c^4} + \frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+b\text{arccosh}(cx))+\frac{1}{8}c^4d^2x^8(a+b\text{arccosh}(cx))$$

output
```
-73/3072*b*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-73/4608*b*d^2*x^3*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/c+43/1152*b*c*d^2*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/6
4*b*c^3*d^2*x^7*(c*x-1)^(1/2)*(c*x+1)^(1/2)-73/3072*b*d^2*arccosh(c*x)/c^4
+1/4*d^2*x^4*(a+b*arccosh(c*x))-1/3*c^2*d^2*x^6*(a+b*arccosh(c*x))+1/8*c^4
*d^2*x^8*(a+b*arccosh(c*x))
```

Mathematica [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int x^3 (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx$$
$$= \frac{d^2\left(2304ac^4x^4-3072ac^6x^6+1152ac^8x^8-219bcx\sqrt{-1+cx}\sqrt{1+cx}-146bc^3x^3\sqrt{-1+cx}\sqrt{1+cx}+3\right.}{}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*(2304*a*c^4*x^4 - 3072*a*c^6*x^6 + 1152*a*c^8*x^8 - 219*b*c*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x] - 146*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 344*b
*c^5*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 144*b*c^7*x^7*Sqrt[-1 + c*x]*Sqrt[
1 + c*x] + 384*b*c^4*x^4*(6 - 8*c^2*x^2 + 3*c^4*x^4)*ArcCosh[c*x] - 438*b*
ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(9216*c^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6336, 27, 1905, 1590, 27, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int \frac{d^2x^4(3c^4x^4-8c^2x^2+6)}{24\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{8}c^4d^2x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{24}bcd^2\int\frac{x^4(3c^4x^4-8c^2x^2+6)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{8}c^4d^2x^8(a+\text{barccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+\text{barccosh}(cx))+\frac{1}{4}d^2x^4(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 1905

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int\frac{x^4(3c^4x^4-8c^2x^2+6)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4d^2x^8(a+\text{barccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+\text{barccosh}(cx))+\frac{1}{4}d^2x^4(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 1590

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{\int\frac{c^2x^4(48-43c^2x^2)}{\sqrt{c^2x^2-1}}dx}{8c^2}+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4d^2x^8(a+\text{barccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+\text{barccosh}(cx))+\frac{1}{4}d^2x^4(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 27

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{1}{8}\int\frac{x^4(48-43c^2x^2)}{\sqrt{c^2x^2-1}}dx+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4d^2x^8(a+\text{barccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+\text{barccosh}(cx))+\frac{1}{4}d^2x^4(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 363

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\int\frac{x^4}{\sqrt{c^2x^2-1}}dx-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4d^2x^8(a+\text{barccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+\text{barccosh}(cx))+\frac{1}{4}d^2x^4(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 262

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\int\frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4d^2x^8(a+\text{barccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+\text{barccosh}(cx))+\frac{1}{4}d^2x^4(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 262

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{1}{8}c^4d^2x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{224}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-}\right.}{24\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}c^4d^2x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}c^4d^2x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2d^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))-$$
$$\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right.}{24\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*d^2*x^6*(a + b*ArcCosh[c*x]))/3 +
(c^4*d^2*x^8*(a + b*ArcCosh[c*x]))/8 - (b*c*d^2*Sqrt[-1 + c^2*x^2]*((3*c^2
*x^7*Sqrt[-1 + c^2*x^2])/8 + ((-43*x^5*Sqrt[-1 + c^2*x^2])/6 + (73*((x^3*S
qrt[-1 + c^2*x^2])/(4*c^2) + (3*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[
(c*x)/Sqrt[-1 + c^2*x^2]]/(2*c^3)))/(4*c^2)))/6)/8))/(24*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

method	result
parts	$d^2a\left(\frac{1}{8}c^4x^8-\frac{1}{3}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^2b\left(\frac{\operatorname{arccosh}(cx)c^8x^8}{8}-\frac{\operatorname{arccosh}(cx)c^6x^6}{3}+\frac{\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(144c^7}{}\right.}{}$
derivativedivides	$\frac{d^2a\left(\frac{1}{8}c^8x^8-\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^8x^8}{8}-\frac{\operatorname{arccosh}(cx)c^6x^6}{3}+\frac{\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(144c^7x^7\sqrt{c^2x^2-}}{}\right.}{c^4}$
default	$\frac{d^2a\left(\frac{1}{8}c^8x^8-\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^8x^8}{8}-\frac{\operatorname{arccosh}(cx)c^6x^6}{3}+\frac{\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(144c^7x^7\sqrt{c^2x^2-}}{}\right.}{c^4}$
orering	$\frac{(2160c^8x^8-5912c^6x^6+4358c^4x^4+1095c^2x^2-876)(-c^2d\,x^2+d)^2(a+b\operatorname{arccosh}(cx))}{9216c^4(cx-1)(cx+1)(c^2x^2-1)}-\frac{(144c^6x^6-344c^4x^4+146c^2x^2}{}$

input
```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/8*c^4*x^8-1/3*c^2*x^6+1/4*x^4)+d^2*b/c^4*(1/8*arccosh(c*x)*c^8*x^
8-1/3*arccosh(c*x)*c^6*x^6+1/4*arccosh(c*x)*c^4*x^4-1/9216*(c*x-1)^(1/2)*(
c*x+1)^(1/2)*(144*c^7*x^7*(c^2*x^2-1)^(1/2)-344*c^5*x^5*(c^2*x^2-1)^(1/2)+
146*(c^2*x^2-1)^(1/2)*c^3*x^3+219*c*x*(c^2*x^2-1)^(1/2)+219*ln(c*x+(c^2*x^
2-1)^(1/2)))/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int x^3(d-c^2dx^2)^2(a+b\operatorname{arccosh}(cx))\,dx = \frac{1152\,ac^8d^2x^8 - 3072\,ac^6d^2x^6 + 2304\,ac^4d^2x^4 + 3\,(384\,bc^8d^2x^8 - 1024\,bc^6d^2x^6 + 768\,bc^4d^2x^4 - 73\,bd^2)\log}{9216\,c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/9216*(1152*a*c^8*d^2*x^8 - 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(
384*b*c^8*d^2*x^8 - 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log
(c*x + sqrt(c^2*x^2 - 1)) - (144*b*c^7*d^2*x^7 - 344*b*c^5*d^2*x^5 + 146*b
*c^3*d^2*x^3 + 219*b*c*d^2*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [F]

$$\int x^3(d-c^2dx^2)^2(a+b\operatorname{arccosh}(cx))\,dx = d^2\left(\int ax^3\,dx + \int(-2ac^2x^5)\,dx + \int ac^4x^7\,dx + \int bx^3\operatorname{acosh}(cx)\,dx + \int(-2bc^2x^5\operatorname{acosh}(cx))\,dx + \int bc^4x^7\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
d**2*(Integral(a*x**3, x) + Integral(-2*a*c**2*x**5, x) + Integral(a*c**4*
x**7, x) + Integral(b*x**3*acosh(c*x), x) + Integral(-2*b*c**2*x**5*acosh(
c*x), x) + Integral(b*c**4*x**7*acosh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(168) = 336$.

Time = 0.03 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.73

$$\int x^3 (d - c^2dx^2)^2 (a + b\operatorname{arccosh}(cx))\,dx = \frac{1}{8}\,ac^4d^2x^8 - \frac{1}{3}\,ac^2d^2x^6$$
$$+ \frac{1}{3072}\left(384\,x^8\operatorname{arcosh}(cx) - \left(\frac{48\sqrt{c^2x^2-1}x^7}{c^2} + \frac{56\sqrt{c^2x^2-1}x^5}{c^4} + \frac{70\sqrt{c^2x^2-1}x^3}{c^6} + \frac{105\sqrt{c^2x^2-1}\ldots}{c^8}\right.\right.$$
$$+ \frac{1}{4}\,ad^2x^4$$
$$- \frac{1}{144}\left(48\,x^6\operatorname{arcosh}(cx) - \left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2} + \frac{10\sqrt{c^2x^2-1}x^3}{c^4} + \frac{15\sqrt{c^2x^2-1}x}{c^6} + \frac{15\log\left(2\,c^2x + 2\sqrt{\ldots}\right.}{c^7}\right.\right.$$
$$+ \frac{1}{32}\left(8\,x^4\operatorname{arcosh}(cx) - \left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2} + \frac{3\sqrt{c^2x^2-1}x}{c^4} + \frac{3\log\left(2\,c^2x + 2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bd^2$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arccosh(c*x) - (48
*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^
2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^
2*x^2 - 1)*c)/c^9)*c)*b*c^4*d^2 + 1/4*a*d^2*x^4 - 1/144*(48*x^6*arccosh(c*
x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt
(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^
2*d^2 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c
^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d - c^2dx^2)^2 (a + b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx = \int x^3\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

$$\int x^3 (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx = \frac{d^2\left(1152 acosh(cx)\,b\,c^8x^8 - 3072 acosh(cx)\,b\,c^6x^6 + 2304 acosh(cx)\,b\,c^4x^4 - 144\sqrt{c^2x^2-1}\,b\,c^7x^7 + 344\sqrt{c$$

input
```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(d**2*(1152*acosh(c*x)*b*c**8*x**8 - 3072*acosh(c*x)*b*c**6*x**6 + 2304*ac
osh(c*x)*b*c**4*x**4 - 144*sqrt(c**2*x**2 - 1)*b*c**7*x**7 + 344*sqrt(c**2
*x**2 - 1)*b*c**5*x**5 - 146*sqrt(c**2*x**2 - 1)*b*c**3*x**3 - 219*sqrt(c*
*2*x**2 - 1)*b*c*x - 219*log(sqrt(c**2*x**2 - 1) + c*x)*b + 1152*a*c**8*x*
*8 - 3072*a*c**6*x**6 + 2304*a*c**4*x**4))/(9216*c**4)
```

3.12 $\int x^2(d-c^2dx^2)^2(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\begin{aligned}\int x^2(d-c^2dx^2)^2(a+b\text{arccosh}(cx))\,dx &= -\frac{8bd^2\sqrt{-1+cx}\sqrt{1+cx}}{105c^3}+\frac{4bd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{315c^3}\\ &\quad-\frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}}{175c^3}-\frac{bd^2(-1+cx)^{7/2}(1+cx)^{7/2}}{49c^3}\\ &\quad+\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))-\frac{2}{5}c^2d^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}c^4d^2x^7(a+b\text{arccosh}(cx))\end{aligned}$$

output

```
-8/105*b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3+4/315*b*d^2*(c*x-1)^(3/2)*(c*
x+1)^(3/2)/c^3-1/175*b*d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^3-1/49*b*d^2*(c*x
-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/3*d^2*x^3*(a+b*arccosh(c*x))-2/5*c^2*d^2*x^5
*(a+b*arccosh(c*x))+1/7*c^4*d^2*x^7*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.66

$$\int x^2 \left(d-c^2 d x^2\right)^2 (a+b\operatorname{arccosh}(cx))\,dx$$
$$= \frac{d^2\left(105ac^3x^3(35-42c^2x^2+15c^4x^4)-b\sqrt{-1+cx}\sqrt{1+cx}(818+409c^2x^2-612c^4x^4+225c^6x^6)+105b\right.}{11025c^3}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*(105*a*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*
(35 - 42*c^2*x^2 + 15*c^4*x^4)*ArcCosh[c*x]))/(11025*c^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6336, 27, 1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^2 (a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \; 6336$$

$$-bc\int \frac{d^2x^3\left(15c^4x^4-42c^2x^2+35\right)}{105\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{7}c^4d^2x^7(a+b\operatorname{arccosh}(cx))-\frac{2}{5}c^2d^2x^5(a+b\operatorname{arccosh}(cx))+\frac{1}{3}d^2x^3(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{105}bcd^2\int \frac{x^3\left(15c^4x^4-42c^2x^2+35\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{7}c^4d^2x^7(a+b\operatorname{arccosh}(cx))-\frac{2}{5}c^2d^2x^5(a+b\operatorname{arccosh}(cx))+\frac{1}{3}d^2x^3(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \text{1905}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{x^3\left(15c^4x^4-42c^2x^2+35\right)}{\sqrt{c^2x^2-1}}dx}{105\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}c^4d^2x^7(a+\text{barccosh}(cx))-\frac{2}{5}c^2d^2x^5(a+\text{barccosh}(cx))+\frac{1}{3}d^2x^3(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{1578}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{x^2\left(15c^4x^4-42c^2x^2+35\right)}{\sqrt{c^2x^2-1}}dx^2}{210\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}c^4d^2x^7(a+\text{barccosh}(cx))-\frac{2}{5}c^2d^2x^5(a+\text{barccosh}(cx))+\frac{1}{3}d^2x^3(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{1195}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \left(\frac{15\left(c^2x^2-1\right)^{5/2}}{c^2}+\frac{3\left(c^2x^2-1\right)^{3/2}}{c^2}-\frac{4\sqrt{c^2x^2-1}}{c^2}+\frac{8}{c^2\sqrt{c^2x^2-1}}\right)dx^2}{210\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}c^4d^2x^7(a+\text{barccosh}(cx))-\frac{2}{5}c^2d^2x^5(a+\text{barccosh}(cx))+\frac{1}{3}d^2x^3(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}c^4d^2x^7(a+\text{barccosh}(cx))-\frac{2}{5}c^2d^2x^5(a+\text{barccosh}(cx))+\frac{1}{3}d^2x^3(a+\text{barccosh}(cx))-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{30\left(c^2x^2-1\right)^{7/2}}{7c^4}+\frac{6\left(c^2x^2-1\right)^{5/2}}{5c^4}-\frac{8\left(c^2x^2-1\right)^{3/2}}{3c^4}+\frac{16\sqrt{c^2x^2-1}}{c^4}\right)}{210\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/210*(b*c*d^2*Sqrt[-1 + c^2*x^2]*((16*Sqrt[-1 + c^2*x^2])/c^4 - (8*(-1 +
 c^2*x^2)^(3/2))/(3*c^4) + (6*(-1 + c^2*x^2)^(5/2))/(5*c^4) + (30*(-1 + c^
2*x^2)^(7/2))/(7*c^4)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*A
rcCosh[c*x]))/3 - (2*c^2*d^2*x^5*(a + b*ArcCosh[c*x]))/5 + (c^4*d^2*x^7*(a
 + b*ArcCosh[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.66

method	result
parts	$d^2a\left(\frac{1}{7}c^4x^7-\frac{2}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^2b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{2\operatorname{arccosh}(cx)c^5x^5}{5}+\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(225\right.}{}\right.}{c^3}$
derivativedivides	$\frac{d^2a\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{2\operatorname{arccosh}(cx)c^5x^5}{5}+\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(225c^6x^6-612c\right.}{11025}\right.}{c^3}$
default	$\frac{d^2a\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{2\operatorname{arccosh}(cx)c^5x^5}{5}+\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(225c^6x^6-612c\right.}{11025}\right.}{c^3}$
orering	$\frac{(2925c^8x^8-8532c^6x^6+7353c^4x^4+4090c^2x^2-1636)(-c^2dx^2+d)^2(a+b\operatorname{arccosh}(cx))}{11025c^4(cx-1)(cx+1)x(c^2x^2-1)}-\frac{(225c^6x^6-612c^4x^4+409c^2x}{}$

input
```
int(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/7*c^4*x^7-2/5*c^2*x^5+1/3*x^3)+d^2*b/c^3*(1/7*arccosh(c*x)*c^7*x^
7-2/5*arccosh(c*x)*c^5*x^5+1/3*c^3*x^3*arccosh(c*x)-1/11025*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*(225*c^6*x^6-612*c^4*x^4+409*c^2*x^2+818))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

$$\int x^2\left(d-c^2dx^2\right)^2\left(a+b\operatorname{arccosh}(cx)\right)dx$$
$$=\frac{1575\,ac^7d^2x^7-4410\,ac^5d^2x^5+3675\,ac^3d^2x^3+105\left(15\,bc^7d^2x^7-42\,bc^5d^2x^5+35\,bc^3d^2x^3\right)\log\left(cx+\sqrt{c^2}\right.}{11025\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/11025*(1575*a*c^7*d^2*x^7 - 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 10
5*(15*b*c^7*d^2*x^7 - 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(
c^2*x^2 - 1)) - (225*b*c^6*d^2*x^6 - 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2
 + 818*b*d^2)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [F]

$$\int x^2 (d - c^2dx^2)^2 (a + b\text{arccosh}(cx))\, dx = d^2\left(\int ax^2\, dx + \int (-2ac^2x^4)\ dx + \int ac^4x^6\, dx + \int bx^2 \operatorname{acosh}(cx)\, dx + \int (-2bc^2x^4 \operatorname{acosh}(cx))\ dx + \int bc^4x^6 \operatorname{acosh}(cx)\, dx\right)$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
d**2*(Integral(a*x**2, x) + Integral(-2*a*c**2*x**4, x) + Integral(a*c**4*
x**6, x) + Integral(b*x**2*acosh(c*x), x) + Integral(-2*b*c**2*x**4*acosh(
c*x), x) + Integral(b*c**4*x**6*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.47

$$\int x^2 (d - c^2dx^2)^2 (a + b\text{arccosh}(cx))\, dx = \frac{1}{7}ac^4d^2x^7 - \frac{2}{5}ac^2d^2x^5 + \frac{1}{245}\left(35\,x^7 \operatorname{arcosh}(cx) - \left(\frac{5\sqrt{c^2x^2 - 1}x^6}{c^2} + \frac{6\sqrt{c^2x^2 - 1}x^4}{c^4} + \frac{8\sqrt{c^2x^2 - 1}x^2}{c^6} + \frac{16\sqrt{c^2x^2 - 1}}{c^8}\right)c\right)bc - \frac{2}{75}\left(15\,x^5 \operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2 - 1}x^4}{c^2} + \frac{4\sqrt{c^2x^2 - 1}x^2}{c^4} + \frac{8\sqrt{c^2x^2 - 1}}{c^6}\right)c\right)bc^2d^2 + \frac{1}{3}ad^2x^3 + \frac{1}{9}\left(3\,x^3 \operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2 - 1}x^2}{c^2} + \frac{2\sqrt{c^2x^2 - 1}}{c^4}\right)\right)bd^2$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^4*d^2*x^7 - 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arccosh(c*x) - (5*sq
rt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1
)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^2 - 2/75*(15*x^5*arccosh(
c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt
(c^2*x^2 - 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x)
- c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^2 (d - c^2 d x^2)^2 (a + b\operatorname{arccosh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 d x^2)^2 (a + b\operatorname{arccosh}(cx))\, dx = \int x^2 (a + b\operatorname{acosh}(c\, x))\, \left(d - c^2\, d\, x^2\right)^2 dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int x^2(d-c^2dx^2)^2(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{d^2\left(1575acosh(cx)\,b\,c^7x^7-4410acosh(cx)\,b\,c^5x^5+3675acosh(cx)\,b\,c^3x^3-225\sqrt{c^2x^2-1}\,b\,c^6x^6+612\sqrt{c}\right.}{11025c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(d**2*(1575*acosh(c*x)*b*c**7*x**7 - 4410*acosh(c*x)*b*c**5*x**5 + 3675*ac
osh(c*x)*b*c**3*x**3 - 225*sqrt(c**2*x**2 - 1)*b*c**6*x**6 + 612*sqrt(c**2
*x**2 - 1)*b*c**4*x**4 - 409*sqrt(c**2*x**2 - 1)*b*c**2*x**2 - 818*sqrt(c*
*2*x**2 - 1)*b + 1575*a*c**7*x**7 - 4410*a*c**5*x**5 + 3675*a*c**3*x**3))/
(11025*c**3)
```

3.13 $\int x(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int x(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))\,dx = -\frac{5bd^2x\sqrt{-1+cx}\sqrt{1+cx}}{96c} + \frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c} - \frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c} + \frac{5bd^2\text{arccosh}(cx)}{96c^2} - \frac{d^2(1-c^2x^2)^3\,(a+b\text{arccosh}(cx))}{6c^2}$$

output
```
-5/96*b*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+5/144*b*d^2*x*(c*x-1)^(3/2)*(c
*x+1)^(3/2)/c-1/36*b*d^2*x*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c+5/96*b*d^2*arccos
h(c*x)/c^2-1/6*d^2*(-c^2*x^2+1)^3*(a+b*arccosh(c*x))/c^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int x(d-c^2dx^2)^2(a+\operatorname{barccosh}(cx))\,dx$$
$$=\frac{d^2\Big(cx\big(b\sqrt{-1+cx}\sqrt{1+cx}(-33+26c^2x^2-8c^4x^4)+48acx(3-3c^2x^2+c^4x^4)\big)+48bc^2x^2(3-3c^2x^2+}{288c^2}$$

input
```
Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-33 + 26*c^2*x^2 - 8*c^4*x^4) +
 48*a*c*x*(3 - 3*c^2*x^2 + c^4*x^4)) + 48*b*c^2*x^2*(3 - 3*c^2*x^2 + c^4*x
^4)*ArcCosh[c*x] - 66*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(288*c^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6329, 40, 40, 40, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d-c^2dx^2)^2(a+\operatorname{barccosh}(cx))\,dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{bd^2\int(cx-1)^{5/2}(cx+1)^{5/2}dx}{6c}-\frac{d^2(1-c^2x^2)^3(a+\operatorname{barccosh}(cx))}{6c^2}$$

$$\Big\downarrow \text{40}$$

$$-\frac{bd^2\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\int(cx-1)^{3/2}(cx+1)^{3/2}dx\right)}{6c}-\frac{d^2(1-c^2x^2)^3(a+\operatorname{barccosh}(cx))}{6c^2}$$

$$\Big\downarrow \text{40}$$

$$-\frac{bd^2\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)\right)}{6c}-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{6c^2}$$

$$\Big\downarrow \; 40$$

$$-\frac{bd^2\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)\right)}{6c}-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{6c^2}$$

$$\Big\downarrow \; 43$$

$$-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{6c^2}-\frac{bd^2\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)}{6c}$$

input
```
Int[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/6*(d^2*(1 - c^2*x^2)^3*(a + b*ArcCosh[c*x]))/c^2 - (b*d^2*((x*(-1 + c*x
)^(5/2)*(1 + c*x)^(5/2))/6 - (5*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 -
(3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/6))/(6*c
)
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{d^2 a \left(c^2 x^2-1\right)^3}{6}+d^2 b \left(\frac{\operatorname{arccosh}(cx) c^6 x^6}{6}-\frac{\operatorname{arccosh}(cx) c^4 x^4}{2}+\frac{c^2 x^2 \operatorname{arccosh}(cx)}{2}-\frac{\operatorname{arccosh}(cx)}{6}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-8c^5x^5\sqrt{c^2x^2-1}\right.}{}\right.}{c^2}$
default	$\frac{\frac{d^2 a \left(c^2 x^2-1\right)^3}{6}+d^2 b \left(\frac{\operatorname{arccosh}(cx) c^6 x^6}{6}-\frac{\operatorname{arccosh}(cx) c^4 x^4}{2}+\frac{c^2 x^2 \operatorname{arccosh}(cx)}{2}-\frac{\operatorname{arccosh}(cx)}{6}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-8c^5x^5\sqrt{c^2x^2-1}\right.}{}\right.}{c^2}$
parts	$\frac{d^2 a (c^2x^2-1)^3}{6c^2}+\frac{d^2 b \left(\frac{\operatorname{arccosh}(cx) c^6 x^6}{6}-\frac{\operatorname{arccosh}(cx) c^4 x^4}{2}+\frac{c^2 x^2 \operatorname{arccosh}(cx)}{2}-\frac{\operatorname{arccosh}(cx)}{6}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-8c^5x^5\sqrt{c^2x^2}\right.}{}\right.}{c^2}$
orering	$\frac{(88c^6x^6-282c^4x^4+335c^2x^2-66)\left(-c^2d\,x^2+d\right)^2 (a+b\operatorname{arccosh}(cx))}{288c^2(cx-1)(cx+1)(c^2x^2-1)}-\frac{(8c^4x^4-26c^2x^2+33)\left(\left(-c^2d\,x^2+d\right)^2(a+b\operatorname{arc}\right.}{}$

input
```
int(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/6*d^2*a*(c^2*x^2-1)^3+d^2*b*(1/6*arccosh(c*x)*c^6*x^6-1/2*arccosh
(c*x)*c^4*x^4+1/2*c^2*x^2*arccosh(c*x)-1/6*arccosh(c*x)+1/288*(c*x-1)^(1/2
)*(c*x+1)^(1/2)*(-8*c^5*x^5*(c^2*x^2-1)^(1/2)+26*(c^2*x^2-1)^(1/2)*c^3*x^3
-33*c*x*(c^2*x^2-1)^(1/2)+15*ln(c*x+(c^2*x^2-1)^(1/2)))/(c^2*x^2-1)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int x(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx$$
$$= \frac{48\,ac^6d^2x^6 - 144\,ac^4d^2x^4 + 144\,ac^2d^2x^2 + 3\,(16\,bc^6d^2x^6 - 48\,bc^4d^2x^4 + 48\,bc^2d^2x^2 - 11\,bd^2)\log\left(cx + \sqrt{\right.}{288\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/288*(48*a*c^6*d^2*x^6 - 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*
c^6*d^2*x^6 - 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 - 11*b*d^2)*log(c*x + sq
rt(c^2*x^2 - 1)) - (8*b*c^5*d^2*x^5 - 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqr
t(c^2*x^2 - 1))/c^2
```

Sympy [F]

$$\int x(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx = d^2\left(\int ax\,dx + \int (-2ac^2x^3)\,dx + \int ac^4x^5\,dx + \int bx\,\text{acosh}\,(cx)\,dx + \int (-2bc^2x^3\,\text{acosh}\,(cx))\,dx + \int bc^4x^5\,\text{acosh}\,(cx)\,dx\right)$$

input
```
integrate(x*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
d**2*(Integral(a*x, x) + Integral(-2*a*c**2*x**3, x) + Integral(a*c**4*x**
5, x) + Integral(b*x*acosh(c*x), x) + Integral(-2*b*c**2*x**3*acosh(c*x),
x) + Integral(b*c**4*x**5*acosh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(113) = 226$.

Time = 0.04 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.11

$$\int x\left(d-c^2dx^2\right)^2\left(a+b\operatorname{arccosh}(cx)\right)dx = \frac{1}{6}ac^4d^2x^6-\frac{1}{2}ac^2d^2x^4 + \frac{1}{288}\left(48x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{\cdots}\right)}{c^7}\right.\right. - \frac{1}{16}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bc^2d^2 + \frac{1}{2}ad^2x^2 + \frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}c\right)}{c^3}\right)\right)bd^2$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arccosh(c*x) - (8*sq
rt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 -
 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^4*d^2 - 1/
16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1
)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*c^2*d^2 + 1/2*a
*d^2*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^
2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x\left(d-c^2dx^2\right)^2\left(a+b\operatorname{arccosh}(cx)\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx = \int x\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int x(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))\,dx = \frac{d^2\left(48acosh(cx)\,b\,c^6x^6-144acosh(cx)\,b\,c^4x^4+144acosh(cx)\,b\,c^2x^2-8\sqrt{c^2x^2-1}\,b\,c^5x^5+26\sqrt{c^2x^2-1}\,b\right.}{288c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(d**2*(48*acosh(c*x)*b*c**6*x**6 - 144*acosh(c*x)*b*c**4*x**4 + 144*acosh(
c*x)*b*c**2*x**2 - 8*sqrt(c**2*x**2 - 1)*b*c**5*x**5 + 26*sqrt(c**2*x**2 -
 1)*b*c**3*x**3 - 33*sqrt(c**2*x**2 - 1)*b*c*x - 33*log(sqrt(c**2*x**2 - 1
) + c*x)*b + 48*a*c**6*x**6 - 144*a*c**4*x**4 + 144*a*c**2*x**2))/(288*c**
2)
```

3.14 $\int (d - c^2dx^2)^2 (a + b\,\mathrm{arccosh}(cx))\, dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int (d - c^2dx^2)^2 (a + b\,\mathrm{arccosh}(cx))\, dx = -\frac{8bd^2\sqrt{-1+cx}\sqrt{1+cx}}{15c} + \frac{4bd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{45c} - \frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}}{25c} + d^2x(a+b\,\mathrm{arccosh}(cx)) - \frac{2}{3}c^2d^2x^3(a+b\,\mathrm{arccosh}(cx)) + \frac{1}{5}c^4d^2x^5(a+b\,\mathrm{arccosh}(cx))$$

output
```
-8/15*b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+4/45*b*d^2*(c*x-1)^(3/2)*(c*x+1)
^(3/2)/c-1/25*b*d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c+d^2*x*(a+b*arccosh(c*x))
-2/3*c^2*d^2*x^3*(a+b*arccosh(c*x))+1/5*c^4*d^2*x^5*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

$$\int \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$
$$=\frac{d^2\left(b\sqrt{-1+cx}\sqrt{1+cx}(-149+38c^2x^2-9c^4x^4)+15acx(15-10c^2x^2+3c^4x^4)+15bcx(15-10c^2x^2+\right.}{225c}$$

input
```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-149 + 38*c^2*x^2 - 9*c^4*x^4) + 15*
a*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^
4)*ArcCosh[c*x]))/(225*c)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6309, 27, 1905, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6309}$$

$$-bc\int \frac{d^2x(3c^4x^4-10c^2x^2+15)}{15\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}c^4d^2x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2d^2x^3(a+b\text{arccosh}(cx))+d^2x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bcd^2\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}c^4d^2x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2d^2x^3(a+b\text{arccosh}(cx))+d^2x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1905}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{c^2x^2-1}}dx}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4d^2x^5(a+\text{barccosh}(cx))-\frac{2}{3}c^2d^2x^3(a+\text{barccosh}(cx))+d^2x(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{c^2x^2-1}}dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4d^2x^5(a+\text{barccosh}(cx))-\frac{2}{3}c^2d^2x^3(a+\text{barccosh}(cx))+d^2x(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{1140}$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \left(3\left(c^2x^2-1\right)^{3/2}-4\sqrt{c^2x^2-1}+\frac{8}{\sqrt{c^2x^2-1}}\right)dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4d^2x^5(a+\text{barccosh}(cx))-\frac{2}{3}c^2d^2x^3(a+\text{barccosh}(cx))+d^2x(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^4d^2x^5(a+\text{barccosh}(cx))-\frac{2}{3}c^2d^2x^3(a+\text{barccosh}(cx))+d^2x(a+\text{barccosh}(cx))-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{6\left(c^2x^2-1\right)^{5/2}}{5c^2}-\frac{8\left(c^2x^2-1\right)^{3/2}}{3c^2}+\frac{16\sqrt{c^2x^2-1}}{c^2}\right)}{30\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/30*(b*c*d^2*Sqrt[-1 + c^2*x^2]*((16*Sqrt[-1 + c^2*x^2])/c^2 - (8*(-1 +
c^2*x^2)^(3/2))/(3*c^2) + (6*(-1 + c^2*x^2)^(5/2))/(5*c^2)))/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) - (2*c^2*d^2*x^3*(a + b*Arc
Cosh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

method	result
parts	$d^2a\left(\frac{1}{5}c^4x^5-\frac{2}{3}c^2x^3+x\right)+\frac{d^2b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{2c^3x^3\operatorname{arccosh}(cx)}{3}+cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(9c^4x^4-\right.}{225}\right.}{c}$
derivativedivides	$\frac{d^2a\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{2c^3x^3\operatorname{arccosh}(cx)}{3}+cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(9c^4x^4-38c^2x^2+149\right.}{225}\right.}{c}$
default	$\frac{d^2a\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{2c^3x^3\operatorname{arccosh}(cx)}{3}+cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(9c^4x^4-38c^2x^2+149\right.}{225}\right.}{c}$
orering	$\frac{x(81c^4x^4-302c^2x^2+821)\left(-c^2dx^2+d\right)^2(a+b\operatorname{arccosh}(cx))}{225(cx-1)(cx+1)(c^2x^2-1)}-\frac{(9c^4x^4-38c^2x^2+149)\left(-4\left(-c^2dx^2+d\right)(a+b\operatorname{arccosh}\right.}{225c^2(cx-1)(cx+1)}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/5*c^4*x^5-2/3*c^2*x^3+x)+d^2*b/c*(1/5*arccosh(c*x)*c^5*x^5-2/3*c^
3*x^3*arccosh(c*x)+c*x*arccosh(c*x)-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c
^4*x^4-38*c^2*x^2+149))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$
$$=\frac{45\,ac^5d^2x^5-150\,ac^3d^2x^3+225\,acd^2x+15\left(3\,bc^5d^2x^5-10\,bc^3d^2x^3+15\,bcd^2x\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-(}{225\,c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*d^2*x^5 - 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*
d^2*x^5 - 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1)) -
(9*b*c^4*d^2*x^4 - 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 - 1))/c
```

Sympy [F]

$$\int \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx = d^2\left(\int a\,dx + \int b\,\text{acosh}\,(cx)\,dx + \int \left(-2ac^2x^2\right)\,dx + \int ac^4x^4\,dx + \int \left(-2bc^2x^2\,\text{acosh}\,(cx)\right)\,dx + \int bc^4x^4\,\text{acosh}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
d**2*(Integral(a, x) + Integral(b*acosh(c*x), x) + Integral(-2*a*c**2*x**2
, x) + Integral(a*c**4*x**4, x) + Integral(-2*b*c**2*x**2*acosh(c*x), x) +
 Integral(b*c**4*x**4*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.36

$$\int \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx = \frac{1}{5}\,ac^4d^2x^5 + \frac{1}{75}\left(15\,x^5\,\text{arcosh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\,\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^4d^2 - \frac{2}{3}\,ac^2d^2x^3 - \frac{2}{9}\left(3\,x^3\,\text{arcosh}\,(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^2d^2 + ad^2x + \frac{\left(cx\,\text{arcosh}\,(cx) - \sqrt{c^2x^2-1}\right)bd^2}{c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c
^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^4*d^2 -
 2/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^
2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arccosh(c*x) - sq
rt(c^2*x^2 - 1))*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int (d-c^2dx^2)^2 (a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d-c^2dx^2)^2 (a+b\operatorname{arccosh}(cx))\,dx = \int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx = \frac{d^2\left(45acosh(cx)\,b\,c^5x^5-150acosh(cx)\,b\,c^3x^3+225acosh(cx)\,bcx-9\sqrt{c^2x^2-1}\,b\,c^4x^4+38\sqrt{c^2x^2-1}\,b\,c^2\right.}{225c}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(d**2*(45*acosh(c*x)*b*c**5*x**5 - 150*acosh(c*x)*b*c**3*x**3 + 225*acosh(
c*x)*b*c*x - 9*sqrt(c**2*x**2 - 1)*b*c**4*x**4 + 38*sqrt(c**2*x**2 - 1)*b*
c**2*x**2 + 76*sqrt(c**2*x**2 - 1)*b - 225*sqrt(c*x + 1)*sqrt(c*x - 1)*b +
 45*a*c**5*x**5 - 150*a*c**3*x**3 + 225*a*c*x))/(225*c)
```

3.15 $\int \frac{(d-c^2dx^2)^2(a+b\,\mathrm{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{(d-c^2dx^2)^2(a+b\,\mathrm{arccosh}(cx))}{x}\,dx = \frac{11}{32}bcd^2x\sqrt{-1+cx}\sqrt{1+cx} - \frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2} - \frac{11}{32}bd^2\,\mathrm{arccosh}(cx) + \frac{1}{2}d^2(1-c^2x^2)(a+b\,\mathrm{arccosh}(cx)) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b\,\mathrm{arccosh}(cx)) - \frac{d^2(a+b\,\mathrm{arccosh}(cx))^2}{2b} + d^2(a+b\,\mathrm{arccosh}(cx))\log\left(1+e^{2\,\mathrm{arccosh}(cx)}\right) + \frac{1}{2}bd^2\,\mathrm{PolyLog}\left(2,-e^{2\,\mathrm{arccosh}(cx)}\right)$$

output
```
11/32*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/16*b*c*d^2*x*(c*x-1)^(3/2)*(
c*x+1)^(3/2)-11/32*b*d^2*arccosh(c*x)+1/2*d^2*(-c^2*x^2+1)*(a+b*arccosh(c*
x))+1/4*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))-1/2*d^2*(a+b*arccosh(c*x))^2
/b+d^2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*
d^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.13

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x}\,dx = \frac{1}{4}d^2\left(-4ac^2x^2+ac^4x^4-4bc^2x^2\text{arccosh}(cx)+bc^4x^4\text{arccosh}(cx)+2b\left(cx\sqrt{-1+cx}\sqrt{1+cx}+2\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)-\frac{1}{8}b\left(cx\sqrt{\frac{-1+cx}{1+cx}}(3+3cx+2c^2x^2+2c^3x^3)+6\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)+2b\text{arccosh}(cx)\left(\text{arccosh}(cx)+2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right)+4a\log(x)-2b\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\right)$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(d^2*(-4*a*c^2*x^2 + a*c^4*x^4 - 4*b*c^2*x^2*ArcCosh[c*x] + b*c^4*x^4*ArcC
osh[c*x] + 2*b*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*
x)/(1 + c*x)]]) - (b*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(3 + 3*c*x + 2*c^2*x^
2 + 2*c^3*x^3) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/8 + 2*b*ArcCosh[c
*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) + 4*a*Log[x] - 2*b*Pol
yLog[2, -E^(-2*ArcCosh[c*x])]))/4
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {6334, 27, 40, 40, 43, 6334, 40, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))}{x} \, dx$$

$$\Big\downarrow \text{6334}$$

$$d\int \frac{d(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bcd^2\int (cx-1)^{3/2}(cx+1)^{3/2}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$d^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bcd^2\int (cx-1)^{3/2}(cx+1)^{3/2}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{40}$$

$$d^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{40}$$

$$d^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{43}$$

$$d^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))-\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

$$\Big\downarrow \text{6334}$$

$$d^2\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{2}(1-c^2x^2)(a+b\text{arccosh}(cx))\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))-\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

$$\Big\downarrow \text{40}$$

$$d^2\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{2}(1-c^2x^2)(a+b\text{arccos}\right.$$
$$\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))-\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

$$\Big\downarrow \text{43}$$

$$d^2\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}(1-c^2x^2)(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))-\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

$$\Big\downarrow \text{6297}$$

$$d^2\left(\frac{\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}(1-c^2x^2)(a+b\text{arccosh}(cx))\right.$$
$$\frac{1}{4}d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))-\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$d^2\left(-\frac{\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-$$
$$\left.\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 3042

$$d^2\left(-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx)\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-$$
$$\left.\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 26

$$d^2\left(\frac{i\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-$$
$$\left.\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 4201

$$d^2\left(\frac{i\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arcco}\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-$$
$$\left.\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 2620

$$d^2\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a\right.}{b}\right.$$

$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-$$

$$\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$d^2\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx)\right)\right.}{b}\right.$$

$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-$$

$$\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$d^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(\right.}{b}\right.$$

$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-$$

$$\frac{1}{4}bcd^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 - (b*c*d^2*((x*(-1 + c*x)^(3/
2)*(1 + c*x)^(3/2))/4 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c
*x]/(2*c)))/4))/4 + d^2*(((1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 + (b*c*((x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/2 + (I*((-1/2*I)*(
a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*
ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6334
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Simp[b*c*((-d
)^p/(2*p))   Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.04

method	result
parts	$d^2a\left(\frac{c^4x^4}{4}-c^2x^2+\ln\left(x\right)\right)+\frac{b\,d^2\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2}+\frac{13b\,d^2\,\mathrm{arccosh}(cx)}{32}-\frac{d^2b\,\mathrm{arcco}}{2}$
derivativedivides	$d^2a\left(\frac{c^4x^4}{4}-c^2x^2+\ln\left(cx\right)\right)+\frac{13b\,d^2\,\mathrm{arccosh}(cx)}{32}-\frac{d^2b\sqrt{cx+1}\,\sqrt{cx-1}\,c^3x^3}{16}+\frac{13bc\,d^2x\sqrt{cx-1}\,\sqrt{cx+1}}{32}$
default	$d^2a\left(\frac{c^4x^4}{4}-c^2x^2+\ln\left(cx\right)\right)+\frac{13b\,d^2\,\mathrm{arccosh}(cx)}{32}-\frac{d^2b\sqrt{cx+1}\,\sqrt{cx-1}\,c^3x^3}{16}+\frac{13bc\,d^2x\sqrt{cx-1}\,\sqrt{cx+1}}{32}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/4*c^4*x^4-c^2*x^2+ln(x))+1/2*b*d^2*polylog(2,-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2)+13/32*b*d^2*arccosh(c*x)-1/2*d^2*b*arccosh(c*x)^2-1/16*d
^2*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^3+13/32*b*c*d^2*x*(c*x-1)^(1/2)*(c*
x+1)^(1/2)+d^2*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/
4*d^2*b*arccosh(c*x)*c^4*x^4-d^2*b*arccosh(c*x)*c^2*x^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(c^2dx^2-d)^2(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))}{x}\,dx = d^2\left(\int \frac{a}{x}\,dx + \int \left(-2ac^2x\right)\,dx + \int ac^4x^3\,dx + \int \frac{b\,\text{acosh}\,(cx)}{x}\,dx + \int \left(-2bc^2x\,\text{acosh}\,(cx)\right)\,dx + \int bc^4x^3\,\text{acosh}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

output
```
d**2*(Integral(a/x, x) + Integral(-2*a*c**2*x, x) + Integral(a*c**4*x**3,
x) + Integral(b*acosh(c*x)/x, x) + Integral(-2*b*c**2*x*acosh(c*x), x) + I
ntegral(b*c**4*x**3*acosh(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(c^2dx^2-d)^2(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*c^4*d^2*x^4 - a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*x*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^2}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))}{x}\,dx$$
$$= \frac{d^2\left(8acosh(cx)\,b\,c^4x^4 - 32acosh(cx)\,b\,c^2x^2 - 2\sqrt{c^2x^2-1}\,b\,c^3x^3 + 13\sqrt{c^2x^2-1}\,bcx + 32\left(\int \frac{acosh(cx)}{x}dx\right) b\right.}{32}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acosh(c*x))/x,x)
```

output
```
(d**2*(8*acosh(c*x)*b*c**4*x**4 - 32*acosh(c*x)*b*c**2*x**2 - 2*sqrt(c**2*
x**2 - 1)*b*c**3*x**3 + 13*sqrt(c**2*x**2 - 1)*b*c*x + 32*int(acosh(c*x)/x
,x)*b + 13*log(sqrt(c**2*x**2 - 1) + c*x)*b + 32*log(x)*a + 8*a*c**4*x**4
- 32*a*c**2*x**2))/32
```

3.16 $\int \frac{(d-c^2dx^2)^2(a+b\mathbf{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))}{x^2}\,dx = {} & \frac{16}{9}bcd^2\sqrt{-1+cx}\sqrt{1+cx} \\ & -\frac{1}{9}bc^3d^2x^2\sqrt{-1+cx}\sqrt{1+cx} \\ & -\frac{d^2(a+b\text{arccosh}(cx))}{x} \\ & -2c^2d^2x(a+b\text{arccosh}(cx)) \\ & +\frac{1}{3}c^4d^2x^3(a+b\text{arccosh}(cx)) \\ & +bcd^2\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\end{aligned}$$

output
```
16/9*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/9*b*c^3*d^2*x^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)-d^2*(a+b*arccosh(c*x))/x-2*c^2*d^2*x*(a+b*arccosh(c*x))+1/3*c
^4*d^2*x^3*(a+b*arccosh(c*x))+b*c*d^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^2}\,dx$$
$$= \frac{d^2\left(-9a-18ac^2x^2+3ac^4x^4+16bcx\sqrt{-1+cx}\sqrt{1+cx}-bc^3x^3\sqrt{-1+cx}\sqrt{1+cx}+3b(-3-6c^2x^2+c\right.}{9x}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
(d^2*(-9*a - 18*a*c^2*x^2 + 3*a*c^4*x^4 + 16*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x] - b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 3*b*(-3 - 6*c^2*x^2 + c^4
*x^4)*ArcCosh[c*x] - 9*b*c*x*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]))/(9
*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6336, 27, 1905, 1578, 1192, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int -\frac{d^2(-c^4x^4+6c^2x^2+3)}{3x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{3}c^4d^2x^3(a+b\text{arccosh}(cx))-2c^2d^2x(a+b\text{arccosh}(cx))-\frac{d^2(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd^2\int \frac{-c^4x^4+6c^2x^2+3}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{3}c^4d^2x^3(a+\text{barccosh}(cx))-2c^2d^2x(a+\text{barccosh}(cx))-\frac{d^2(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{1905}$$

$$\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{-c^4x^4+6c^2x^2+3}{x\sqrt{c^2x^2-1}}dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4d^2x^3(a+\text{barccosh}(cx))-2c^2d^2x(a+\text{barccosh}(cx))-\frac{d^2(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{1578}$$

$$\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{-c^4x^4+6c^2x^2+3}{x^2\sqrt{c^2x^2-1}}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4d^2x^3(a+\text{barccosh}(cx))-2c^2d^2x(a+\text{barccosh}(cx))-\frac{d^2(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{1192}$$

$$\frac{bd^2\sqrt{c^2x^2-1}\int \frac{-c^4x^8+4c^4x^4+8c^4}{x^4+1}d\sqrt{c^2x^2-1}}{3c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4d^2x^3(a+\text{barccosh}(cx))-2c^2d^2x(a+\text{barccosh}(cx))-\frac{d^2(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{1467}$$

$$\frac{bd^2\sqrt{c^2x^2-1}\int \left(-x^4c^4+\frac{3c^4}{x^4+1}+5c^4\right)d\sqrt{c^2x^2-1}}{3c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4d^2x^3(a+\text{barccosh}(cx))-2c^2d^2x(a+\text{barccosh}(cx))-\frac{d^2(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^4d^2x^3(a+\text{barccosh}(cx))-2c^2d^2x(a+\text{barccosh}(cx))-\frac{d^2(a+\text{barccosh}(cx))}{x}+\frac{bd^2\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x^2-1}\right)-\frac{1}{3}c^4x^6+5c^4\sqrt{c^2x^2-1}\right)}{3c^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcCosh[c*x]))/x) - 2*c^2*d^2*x*(a + b*ArcCosh[c*x]) + (c^4*
d^2*x^3*(a + b*ArcCosh[c*x]))/3 + (b*d^2*Sqrt[-1 + c^2*x^2]*(-1/3*(c^4*x^6
) + 5*c^4*Sqrt[-1 + c^2*x^2] + 3*c^4*ArcTan[Sqrt[-1 + c^2*x^2]]))/(3*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

method	result
parts	$d^2a\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)+d^2bc\left(\frac{c^3x^3\,\mathrm{arccosh}(cx)}{3}-2cx\,\mathrm{arccosh}\left(cx\right)-\frac{\mathrm{arccosh}(cx)}{cx}-\frac{\sqrt{cx-1}\,\sqrt{}}{}\right.$
derivativedivides	$c\left(d^2a\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+d^2b\left(\frac{c^3x^3\,\mathrm{arccosh}(cx)}{3}-2cx\,\mathrm{arccosh}\left(cx\right)-\frac{\mathrm{arccosh}(cx)}{cx}-\frac{\sqrt{cx-1}}{}\right.\right.$
default	$c\left(d^2a\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+d^2b\left(\frac{c^3x^3\,\mathrm{arccosh}(cx)}{3}-2cx\,\mathrm{arccosh}\left(cx\right)-\frac{\mathrm{arccosh}(cx)}{cx}-\frac{\sqrt{cx-1}}{}\right.\right.$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/3*c^4*x^3-2*c^2*x-1/x)+d^2*b*c*(1/3*c^3*x^3*arccosh(c*x)-2*c*x*ar
ccosh(c*x)-arccosh(c*x)/c/x-1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*x^2*(c^2*
x^2-1)^(1/2)+9*arctan(1/(c^2*x^2-1)^(1/2))-16*(c^2*x^2-1)^(1/2))/(c^2*x^2-
1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.44

$$\int \frac{(d-c^2dx^2)^2\,(a+b\operatorname{arccosh}(cx))}{x^2}\,dx$$
$$= \frac{3\,ac^4d^2x^4 - 18\,ac^2d^2x^2 + 18\,bcd^2x\arctan\left(-cx+\sqrt{c^2x^2-1}\right) - 3\,(bc^4-6\,bc^2-3\,b)d^2x\log\left(-cx+\sqrt{c^2x}\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/9*(3*a*c^4*d^2*x^4 - 18*a*c^2*d^2*x^2 + 18*b*c*d^2*x*arctan(-c*x + sqrt(
c^2*x^2 - 1)) - 3*(b*c^4 - 6*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 -
1)) - 9*a*d^2 + 3*(b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (b*c^4 - 6*b*c^2 - 3*
b)*d^2*x - 3*b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^3*d^2*x^3 - 16*b*c
*d^2*x)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2\,(a+b\operatorname{arccosh}(cx))}{x^2}\,dx = d^2\left(\int \left(-2ac^2\right)\,dx + \int \frac{a}{x^2}\,dx + \int ac^4x^2\,dx + \int \left(-2bc^2\operatorname{acosh}(cx)\right)\,dx + \int \frac{b\operatorname{acosh}(cx)}{x^2}\,dx + \int bc^4x^2\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
```

output
```
d**2*(Integral(-2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2,
 x) + Integral(-2*b*c**2*acosh(c*x), x) + Integral(b*acosh(c*x)/x**2, x) +
 Integral(b*c**4*x**2*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \frac{(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))}{x^2} dx$$
$$= \frac{1}{3} ac^4d^2x^3 + \frac{1}{9}\left(3x^3 \text{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^4d^2$$
$$- 2ac^2d^2x - 2\left(cx \text{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bcd^2$$
$$- \left(c \arcsin\left(\frac{1}{c|x|}\right) + \frac{\text{arcosh}(cx)}{x}\right)bd^2 - \frac{ad^2}{x}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^2 - 2*a*c^2*d^2*x - 2*(c*x*arccosh(c*
x) - sqrt(c^2*x^2 - 1))*b*c*d^2 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x
)*b*d^2 - a*d^2/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\text{arccosh}(cx)\right)}{x^2}\,dx=\int \frac{\left(a+b\,\text{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^2}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\text{arccosh}(cx)\right)}{x^2}\,dx$$
$$=\frac{d^2\left(3acosh(cx)\,b\,c^4x^4-18acosh(cx)\,b\,c^2x^2-9acosh(cx)\,b-18atan\left(\sqrt{c^2x^2-1}+cx\right)bcx-\sqrt{c^2x^2-1}\,b\right.}{9x}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acosh(c*x))/x^2,x)
```

output
```
(d**2*(3*acosh(c*x)*b*c**4*x**4 - 18*acosh(c*x)*b*c**2*x**2 - 9*acosh(c*x)
*b - 18*atan(sqrt(c**2*x**2 - 1) + c*x)*b*c*x - sqrt(c**2*x**2 - 1)*b*c**3
*x**3 - 2*sqrt(c**2*x**2 - 1)*b*c*x + 18*sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x
 + 3*a*c**4*x**4 - 18*a*c**2*x**2 - 9*a))/(9*x)
```

3.17 $\int \frac{(d-c^2dx^2)^2(a+b\,\textbf{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^2\,(a+b\operatorname{arccosh}(cx))}{x^3}\,dx &= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx} \\
&\quad - \frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x} \\
&\quad - \frac{1}{4}bc^2d^2\operatorname{arccosh}(cx) \\
&\quad - c^2d^2\left(1-c^2x^2\right)(a+b\operatorname{arccosh}(cx)) \\
&\quad - \frac{d^2(1-c^2x^2)^2\,(a+b\operatorname{arccosh}(cx))}{2x^2} \\
&\quad + \frac{c^2d^2(a+b\operatorname{arccosh}(cx))^2}{b} \\
&\quad - 2c^2d^2(a+b\operatorname{arccosh}(cx))\log\left(1+e^{2\operatorname{arccosh}(cx)}\right) \\
&\quad - bc^2d^2\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)
\end{aligned}$$

output
```
1/4*b*c^3*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*b*c*d^2*(c*x-1)^(3/2)*(c*x
+1)^(3/2)/x-1/4*b*c^2*d^2*arccosh(c*x)-c^2*d^2*(-c^2*x^2+1)*(a+b*arccosh(c
*x))-1/2*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))/x^2+c^2*d^2*(a+b*arccosh(c*
x))^2/b-2*c^2*d^2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2)-b*c^2*d^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$= \frac{d^2\left(-2a+2ac^4x^4+2bcx\sqrt{-1+cx}\sqrt{1+cx}-bc^3x^3\sqrt{-1+cx}\sqrt{1+cx}-4bc^2x^2\text{arccosh}(cx)^2-2bc^2x^2\text{a}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
(d^2*(-2*a + 2*a*c^4*x^4 + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - b*c^3*x^
3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 4*b*c^2*x^2*ArcCosh[c*x]^2 - 2*b*c^2*x^2*
ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 2*b*ArcCosh[c*x]*(-1 + c^4*x^4 - 4*c
^2*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*c^2*x^2*Log[x] + 4*b*c^2*x^2*Po
lyLog[2, -E^(-2*ArcCosh[c*x])]))/(4*x^2)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.24, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {6335, 27, 108, 27, 40, 43, 6334, 40, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{6335}$$
$$-2c^2d\int \frac{d(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{2}bcd^2\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2}dx-\frac{d^2(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 27$$

$$-2c^2d^2\int \frac{(1-c^2x^2)\,(a+b\operatorname{arccosh}(cx))}{x}dx+\frac{1}{2}bcd^2\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\operatorname{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 108$$

$$-2c^2d^2\int \frac{(1-c^2x^2)\,(a+b\operatorname{arccosh}(cx))}{x}dx+\frac{1}{2}bcd^2\left(\int 3c^2\sqrt{cx-1}\sqrt{cx+1}dx-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)-\frac{d^2\left(1-c^2x^2\right)^2(a+b\operatorname{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 27$$

$$-2c^2d^2\int \frac{(1-c^2x^2)\,(a+b\operatorname{arccosh}(cx))}{x}dx+\frac{1}{2}bcd^2\left(3c^2\int \sqrt{cx-1}\sqrt{cx+1}dx-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)-\frac{d^2\left(1-c^2x^2\right)^2(a+b\operatorname{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 40$$

$$-2c^2d^2\int \frac{(1-c^2x^2)\,(a+b\operatorname{arccosh}(cx))}{x}dx+\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)-\frac{d^2\left(1-c^2x^2\right)^2(a+b\operatorname{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 43$$

$$-2c^2d^2\int \frac{(1-c^2x^2)\,(a+b\operatorname{arccosh}(cx))}{x}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\operatorname{arccosh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\operatorname{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \; 6334$$

$$-2c^2d^2\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))\right)-\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \text{40}$$

$$-2c^2d^2\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{2}\left(1-c^2x^2\right)(a+ba\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \text{43}$$

$$-2c^2d^2\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \text{6297}$$

$$-2c^2d^2\left(\frac{\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$-2c^2d^2\left(-\frac{\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arccosh}(cx\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$\big\downarrow$ 3042

$$-2c^2d^2\left(-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arccos}\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$\big\downarrow$ 26

$$-2c^2d^2\left(\frac{i\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arccosh}(c\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$\big\downarrow$ 4201

$$-2c^2d^2\left(\frac{i\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$\big\downarrow$ 2620

$$-2c^2d^2\left(\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\right.}{b}\right.$$

$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \text{2715}$$

$$-2c^2d^2\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}\right.\right.}{b}\right.$$

$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$-2c^2d^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.$$

$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/x^2 + (b*c*d^2*(-(((-1 + c
*x)^(3/2)*(1 + c*x)^(3/2))/x) + 3*c^2*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2
- ArcCosh[c*x]/(2*c))))/2 - 2*c^2*d^2*(((1 - c^2*x^2)*(a + b*ArcCosh[c*x])
)/2 + (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/2 +
(I*((-1/2*I)*(a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*
Log[1 + E^(-2*ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))
/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6334
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Simp[b*c*((-d
)^p/(2*p))   Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6335
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*((-d)^p/(f*(m + 1)))   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(
m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

method	result
derivativedivides	$c^2\left(d^2a\left(\frac{c^2x^2}{2}-2\ln(cx)-\frac{1}{2c^2x^2}\right)+d^2b\operatorname{arccosh}(cx)^2-\frac{bc\,d^2x\sqrt{cx-1}\,\sqrt{cx+1}}{4}+\frac{d^2b\operatorname{arccosh}(cx}{2}\right.$
default	$c^2\left(d^2a\left(\frac{c^2x^2}{2}-2\ln(cx)-\frac{1}{2c^2x^2}\right)+d^2b\operatorname{arccosh}(cx)^2-\frac{bc\,d^2x\sqrt{cx-1}\,\sqrt{cx+1}}{4}+\frac{d^2b\operatorname{arccosh}(cx}{2}\right.$
parts	$d^2a\left(\frac{c^4x^2}{2}-2c^2\ln(x)-\frac{1}{2x^2}\right)+d^2b\,c^2\operatorname{arccosh}(cx)^2+\frac{d^2b\,c^4\operatorname{arccosh}(cx)x^2}{2}-\frac{b\,c^3d^2x\sqrt{cx-1}\,\sqrt{c}}{4}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^2*a*(1/2*c^2*x^2-2*ln(c*x)-1/2/c^2/x^2)+d^2*b*arccosh(c*x)^2-1/4*b*
c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/2*d^2*b*arccosh(c*x)*c^2*x^2-1/4*b*d
^2*arccosh(c*x)-1/2*d^2*b+1/2*d^2*b/c/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/2*d^
2*b*arccosh(c*x)/c^2/x^2-2*d^2*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)-b*d^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2))
```

Fricas [F]

$$\int\frac{(d-c^2dx^2)^2\,(a+b\operatorname{arccosh}(cx))}{x^3}\,dx=\int\frac{(c^2dx^2-d)^2(b\operatorname{arcosh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{\left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))}{x^3}\,dx = d^2\left(\int \frac{a}{x^3}\,dx + \int \left(-\frac{2ac^2}{x}\right)dx + \int ac^4x\,dx + \int \frac{b\,\text{acosh}\,(cx)}{x^3}\,dx + \int \left(-\frac{2bc^2\,\text{acosh}\,(cx)}{x}\right)dx + \int bc^4x\,\text{acosh}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

output
```
d**2*(Integral(a/x**3, x) + Integral(-2*a*c**2/x, x) + Integral(a*c**4*x,
x) + Integral(b*acosh(c*x)/x**3, x) + Integral(-2*b*c**2*acosh(c*x)/x, x)
+ Integral(b*c**4*x*acosh(c*x), x))
```

Maxima [F]

$$\int \frac{\left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{\left(c^2dx^2-d\right)^2 (b\,\text{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*c^4*d^2*x^2 - 2*a*c^2*d^2*log(x) + 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x
- arccosh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^2}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$= \frac{d^2\left(2acosh(cx)\,b\,c^4x^4 - 2acosh(cx)\,b - \sqrt{c^2x^2-1}\,b\,c^3x^3 - 2\sqrt{c^2x^2-1}\,bcx - 8\left(\int \frac{acosh(cx)}{x}dx\right)b\,c^2x^2 - \text{lo}\right.}{4x^2}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acosh(c*x))/x^3,x)
```

output
```
(d**2*(2*acosh(c*x)*b*c**4*x**4 - 2*acosh(c*x)*b - sqrt(c**2*x**2 - 1)*b*c
**3*x**3 - 2*sqrt(c**2*x**2 - 1)*b*c*x - 8*int(acosh(c*x)/x,x)*b*c**2*x**2
 - log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*x**2 - 8*log(x)*a*c**2*x**2 + 2*a
*c**4*x**4 - 2*a - 2*b*c**2*x**2))/(4*x**2)
```

3.18 $\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{(d-c^2dx^2)^2(a+b\text{arccosh}(cx))}{x^4}\,dx = -bc^3d^2\sqrt{-1+cx}\sqrt{1+cx}+\frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d^2(a+b\text{arccosh}(cx))}{3x^3}+\frac{2c^2d^2(a+b\text{arccosh}(cx))}{x} + c^4d^2x(a+b\text{arccosh}(cx))-\frac{11}{6}bc^3d^2\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
-b*c^3*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/x^2-1/3*d^2*(a+b*arccosh(c*x))/x^3+2*c^2*d^2*(a+b*arccosh(c*x))/x+c^4
*d^2*x*(a+b*arccosh(c*x))-11/6*b*c^3*d^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2
))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))}{x^4}\,dx$$
$$= \frac{d^2\left(-2a+12ac^2x^2+6ac^4x^4+bcx\sqrt{-1+cx}\sqrt{1+cx}-6bc^3x^3\sqrt{-1+cx}\sqrt{1+cx}+2b(-1+6c^2x^2+3c\right.}{6x^3}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
(d^2*(-2*a + 12*a*c^2*x^2 + 6*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x] - 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(-1 + 6*c^2*x^2 + 3*c^
4*x^4)*ArcCosh[c*x] + 11*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
]))/(6*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {6336, 27, 1905, 1578, 1192, 25, 1471, 25, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2 (a+b\text{arccosh}(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{6336}$$
$$-bc\int -\frac{d^2(-3c^4x^4-6c^2x^2+1)}{3x^3\sqrt{cx-1}\sqrt{cx+1}}dx+c^4d^2x(a+b\text{arccosh}(cx))+\frac{2c^2d^2(a+b\text{arccosh}(cx))}{x}-\frac{d^2(a+b\text{arccosh}(cx))}{3x^3}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd^2\int \frac{-3c^4x^4-6c^2x^2+1}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx+c^4d^2x(a+b\operatorname{arccosh}(cx))+\frac{2c^2d^2(a+b\operatorname{arccosh}(cx))}{x}-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 1905

$$\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{-3c^4x^4-6c^2x^2+1}{x^3\sqrt{c^2x^2-1}}dx}{3\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\operatorname{arccosh}(cx))+\frac{2c^2d^2(a+b\operatorname{arccosh}(cx))}{x}-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 1578

$$\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{-3c^4x^4-6c^2x^2+1}{x^4\sqrt{c^2x^2-1}}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\operatorname{arccosh}(cx))+\frac{2c^2d^2(a+b\operatorname{arccosh}(cx))}{x}-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 1192

$$\frac{bd^2\sqrt{c^2x^2-1}\int -\frac{3c^4x^8+12c^4x^4+8c^4}{(x^4+1)^2}d\sqrt{c^2x^2-1}}{3c\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\operatorname{arccosh}(cx))+\frac{2c^2d^2(a+b\operatorname{arccosh}(cx))}{x}-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 25

$$-\frac{bd^2\sqrt{c^2x^2-1}\int \frac{3c^4x^8+12c^4x^4+8c^4}{(x^4+1)^2}d\sqrt{c^2x^2-1}}{3c\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\operatorname{arccosh}(cx))+\frac{2c^2d^2(a+b\operatorname{arccosh}(cx))}{x}-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 1471

$$\frac{bd^2\sqrt{c^2x^2-1}\left(\frac{1}{2}\int -\frac{c^4(6x^4+17)}{x^4+1}d\sqrt{c^2x^2-1}+\frac{c^4\sqrt{c^2x^2-1}}{2(x^4+1)}\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\operatorname{arccosh}(cx))+\frac{2c^2d^2(a+b\operatorname{arccosh}(cx))}{x}-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 25

$$\frac{bd^2\sqrt{c^2x^2-1}\left(\frac{c^4\sqrt{c^2x^2-1}}{2(x^4+1)}-\frac{1}{2}\int \frac{c^4(6x^4+17)}{x^4+1}d\sqrt{c^2x^2-1}\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\operatorname{arccosh}(cx))+\frac{2c^2d^2(a+b\operatorname{arccosh}(cx))}{x}-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 27

$$\frac{bd^2\sqrt{c^2x^2-1}\left(\frac{c^4\sqrt{c^2x^2-1}}{2(x^4+1)}-\frac{1}{2}c^4\int\frac{6x^4+17}{x^4+1}d\sqrt{c^2x^2-1}\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\text{arccosh}(cx))+\frac{2c^2d^2(a+b\text{arccosh}(cx))}{x}-\frac{d^2(a+b\text{arccosh}(cx))}{3x^3}$$

$$\downarrow 299$$

$$\frac{bd^2\sqrt{c^2x^2-1}\left(\frac{c^4\sqrt{c^2x^2-1}}{2(x^4+1)}-\frac{1}{2}c^4\left(11\int\frac{1}{x^4+1}d\sqrt{c^2x^2-1}+6\sqrt{c^2x^2-1}\right)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}+c^4d^2x(a+b\text{arccosh}(cx))+\frac{2c^2d^2(a+b\text{arccosh}(cx))}{x}-\frac{d^2(a+b\text{arccosh}(cx))}{3x^3}$$

$$\downarrow 216$$

$$c^4d^2x(a+b\text{arccosh}(cx))+\frac{2c^2d^2(a+b\text{arccosh}(cx))}{x}-\frac{d^2(a+b\text{arccosh}(cx))}{3x^3}+\frac{bd^2\sqrt{c^2x^2-1}\left(\frac{c^4\sqrt{c^2x^2-1}}{2(x^4+1)}-\frac{1}{2}c^4\left(11\arctan\left(\sqrt{c^2x^2-1}\right)+6\sqrt{c^2x^2-1}\right)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcCosh[c*x]))/x^3 + (2*c^2*d^2*(a + b*ArcCosh[c*x]))/x +
 c^4*d^2*x*(a + b*ArcCosh[c*x]) + (b*d^2*Sqrt[-1 + c^2*x^2]*((c^4*Sqrt[-1
+ c^2*x^2])/(2*(1 + x^4)) - (c^4*(6*Sqrt[-1 + c^2*x^2] + 11*ArcTan[Sqrt[-1
 + c^2*x^2]]))/2))/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

method	result
parts	$d^2a\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)+d^2b\,c^3\left(cx\operatorname{arccosh}(cx)+\frac{2\operatorname{arccosh}(cx)}{cx}-\frac{\operatorname{arccosh}(cx)}{3c^3x^3}+\frac{\sqrt{cx-1}\,\sqrt{cx-}}{}\right.$
derivativedivides	$c^3\left(d^2a\left(cx+\frac{2}{cx}-\frac{1}{3c^3x^3}\right)+d^2b\left(cx\operatorname{arccosh}(cx)+\frac{2\operatorname{arccosh}(cx)}{cx}-\frac{\operatorname{arccosh}(cx)}{3c^3x^3}+\frac{\sqrt{cx-1}\,\sqrt{cx}}{}\right.\right.$
default	$c^3\left(d^2a\left(cx+\frac{2}{cx}-\frac{1}{3c^3x^3}\right)+d^2b\left(cx\operatorname{arccosh}(cx)+\frac{2\operatorname{arccosh}(cx)}{cx}-\frac{\operatorname{arccosh}(cx)}{3c^3x^3}+\frac{\sqrt{cx-1}\,\sqrt{cx}}{}\right.\right.$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(c^4*x+2*c^2/x-1/3/x^3)+d^2*b*c^3*(c*x*arccosh(c*x)+2*arccosh(c*x)/c
/x-1/3*arccosh(c*x)/c^3/x^3+1/6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(11*arctan(1/(
c^2*x^2-1)^(1/2))*c^2*x^2-6*c^2*x^2*(c^2*x^2-1)^(1/2)+(c^2*x^2-1)^(1/2))/c
^2/x^2/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.50

$$\int\frac{\left(d-c^2dx^2\right)^2(a+b\operatorname{arccosh}(cx))}{x^4}\,dx$$
$$=\frac{6\,ac^4d^2x^4-22\,bc^3d^2x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+12\,ac^2d^2x^2-2\left(3\,bc^4+6\,bc^2-b\right)d^2x^3\log\left(-cx+\sqrt{}\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a*c^4*d^2*x^4 - 22*b*c^3*d^2*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
 12*a*c^2*d^2*x^2 - 2*(3*b*c^4 + 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*
x^2 - 1)) - 2*a*d^2 + 2*(3*b*c^4*d^2*x^4 + 6*b*c^2*d^2*x^2 - (3*b*c^4 + 6*
b*c^2 - b)*d^2*x^3 - b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^3*d^2*x^
3 - b*c*d^2*x)*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))}{x^4}\,dx = d^2\left(\int ac^4\,dx+\int \frac{a}{x^4}\,dx+\int \left(-\frac{2ac^2}{x^2}\right)dx + \int bc^4\operatorname{acosh}(cx)\,dx+\int \frac{b\operatorname{acosh}(cx)}{x^4}\,dx + \int \left(-\frac{2bc^2\operatorname{acosh}(cx)}{x^2}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
```

output
```
d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(-2*a*c**2/x**2,
 x) + Integral(b*c**4*acosh(c*x), x) + Integral(b*acosh(c*x)/x**4, x) + In
tegral(-2*b*c**2*acosh(c*x)/x**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))}{x^4}\,dx &= ac^4d^2x+\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bc^3d^2 \\ &\quad +2\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)bc^2d^2 \\ &\quad -\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\operatorname{arcosh}(cx)}{x^3}\right)bd^2+\frac{2\,ac^2d^2}{x}-\frac{ad^2}{3\,x^3}\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

output
```
a*c^4*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c^3*d^2 + 2*(c*arcs
in(1/(c*abs(x))) + arccosh(c*x)/x)*b*c^2*d^2 - 1/6*((c^2*arcsin(1/(c*abs(x
))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d^2 + 2*a*c^2*d^2/x
 - 1/3*a*d^2/x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))}{x^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2\,(a+b\text{arccosh}(cx))}{x^4}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{(d-c^2dx^2)^2 (a+b\operatorname{arccosh}(cx))}{x^4} \, dx = \frac{d^2 \left(6acosh(cx)\, b\, c^4x^4+12acosh(cx)\, b\, c^2x^2-2acosh(cx)\, b+22atan\left(\sqrt{c^2x^2-1}+cx\right) b\, c^3x^3-\sqrt{c^2x^2-1}\right.}{6x^3}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acosh(c*x))/x^4,x)
```

output
```
(d**2*(6*acosh(c*x)*b*c**4*x**4 + 12*acosh(c*x)*b*c**2*x**2 - 2*acosh(c*x)
*b + 22*atan(sqrt(c**2*x**2 - 1) + c*x)*b*c**3*x**3 - sqrt(c**2*x**2 - 1)*
b*c*x - 6*sqrt(c*x + 1)*sqrt(c*x - 1)*b*c**3*x**3 + 6*a*c**4*x**4 + 12*a*c
**2*x**2 - 2*a))/(6*x**3)
```

3.19 $\int x^4(d-c^2dx^2)^3(a+b\,\mathrm{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 256

$$\begin{aligned}\int x^4(d-c^2dx^2)^3(a+b\,\mathrm{arccosh}(cx))\,dx &= -\frac{16bd^3\sqrt{-1+cx}\sqrt{1+cx}}{1155c^5}+\frac{8bd^3(-1+cx)^{3/2}(1+cx)^{3/2}}{3465c^5}\\ &\quad-\frac{2bd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{1925c^5}+\frac{bd^3(-1+cx)^{7/2}(1+cx)^{7/2}}{1617c^5}\\ &\quad+\frac{4bd^3(-1+cx)^{9/2}(1+cx)^{9/2}}{297c^5}+\frac{bd^3(-1+cx)^{11/2}(1+cx)^{11/2}}{121c^5}\\ &\quad+\frac{1}{5}d^3x^5(a+b\,\mathrm{arccosh}(cx))-\frac{3}{7}c^2d^3x^7(a+b\,\mathrm{arccosh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\,\mathrm{arccosh}(cx))-\frac{1}{11}c^6d^3x^{11}(a+\mathrm{barcc}\end{aligned}$$

output
```
-16/1155*b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5+8/3465*b*d^3*(c*x-1)^(3/2)*
(c*x+1)^(3/2)/c^5-2/1925*b*d^3*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^5+1/1617*b*d^
3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^5+4/297*b*d^3*(c*x-1)^(9/2)*(c*x+1)^(9/2)/
c^5+1/121*b*d^3*(c*x-1)^(11/2)*(c*x+1)^(11/2)/c^5+1/5*d^3*x^5*(a+b*arccosh
(c*x))-3/7*c^2*d^3*x^7*(a+b*arccosh(c*x))+1/3*c^4*d^3*x^9*(a+b*arccosh(c*x
))-1/11*c^6*d^3*x^11*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.57

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{d^3\left(3465ac^5x^5(-231+495c^2x^2-385c^4x^4+105c^6x^6)+b\sqrt{-1+cx}\sqrt{1+cx}(50488+25244c^2x^2+18\cdots\right.}{}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/4002075*(d^3*(3465*a*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^
6*x^6) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(50488 + 25244*c^2*x^2 + 18933*c^4
*x^4 - 117625*c^6*x^6 + 111475*c^8*x^8 - 33075*c^10*x^10) + 3465*b*c^5*x^5
*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6)*ArcCosh[c*x]))/c^5
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6336, 27, 2113, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6336}$$

$$-bc\int \frac{d^3x^5\left(-105c^6x^6+385c^4x^4-495c^2x^2+231\right)}{1155\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{11}c^6d^3x^{11}(a+b\operatorname{arccosh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\operatorname{arccosh}(cx))-\frac{3}{7}c^2d^3x^7(a+b\operatorname{arccosh}(cx))+\frac{1}{5}d^3x^5(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcd^3\int \frac{x^5\left(-105c^6x^6+385c^4x^4-495c^2x^2+231\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{1155}-\frac{1}{11}c^6d^3x^{11}(a+b\operatorname{arccosh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\operatorname{arccosh}(cx))-\frac{3}{7}c^2d^3x^7(a+b\operatorname{arccosh}(cx))+\frac{1}{5}d^3x^5(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \text{2113}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{x^5\left(-105c^6x^6+385c^4x^4-495c^2x^2+231\right)}{\sqrt{c^2x^2-1}}dx}{1155\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{11}c^6d^3x^{11}(a+\text{barccosh}(cx))+\frac{1}{3}c^4d^3x^9(a+\text{barccosh}(cx))-\frac{3}{7}c^2d^3x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^3x^5(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{x^4\left(-105c^6x^6+385c^4x^4-495c^2x^2+231\right)}{\sqrt{c^2x^2-1}}dx^2}{2310\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{11}c^6d^3x^{11}(a+\text{barccosh}(cx))+\frac{1}{3}c^4d^3x^9(a+\text{barccosh}(cx))-\frac{3}{7}c^2d^3x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^3x^5(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{2123}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int \left(-\frac{105\left(c^2x^2-1\right)^{9/2}}{c^4}-\frac{140\left(c^2x^2-1\right)^{7/2}}{c^4}-\frac{5\left(c^2x^2-1\right)^{5/2}}{c^4}+\frac{6\left(c^2x^2-1\right)^{3/2}}{c^4}-\frac{8\sqrt{c^2x^2-1}}{c^4}+\frac{16}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{2310\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{11}c^6d^3x^{11}(a+\text{barccosh}(cx))+\frac{1}{3}c^4d^3x^9(a+\text{barccosh}(cx))-\frac{3}{7}c^2d^3x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^3x^5(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{11}c^6d^3x^{11}(a+\text{barccosh}(cx))+\frac{1}{3}c^4d^3x^9(a+\text{barccosh}(cx))-\frac{3}{7}c^2d^3x^7(a+\text{barccosh}(cx))+\frac{1}{5}d^3x^5(a+\text{barccosh}(cx))-\frac{bcd^3\sqrt{c^2x^2-1}\left(-\frac{210\left(c^2x^2-1\right)^{11/2}}{11c^6}-\frac{280\left(c^2x^2-1\right)^{9/2}}{9c^6}-\frac{10\left(c^2x^2-1\right)^{7/2}}{7c^6}+\frac{12\left(c^2x^2-1\right)^{5/2}}{5c^6}-\frac{16\left(c^2x^2-1\right)^{3/2}}{3c^6}+\frac{32\sqrt{c^2x^2-1}}{c^6}\right)}{2310\sqrt{cx-1}\sqrt{cx+1}}$$

input

```
Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output

```
-1/2310*(b*c*d^3*Sqrt[-1 + c^2*x^2]*((32*Sqrt[-1 + c^2*x^2])/c^6 - (16*(-1
 + c^2*x^2)^(3/2))/(3*c^6) + (12*(-1 + c^2*x^2)^(5/2))/(5*c^6) - (10*(-1 +
 c^2*x^2)^(7/2))/(7*c^6) - (280*(-1 + c^2*x^2)^(9/2))/(9*c^6) - (210*(-1 +
 c^2*x^2)^(11/2))/(11*c^6)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^5*(a
+ b*ArcCosh[c*x]))/5 - (3*c^2*d^3*x^7*(a + b*ArcCosh[c*x]))/7 + (c^4*d^3*x
^9*(a + b*ArcCosh[c*x]))/3 - (c^6*d^3*x^11*(a + b*ArcCosh[c*x]))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.60

method	result
parts	$-d^3a\left(\frac{1}{11}c^6x^{11}-\frac{1}{3}c^4x^9+\frac{3}{7}c^2x^7-\frac{1}{5}x^5\right)-\frac{d^3b\left(\frac{\operatorname{arccosh}(cx)c^{11}x^{11}}{11}-\frac{\operatorname{arccosh}(cx)c^9x^9}{3}+\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}\right.}{}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^{11}x^{11}}{11}-\frac{\operatorname{arccosh}(cx)c^9x^9}{3}+\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)c^5}{5}\right.}{c^5}$
default	$\frac{-d^3a\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^{11}x^{11}}{11}-\frac{\operatorname{arccosh}(cx)c^9x^9}{3}+\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)c^5}{5}\right.}{c^5}$
orering	$\frac{(694575c^{12}x^{12}-2581075c^{10}x^{10}+3337325c^8x^8-1460245c^6x^6-176708c^4x^4-403904c^2x^2+201952)(-c^2dx^2+d)^3(a+}{4002075c^6(cx-1)^2(cx+1)^2x(c^2x^2-1)}$

input
```
int(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/11*c^6*x^11-1/3*c^4*x^9+3/7*c^2*x^7-1/5*x^5)-d^3*b/c^5*(1/11*arc
cosh(c*x)*c^11*x^11-1/3*arccosh(c*x)*c^9*x^9+3/7*arccosh(c*x)*c^7*x^7-1/5*
arccosh(c*x)*c^5*x^5-1/4002075*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(33075*c^10*x^1
0-111475*c^8*x^8+117625*c^6*x^6-18933*c^4*x^4-25244*c^2*x^2-50488))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.79

$$\int x^4\left(d-c^2dx^2\right)^3(a+b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{363825\,ac^{11}d^3x^{11}-1334025\,ac^9d^3x^9+1715175\,ac^7d^3x^7-800415\,ac^5d^3x^5+3465\,(105\,bc^{11}d^3x^{11}-38}{}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/4002075*(363825*a*c^11*d^3*x^11 - 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7
*d^3*x^7 - 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 - 385*b*c^9*d^
3*x^9 + 495*b*c^7*d^3*x^7 - 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 - 1)
) - (33075*b*c^10*d^3*x^10 - 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 -
 18933*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 - 50488*b*d^3)*sqrt(c^2*x^2 - 1
))/c^5
```

Sympy [F(-1)]

Timed out.

$$\int x^4 (d - c^2dx^2)^3 (a + b\text{arccosh}(cx))\, dx = \text{Timed out}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(212) = 424.

Time = 0.04 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.82

$$\int x^4 (d - c^2dx^2)^3 (a + b\text{arccosh}(cx))\, dx = -\frac{1}{11}\, ac^6d^3x^{11} + \frac{1}{3}\, ac^4d^3x^9 - \frac{3}{7}\, ac^2d^3x^7$$
$$-\frac{1}{7623}\left(693\, x^{11}\, \text{arcosh}\,(cx) - \left(\frac{63\,\sqrt{c^2x^2 - 1}x^{10}}{c^2} + \frac{70\,\sqrt{c^2x^2 - 1}x^8}{c^4} + \frac{80\,\sqrt{c^2x^2 - 1}x^6}{c^6} + \frac{96\,\sqrt{c^2x^2 - 1}}{c^8}\right.\right.$$
$$+\frac{1}{945}\left(315\, x^9\, \text{arcosh}\,(cx) - \left(\frac{35\,\sqrt{c^2x^2 - 1}x^8}{c^2} + \frac{40\,\sqrt{c^2x^2 - 1}x^6}{c^4} + \frac{48\,\sqrt{c^2x^2 - 1}x^4}{c^6} + \frac{64\,\sqrt{c^2x^2 - 1}x^2}{c^8}\right.\right.$$
$$+\frac{1}{5}\, ad^3x^5$$
$$-\frac{3}{245}\left(35\, x^7\, \text{arcosh}\,(cx) - \left(\frac{5\,\sqrt{c^2x^2 - 1}x^6}{c^2} + \frac{6\,\sqrt{c^2x^2 - 1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2 - 1}x^2}{c^6} + \frac{16\,\sqrt{c^2x^2 - 1}}{c^8}\right)c\right)bc$$
$$+\frac{1}{75}\left(15\, x^5\, \text{arcosh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2 - 1}x^4}{c^2} + \frac{4\,\sqrt{c^2x^2 - 1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2 - 1}}{c^6}\right)c\right)bd^3$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 - 1/7623*(693
*x^11*arccosh(c*x) - (63*sqrt(c^2*x^2 - 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)
*x^8/c^4 + 80*sqrt(c^2*x^2 - 1)*x^6/c^6 + 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 1
28*sqrt(c^2*x^2 - 1)*x^2/c^10 + 256*sqrt(c^2*x^2 - 1)/c^12)*c)*b*c^6*d^3 +
 1/945*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2
*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^
2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 - 3/245*(
35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x
^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*
d^3 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^
2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3 \,d x$$

input
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.80

$$\int x^4 (d - c^2 d x^2)^3 (a + b\text{arccosh}(cx))\, dx$$

$$= \frac{d^3 (-363825 acosh(cx)\, b\, c^{11} x^{11} + 1334025 acosh(cx)\, b\, c^9 x^9 - 1715175 acosh(cx)\, b\, c^7 x^7 + 800415 acosh(cx)$$

input
```
int(x^4*(-c^2*d*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(d**3*( - 363825*acosh(c*x)*b*c**11*x**11 + 1334025*acosh(c*x)*b*c**9*x**9
 - 1715175*acosh(c*x)*b*c**7*x**7 + 800415*acosh(c*x)*b*c**5*x**5 + 33075*
sqrt(c**2*x**2 - 1)*b*c**10*x**10 - 111475*sqrt(c**2*x**2 - 1)*b*c**8*x**8
 + 117625*sqrt(c**2*x**2 - 1)*b*c**6*x**6 - 18933*sqrt(c**2*x**2 - 1)*b*c*
*4*x**4 - 25244*sqrt(c**2*x**2 - 1)*b*c**2*x**2 - 50488*sqrt(c**2*x**2 - 1
)*b - 363825*a*c**11*x**11 + 1334025*a*c**9*x**9 - 1715175*a*c**7*x**7 + 8
00415*a*c**5*x**5))/(4002075*c**5)
```

3.20 $\int x^3(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 226

$$\int x^3(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx = -\frac{49bd^3x\sqrt{-1+cx}\sqrt{1+cx}}{5120c^3} + \frac{49bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{7680c^3} - \frac{49bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{9600c^3} + \frac{7bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{1600c^3} + \frac{bd^3x(-1+cx)^{9/2}(1+cx)^{9/2}}{100c^3} + \frac{49bd^3\text{arccosh}(cx)}{5120c^4} - \frac{d^3(1-c^2x^2)^4(a+b\text{arccosh}(cx))}{8c^4} + \frac{d^3(1-c^2x^2)^5(a+b\text{arccosh}(cx))}{10c^4}$$

output
```
-49/5120*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3+49/7680*b*d^3*x*(c*x-1)^(
3/2)*(c*x+1)^(3/2)/c^3-49/9600*b*d^3*x*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^3+7/1
600*b*d^3*x*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/100*b*d^3*x*(c*x-1)^(9/2)*(c
*x+1)^(9/2)/c^3+49/5120*b*d^3*arccosh(c*x)/c^4-1/8*d^3*(-c^2*x^2+1)^4*(a+b
*arccosh(c*x))/c^4+1/10*d^3*(-c^2*x^2+1)^5*(a+b*arccosh(c*x))/c^4
```

Mathematica [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72

$$\int x^3(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx =$$
$$-\frac{d^3\Big(1920ac^4x^4(-10+20c^2x^2-15c^4x^4+4c^6x^6)+bcx\sqrt{-1+cx}\sqrt{1+cx}(1185+790c^2x^2-3208c^4x^4$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/76800*(d^3*(1920*a*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6)
+ b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1185 + 790*c^2*x^2 - 3208*c^4*x^4 +
2736*c^6*x^6 - 768*c^8*x^8) + 1920*b*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^
4 + 4*c^6*x^6)*ArcCosh[c*x] + 2370*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))
/c^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6336, 27, 2003, 35, 646, 40, 40, 40, 40, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int -\frac{d^3\left(1-c^2x^2\right)^4\left(4c^2x^2+1\right)}{40c^4\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{d^3\left(1-c^2x^2\right)^5(a+b\text{arccosh}(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\text{arccosh}(cx))}{8c^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bd^3\int \frac{\left(1-c^2x^2\right)^4\left(4c^2x^2+1\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\text{arccosh}(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\text{arccosh}(cx))}{8c^4}$$

$$\Big\downarrow \text{ 2003}$$

$$\frac{bd^3\int \frac{(-cx-1)^4(cx-1)^{7/2}\left(4c^2x^2+1\right)}{\sqrt{cx+1}}dx}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\text{arccosh}(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\text{arccosh}(cx))}{8c^4}$$

$$\Big\downarrow \text{ 35}$$

$$\frac{bd^3\int (cx-1)^{7/2}(cx+1)^{7/2}\left(4c^2x^2+1\right)dx}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\text{arccosh}(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\text{arccosh}(cx))}{8c^4}$$

$$\Big\downarrow \text{ 646}$$

$$\frac{bd^3\left(\frac{7}{5}\int (cx-1)^{7/2}(cx+1)^{7/2}dx+\frac{2}{5}x(cx-1)^{9/2}(cx+1)^{9/2}\right)}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\text{arccosh}(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\text{arccosh}(cx))}{8c^4}$$

$$\Big\downarrow \text{ 40}$$

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\int (cx-1)^{5/2}(cx+1)^{5/2}dx\right)+\frac{2}{5}x(cx-1)^{9/2}(cx+1)^{9/2}\right)}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\text{arccosh}(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\text{arccosh}(cx))}{8c^4}$$

$$\Big\downarrow \text{ 40}$$

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\int (cx-1)^{3/2}(cx+1)^{3/2}dx\right)\right)+\frac{2}{5}x(cx-1)^{9/2}\right.}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\text{arccosh}(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\text{arccosh}(cx))}{8c^4}$$

$$\Big\downarrow \text{ 40}$$

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int\sqrt{cx-1}\sqrt{cx}\right.\right.\right.\right.}{40c^3}$$
$$\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\text{arccosh}(cx)\right)}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\text{arccosh}(cx)\right)}{8c^4}$$

$\downarrow$ 40

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\right.\right.\right.\right.\right.}{40c^3}$$
$$\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\text{arccosh}(cx)\right)}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\text{arccosh}(cx)\right)}{8c^4}$$

$\downarrow$ 43

$$\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\text{arccosh}(cx)\right)}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\text{arccosh}(cx)\right)}{8c^4}+$$
$$\frac{bd^3\left(\frac{7}{5}\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\right.\right.\right.\right.\right.}{40c^3}$$

input
```
Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcCosh[c*x]))/c^4 + (d^3*(1 - c^2*x^2)^5
*(a + b*ArcCosh[c*x]))/(10*c^4) + (b*d^3*((2*x*(-1 + c*x)^(9/2)*(1 + c*x)^
(9/2))/5 + (7*((x*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/8 - (7*((x*(-1 + c*x)^
(5/2)*(1 + c*x)^(5/2))/6 - (5*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 - (3
*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/6))/8))/5)
)/(40*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

method	result
parts	$-d^3a\left(\frac{1}{10}c^6x^{10}-\frac{3}{8}c^4x^8+\frac{1}{2}c^2x^6-\frac{1}{4}x^4\right)-\frac{d^3b\left(\frac{\operatorname{arccosh}(cx)c^{10}x^{10}}{10}-\frac{3\operatorname{arccosh}(cx)c^8x^8}{8}+\frac{\operatorname{arccosh}(cx)c^6x^6}{2}-\right.}{}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{10}c^{10}x^{10}-\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6-\frac{1}{4}c^4x^4\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^{10}x^{10}}{10}-\frac{3\operatorname{arccosh}(cx)c^8x^8}{8}+\frac{\operatorname{arccosh}(cx)c^6x^6}{2}-\frac{\operatorname{arccosh}(cx)c^4}{4}\right.}{}$
default	$\frac{-d^3a\left(\frac{1}{10}c^{10}x^{10}-\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6-\frac{1}{4}c^4x^4\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^{10}x^{10}}{10}-\frac{3\operatorname{arccosh}(cx)c^8x^8}{8}+\frac{\operatorname{arccosh}(cx)c^6x^6}{2}-\frac{\operatorname{arccosh}(cx)c^4}{4}\right.}{}$
orering	$\frac{(4864c^{10}x^{10}-18576c^8x^8+25160c^6x^6-11978c^4x^4-2765c^2x^2+1580)\left(-c^2d\,x^2+d\right)^3(a+b\operatorname{arccosh}(cx))}{25600c^4(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{(768c^8x^8-}{}$

input
```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/10*c^6*x^10-3/8*c^4*x^8+1/2*c^2*x^6-1/4*x^4)-d^3*b/c^4*(1/10*arc
cosh(c*x)*c^10*x^10-3/8*arccosh(c*x)*c^8*x^8+1/2*arccosh(c*x)*c^6*x^6-1/4*
arccosh(c*x)*c^4*x^4+1/76800*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-768*c^9*x^9*(c^
2*x^2-1)^(1/2)+2736*c^7*x^7*(c^2*x^2-1)^(1/2)-3208*c^5*x^5*(c^2*x^2-1)^(1/
2)+790*(c^2*x^2-1)^(1/2)*c^3*x^3+1185*c*x*(c^2*x^2-1)^(1/2)+1185*ln(c*x+(c
^2*x^2-1)^(1/2)))/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.87

$$\int x^3\left(d-c^2dx^2\right)^3(a+b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{7680\,ac^{10}d^3x^{10}-28800\,ac^8d^3x^8+38400\,ac^6d^3x^6-19200\,ac^4d^3x^4+15\,(512\,bc^{10}d^3x^{10}-1920\,bc^8d^3x^8}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/76800*(7680*a*c^10*d^3*x^10 - 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6
 - 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 - 1920*b*c^8*d^3*x^8 + 25
60*b*c^6*d^3*x^6 - 1280*b*c^4*d^3*x^4 + 79*b*d^3)*log(c*x + sqrt(c^2*x^2 -
 1)) - (768*b*c^9*d^3*x^9 - 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 - 790*
b*c^3*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (d - c^2dx^2)^3 (a + b\operatorname{arccosh}(cx))\, dx = \text{Timed out}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(188) = 376$.

Time = 0.04 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.22

$$\int x^3 (d - c^2dx^2)^3 (a + b\operatorname{arccosh}(cx))\, dx = -\frac{1}{10}ac^6d^3x^{10} + \frac{3}{8}ac^4d^3x^8 - \frac{1}{2}ac^2d^3x^6$$
$$-\frac{1}{12800}\left(1280\,x^{10}\operatorname{arcosh}(cx) - \left(\frac{128\sqrt{c^2x^2-1}x^9}{c^2} + \frac{144\sqrt{c^2x^2-1}x^7}{c^4} + \frac{168\sqrt{c^2x^2-1}x^5}{c^6} + \frac{210\sqrt{c^2}}{c}\right.\right.$$
$$+\frac{1}{1024}\left(384\,x^8\operatorname{arcosh}(cx) - \left(\frac{48\sqrt{c^2x^2-1}x^7}{c^2} + \frac{56\sqrt{c^2x^2-1}x^5}{c^4} + \frac{70\sqrt{c^2x^2-1}x^3}{c^6} + \frac{105\sqrt{c^2x^2-1}}{c^8}\right.\right.$$
$$+\frac{1}{4}ad^3x^4$$
$$-\frac{1}{96}\left(48\,x^6\operatorname{arcosh}(cx) - \left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2} + \frac{10\sqrt{c^2x^2-1}x^3}{c^4} + \frac{15\sqrt{c^2x^2-1}x}{c^6} + \frac{15\log\left(2c^2x+2\sqrt{c}\right.}{c^7}\right.\right.$$
$$+\frac{1}{32}\left(8\,x^4\operatorname{arcosh}(cx) - \left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2} + \frac{3\sqrt{c^2x^2-1}x}{c^4} + \frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bd^3$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 - 1/12800*(12
80*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2 + 144*sqrt(c^2*x^2 -
 1)*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqrt(c^2*x^2 - 1)*x^3/c^
8 + 315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c
)/c^11)*c)*b*c^6*d^3 + 1/1024*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1
)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 +
105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9
)*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 - 1/96*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x
^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^
6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x^
4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4
+ 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d - c^2 dx^2)^3 (a + b\text{arccosh}(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d-c^2dx^2)^3 (a+b\text{arccosh}(cx))\,dx = \int x^3\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.91

$$\int x^3 (d-c^2dx^2)^3 (a+b\text{arccosh}(cx))\,dx$$
$$= \frac{d^3\left(-7680acosh(cx)\,b\,c^{10}x^{10}+28800acosh(cx)\,b\,c^8x^8-38400acosh(cx)\,b\,c^6x^6+19200acosh(cx)\,b\,c^4x^4+\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(d**3*( - 7680*acosh(c*x)*b*c**10*x**10 + 28800*acosh(c*x)*b*c**8*x**8 - 3
8400*acosh(c*x)*b*c**6*x**6 + 19200*acosh(c*x)*b*c**4*x**4 + 768*sqrt(c**2
*x**2 - 1)*b*c**9*x**9 - 2736*sqrt(c**2*x**2 - 1)*b*c**7*x**7 + 3208*sqrt(
c**2*x**2 - 1)*b*c**5*x**5 - 790*sqrt(c**2*x**2 - 1)*b*c**3*x**3 - 1185*sq
rt(c**2*x**2 - 1)*b*c*x - 1185*log(sqrt(c**2*x**2 - 1) + c*x)*b - 7680*a*c
**10*x**10 + 28800*a*c**8*x**8 - 38400*a*c**6*x**6 + 19200*a*c**4*x**4))/(
76800*c**4)
```

3.21 $\int x^2(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 227

$$\int x^2(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx = -\frac{16bd^3\sqrt{-1+cx}\sqrt{1+cx}}{315c^3} + \frac{8bd^3(-1+cx)^{3/2}(1+cx)^{3/2}}{945c^3} - \frac{2bd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{525c^3} + \frac{bd^3(-1+cx)^{7/2}(1+cx)^{7/2}}{441c^3} + \frac{bd^3(-1+cx)^{9/2}(1+cx)^{9/2}}{81c^3} + \frac{1}{3}d^3x^3(a+b\text{arccosh}(cx)) - \frac{3}{5}c^2d^3x^5(a+b\text{arccosh}(cx)) + \frac{3}{7}c^4d^3x^7(a+b\text{arccosh}(cx)) - \frac{1}{9}c^6d^3x^9(a+b\text{arccos}$$

output
```
-16/315*b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3+8/945*b*d^3*(c*x-1)^(3/2)*(c
*x+1)^(3/2)/c^3-2/525*b*d^3*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^3+1/441*b*d^3*(c
*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/81*b*d^3*(c*x-1)^(9/2)*(c*x+1)^(9/2)/c^3+1
/3*d^3*x^3*(a+b*arccosh(c*x))-3/5*c^2*d^3*x^5*(a+b*arccosh(c*x))+3/7*c^4*d
^3*x^7*(a+b*arccosh(c*x))-1/9*c^6*d^3*x^9*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.61

$$\int x^2\left(d-c^2dx^2\right)^3(a+b\operatorname{arccosh}(cx))\,dx = -\frac{d^3\left(315ac^3x^3(-105+189c^2x^2-135c^4x^4+35c^6x^6)+b\sqrt{-1+cx}\sqrt{1+cx}(5258+2629c^2x^2-6297c^4x\right.}{99225c^3}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/99225*(d^3*(315*a*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^
6) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5258 + 2629*c^2*x^2 - 6297*c^4*x^4 +
4675*c^6*x^6 - 1225*c^8*x^8) + 315*b*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4
*x^4 + 35*c^6*x^6)*ArcCosh[c*x]))/c^3
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6336, 27, 2113, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d-c^2dx^2\right)^3(a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \ 6336$$

$$-bc\int \frac{d^3x^3\left(-35c^6x^6+135c^4x^4-189c^2x^2+105\right)}{315\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{9}c^6d^3x^9(a+b\operatorname{arccosh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\operatorname{arccosh}(cx))-\frac{3}{5}c^2d^3x^5(a+b\operatorname{arccosh}(cx))+\frac{1}{3}d^3x^3(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{315}bcd^3\int \frac{x^3\left(-35c^6x^6+135c^4x^4-189c^2x^2+105\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{9}c^6d^3x^9(a+b\operatorname{arccosh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\operatorname{arccosh}(cx))-\frac{3}{5}c^2d^3x^5(a+b\operatorname{arccosh}(cx))+\frac{1}{3}d^3x^3(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 2113$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{x^3\left(-35c^6x^6+135c^4x^4-189c^2x^2+105\right)}{\sqrt{c^2x^2-1}}dx}{315\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{9}c^6d^3x^9(a+b\text{arccosh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arccosh}(cx))-\frac{3}{5}c^2d^3x^5(a+b\text{arccosh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 2331$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{x^2\left(-35c^6x^6+135c^4x^4-189c^2x^2+105\right)}{\sqrt{c^2x^2-1}}dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{9}c^6d^3x^9(a+b\text{arccosh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arccosh}(cx))-\frac{3}{5}c^2d^3x^5(a+b\text{arccosh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 2123$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int \left(-\frac{35\left(c^2x^2-1\right)^{7/2}}{c^2}-\frac{5\left(c^2x^2-1\right)^{5/2}}{c^2}+\frac{6\left(c^2x^2-1\right)^{3/2}}{c^2}-\frac{8\sqrt{c^2x^2-1}}{c^2}+\frac{16}{c^2\sqrt{c^2x^2-1}}\right)dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{9}c^6d^3x^9(a+b\text{arccosh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arccosh}(cx))-\frac{3}{5}c^2d^3x^5(a+b\text{arccosh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{9}c^6d^3x^9(a+b\text{arccosh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arccosh}(cx))-\frac{3}{5}c^2d^3x^5(a+b\text{arccosh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arccosh}(cx))-\frac{bcd^3\sqrt{c^2x^2-1}\left(-\frac{70\left(c^2x^2-1\right)^{9/2}}{9c^4}-\frac{10\left(c^2x^2-1\right)^{7/2}}{7c^4}+\frac{12\left(c^2x^2-1\right)^{5/2}}{5c^4}-\frac{16\left(c^2x^2-1\right)^{3/2}}{3c^4}+\frac{32\sqrt{c^2x^2-1}}{c^4}\right)}{630\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/630*(b*c*d^3*Sqrt[-1 + c^2*x^2]*((32*Sqrt[-1 + c^2*x^2])/c^4 - (16*(-1
+ c^2*x^2)^(3/2))/(3*c^4) + (12*(-1 + c^2*x^2)^(5/2))/(5*c^4) - (10*(-1 +
c^2*x^2)^(7/2))/(7*c^4) - (70*(-1 + c^2*x^2)^(9/2))/(9*c^4)))/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (3*c^2*d^3*x^5*(a
+ b*ArcCosh[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcCosh[c*x]))/7 - (c^6*d^3*x
^9*(a + b*ArcCosh[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.64

method	result
parts	$-d^3a\left(\frac{1}{9}c^6x^9-\frac{3}{7}c^4x^7+\frac{3}{5}c^2x^5-\frac{1}{3}x^3\right)-\frac{d^3b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3}{}\right.}{}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\right.}{c^3}$
default	$\frac{-d^3a\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\right.}{c^3}$
orering	$\frac{(20825c^{10}x^{10}-82375c^8x^8+119261c^6x^6-66701c^4x^4-36806c^2x^2+10516)(-c^2d\,x^2+d)^3(a+b\operatorname{arccosh}(cx))}{99225c^4(cx-1)^2(cx+1)^2x(c^2x^2-1)}-\frac{(1225c}{}$

input
```
int(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/9*c^6*x^9-3/7*c^4*x^7+3/5*c^2*x^5-1/3*x^3)-d^3*b/c^3*(1/9*arccos
h(c*x)*c^9*x^9-3/7*arccosh(c*x)*c^7*x^7+3/5*arccosh(c*x)*c^5*x^5-1/3*c^3*x
^3*arccosh(c*x)-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*x^8-4675*c^6
*x^6+6297*c^4*x^4-2629*c^2*x^2-5258))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int x^2\left(d-c^2dx^2\right)^3\left(a+b\operatorname{arccosh}(cx)\right)dx =$$
$$-\frac{11025\,ac^9d^3x^9-42525\,ac^7d^3x^7+59535\,ac^5d^3x^5-33075\,ac^3d^3x^3+315\,(35\,bc^9d^3x^9-135\,bc^7d^3x^7+1}{}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/99225*(11025*a*c^9*d^3*x^9 - 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5
- 33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 - 135*b*c^7*d^3*x^7 + 189*b*
c^5*d^3*x^5 - 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*b*c^
8*d^3*x^8 - 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 - 2629*b*c^2*d^3*x^2 -
 5258*b*d^3)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [F]

$$\int x^2(d-c^2dx^2)^3(a+b\operatorname{arccosh}(cx))\,dx = -d^3\left(\int(-ax^2)\,dx+\int 3ac^2x^4\,dx+\int(-3ac^4x^6)\,dx+\int ac^6x^8\,dx+\int(-bx^2\operatorname{acosh}(cx))\,dx+\int 3bc^2x^4\operatorname{acosh}(cx)\,dx+\int(-3bc^4x^6\operatorname{acosh}(cx))\,dx+\int bc^6x^8\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
-d**3*(Integral(-a*x**2, x) + Integral(3*a*c**2*x**4, x) + Integral(-3*a*c
**4*x**6, x) + Integral(a*c**6*x**8, x) + Integral(-b*x**2*acosh(c*x), x)
+ Integral(3*b*c**2*x**4*acosh(c*x), x) + Integral(-3*b*c**4*x**6*acosh(c*
x), x) + Integral(b*c**6*x**8*acosh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(189) = 378$.

Time = 0.04 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.71

$$\int x^2 (d - c^2dx^2)^3 (a + b\operatorname{arccosh}(cx))\, dx = -\frac{1}{9}\, ac^6d^3x^9 + \frac{3}{7}\, ac^4d^3x^7$$

$$-\frac{1}{2835}\left(315\, x^9 \operatorname{arcosh}(cx) - \left(\frac{35\sqrt{c^2x^2-1}x^8}{c^2} + \frac{40\sqrt{c^2x^2-1}x^6}{c^4} + \frac{48\sqrt{c^2x^2-1}x^4}{c^6} + \frac{64\sqrt{c^2x^2-1}x}{c^8}\right.\right.$$

$$-\frac{3}{5}\, ac^2d^3x^5$$

$$+\frac{3}{245}\left(35\, x^7 \operatorname{arcosh}(cx) - \left(\frac{5\sqrt{c^2x^2-1}x^6}{c^2} + \frac{6\sqrt{c^2x^2-1}x^4}{c^4} + \frac{8\sqrt{c^2x^2-1}x^2}{c^6} + \frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc$$

$$-\frac{1}{25}\left(15\, x^5 \operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^2d^3$$

$$+\frac{1}{3}\, ad^3x^3 + \frac{1}{9}\left(3\, x^3 \operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 1/2835*(315*x^9*arccosh(c*x) - (3
5*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x
^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^1
0)*c)*b*c^6*d^3 - 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arccosh(c*x) - (5*sqrt
(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*
x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^3 - 1/25*(15*x^5*arccosh(c*
x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c
^2*x^2 - 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arccosh(c*x) -
c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.82

$$\int x^2 \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \frac{d^3 \left(-11025 acosh(cx)\, b\, c^9 x^9 + 42525 acosh(cx)\, b\, c^7 x^7 - 59535 acosh(cx)\, b\, c^5 x^5 + 33075 acosh(cx)\, b\, c^3 x^3 + \right.}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(d**3*( - 11025*acosh(c*x)*b*c**9*x**9 + 42525*acosh(c*x)*b*c**7*x**7 - 59
535*acosh(c*x)*b*c**5*x**5 + 33075*acosh(c*x)*b*c**3*x**3 + 1225*sqrt(c**2
*x**2 - 1)*b*c**8*x**8 - 4675*sqrt(c**2*x**2 - 1)*b*c**6*x**6 + 6297*sqrt(
c**2*x**2 - 1)*b*c**4*x**4 - 2629*sqrt(c**2*x**2 - 1)*b*c**2*x**2 - 5258*s
qrt(c**2*x**2 - 1)*b - 11025*a*c**9*x**9 + 42525*a*c**7*x**7 - 59535*a*c**
5*x**5 + 33075*a*c**3*x**3))/(99225*c**3)
```

3.22 $\int x(d-c^2dx^2)^3 (a+b\text{arccosh}(cx)) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int x(d-c^2dx^2)^3 (a+b\text{arccosh}(cx)) \, dx = -\frac{35bd^3x\sqrt{-1+cx}\sqrt{1+cx}}{1024c} + \frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c} - \frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c} + \frac{bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{64c} + \frac{35bd^3\text{arccosh}(cx)}{1024c^2} - \frac{d^3(1-c^2x^2)^4 (a+b\text{arccosh}(cx))}{8c^2}$$

output
```
-35/1024*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+35/1536*b*d^3*x*(c*x-1)^(3/
2)*(c*x+1)^(3/2)/c-7/384*b*d^3*x*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c+1/64*b*d^3*
x*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c+35/1024*b*d^3*arccosh(c*x)/c^2-1/8*d^3*(-c
^2*x^2+1)^4*(a+b*arccosh(c*x))/c^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90

$$\int x(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx =$$

$$-\frac{d^3\left(cx\left(b\sqrt{-1+cx}\sqrt{1+cx}(279-326c^2x^2+200c^4x^4-48c^6x^6)+384acx(-4+6c^2x^2-4c^4x^4+c^6x^6\right.\right.}{3072c^2}$$

input
```
Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/3072*(d^3*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(279 - 326*c^2*x^2 + 200
*c^4*x^4 - 48*c^6*x^6) + 384*a*c*x*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6))
 + 384*b*c^2*x^2*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)*ArcCosh[c*x] + 558
*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6329, 40, 40, 40, 40, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6329}$$

$$\frac{bd^3\int(cx-1)^{7/2}(cx+1)^{7/2}dx}{8c}-\frac{d^3(1-c^2x^2)^4(a+b\text{arccosh}(cx))}{8c^2}$$

$$\Big\downarrow \text{40}$$

$$\frac{bd^3\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\int(cx-1)^{5/2}(cx+1)^{5/2}dx\right)}{8c}-\frac{d^3(1-c^2x^2)^4(a+b\text{arccosh}(cx))}{8c^2}$$

$\Big\downarrow$ 40

$$\frac{bd^3\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\int(cx-1)^{3/2}(cx+1)^{3/2}dx\right)\right)}{8c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\operatorname{arccosh}(cx))}{8c^2}$$

$\Big\downarrow$ 40

$$\frac{bd^3\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int\sqrt{cx-1}\sqrt{cx+\right.\right.\right.}{8c}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\operatorname{arccosh}(cx))}{8c^2}$$

$\Big\downarrow$ 40

$$\frac{bd^3\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx\right.\right.\right.\right.}{8c}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\operatorname{arccosh}(cx))}{8c^2}$$

$\Big\downarrow$ 43

$$\frac{bd^3\left(\frac{1}{8}x(cx-1)^{7/2}(cx+1)^{7/2}-\frac{7}{8}\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx\right.\right.\right.\right.}{8c}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\operatorname{arccosh}(cx))}{8c^2}$$

input
```
Int[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcCosh[c*x]))/c^2 + (b*d^3*((x*(-1 + c*x
)^(7/2)*(1 + c*x)^(7/2))/8 - (7*((x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/6 -
(5*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/6))/8))/(8*c)
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{d^3 a \left(c^2x^2-1\right)^4}{8}-d^3 b\left(\frac{\operatorname{arccosh}(cx)c^8x^8}{8}-\frac{\operatorname{arccosh}(cx)c^6x^6}{2}+\frac{3\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{c^2x^2\operatorname{arccosh}(cx)}{2}+\frac{\operatorname{arccosh}(cx)}{8}-\frac{\sqrt{cx-1}\,\sqrt{c\cdots}}{\cdots}\right.}{c^2}$
default	$\frac{-\frac{d^3 a \left(c^2x^2-1\right)^4}{8}-d^3 b\left(\frac{\operatorname{arccosh}(cx)c^8x^8}{8}-\frac{\operatorname{arccosh}(cx)c^6x^6}{2}+\frac{3\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{c^2x^2\operatorname{arccosh}(cx)}{2}+\frac{\operatorname{arccosh}(cx)}{8}-\frac{\sqrt{cx-1}\,\sqrt{c\cdots}}{\cdots}\right.}{c^2}$
parts	$-\frac{d^3 a(c^2x^2-1)^4}{8c^2}-\frac{d^3 b\left(\frac{\operatorname{arccosh}(cx)c^8x^8}{8}-\frac{\operatorname{arccosh}(cx)c^6x^6}{2}+\frac{3\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{c^2x^2\operatorname{arccosh}(cx)}{2}+\frac{\operatorname{arccosh}(cx)}{8}-\frac{\sqrt{cx-1}\cdots}{\cdots}\right.}{\cdots}$
orering	$\frac{(720c^8x^8-2984c^6x^6+4786c^4x^4-3815c^2x^2+558)\left(-c^2d\,x^2+d\right)^3(a+b\operatorname{arccosh}(cx))}{3072c^2(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{(48c^6x^6-200c^4x^4+326c^2x^2-\cdots}{\cdots}$

input
```
int(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/8*d^3*a*(c^2*x^2-1)^4-d^3*b*(1/8*arccosh(c*x)*c^8*x^8-1/2*arccos
h(c*x)*c^6*x^6+3/4*arccosh(c*x)*c^4*x^4-1/2*c^2*x^2*arccosh(c*x)+1/8*arcco
sh(c*x)-1/3072*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(48*c^7*x^7*(c^2*x^2-1)^(1/2)-2
00*c^5*x^5*(c^2*x^2-1)^(1/2)+326*(c^2*x^2-1)^(1/2)*c^3*x^3-279*c*x*(c^2*x^
2-1)^(1/2)+105*ln(c*x+(c^2*x^2-1)^(1/2)))/(c^2*x^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

$$\int x\left(d-c^2dx^2\right)^3(a+b\text{arccosh}(cx))\,dx =$$
$$-\frac{384\,ac^8d^3x^8-1536\,ac^6d^3x^6+2304\,ac^4d^3x^4-1536\,ac^2d^3x^2+3\,(128\,bc^8d^3x^8-512\,bc^6d^3x^6+768\,bc^4d}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/3072*(384*a*c^8*d^3*x^8 - 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 - 153
6*a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 - 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3
*x^4 - 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (48*b*
c^7*d^3*x^7 - 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 - 279*b*c*d^3*x)*sqrt(
c^2*x^2 - 1))/c^2
```

Sympy [F]

$$\int x\left(d-c^2dx^2\right)^3(a+b\text{arccosh}(cx))\,dx = -d^3\left(\int(-ax)\,dx+\int 3ac^2x^3\,dx+\int\left(-3ac^4x^5\right)dx+\int ac^6x^7\,dx+\int(-bx\,\text{acosh}(cx))\,dx+\int 3bc^2x^3\,\text{acosh}(cx)\,dx+\int\left(-3bc^4x^5\,\text{acosh}(cx)\right)dx+\int bc^6x^7\,\text{acosh}(cx)\,dx\right)$$

input
```
integrate(x*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
-d**3*(Integral(-a*x, x) + Integral(3*a*c**2*x**3, x) + Integral(-3*a*c**4
*x**5, x) + Integral(a*c**6*x**7, x) + Integral(-b*x*acosh(c*x), x) + Inte
gral(3*b*c**2*x**3*acosh(c*x), x) + Integral(-3*b*c**4*x**5*acosh(c*x), x)
 + Integral(b*c**6*x**7*acosh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(137) = 274.

Time = 0.04 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.55

$$\int x(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx = -\frac{1}{8}ac^6d^3x^8+\frac{1}{2}ac^4d^3x^6$$
$$-\frac{1}{3072}\left(384\,x^8\,\text{arcosh}\,(cx)-\left(\frac{48\sqrt{c^2x^2-1}x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}}{c^8}\right.\right.$$
$$-\frac{3}{4}ac^2d^3x^4$$
$$+\frac{1}{96}\left(48\,x^6\,\text{arcosh}\,(cx)-\left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}x}{c^6}+\frac{15\,\log\left(2\,c^2x+2\sqrt{c}\right.}{c^7}\right.\right.$$
$$-\frac{3}{32}\left(8\,x^4\,\text{arcosh}\,(cx)-\left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bc^2d^3$$
$$+\frac{1}{2}ad^3x^2$$
$$+\frac{1}{4}\left(2\,x^2\,\text{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\sqrt{c^2x^2-1}c\right)}{c^3}\right)\right)bd^3$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 1/3072*(384*x^8*arccosh(c*x) - (4
8*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x
^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c
^2*x^2 - 1)*c)/c^9)*c)*b*c^6*d^3 - 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arccos
h(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*
sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*
b*c^4*d^3 - 3/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sq
rt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*c
^2*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/
c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^3(a+b\text{arccosh}(cx))\,dx = \int x\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int x\left(d-c^2dx^2\right)^3\left(a+b\text{arccosh}(cx)\right)dx$$
$$=\frac{d^3\left(-384acosh(cx)\,b\,c^8x^8+1536acosh(cx)\,b\,c^6x^6-2304acosh(cx)\,b\,c^4x^4+1536acosh(cx)\,b\,c^2x^2+48\sqrt{c\right.}}{}$$

input
```
int(x*(-c^2*d*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(d**3*( - 384*acosh(c*x)*b*c**8*x**8 + 1536*acosh(c*x)*b*c**6*x**6 - 2304*
acosh(c*x)*b*c**4*x**4 + 1536*acosh(c*x)*b*c**2*x**2 + 48*sqrt(c**2*x**2 -
 1)*b*c**7*x**7 - 200*sqrt(c**2*x**2 - 1)*b*c**5*x**5 + 326*sqrt(c**2*x**2
 - 1)*b*c**3*x**3 - 279*sqrt(c**2*x**2 - 1)*b*c*x - 279*log(sqrt(c**2*x**2
 - 1) + c*x)*b - 384*a*c**8*x**8 + 1536*a*c**6*x**6 - 2304*a*c**4*x**4 + 1
536*a*c**2*x**2))/(3072*c**2)
```

3.23 $\int (d - c^2 dx^2)^3 (a + b\text{arccosh}(cx))\, dx$

Optimal result

Integrand size = 22, antiderivative size = 191

$$\begin{aligned}\int (d-c^2dx^2)^3 (a+b\text{arccosh}(cx))\,dx &= -\frac{16bd^3\sqrt{-1+cx}\sqrt{1+cx}}{35c}+\frac{8bd^3(-1+cx)^{3/2}(1+cx)^{3/2}}{105c}\\&\quad-\frac{6bd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{175c}+\frac{bd^3(-1+cx)^{7/2}(1+cx)^{7/2}}{49c}\\&\quad+d^3x(a+b\text{arccosh}(cx))-c^2d^3x^3(a+b\text{arccosh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arccosh}(cx))-\frac{1}{7}c^6d^3x^7(a+b\text{arccosh}(c\end{aligned}$$

output
```
-16/35*b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+8/105*b*d^3*(c*x-1)^(3/2)*(c*x+
1)^(3/2)/c-6/175*b*d^3*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c+1/49*b*d^3*(c*x-1)^(7
/2)*(c*x+1)^(7/2)/c+d^3*x*(a+b*arccosh(c*x))-c^2*d^3*x^3*(a+b*arccosh(c*x)
)+3/5*c^4*d^3*x^5*(a+b*arccosh(c*x))-1/7*c^6*d^3*x^7*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx =$$
$$-\frac{d^3\left(b\sqrt{-1+cx}\sqrt{1+cx}(2161-757c^2x^2+351c^4x^4-75c^6x^6)+105acx(-35+35c^2x^2-21c^4x^4+5c^6}{3675c}$$

input
```
Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/3675*(d^3*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2161 - 757*c^2*x^2 + 351*c^4
*x^4 - 75*c^6*x^6) + 105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)
 + 105*b*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCosh[c*x]))/c
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6309, 27, 2113, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6309}$$

$$-bc\int \frac{d^3x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{35\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^6d^3x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arccosh}(cx))-c^2d^3x^3(a+b\text{arccosh}(cx))+d^3x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{35}bcd^3\int \frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^6d^3x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arccosh}(cx))-c^2d^3x^3(a+b\text{arccosh}(cx))+d^3x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 2113}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{c^2x^2-1}}dx}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{7}c^6d^3x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arccosh}(cx))-c^2d^3x^3(a+b\text{arccosh}(cx))+d^3x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 2331}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int\frac{-5c^6x^6+21c^4x^4-35c^2x^2+35}{\sqrt{c^2x^2-1}}dx^2}{70\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{7}c^6d^3x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arccosh}(cx))-c^2d^3x^3(a+b\text{arccosh}(cx))+d^3x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 2389}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int\left(-5(c^2x^2-1)^{5/2}+6(c^2x^2-1)^{3/2}-8\sqrt{c^2x^2-1}+\frac{16}{\sqrt{c^2x^2-1}}\right)dx^2}{70\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{7}c^6d^3x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arccosh}(cx))-c^2d^3x^3(a+b\text{arccosh}(cx))+d^3x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{7}c^6d^3x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arccosh}(cx))-c^2d^3x^3(a+b\text{arccosh}(cx))+d^3x(a+b\text{arccosh}(cx))-\frac{bcd^3\sqrt{c^2x^2-1}\left(-\frac{10(c^2x^2-1)^{7/2}}{7c^2}+\frac{12(c^2x^2-1)^{5/2}}{5c^2}-\frac{16(c^2x^2-1)^{3/2}}{3c^2}+\frac{32\sqrt{c^2x^2-1}}{c^2}\right)}{70\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/70*(b*c*d^3*Sqrt[-1 + c^2*x^2]*((32*Sqrt[-1 + c^2*x^2])/c^2 - (16*(-1 +
 c^2*x^2)^(3/2))/(3*c^2) + (12*(-1 + c^2*x^2)^(5/2))/(5*c^2) - (10*(-1 + c
^2*x^2)^(7/2))/(7*c^2)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*Arc
Cosh[c*x]) - c^2*d^3*x^3*(a + b*ArcCosh[c*x]) + (3*c^4*d^3*x^5*(a + b*ArcC
osh[c*x]))/5 - (c^6*d^3*x^7*(a + b*ArcCosh[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

method	result
parts	$-d^3a\left(\frac{1}{7}c^6x^7-\frac{3}{5}c^4x^5+c^2x^3-x\right)-\frac{d^3b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}+c^3x^3\operatorname{arccosh}(cx)-cx\ \operatorname{arcc}\right.}{c}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}+c^3x^3\operatorname{arccosh}(cx)-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-}}{}\right.}{c}$
default	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}+c^3x^3\operatorname{arccosh}(cx)-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-}}{}\right.}{c}$
orering	$\frac{x\left(325c^6x^6-1437c^4x^4+2739c^2x^2-5547\right)\left(-c^2d\,x^2+d\right)^3\left(a+b\operatorname{arccosh}(cx)\right)}{1225(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{\left(75c^6x^6-351c^4x^4+757c^2x^2-2161\right)\left(-\right.}{3}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/7*c^6*x^7-3/5*c^4*x^5+c^2*x^3-x)-d^3*b/c*(1/7*arccosh(c*x)*c^7*x
^7-3/5*arccosh(c*x)*c^5*x^5+c^3*x^3*arccosh(c*x)-c*x*arccosh(c*x)-1/3675*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-351*c^4*x^4+757*c^2*x^2-2161))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \left(d-c^2dx^2\right)^3\left(a+b\operatorname{arccosh}(cx)\right)dx =$$

$$-\frac{525\,ac^7d^3x^7-2205\,ac^5d^3x^5+3675\,ac^3d^3x^3-3675\,acd^3x+105\left(5\,bc^7d^3x^7-21\,bc^5d^3x^5+35\,bc^3d^3x^3\right.}{3675}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/3675*(525*a*c^7*d^3*x^7 - 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 - 367
5*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 -
 35*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d^3*x^6 - 351*b*c^
4*d^3*x^4 + 757*b*c^2*d^3*x^2 - 2161*b*d^3)*sqrt(c^2*x^2 - 1))/c
```

Sympy [F]

$$\int \left(d-c^{2}dx^{2}\right)^{3}\left(a+b\operatorname{arccosh}(cx)\right)dx=-d^{3}\left(\int\left(-a\right)\,dx+\int\left(-b\operatorname{acosh}\left(cx\right)\right)\,dx+\int 3ac^{2}x^{2}\,dx+\int\left(-3ac^{4}x^{4}\right)\,dx+\int ac^{6}x^{6}\,dx+\int 3bc^{2}x^{2}\operatorname{acosh}\left(cx\right)dx+\int\left(-3bc^{4}x^{4}\operatorname{acosh}\left(cx\right)\right)\,dx+\int bc^{6}x^{6}\operatorname{acosh}\left(cx\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
-d**3*(Integral(-a, x) + Integral(-b*acosh(c*x), x) + Integral(3*a*c**2*x**
*2, x) + Integral(-3*a*c**4*x**4, x) + Integral(a*c**6*x**6, x) + Integral
(3*b*c**2*x**2*acosh(c*x), x) + Integral(-3*b*c**4*x**4*acosh(c*x), x) + I
ntegral(b*c**6*x**6*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.58

$$\int \left(d-c^{2}dx^{2}\right)^{3}\left(a+b\operatorname{arccosh}(cx)\right)dx=-\frac{1}{7}ac^{6}d^{3}x^{7}+\frac{3}{5}ac^{4}d^{3}x^{5}-\frac{1}{245}\left(35\,x^{7}\operatorname{arcosh}\left(cx\right)-\left(\frac{5\sqrt{c^{2}x^{2}-1}x^{6}}{c^{2}}+\frac{6\sqrt{c^{2}x^{2}-1}x^{4}}{c^{4}}+\frac{8\sqrt{c^{2}x^{2}-1}x^{2}}{c^{6}}+\frac{16\sqrt{c^{2}x^{2}-1}}{c^{8}}\right)c\right)bc+\frac{1}{25}\left(15\,x^{5}\operatorname{arcosh}\left(cx\right)-\left(\frac{3\sqrt{c^{2}x^{2}-1}x^{4}}{c^{2}}+\frac{4\sqrt{c^{2}x^{2}-1}x^{2}}{c^{4}}+\frac{8\sqrt{c^{2}x^{2}-1}}{c^{6}}\right)c\right)bc^{4}d^{3}-ac^{2}d^{3}x^{3}-\frac{1}{3}\left(3\,x^{3}\operatorname{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^{2}x^{2}-1}x^{2}}{c^{2}}+\frac{2\sqrt{c^{2}x^{2}-1}}{c^{4}}\right)\right)bc^{2}d^{3}+ad^{3}x+\frac{\left(cx\operatorname{arcosh}\left(cx\right)-\sqrt{c^{2}x^{2}-1}\right)bd^{3}}{c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*s
qrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 -
1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arccosh
(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqr
t(c^2*x^2 - 1)/c^6)*c)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*arccosh(c*x)
 - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + a*
d^3*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \int (a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\int \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$
$$=\frac{d^3\left(-525acosh(cx)\,b\,c^7x^7+2205acosh(cx)\,b\,c^5x^5-3675acosh(cx)\,b\,c^3x^3+3675acosh(cx)\,bcx+75\sqrt{c^2x}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(d**3*( - 525*acosh(c*x)*b*c**7*x**7 + 2205*acosh(c*x)*b*c**5*x**5 - 3675*
acosh(c*x)*b*c**3*x**3 + 3675*acosh(c*x)*b*c*x + 75*sqrt(c**2*x**2 - 1)*b*
c**6*x**6 - 351*sqrt(c**2*x**2 - 1)*b*c**4*x**4 + 757*sqrt(c**2*x**2 - 1)*
b*c**2*x**2 + 1514*sqrt(c**2*x**2 - 1)*b - 3675*sqrt(c*x + 1)*sqrt(c*x - 1
)*b - 525*a*c**7*x**7 + 2205*a*c**5*x**5 - 3675*a*c**3*x**3 + 3675*a*c*x))
/(3675*c)
```

3.24 $\int \frac{(d-c^2dx^2)^3(a+b\mathbf{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \frac{(d-c^2dx^2)^3\,(a+b\text{arccosh}(cx))}{x}\,dx$$
$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx} - \frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}$$
$$+ \frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/2} - \frac{19}{48}bd^3\text{arccosh}(cx) + \frac{1}{2}d^3\left(1-c^2x^2\right)(a+b\text{arccosh}(cx)) + \frac{1}{4}d^3\left(1-c^2x^2\right)^2$$

output
```
19/48*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-7/72*b*c*d^3*x*(c*x-1)^(3/2)*(
c*x+1)^(3/2)+1/36*b*c*d^3*x*(c*x-1)^(5/2)*(c*x+1)^(5/2)-19/48*b*d^3*arccos
h(c*x)+1/2*d^3*(-c^2*x^2+1)*(a+b*arccosh(c*x))+1/4*d^3*(-c^2*x^2+1)^2*(a+b
*arccosh(c*x))+1/6*d^3*(-c^2*x^2+1)^3*(a+b*arccosh(c*x))-1/2*d^3*(a+b*arcc
osh(c*x))^2/b+d^3*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2)+1/2*b*d^3*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.28

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x} \, dx = -\frac{1}{144}d^3\Bigg(216ac^2x^2-108ac^4x^4+24ac^6x^6+33bcx\sqrt{\frac{-1+cx}{1+cx}}+33bc^2x^2\sqrt{\frac{-1+cx}{1+cx}}+22bc^3x^3\sqrt{\frac{-1+cx}{1+cx}}+22bc^4x^4\sqrt{\frac{-1+cx}{1+cx}}-4bc^5x^5\sqrt{\frac{-1+cx}{1+cx}}-4bc^6x^6\sqrt{\frac{-1+cx}{1+cx}}-108bcx\sqrt{-1+cx}\sqrt{1+cx}-72b\text{arccosh}(cx)^2-150b\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)+12b\text{arccosh}(cx)\left(18c^2x^2-9c^4x^4+2c^6x^6-12\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right)-144a\log(x)+72b\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\Bigg)$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
-1/144*(d^3*(216*a*c^2*x^2 - 108*a*c^4*x^4 + 24*a*c^6*x^6 + 33*b*c*x*Sqrt[
(-1 + c*x)/(1 + c*x)] + 33*b*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 22*b*c^3
*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 22*b*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]
- 4*b*c^5*x^5*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*c^6*x^6*Sqrt[(-1 + c*x)/(1
+ c*x)] - 108*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 72*b*ArcCosh[c*x]^2 - 1
50*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 12*b*ArcCosh[c*x]*(18*c^2*x^2 -
 9*c^4*x^4 + 2*c^6*x^6 - 12*Log[1 + E^(-2*ArcCosh[c*x])]) - 144*a*Log[x] +
 72*b*PolyLog[2, -E^(-2*ArcCosh[c*x])]))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.51, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {6334, 27, 40, 40, 40, 43, 6334, 40, 40, 43, 6334, 40, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x} dx$$

$$\Big\downarrow \text{6334}$$

$$d\int \frac{d^2(1-c^2x^2)^2 (a+b\text{arccosh}(cx))}{x}dx+\frac{1}{6}bcd^3\int (cx-1)^{5/2}(cx+1)^{5/2}dx+\frac{1}{6}d^3(1-c^2x^2)^3 (a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$d^3\int \frac{(1-c^2x^2)^2 (a+b\text{arccosh}(cx))}{x}dx+\frac{1}{6}bcd^3\int (cx-1)^{5/2}(cx+1)^{5/2}dx+\frac{1}{6}d^3(1-c^2x^2)^3 (a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{40}$$

$$d^3\int \frac{(1-c^2x^2)^2 (a+b\text{arccosh}(cx))}{x}dx+\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\int (cx-1)^{3/2}(cx+1)^{3/2}dx\right)+\frac{1}{6}d^3(1-c^2x^2)^3 (a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{40}$$

$$d^3\int \frac{(1-c^2x^2)^2 (a+b\text{arccosh}(cx))}{x}dx+\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)\right)+\frac{1}{6}d^3(1-c^2x^2)^3 (a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{40}$$

$$d^3 \int \frac{\left(1-c^2x^2\right)^2 (a+b\text{arccosh}(cx))}{x}dx+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+}}\right.\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))$$

$\Big\downarrow$ 43

$$d^3 \int \frac{\left(1-c^2x^2\right)^2 (a+b\text{arccosh}(cx))}{x}dx+\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 6334

$$d^3\left(\int \frac{\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bc\int (cx-1)^{3/2}(cx+1)^{3/2}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))\right)+$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 40

$$d^3\left(\int \frac{\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)+\frac{1}{4}(1-\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 40

$$d^3\left(\int \frac{\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{}{\sqrt{cx}}\right.\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\Big\downarrow$ 43

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\right.\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 6334

$$d^3\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\text{arccosh}(cx))+\frac{1}{2}(1-c^2x^2)(a+\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 40

$$d^3\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{4}(1-c^2x^2)^2(a+b\text{arcco}\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 43

$$d^3\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\text{arccosh}(cx))+\frac{1}{2}(1-c^2x^2)(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx}\right.\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 6297

$$d^3\left(\frac{\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}(1-c^2x^2)^2(a+b\text{arccosh}(cx)\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 25

$$d^3\left(-\frac{\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))+\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 3042

$$d^3\left(-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 26

$$d^3\left(\frac{i\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))+\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 4201

$$d^3\left(\frac{i\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+\text{barc}\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$\downarrow$ 2620

$$d^3\left(\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$

$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$d^3\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx)\right)\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$

$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$d^3\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))+$$

$$\frac{1}{6}bcd^3\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)\right)$$

input

```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

output

```
(d^3*(1 - c^2*x^2)^3*(a + b*ArcCosh[c*x]))/6 + (b*c*d^3*((x*(-1 + c*x)^(5/
2)*(1 + c*x)^(5/2))/6 - (5*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 - (3*((
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/6))/6 + d^3*(
((1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 + ((1 - c^2*x^2)^2*(a + b*ArcCosh[c
*x]))/4 + (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/
2 - (b*c*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 - (3*((x*Sqrt[-1 + c*x]*S
qrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/4 + (I*((-1/2*I)*(a + b*ArcCosh
[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])
]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6334
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Simp[b*c*((-d
)^p/(2*p))   Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03

method	result
parts	$-d^3 a\left(\frac{c^6x^6}{6}-\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}-\ln\left(x\right)\right)+\frac{b\,d^3\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2}+\frac{25b\,d^3\,\mathrm{arccosh}(cx)}{48}$
derivativedivides	$-d^3 a\left(\frac{c^6x^6}{6}-\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}-\ln\left(cx\right)\right)+\frac{25b\,d^3\,\mathrm{arccosh}(cx)}{48}+\frac{d^3b\sqrt{cx+1}\,\sqrt{cx-1}\,c^5x^5}{36}-\frac{11d^3b\sqrt{cx+}}{}$
default	$-d^3 a\left(\frac{c^6x^6}{6}-\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}-\ln\left(cx\right)\right)+\frac{25b\,d^3\,\mathrm{arccosh}(cx)}{48}+\frac{d^3b\sqrt{cx+1}\,\sqrt{cx-1}\,c^5x^5}{36}-\frac{11d^3b\sqrt{cx+}}{}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/6*c^6*x^6-3/4*c^4*x^4+3/2*c^2*x^2-ln(x))+1/2*b*d^3*polylog(2,-(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+25/48*b*d^3*arccosh(c*x)-1/6*d^3*b*arcc
osh(c*x)*c^6*x^6+3/4*d^3*b*arccosh(c*x)*c^4*x^4-3/2*d^3*b*arccosh(c*x)*c^2
*x^2+1/36*d^3*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5*x^5-11/72*d^3*b*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*c^3*x^3+25/48*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*
d^3*b*arccosh(c*x)^2+d^3*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\text{arccosh}(cx))}{x}\,dx=\int -\frac{(c^2dx^2-d)^3(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*
c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x,
x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\text{arccosh}(cx))}{x}\,dx=-d^3\left(\int\left(-\frac{a}{x}\right)dx+\int 3ac^2x\,dx+\int\left(-3ac^4x^3\right)dx+\int ac^6x^5\,dx+\int\left(-\frac{b\,\text{acosh}\,(cx)}{x}\right)dx+\int 3bc^2x\,\text{acosh}\,(cx)\,dx+\int\left(-3bc^4x^3\,\text{acosh}\,(cx)\right)dx+\int bc^6x^5\,\text{acosh}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

output
```
-d**3*(Integral(-a/x, x) + Integral(3*a*c**2*x, x) + Integral(-3*a*c**4*x**3, x) + Integral(a*c**6*x**5, x) + Integral(-b*acosh(c*x)/x, x) + Integral(3*b*c**2*x*acosh(c*x), x) + Integral(-3*b*c**4*x**3*acosh(c*x), x) + Integral(b*c**6*x**5*acosh(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x}\,dx = \int -\frac{(c^2dx^2-d)^3(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
-1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 - 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) - integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\operatorname{arccosh}(cx))}{x} \, dx = \int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3}{x} \, dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\operatorname{arccosh}(cx))}{x} \, dx$$
$$= \frac{d^3\left(-24acosh(cx)\,b\,c^6x^6+108acosh(cx)\,b\,c^4x^4-216acosh(cx)\,b\,c^2x^2+4\sqrt{c^2x^2-1}\,b\,c^5x^5-22\sqrt{c^2x^2-\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acosh(c*x))/x,x)
```

output
```
(d**3*( - 24*acosh(c*x)*b*c**6*x**6 + 108*acosh(c*x)*b*c**4*x**4 - 216*aco
sh(c*x)*b*c**2*x**2 + 4*sqrt(c**2*x**2 - 1)*b*c**5*x**5 - 22*sqrt(c**2*x**
2 - 1)*b*c**3*x**3 + 75*sqrt(c**2*x**2 - 1)*b*c*x + 144*int(acosh(c*x)/x,x
)*b + 75*log(sqrt(c**2*x**2 - 1) + c*x)*b + 144*log(x)*a - 24*a*c**6*x**6
+ 108*a*c**4*x**4 - 216*a*c**2*x**2))/144
```

3.25 $\int \frac{(d-c^2dx^2)^3(a+b\,\text{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 190

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3(a+b\,\text{arccosh}(cx))}{x^2}\,dx &= \frac{61}{25}bcd^3\sqrt{-1+cx}\sqrt{1+cx} \\ &- \frac{7}{25}bc^3d^3x^2\sqrt{-1+cx}\sqrt{1+cx} \\ &+ \frac{1}{25}bc^5d^3x^4\sqrt{-1+cx}\sqrt{1+cx} \\ &- \frac{d^3(a+b\,\text{arccosh}(cx))}{x} \\ &- 3c^2d^3x(a+b\,\text{arccosh}(cx)) \\ &+ c^4d^3x^3(a+b\,\text{arccosh}(cx)) \\ &- \frac{1}{5}c^6d^3x^5(a+b\,\text{arccosh}(cx)) \\ &+ bcd^3\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\end{aligned}$$

output
```
61/25*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-7/25*b*c^3*d^3*x^2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)+1/25*b*c^5*d^3*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)-d^3*(a+b*arc
cosh(c*x))/x-3*c^2*d^3*x*(a+b*arccosh(c*x))+c^4*d^3*x^3*(a+b*arccosh(c*x))
-1/5*c^6*d^3*x^5*(a+b*arccosh(c*x))+b*c*d^3*arctan((c*x-1)^(1/2)*(c*x+1)^(
1/2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^2} dx = \frac{1}{25}d^3\left(-\frac{25a}{x} - 75ac^2x + 25ac^4x^3 - 5ac^6x^5 + bc\sqrt{-1+cx}\sqrt{1+cx}(61-7c^2x^2+c^4x^4) - \frac{5b(5+15c^2x^2-5c^4x^4+c^6x^6)\text{arccosh}(cx)}{x} - 25bc\arctan\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)\right)$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
(d^3*((-25*a)/x - 75*a*c^2*x + 25*a*c^4*x^3 - 5*a*c^6*x^5 + b*c*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(61 - 7*c^2*x^2 + c^4*x^4) - (5*b*(5 + 15*c^2*x^2 - 5*c
^4*x^4 + c^6*x^6)*ArcCosh[c*x])/x - 25*b*c*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1
 + c*x])]))/25
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6336, 27, 2113, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^2} dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int -\frac{d^3(c^6x^6-5c^4x^4+15c^2x^2+5)}{5x\sqrt{cx-1}\sqrt{cx+1}}dx - \frac{1}{5}c^6d^3x^5(a+b\text{arccosh}(cx)) + c^4d^3x^3(a+b\text{arccosh}(cx)) - 3c^2d^3x(a+b\text{arccosh}(cx)) - \frac{d^3(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}bcd^3\int \frac{c^6x^6-5c^4x^4+15c^2x^2+5}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{5}c^6d^3x^5(a+\text{barccosh}(cx))+c^4d^3x^3(a+\text{barccosh}(cx))-3c^2d^3x(a+\text{barccosh}(cx))-\frac{d^3(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{2113}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{c^6x^6-5c^4x^4+15c^2x^2+5}{x\sqrt{c^2x^2-1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{5}c^6d^3x^5(a+\text{barccosh}(cx))+c^4d^3x^3(a+\text{barccosh}(cx))-3c^2d^3x(a+\text{barccosh}(cx))-\frac{d^3(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{2331}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{c^6x^6-5c^4x^4+15c^2x^2+5}{x^2\sqrt{c^2x^2-1}}dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{5}c^6d^3x^5(a+\text{barccosh}(cx))+c^4d^3x^3(a+\text{barccosh}(cx))-3c^2d^3x(a+\text{barccosh}(cx))-\frac{d^3(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{2123}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\int \left(\left(c^2x^2-1\right)^{3/2}c^2-3\sqrt{c^2x^2-1}c^2+\frac{11c^2}{\sqrt{c^2x^2-1}}+\frac{5}{x^2\sqrt{c^2x^2-1}}\right)dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{5}c^6d^3x^5(a+\text{barccosh}(cx))+c^4d^3x^3(a+\text{barccosh}(cx))-3c^2d^3x(a+\text{barccosh}(cx))-\frac{d^3(a+\text{barccosh}(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}c^6d^3x^5(a+\text{barccosh}(cx))+c^4d^3x^3(a+\text{barccosh}(cx))-3c^2d^3x(a+\text{barccosh}(cx))-\frac{d^3(a+\text{barccosh}(cx))}{x}+\frac{bcd^3\sqrt{c^2x^2-1}\left(10\arctan\left(\sqrt{c^2x^2-1}\right)+\frac{2}{5}\left(c^2x^2-1\right)^{5/2}-2\left(c^2x^2-1\right)^{3/2}+22\sqrt{c^2x^2-1}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((d^3*(a + b*ArcCosh[c*x]))/x) - 3*c^2*d^3*x*(a + b*ArcCosh[c*x]) + c^4*d
^3*x^3*(a + b*ArcCosh[c*x]) - (c^6*d^3*x^5*(a + b*ArcCosh[c*x]))/5 + (b*c*
d^3*Sqrt[-1 + c^2*x^2]*(22*Sqrt[-1 + c^2*x^2] - 2*(-1 + c^2*x^2)^(3/2) + (
2*(-1 + c^2*x^2)^(5/2))/5 + 10*ArcTan[Sqrt[-1 + c^2*x^2]]))/(10*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94

method	result
parts	$-d^3a\left(\frac{c^6x^5}{5}-c^4x^3+3c^2x+\frac{1}{x}\right)-d^3bc\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-c^3x^3\operatorname{arccosh}(cx)+3cx\ \operatorname{arcco}\right.$
derivativedivides	$c\left(-d^3a\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-c^3x^3\operatorname{arccosh}(cx)+3cx\ \operatorname{arcc}\right.\right.$
default	$c\left(-d^3a\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-c^3x^3\operatorname{arccosh}(cx)+3cx\ \operatorname{arcc}\right.\right.$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/5*c^6*x^5-c^4*x^3+3*c^2*x+1/x)-d^3*b*c*(1/5*arccosh(c*x)*c^5*x^5
-c^3*x^3*arccosh(c*x)+3*c*x*arccosh(c*x)+arccosh(c*x)/c/x+1/25*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*(-c^4*x^4*(c^2*x^2-1)^(1/2)+7*c^2*x^2*(c^2*x^2-1)^(1/2)+2
5*arctan(1/(c^2*x^2-1)^(1/2))-61*(c^2*x^2-1)^(1/2))/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.31

$$\int\frac{(d-c^2dx^2)^3\,(a+b\operatorname{arccosh}(cx))}{x^2}\,dx=$$
$$-\frac{5\,ac^6d^3x^6-25\,ac^4d^3x^4+75\,ac^2d^3x^2-50\,bcd^3x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-5\left(bc^6-5\,bc^4+15\,bc^2+\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

output
```
-1/25*(5*a*c^6*d^3*x^6 - 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 50*b*c*d^3*
x*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*
d^3*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 25*a*d^3 + 5*(b*c^6*d^3*x^6 - 5*b*c^
4*d^3*x^4 + 15*b*c^2*d^3*x^2 - (b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^3*x +
5*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^5*d^3*x^5 - 7*b*c^3*d^3*x^3 +
 61*b*c*d^3*x)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F]

$$\int \frac{(d - c^2dx^2)^3 (a + b\operatorname{arccosh}(cx))}{x^2}\,dx = -d^3\left(\int 3ac^2\,dx + \int \left(-\frac{a}{x^2}\right)dx + \int \left(-3ac^4x^2\right)dx + \int ac^6x^4\,dx + \int 3bc^2\operatorname{acosh}(cx)\,dx + \int \left(-\frac{b\operatorname{acosh}(cx)}{x^2}\right)dx + \int \left(-3bc^4x^2\operatorname{acosh}(cx)\right)dx + \int bc^6x^4\operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
```

output
```
-d**3*(Integral(3*a*c**2, x) + Integral(-a/x**2, x) + Integral(-3*a*c**4*x
**2, x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*acosh(c*x), x) + In
tegral(-b*acosh(c*x)/x**2, x) + Integral(-3*b*c**4*x**2*acosh(c*x), x) + I
ntegral(b*c**6*x**4*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.22

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{1}{5}\,ac^6d^3x^5 - \frac{1}{75}\left(15\,x^5\,\text{arcosh}\,(cx) - \left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^6d^3 + ac^4d^3x^3 + \frac{1}{3}\left(3\,x^3\,\text{arcosh}\,(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^4d^3 - 3\,ac^2d^3x - 3\left(cx\,\text{arcosh}\,(cx) - \sqrt{c^2x^2-1}\right)bcd^3 - \left(c\arcsin\left(\frac{1}{c|x|}\right) + \frac{\text{arcosh}\,(cx)}{x}\right)bd^3 - \frac{ad^3}{x}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/5*a*c^6*d^3*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/
c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^6*d^3
+ a*c^4*d^3*x^3 + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^3 - 3*a*c^2*d^3*x - 3*(c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*c*d^3 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*
b*d^3 - a*d^3/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^2} dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^3}{x^2} dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^2} dx$$
$$= \frac{d^3(-5acosh(cx)\,b\,c^6x^6+25acosh(cx)\,b\,c^4x^4-75acosh(cx)\,b\,c^2x^2-25acosh(cx)\,b-50atan(\sqrt{c^2x^2-1}}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acosh(c*x))/x^2,x)
```

output
```
(d**3*( - 5*acosh(c*x)*b*c**6*x**6 + 25*acosh(c*x)*b*c**4*x**4 - 75*acosh(
c*x)*b*c**2*x**2 - 25*acosh(c*x)*b - 50*atan(sqrt(c**2*x**2 - 1) + c*x)*b*
c*x + sqrt(c**2*x**2 - 1)*b*c**5*x**5 - 7*sqrt(c**2*x**2 - 1)*b*c**3*x**3
- 14*sqrt(c**2*x**2 - 1)*b*c*x + 75*sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - 5*
a*c**6*x**6 + 25*a*c**4*x**4 - 75*a*c**2*x**2 - 25*a))/(25*x)
```

3.26 $\int \frac{(d-c^2dx^2)^3(a+b\,\mathbf{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{(d-c^2dx^2)^3\,(a+b\operatorname{arccosh}(cx))}{x^3}\,dx = -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x} + \frac{3}{32}bc^2d^3\operatorname{arccosh}(cx) - \frac{3}{2}c^2d^3\left(1-c^2x^2\right)(a+b\operatorname{arccosh}(cx)) - \frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2(a+b\operatorname{arccosh}(cx)) - \frac{d^3(1-$$

output

```
-3/32*b*c^3*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-7/16*b*c^3*d^3*x*(c*x-1)^(3/
2)*(c*x+1)^(3/2)+1/2*b*c*d^3*(c*x-1)^(5/2)*(c*x+1)^(5/2)/x+3/32*b*c^2*d^3*
arccosh(c*x)-3/2*c^2*d^3*(-c^2*x^2+1)*(a+b*arccosh(c*x))-3/4*c^2*d^3*(-c^2
*x^2+1)^2*(a+b*arccosh(c*x))-1/2*d^3*(-c^2*x^2+1)^3*(a+b*arccosh(c*x))/x^2
+3/2*c^2*d^3*(a+b*arccosh(c*x))^2/b-3*c^2*d^3*(a+b*arccosh(c*x))*ln(1+(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/2*b*c^2*d^3*polylog(2,-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.13

$$\int \frac{(d-c^2dx^2)^3(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$= \frac{d^3\left(-16a+48ac^4x^4-8ac^6x^6+3bc^3x^3\sqrt{\frac{-1+cx}{1+cx}}+3bc^4x^4\sqrt{\frac{-1+cx}{1+cx}}+2bc^5x^5\sqrt{\frac{-1+cx}{1+cx}}+2bc^6x^6\sqrt{\frac{-1+cx}{1+cx}}+1\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
(d^3*(-16*a + 48*a*c^4*x^4 - 8*a*c^6*x^6 + 3*b*c^3*x^3*Sqrt[(-1 + c*x)/(1
+ c*x)] + 3*b*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*b*c^5*x^5*Sqrt[(-1 +
c*x)/(1 + c*x)] + 2*b*c^6*x^6*Sqrt[(-1 + c*x)/(1 + c*x)] + 16*b*c*x*Sqrt[-
1 + c*x]*Sqrt[1 + c*x] - 24*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 48*b*
c^2*x^2*ArcCosh[c*x]^2 - 42*b*c^2*x^2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]
- 8*b*ArcCosh[c*x]*(2 - 6*c^4*x^4 + c^6*x^6 + 12*c^2*x^2*Log[1 + E^(-2*Arc
Cosh[c*x])]) - 96*a*c^2*x^2*Log[x] + 48*b*c^2*x^2*PolyLog[2, -E^(-2*ArcCos
h[c*x])]))/(32*x^2)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.39, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {6335, 27, 108, 27, 40, 40, 43, 6334, 40, 40, 43, 6334, 40, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{6335}$$

$$-3c^2d\int \frac{d^2\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{2}bcd^3\int \frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x^2}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 27$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{2}bcd^3\int \frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x^2}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 108$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{2}bcd^3\left(\int 5c^2(cx-1)^{3/2}(cx+1)^{3/2}dx-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)-\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 27$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{2}bcd^3\left(5c^2\int (cx-1)^{3/2}(cx+1)^{3/2}dx-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)-\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 40$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)-\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 40$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(cx)\right)}{x}dx-$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)-\frac{(cx-1)^{5/2}(cx}{x}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{2x^2}$$

$\Big\downarrow$ 43

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(cx)\right)}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$\Big\downarrow$ 6334

$$-3c^2d^3\left(\int\frac{\left(1-c^2x^2\right)\left(a+b\text{arccosh}(cx)\right)}{x}dx-\frac{1}{4}bc\int(cx-1)^{3/2}(cx+1)^{3/2}dx+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(cx\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$\Big\downarrow$ 40

$$-3c^2d^3\left(\int\frac{\left(1-c^2x^2\right)\left(a+b\text{arccosh}(cx)\right)}{x}dx-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int\sqrt{cx-1}\sqrt{cx+1}dx\right)+\frac{1}{4}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$\Big\downarrow$ 40

$$-3c^2d^3\left(\int\frac{\left(1-c^2x^2\right)\left(a+b\text{arccosh}(cx)\right)}{x}dx-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\right.\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow 43$$

$$-3c^2d^3\left(\int \frac{(1-c^2x^2)\,(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{4}(1-c^2x^2)^2\,(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/\right.\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow 6334$$

$$-3c^2d^3\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{4}(1-c^2x^2)^2\,(a+b\text{arccosh}(cx))+\frac{1}{2}(1-c^2x^2)\,(\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow 40$$

$$-3c^2d^3\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{4}(1-c^2x^2)^2\,(a+ba\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow 43$$

$$-3c^2d^3\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{4}(1-c^2x^2)^2\,(a+b\text{arccosh}(cx))+\frac{1}{2}(1-c^2x^2)\,(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(\frac{1}{2}x\right.\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow 6297$$

$$-3c^2d^3\left(\frac{\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccos}$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$\Big\downarrow$ 25

$$-3c^2d^3\left(-\frac{\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(c$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$\Big\downarrow$ 3042

$$-3c^2d^3\left(-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arcco}$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$\Big\downarrow$ 26

$$-3c^2d^3\left(\frac{i\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}($$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$\Big\downarrow$ 4201

$$-3c^2d^3\left(\frac{i\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2(a+\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow \; 2620$$

$$-3c^2d^3\left(\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\right.}{b}\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow \; 2715$$

$$-3c^2d^3\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}\right.\right.}{b}\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

$$\Big\downarrow \; 2838$$

$$-3c^2d^3\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)-\frac{(cx-1)^{5/2}(cx+1)^{5/2}}{x}\right)$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(d^3*(1 - c^2*x^2)^3*(a + b*ArcCosh[c*x]))/x^2 - (b*c*d^3*(-(((-1 + c
*x)^(5/2)*(1 + c*x)^(5/2))/x) + 5*c^2*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)
)/4 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4)))/2
 - 3*c^2*d^3*(((1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 + ((1 - c^2*x^2)^2*(a
 + b*ArcCosh[c*x]))/4 + (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh
[c*x]/(2*c)))/2 - (b*c*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 - (3*((x*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/4 + (I*((-1/2*I)*
(a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2
*ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6334
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Simp[b*c*((-d
)^p/(2*p))   Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6335
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*((-d)^p/(f*(m + 1)))   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(
m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.97

method	result
derivativedivides	$c^2\left(-d^3a\left(\frac{c^4x^4}{4}-\frac{3c^2x^2}{2}+3\ln\left(cx\right)+\frac{1}{2c^2x^2}\right)-\frac{d^3b}{2}-\frac{d^3b\,\mathrm{arccosh}(cx)c^4x^4}{4}+\frac{3d^3b\,\mathrm{arccosh}(cx)c^2x^2}{2}\right.$
default	$c^2\left(-d^3a\left(\frac{c^4x^4}{4}-\frac{3c^2x^2}{2}+3\ln\left(cx\right)+\frac{1}{2c^2x^2}\right)-\frac{d^3b}{2}-\frac{d^3b\,\mathrm{arccosh}(cx)c^4x^4}{4}+\frac{3d^3b\,\mathrm{arccosh}(cx)c^2x^2}{2}\right.$
parts	$-d^3a\left(\frac{c^6x^4}{4}-\frac{3c^4x^2}{2}+3c^2\ln\left(x\right)+\frac{1}{2x^2}\right)-\frac{d^3b\,c^2}{2}-\frac{21b\,c^2d^3\,\mathrm{arccosh}(cx)}{32}+\frac{d^3b\,c^5\sqrt{cx+1}\,\sqrt{cx-1}\,x^3}{16}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-d^3*a*(1/4*c^4*x^4-3/2*c^2*x^2+3*ln(c*x)+1/2/c^2/x^2)-1/2*d^3*b-1/4*
d^3*b*arccosh(c*x)*c^4*x^4+3/2*d^3*b*arccosh(c*x)*c^2*x^2-1/2*d^3*b*arccos
h(c*x)/c^2/x^2-21/32*b*d^3*arccosh(c*x)-21/32*b*c*d^3*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)-3*d^3*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1
/16*d^3*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^3-3/2*b*d^3*polylog(2,-(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))^2)+3/2*d^3*b*arccosh(c*x)^2+1/2*d^3*b/c/x*(c*x+
1)^(1/2)*(c*x-1)^(1/2))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\text{arccosh}(cx))}{x^3}\,dx = \int -\frac{(c^2dx^2-d)^3(b\,\text{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*
c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x^3
, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\text{arccosh}(cx))}{x^3}\,dx = -d^3\left(\int\left(-\frac{a}{x^3}\right)dx+\int\frac{3ac^2}{x}\,dx + \int\left(-3ac^4x\right)dx+\int ac^6x^3\,dx + \int\left(-\frac{b\,\text{acosh}\,(cx)}{x^3}\right)dx + \int\frac{3bc^2\,\text{acosh}\,(cx)}{x}\,dx + \int\left(-3bc^4x\,\text{acosh}\,(cx)\right)dx + \int bc^6x^3\,\text{acosh}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
```

output
```
-d**3*(Integral(-a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(-3*a*c**4
*x, x) + Integral(a*c**6*x**3, x) + Integral(-b*acosh(c*x)/x**3, x) + Inte
gral(3*b*c**2*acosh(c*x)/x, x) + Integral(-3*b*c**4*x*acosh(c*x), x) + Int
egral(b*c**6*x**3*acosh(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)^3 (a + b\operatorname{arccosh}(cx))}{x^3} \, dx = \int -\frac{(c^2dx^2 - d)^3 (b \operatorname{arcosh}(cx) + a)}{x^3} \, dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 - 3*a*c^2*d^3*log(x) + 1/2*b*d^3*(s
qrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d^3/x^2 - integrate(b*c^6
*d^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d^3*x*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)^3 (a + b\operatorname{arccosh}(cx))}{x^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)^3 (a + b\operatorname{arccosh}(cx))}{x^3} \, dx = \int \frac{(a + b \operatorname{acosh}(c\,x)) \, (d - c^2\,d\,x^2)^3}{x^3} \, dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(d - c^2dx^2)^3 (a + b\text{arccosh}(cx))}{x^3} dx$$
$$= \frac{d^3\left(-8acosh(cx)\, b\, c^6x^6 + 48acosh(cx)\, b\, c^4x^4 - 16acosh(cx)\, b + 2\sqrt{c^2x^2-1}\, b\, c^5x^5 - 21\sqrt{c^2x^2-1}\, b\, c^3x^3 \right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acosh(c*x))/x^3,x)
```

output
```
(d**3*( - 8*acosh(c*x)*b*c**6*x**6 + 48*acosh(c*x)*b*c**4*x**4 - 16*acosh(
c*x)*b + 2*sqrt(c**2*x**2 - 1)*b*c**5*x**5 - 21*sqrt(c**2*x**2 - 1)*b*c**3
*x**3 - 16*sqrt(c**2*x**2 - 1)*b*c*x - 96*int(acosh(c*x)/x,x)*b*c**2*x**2
- 21*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*x**2 - 96*log(x)*a*c**2*x**2 -
8*a*c**6*x**6 + 48*a*c**4*x**4 - 16*a - 16*b*c**2*x**2))/(32*x**2)
```

3.27 $\int \frac{(d-c^2dx^2)^3(a+b\,\text{arccosh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 198

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3(a+b\,\text{arccosh}(cx))}{x^4}\,dx &= -\frac{25}{9}bc^3d^3\sqrt{-1+cx}\sqrt{1+cx}+\frac{bcd^3\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}+\frac{1}{9}bc^5d^3x^2\sqrt{-1+cx}\sqrt{1+cx}\\&\quad-\frac{d^3(a+b\,\text{arccosh}(cx))}{3x^3}+\frac{3c^2d^3(a+b\,\text{arccosh}(cx))}{x}+3c^4d^3x(a+b\,\text{arccosh}(cx))\\&\quad-\frac{1}{3}c^6d^3x^3(a+b\,\text{arccosh}(cx))-\frac{17}{6}bc^3d^3\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\end{aligned}$$

output
```
-25/9*b*c^3*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*b*c*d^3*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/x^2+1/9*b*c^5*d^3*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/3*d^3*(a+b*a
rccosh(c*x))/x^3+3*c^2*d^3*(a+b*arccosh(c*x))/x+3*c^4*d^3*x*(a+b*arccosh(c
*x))-1/3*c^6*d^3*x^3*(a+b*arccosh(c*x))-17/6*b*c^3*d^3*arctan((c*x-1)^(1/2
)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.72

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^4} dx$$
$$= \frac{d^3\left(-6a+54ac^2x^2+54ac^4x^4-6ac^6x^6+bcx\sqrt{-1+cx}\sqrt{1+cx}(3-50c^2x^2+2c^4x^4)-6b(1-9c^2x^2-\right.}{18x^3}$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
(d^3*(-6*a + 54*a*c^2*x^2 + 54*a*c^4*x^4 - 6*a*c^6*x^6 + b*c*x*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(3 - 50*c^2*x^2 + 2*c^4*x^4) - 6*b*(1 - 9*c^2*x^2 - 9*c^
4*x^4 + c^6*x^6)*ArcCosh[c*x] + 51*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt
[1 + c*x])]))/(18*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6336, 27, 2113, 2331, 2124, 27, 1192, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^4} dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int -\frac{d^3(c^6x^6-9c^4x^4-9c^2x^2+1)}{3x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^6d^3x^3(a+b\text{arccosh}(cx))+3c^4d^3x(a+b\text{arccosh}(cx))+\frac{3c^2d^3(a+b\text{arccosh}(cx))}{x}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd^3\int \frac{c^6x^6-9c^4x^4-9c^2x^2+1}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^6d^3x^3(a+\text{barccosh}(cx))+3c^4d^3x(a+\text{barccosh}(cx))+\frac{3c^2d^3(a+\text{barccosh}(cx))}{x}-\frac{d^3(a+\text{barccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2113}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{c^6x^6-9c^4x^4-9c^2x^2+1}{x^3\sqrt{c^2x^2-1}}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3}c^6d^3x^3(a+\text{barccosh}(cx))+3c^4d^3x(a+\text{barccosh}(cx))+\frac{3c^2d^3(a+\text{barccosh}(cx))}{x}-\frac{d^3(a+\text{barccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2331}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{c^6x^6-9c^4x^4-9c^2x^2+1}{x^4\sqrt{c^2x^2-1}}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3}c^6d^3x^3(a+\text{barccosh}(cx))+3c^4d^3x(a+\text{barccosh}(cx))+\frac{3c^2d^3(a+\text{barccosh}(cx))}{x}-\frac{d^3(a+\text{barccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2124}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\left(\int -\frac{-2x^4c^6+18x^2c^4+17c^2}{2x^2\sqrt{c^2x^2-1}}dx^2+\frac{\sqrt{c^2x^2-1}}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3}c^6d^3x^3(a+\text{barccosh}(cx))+3c^4d^3x(a+\text{barccosh}(cx))+\frac{3c^2d^3(a+\text{barccosh}(cx))}{x}-\frac{d^3(a+\text{barccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\sqrt{c^2x^2-1}}{x^2}-\frac{1}{2}\int \frac{-2x^4c^6+18x^2c^4+17c^2}{x^2\sqrt{c^2x^2-1}}dx^2\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3}c^6d^3x^3(a+\text{barccosh}(cx))+3c^4d^3x(a+\text{barccosh}(cx))+\frac{3c^2d^3(a+\text{barccosh}(cx))}{x}-\frac{d^3(a+\text{barccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{1192}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\sqrt{c^2x^2-1}}{x^2}-\frac{\int \frac{-2c^6x^8+14c^6x^4+33c^6}{x^4+1}d\sqrt{c^2x^2-1}}{c^4}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3}c^6d^3x^3(a+\text{barccosh}(cx))+3c^4d^3x(a+\text{barccosh}(cx))+\frac{3c^2d^3(a+\text{barccosh}(cx))}{x}-\frac{d^3(a+\text{barccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{1467}$$

$$\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\sqrt{c^2x^2-1}}{x^2}-\frac{\int \left(-2x^4c^6+\frac{17c^6}{x^4+1}+16c^6\right)d\sqrt{c^2x^2-1}}{c^4}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3}c^6d^3x^3(a+\text{barccosh}(cx))+3c^4d^3x(a+\text{barccosh}(cx))+\frac{3c^2d^3(a+\text{barccosh}(cx))}{x}-\frac{d^3(a+\text{barccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}c^6d^3x^3(a+b\text{arccosh}(cx))+3c^4d^3x(a+b\text{arccosh}(cx))+\frac{3c^2d^3(a+b\text{arccosh}(cx))}{x}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\sqrt{c^2x^2-1}}{x^2}-\frac{17c^6\arctan\left(\sqrt{c^2x^2-1}\right)-\frac{2}{3}c^6x^6+16c^6\sqrt{c^2x^2-1}}{c^4}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(d^3*(a + b*ArcCosh[c*x]))/x^3 + (3*c^2*d^3*(a + b*ArcCosh[c*x]))/x +
 3*c^4*d^3*x*(a + b*ArcCosh[c*x]) - (c^6*d^3*x^3*(a + b*ArcCosh[c*x]))/3 +
 (b*c*d^3*Sqrt[-1 + c^2*x^2]*(Sqrt[-1 + c^2*x^2]/x^2 - ((-2*c^6*x^6)/3 + 1
6*c^6*Sqrt[-1 + c^2*x^2] + 17*c^6*ArcTan[Sqrt[-1 + c^2*x^2]])/c^4))/(6*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98

method	result
parts	$-d^3a\left(\frac{c^6x^3}{3}-3c^4x-\frac{3c^2}{x}+\frac{1}{3x^3}\right)-d^3b\,c^3\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-3cx\operatorname{arccosh}(cx)-\frac{3\operatorname{arccosh}(c}{cx}\right.$
derivativedivides	$c^3\left(-d^3a\left(\frac{c^3x^3}{3}-3cx-\frac{3}{cx}+\frac{1}{3c^3x^3}\right)-d^3b\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-3cx\operatorname{arccosh}(cx)-\frac{3\operatorname{arccosh}}{cx}\right.\right.$
default	$c^3\left(-d^3a\left(\frac{c^3x^3}{3}-3cx-\frac{3}{cx}+\frac{1}{3c^3x^3}\right)-d^3b\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-3cx\operatorname{arccosh}(cx)-\frac{3\operatorname{arccosh}}{cx}\right.\right.$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/3*c^6*x^3-3*c^4*x-3*c^2/x+1/3/x^3)-d^3*b*c^3*(1/3*c^3*x^3*arccos
h(c*x)-3*c*x*arccosh(c*x)-3*arccosh(c*x)/c/x+1/3*arccosh(c*x)/c^3/x^3-1/18
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(2*c^4*x^4*(c^2*x^2-1)^(1/2)+51*arctan(1/(c^2
*x^2-1)^(1/2))*c^2*x^2-50*c^2*x^2*(c^2*x^2-1)^(1/2)+3*(c^2*x^2-1)^(1/2))/c
^2/x^2/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.28

$$\int\frac{(d-c^2dx^2)^3\,(a+b\operatorname{arccosh}(cx))}{x^4}\,dx=$$
$$-\frac{6\,ac^6d^3x^6-54\,ac^4d^3x^4+102\,bc^3d^3x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-54\,ac^2d^3x^2-6\,(bc^6-9\,bc^4-9\,bc^2}{}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/18*(6*a*c^6*d^3*x^6 - 54*a*c^4*d^3*x^4 + 102*b*c^3*d^3*x^3*arctan(-c*x
+ sqrt(c^2*x^2 - 1)) - 54*a*c^2*d^3*x^2 - 6*(b*c^6 - 9*b*c^4 - 9*b*c^2 + b
)*d^3*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) + 6*a*d^3 + 6*(b*c^6*d^3*x^6 - 9*b
*c^4*d^3*x^4 - 9*b*c^2*d^3*x^2 - (b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*d^3*x^3 +
 b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d^3*x^5 - 50*b*c^3*d^3*x^3
 + 3*b*c*d^3*x)*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^4} \, dx = -d^3\left(\int (-3ac^4)\, dx + \int \left(-\frac{a}{x^4}\right) dx + \int \frac{3ac^2}{x^2}\, dx + \int ac^6x^2\, dx + \int (-3bc^4 \operatorname{acosh}(cx))\, dx + \int \left(-\frac{b\operatorname{acosh}(cx)}{x^4}\right) dx + \int \frac{3bc^2\operatorname{acosh}(cx)}{x^2}\, dx + \int bc^6x^2\operatorname{acosh}(cx)\, dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

output
```
-d**3*(Integral(-3*a*c**4, x) + Integral(-a/x**4, x) + Integral(3*a*c**2/x
**2, x) + Integral(a*c**6*x**2, x) + Integral(-3*b*c**4*acosh(c*x), x) + I
ntegral(-b*acosh(c*x)/x**4, x) + Integral(3*b*c**2*acosh(c*x)/x**2, x) + I
ntegral(b*c**6*x**2*acosh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.05

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^4} \, dx = -\frac{1}{3}\,ac^6d^3x^3 - \frac{1}{9}\left(3\,x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^6d^3 + 3\,ac^4d^3x + 3\left(cx\operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bc^3d^3 + 3\left(c\arcsin\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)bc^2d^3 - \frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2-1}}{x^2}\right)c + \frac{2\operatorname{arcosh}(cx)}{x^3}\right)bd^3 + \frac{3\,ac^2d^3}{x} - \frac{ad^3}{3\,x^3}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/3*a*c^6*d^3*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^
2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arccosh(c
*x) - sqrt(c^2*x^2 - 1))*b*c^3*d^3 + 3*(c*arcsin(1/(c*abs(x))) + arccosh(c
*x)/x)*b*c^2*d^3 - 1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)
*c + 2*arccosh(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{(d-c^2dx^2)^3 (a+b\text{arccosh}(cx))}{x^4}\,dx$$
$$= \frac{d^3(-6acosh(cx)\,b\,c^6x^6+54acosh(cx)\,b\,c^4x^4+54acosh(cx)\,b\,c^2x^2-6acosh(cx)\,b+102atan\left(\sqrt{c^2x^2-1}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acosh(c*x))/x^4,x)
```

output
```
(d**3*( - 6*acosh(c*x)*b*c**6*x**6 + 54*acosh(c*x)*b*c**4*x**4 + 54*acosh(
c*x)*b*c**2*x**2 - 6*acosh(c*x)*b + 102*atan(sqrt(c**2*x**2 - 1) + c*x)*b*
c**3*x**3 + 2*sqrt(c**2*x**2 - 1)*b*c**5*x**5 + 4*sqrt(c**2*x**2 - 1)*b*c*
*3*x**3 - 3*sqrt(c**2*x**2 - 1)*b*c*x - 54*sqrt(c*x + 1)*sqrt(c*x - 1)*b*c
**3*x**3 - 6*a*c**6*x**6 + 54*a*c**4*x**4 + 54*a*c**2*x**2 - 6*a))/(18*x**
3)
```

3.28 $\int \frac{x^4(a+b\,\mathbf{arccosh}(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \frac{11b\sqrt{-1+cx}\sqrt{1+cx}}{9c^5d}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d} - \frac{x(a+b\text{arccosh}(cx))}{c^4d}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d} + \frac{2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{c^5d} + \frac{b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{c^5d}-\frac{b\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{c^5d}$$

output
```
11/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d+1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c^3/d-x*(a+b*arccosh(c*x))/c^4/d-1/3*x^3*(a+b*arccosh(c*x))/c^2/d+2*(
a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d+b*polylog
(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/c^5/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx =$$
$$-\frac{18acx+6ac^3x^3-18b\sqrt{\frac{-1+cx}{1+cx}}-18bcx\sqrt{\frac{-1+cx}{1+cx}}-4b\sqrt{-1+cx}\sqrt{1+cx}-2bc^2x^2\sqrt{-1+cx}\sqrt{1+cx}+\cdots}{\cdots}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/18*(18*a*c*x + 6*a*c^3*x^3 - 18*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 18*b*c*x
*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 18*b*c*x*ArcCosh[c*x] + 6*b*c^3*x^3*ArcC
osh[c*x] - 9*b*ArcCosh[c*x]^2 - 18*b*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x]
)] + 18*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 9*a*Log[1 - c*x] - 9*a*Lo
g[1 + c*x] + 18*b*PolyLog[2, -E^(-ArcCosh[c*x])] + 18*b*PolyLog[2, E^ArcCo
sh[c*x]])/(c^5*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {6353, 27, 111, 27, 83, 6353, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{\int \frac{x^2(a+b\text{arccosh}(cx))}{d(1-c^2x^2)}dx}{c^2}+\frac{b\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3cd}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d}+\frac{b\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3cd}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{111}$$

$$\frac{\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d}+\frac{b\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d}+\frac{b\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{83}$$

$$\frac{\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$$\Big\downarrow \text{6353}$$

$$\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2}+\frac{b\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{x(a+b\text{arccosh}(cx))}{c^2}}{c^2d}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$$\Big\downarrow \text{83}$$

$$\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}}{c^2d}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$$\Big\downarrow \text{6318}$$

$$\frac{-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}}{c^2d}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}}{c^2d}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$\big\downarrow$ 26

$$\frac{-\frac{i\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}}{c^2d}-\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$\big\downarrow$ 4670

$$\frac{-\frac{i\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)}{c^3}-\frac{x}{}}{c^2d}\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$\big\downarrow$ 2715

$$\frac{-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(\right.}{c^3}}{c^2d}\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

$\big\downarrow$ 2838

$$\frac{-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\text{ PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\text{ PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+}{c^2d}\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{3cd}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
(b*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(3*c^2)))/(3*c*d) - (x^3*(a + b*ArcCosh[c*x]))/(3*c^2*d) + ((b*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/c^3 - (x*(a + b*ArcCosh[c*x]))/c^2 - (I*((2*I)*
(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c
*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/c^3)/(c^2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.47

method	result
derivativedivides	$-\frac{a\left(\frac{c^3x^3}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{11b\sqrt{cx-1}\,\sqrt{cx+1}}{9d}-\frac{b\ \mathrm{arccosh}(cx)cx}{d}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{9d}-\frac{b\ \mathrm{arccosh}(cx)c^3x^3}{3d}+\frac{b\,\mathrm{pol}}{}$
default	$-\frac{a\left(\frac{c^3x^3}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{11b\sqrt{cx-1}\,\sqrt{cx+1}}{9d}-\frac{b\ \mathrm{arccosh}(cx)cx}{d}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{9d}-\frac{b\ \mathrm{arccosh}(cx)c^3x^3}{3d}+\frac{b\,\mathrm{pol}}{}$
parts	$-\frac{a\left(\frac{\frac{1}{3}c^2x^3+x}{c^4}-\frac{\ln(cx+1)}{2c^5}+\frac{\ln(cx-1)}{2c^5}\right)}{d}-\frac{b\ \mathrm{arccosh}(cx)x^3}{3d\,c^2}+\frac{b\ \mathrm{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d\,c^5}+\frac{11b\sqrt{cx-}}{9c}$

input
```
int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-a/d*(1/3*c^3*x^3+c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))+11/9*b/d*(c*x-1)
^(1/2)*(c*x+1)^(1/2)-b/d*arccosh(c*x)*c*x+1/9*b/d*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*c^2*x^2-1/3*b/d*arccosh(c*x)*c^3*x^3+b/d*polylog(2,-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))-b/d*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*arccosh(
c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*arccosh(c*x)*ln(1+c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\int -\frac{(b\operatorname{arcosh}(cx)+a)x^4}{c^2dx^2-d}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=-\frac{\int \frac{ax^4}{c^2x^2-1}\,dx+\int \frac{bx^4\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x**4/(c**2*x**2 - 1), x) + Integral(b*x**4*acosh(c*x)/(c**2*x
**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{d - c^2dx^2}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx) + a)x^4}{c^2dx^2 - d}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/72*(4*c^4*(2*(c^2*x^3 + 3*x)/(c^8*d) - 3*log(c*x + 1)/(c^9*d) + 3*log(c*
x - 1)/(c^9*d)) + 36*c^2*(2*x/(c^6*d) - log(c*x + 1)/(c^7*d) + log(c*x - 1
)/(c^7*d)) + 648*c*integrate(1/12*x*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x) -
 3*(4*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c*x - 1))*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) + 3*log(c*x + 1)^2 + 6*log(c*x + 1)*log(c*x - 1))
/(c^5*d) + 72*integrate(-1/6*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c
*x - 1))/(c^7*d*x^3 - c^5*d*x + (c^6*d*x^2 - c^4*d)*sqrt(c*x + 1)*sqrt(c*x
 - 1)), x) - 216*integrate(1/12*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x))*b -
1/6*a*(2*(c^2*x^3 + 3*x)/(c^4*d) - 3*log(c*x + 1)/(c^5*d) + 3*log(c*x - 1)
/(c^5*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{d - c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acosh}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \frac{-6\left(\int \frac{acosh(cx)x^4}{c^2x^2-1}dx\right)b\,c^5-3\log(c^2x-c)\,a+3\log(c^2x+c)\,a-2a\,c^3x^3-6acx}{6c^5d}$$

input
```
int(x^4*(a+b*acosh(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 6*int((acosh(c*x)*x**4)/(c**2*x**2 - 1),x)*b*c**5 - 3*log(c**2*x - c)*
a + 3*log(c**2*x + c)*a - 2*a*c**3*x**3 - 6*a*c*x)/(6*c**5*d)
```

3.29 $\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d}+\frac{b\text{arccosh}(cx)}{4c^4d}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{(a+b\text{arccosh}(cx))^2}{2bc^4d}-\frac{(a+b\text{arccosh}(cx))\log\left(1-e^{2\text{arccosh}(cx)}\right)}{c^4d}-\frac{b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2c^4d}$$

output
```
1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d+1/4*b*arccosh(c*x)/c^4/d-1/2*x^2
*(a+b*arccosh(c*x))/c^2/d+1/2*(a+b*arccosh(c*x))^2/b/c^4/d-(a+b*arccosh(c*
x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^4/d-1/2*b*polylog(2,(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^4/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx =$$
$$-\frac{2c^2x^2(a+b\text{arccosh}(cx)) - \frac{2(a+b\text{arccosh}(cx))^2}{b} - b\left(cx\sqrt{-1+cx}\sqrt{1+cx} + 2\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right) + 4(a$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/4*(2*c^2*x^2*(a + b*ArcCosh[c*x]) - (2*(a + b*ArcCosh[c*x])^2)/b - b*(c
*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]) +
 4*(a + b*ArcCosh[c*x])*Log[1 - E^ArcCosh[c*x]] + 4*(a + b*ArcCosh[c*x])*L
og[1 + E^ArcCosh[c*x]] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] + 4*b*PolyLog[2,
E^ArcCosh[c*x]])/(c^4*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {6353, 27, 101, 43, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{\int \frac{x(a+b\text{arccosh}(cx))}{d(1-c^2x^2)}dx}{c^2} + \frac{b\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2cd} - \frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d} + \frac{b\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2cd} - \frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}$$

$$\Big\downarrow \ 101$$

$$\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d}+\frac{b\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}$$

$$\Big\downarrow \ 43$$

$$\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \ 6328$$

$$-\frac{\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^4d}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4d}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \ 26$$

$$\frac{i\int (a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4d}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \ 4199$$

$$\frac{i\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \ 25$$

$$\frac{i\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \ 2620$$

$$\frac{i\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d}$$
$$\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$\Big\downarrow$ 2715

$$\frac{i\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d}$$
$$\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

$\Big\downarrow$ 2838

$$\frac{i\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d}-$$
$$\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2cd}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/2*(x^2*(a + b*ArcCosh[c*x]))/(c^2*d) + (b*((x*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/(2*c*d) + (I*(((-1/2*I)*(a + b*ArcCo
sh[c*x])^2)/b - (2*I)*(-1/2*((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x
])]) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/4)))/(c^4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{c^2x^2}{2}+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}+\frac{b\operatorname{arccosh}(cx)^2}{2d}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,cx}{4d}-\frac{b\operatorname{arccosh}(cx)c^2x^2}{2d}+\frac{b\operatorname{arccosh}(cx)}{4d}-\frac{b\operatorname{arccosh}(cx)\ln(1}{}$
default	$\frac{-\frac{a\left(\frac{c^2x^2}{2}+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}+\frac{b\operatorname{arccosh}(cx)^2}{2d}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,cx}{4d}-\frac{b\operatorname{arccosh}(cx)c^2x^2}{2d}+\frac{b\operatorname{arccosh}(cx)}{4d}-\frac{b\operatorname{arccosh}(cx)\ln(1}{}$
parts	$-\frac{a\,x^2}{2d\,c^2}-\frac{a\ln(c^2x^2-1)}{2d\,c^4}+\frac{b\operatorname{arccosh}(cx)^2}{2d\,c^4}-\frac{b\operatorname{arccosh}(cx)x^2}{2d\,c^2}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{4c^3d}+\frac{b\operatorname{arccosh}(cx)}{4c^4d}-\frac{b\operatorname{arcc}}{}$

input
```
int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-a/d*(1/2*c^2*x^2+1/2*ln(c*x-1)+1/2*ln(c*x+1))+1/2*b/d*arccosh(c*x)
^2+1/4*b/d*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x-1/2*b/d*arccosh(c*x)*c^2*x^2+1/
4*b/d*arccosh(c*x)-b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-
b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*arccosh(c*x)*ln(1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx)+a)x^3}{c^2dx^2-d}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x^3*arccosh(c*x) + a*x^3)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = -\frac{\int \frac{ax^3}{c^2x^2-1}\,dx + \int \frac{bx^3\,\text{acosh}\,(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x**3/(c**2*x**2 - 1), x) + Integral(b*x**3*acosh(c*x)/(c**2*x
**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{d - c^2dx^2} \, dx = \int -\frac{(b \operatorname{arcosh}(cx) + a)x^3}{c^2dx^2 - d} \, dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) + 1/8*b*((2*c^2*x^2 - 4*(c
^2*x^2 + log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)) + 2*(log(c*x - 1) + 1)*log(c*x + 1) + log(c*x + 1)^2 + log(c*x - 1)^2 +
 2*log(c*x - 1))/(c^4*d) - 8*integrate(1/2*(c^2*x^2 + log(c*x + 1) + log(c
*x - 1))/(c^6*d*x^3 - c^4*d*x + (c^5*d*x^2 - c^3*d)*e^(1/2*log(c*x + 1) +
1/2*log(c*x - 1))), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{d - c^2dx^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acosh}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \frac{-2\left(\int \frac{acosh(cx)x^3}{c^2x^2-1}dx\right)b\,c^4-\log(c^2x-c)\,a-\log(c^2x+c)\,a-a\,c^2x^2}{2c^4d}$$

input
```
int(x^3*(a+b*acosh(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int((acosh(c*x)*x**3)/(c**2*x**2 - 1),x)*b*c**4 - log(c**2*x - c)*a
- log(c**2*x + c)*a - a*c**2*x**2)/(2*c**4*d)
```

3.30 $\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^3d}-\frac{x(a+b\text{arccosh}(cx))}{c^2d}+\frac{2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{c^3d}+\frac{b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{c^3d}-\frac{b\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{c^3d}$$

output
```
b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d-x*(a+b*arccosh(c*x))/c^2/d+2*(a+b*arcc
osh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d+b*polylog(2,-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/c^3/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$$
$$= \frac{-2acx+2b\sqrt{\frac{-1+cx}{1+cx}}+2bcx\sqrt{\frac{-1+cx}{1+cx}}-2bcx\text{arccosh}(cx)+b\text{arccosh}(cx)^2+2b\text{arccosh}(cx)\log\left(1+e^{-\text{arcco}}\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
(-2*a*c*x + 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*b*c*x*Sqrt[(-1 + c*x)/(1 +
c*x)] - 2*b*c*x*ArcCosh[c*x] + b*ArcCosh[c*x]^2 + 2*b*ArcCosh[c*x]*Log[1 +
 E^(-ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] - a*Log[1 -
 c*x] + a*Log[1 + c*x] - 2*b*PolyLog[2, -E^(-ArcCosh[c*x])] - 2*b*PolyLog[
2, E^ArcCosh[c*x]])/(2*c^3*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {6353, 27, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \text{ 6353}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{d(1-c^2x^2)}dx}{c^2}+\frac{b\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{cd}-\frac{x(a+b\text{arccosh}(cx))}{c^2d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2d}+\frac{b\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{cd}-\frac{x(a+b\text{arccosh}(cx))}{c^2d}$$

$$\Big\downarrow \text{83}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2d} - \frac{x(a+b\text{arccosh}(cx))}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3d}$$

$$\Big\downarrow \text{6318}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^3d} - \frac{x(a+b\text{arccosh}(cx))}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3d} - \frac{x(a+b\text{arccosh}(cx))}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3d} - \frac{x(a+b\text{arccosh}(cx))}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3d}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx) - ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx) + 2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+}{c^3d} \frac{x(a+b\text{arccosh}(cx))}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3d}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)} - ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)} + 2i\text{arct}}{c^3d} \frac{x(a+b\text{arccosh}(cx))}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3d}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx)) + ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right) - ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{c^3d} - \frac{x(a+b\text{arccosh}(cx))}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3d}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^2*d
) - (I*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2
, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/(c^3*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{-\frac{a\left(cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d}-\frac{b\,\operatorname{arccosh}(cx)\ln\left(1-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\,\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx}\cdots\right)}{d}}{c^3}$
default	$\frac{-\frac{a\left(cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d}-\frac{b\,\operatorname{arccosh}(cx)\ln\left(1-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\,\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx}\cdots\right)}{d}}{c^3}$
parts	$-\frac{a\left(\frac{x}{c^2}-\frac{\ln(cx+1)}{2c^3}+\frac{\ln(cx-1)}{2c^3}\right)}{d}-\frac{b\,\operatorname{arccosh}(cx)x}{d\,c^2}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^3d}+\frac{b\,\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d\,c^3}$

input
```
int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a/d*(c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))+b/d*(c*x-1)^(1/2)*(c*x+1)^(1
/2)-b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*arccosh(c*x
)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*arccosh(c*x)*c*x-b/d*polylog(2
,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{d - c^2dx^2}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx) + a)x^2}{c^2dx^2 - d}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{d - c^2dx^2}\,dx = -\frac{\int \frac{ax^2}{c^2x^2-1}\,dx + \int \frac{bx^2\,\text{acosh}\,(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x**2/(c**2*x**2 - 1), x) + Integral(b*x**2*acosh(c*x)/(c**2*x
**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx)+a)x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/8*(4*c^2*(2*x/(c^4*d) - log(c*x + 1)/(c^5*d) + log(c*x - 1)/(c^5*d)) + 2
4*c*integrate(1/4*x*log(c*x - 1)/(c^4*d*x^2 - c^2*d), x) - (4*(2*c*x - log
(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + log(c*x
 + 1)^2 + 2*log(c*x + 1)*log(c*x - 1))/(c^3*d) + 8*integrate(-1/2*(2*c*x -
 log(c*x + 1) + log(c*x - 1))/(c^5*d*x^3 - c^3*d*x + (c^4*d*x^2 - c^2*d)*s
qrt(c*x + 1)*sqrt(c*x - 1)), x) - 8*integrate(1/4*log(c*x - 1)/(c^4*d*x^2
- c^2*d), x))*b - 1/2*a*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)
/(c^3*d))
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx)+a)x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \int \frac{x^2\,(a+b\,\text{acosh}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \frac{-2\left(\int \frac{acosh(cx)x^2}{c^2x^2-1}dx\right)b\,c^3-\log(c^2x-c)\,a+\log(c^2x+c)\,a-2acx}{2c^3d}$$

input
```
int(x^2*(a+b*acosh(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int((acosh(c*x)*x**2)/(c**2*x**2 - 1),x)*b*c**3 - log(c**2*x - c)*a
+ log(c**2*x + c)*a - 2*a*c*x)/(2*c**3*d)
```

3.31 $\int \frac{x(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{x(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \frac{(a+b\text{arccosh}(cx))^2}{2bc^2d} - \frac{(a+b\text{arccosh}(cx))\log\left(1-e^{2\text{arccosh}(cx)}\right)}{c^2d} - \frac{b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2c^2d}$$

output
```
1/2*(a+b*arccosh(c*x))^2/b/c^2/d-(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)/c^2/d-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2)/c^2/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{x(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \frac{(a+b\text{arccosh}(cx))\left(a+b\text{arccosh}(cx)-2b\log\left(1-e^{\text{arccosh}(cx)}\right)-2b\log\left(1+e^{\text{arccosh}(cx)}\right)\right)-2b^2\,\text{PolyLog}}{2bc^2d}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
((a + b*ArcCosh[c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] -
2*b*Log[1 + E^ArcCosh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*P
olyLog[2, E^ArcCosh[c*x]])/(2*b*c^2*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{d-c^2dx^2}dx$$

$$\Big\downarrow \text{ 6328}$$

$$-\frac{\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^2d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^2d}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^2d}$$

$$\Big\downarrow \text{ 4199}$$

$$\frac{i\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^2d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^2d}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{i\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right.}{c^2d}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{i\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\right.}{c^2d}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{i\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^2d}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
(I*(((-1/2*I)*(a + b*ArcCosh[c*x])^2)/b - (2*I)*(-1/2*((a + b*ArcCosh[c*x]
)*Log[1 - E^(2*ArcCosh[c*x])]) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/4)))/(
c^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.85

method	result
parts	$-\frac{a\ln(c^2x^2-1)}{2d\,c^2}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)^2}{2}+\operatorname{arccosh}(cx)\ln(1+cx+\sqrt{cx-1}\,\sqrt{cx+1})+\operatorname{polylog}(2,-cx-\sqrt{cx-1}\,\sqrt{cx+1})+\operatorname{arc}\right.}{d\,c^2}$
derivativedivides	$\frac{-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)^2}{2}+\operatorname{arccosh}(cx)\ln(1+cx+\sqrt{cx-1}\,\sqrt{cx+1})+\operatorname{polylog}(2,-cx-\sqrt{cx-1}\,\sqrt{cx+1})+\operatorname{arccosh}(\right.}{d}}{c^2}$
default	$\frac{-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)^2}{2}+\operatorname{arccosh}(cx)\ln(1+cx+\sqrt{cx-1}\,\sqrt{cx+1})+\operatorname{polylog}(2,-cx-\sqrt{cx-1}\,\sqrt{cx+1})+\operatorname{arccosh}(\right.}{d}}{c^2}$

input
```
int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/c^2*ln(c^2*x^2-1)-b/d/c^2*(-1/2*arccosh(c*x)^2+arccosh(c*x)*ln(1+
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))+arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+polylog(2,c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{d-c^2dx^2}\,dx=\int -\frac{(b\operatorname{arcosh}(cx)+a)x}{c^2dx^2-d}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x*arccosh(c*x) + a*x)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{d-c^2dx^2}\,dx = -\frac{\int \frac{ax}{c^2x^2-1}\,dx + \int \frac{bx\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x/(c**2*x**2 - 1), x) + Integral(b*x*acosh(c*x)/(c**2*x**2 -
1), x))/d
```

Maxima [F]

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\operatorname{arcosh}(cx)+a)x}{c^2dx^2-d}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/8*b*((4*(log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) - log(c*x - 1)^2)/(c^
2*d) + 8*integrate(1/2*(log(c*x + 1) + log(c*x - 1))/(c^4*d*x^3 - c^2*d*x
+ (c^3*d*x^2 - c*d)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) - 1/2*a*
log(c^2*d*x^2 - d)/(c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\operatorname{arcosh}(cx)+a)x}{c^2dx^2-d}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\int \frac{x\,(a+b\,\mathrm{acosh}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\frac{-2\left(\int \frac{acosh(cx)x}{c^2x^2-1}dx\right)b\,c^2-\log(c^2x-c)\,a-\log(c^2x+c)\,a}{2c^2d}$$

input
```
int(x*(a+b*acosh(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int((acosh(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2 - log(c**2*x - c)*a - l
og(c**2*x + c)*a)/(2*c**2*d)
```

3.32 $\int \frac{a+b\text{arccosh}(cx)}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{a+b\text{arccosh}(cx)}{d-c^2dx^2}\,dx=\frac{2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{cd}+\frac{b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{cd}-\frac{b\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{cd}$$

output
```
2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d+b*polylo
g(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/c/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{arccosh}(cx)}{d-c^2dx^2}\,dx$$
$$=\frac{-\left((a+b\text{arccosh}(cx))\left(\log\left(1-e^{\text{arccosh}(cx)}\right)-\log\left(1+e^{\text{arccosh}(cx)}\right)\right)\right)+b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-b\,\text{Po}}{cd}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
```

output
```
(-((a + b*ArcCosh[c*x])*(Log[1 - E^ArcCosh[c*x]] - Log[1 + E^ArcCosh[c*x]]
)) + b*PolyLog[2, -E^ArcCosh[c*x]] - b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{d-c^2dx^2}dx$$

$$\Big\downarrow \text{6318}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{cd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{cd}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{cd}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arct}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(2i\operatorname{arctanh}\left(e^{\operatorname{arccosh}(cx)}\right)(a+b\operatorname{arccosh}(cx))+ib\operatorname{PolyLog}\left(2,-e^{\operatorname{arccosh}(cx)}\right)-ib\operatorname{PolyLog}\left(2,e^{\operatorname{arccosh}(cx)}\right)\right)}{cd}$$

input
```
Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
```

output
```
((-I)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2,
 -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/(c*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

method	result
derivativedivides	$\frac{\frac{a\,\mathrm{arctanh}(cx)}{d}-\frac{b\left(-\mathrm{arctanh}(cx)\,\mathrm{arccosh}(cx)-\frac{2i\left(\mathrm{arctanh}(cx)\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\mathrm{arctanh}(cx)\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)+\mathrm{dilog}\left(1+\right.\right.}{c^2x^2-1}\right.}{d}}{c}$
default	$\frac{\frac{a\,\mathrm{arctanh}(cx)}{d}-\frac{b\left(-\mathrm{arctanh}(cx)\,\mathrm{arccosh}(cx)-\frac{2i\left(\mathrm{arctanh}(cx)\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\mathrm{arctanh}(cx)\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)+\mathrm{dilog}\left(1+\right.\right.}{c^2x^2-1}\right.}{d}}{c}$
parts	$\frac{a\ln(cx+1)}{2dc}-\frac{a\ln(cx-1)}{2dc}-\frac{b\left(-\mathrm{arctanh}(cx)\,\mathrm{arccosh}(cx)-\frac{2i\left(\mathrm{arctanh}(cx)\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\mathrm{arctanh}(cx)\ln\left(1-\right.\right.}{}\right.}{dc}$

input
```
int((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d*arctanh(c*x)-b/d*(-arctanh(c*x)*arccosh(c*x)-2*I*(arctanh(c*x)*ln
(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^
(1/2))+dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-dilog(1-I*(c*x+1)/(-c^2*x^2+1
)^(1/2)))*(-c^2*x^2+1)^(1/2)*(1/2*c*x+1/2)^(1/2)*(1/2*c*x-1/2)^(1/2)/(c^2*
x^2-1)))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{d - c^2dx^2} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{c^2dx^2 - d} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{d - c^2dx^2} \, dx = -\frac{\int \frac{a}{c^2x^2-1} \, dx + \int \frac{b \operatorname{acosh}(cx)}{c^2x^2-1} \, dx}{d}$$

input
```
integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**2 - 1), x) + Integral(b*acosh(c*x)/(c**2*x**2 - 1),
x))/d
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{d - c^2dx^2} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{c^2dx^2 - d} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/8*b*((4*(log(c*x + 1) - log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1))/(c*d) + 8*integrate(1
/4*(3*c*x - 1)*log(c*x - 1)/(c^2*d*x^2 - d), x) + 8*integrate(1/2*(log(c*x
 + 1) - log(c*x - 1))/(c^3*d*x^3 - c*d*x + (c^2*d*x^2 - d)*sqrt(c*x + 1)*s
qrt(c*x - 1)), x)) + 1/2*a*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d))
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{d-c^2dx^2}\,dx=\int -\frac{b\,\text{arcosh}\,(cx)+a}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{d-c^2dx^2}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2),x)
```

output
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{d-c^2dx^2}\,dx=\frac{-2\left(\int \frac{acosh(cx)}{c^2x^2-1}dx\right)bc-\log(c^2x-c)\,a+\log(c^2x+c)\,a}{2cd}$$

input
```
int((a+b*acosh(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acosh(c*x)/(c**2*x**2 - 1),x)*b*c - log(c**2*x - c)*a + log(c**2
*x + c)*a)/(2*c*d)
```

3.33 $\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)}\,dx = \frac{2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)}{d} + \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2d}$$

output
```
2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*b*
polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-1/2*b*polylog(2,(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(61) = 122.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.11

$$\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)}\,dx = -\frac{-b\text{arccosh}(cx)^2 - 2b\text{arccosh}(cx)\log\left(1+e^{-2\text{arccosh}(cx)}\right) + 2b\text{arccosh}(cx)\log\left(1+e^{-\text{arccosh}(cx)}\right) + 2b\text{arc}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

output
```
-1/2*(-(b*ArcCosh[c*x]^2) - 2*b*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])]
+ 2*b*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 2*b*ArcCosh[c*x]*Log[1 - E
^ArcCosh[c*x]] - 2*a*Log[x] + a*Log[1 - c^2*x^2] + b*PolyLog[2, -E^(-2*Arc
Cosh[c*x])] - 2*b*PolyLog[2, -E^(-ArcCosh[c*x])] + 2*b*PolyLog[2, E^ArcCos
h[c*x]])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)}dx$$

$$\Big\downarrow \text{6331}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d}$$

$$\Big\downarrow \text{5984}$$

$$-\frac{2\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2i\int (a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right.}{d}$$

$\Big\downarrow$ 2715

$$-\frac{2i\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx}\right.}{d}$$

$\Big\downarrow$ 2838

$$-\frac{2i\left(i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)}{d}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

output
```
((-2*I)*(I*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])] + (I/4)*b*Poly
Log[2, -E^(2*ArcCosh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcCosh[c*x])]))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.13

method	result
parts	$-\frac{a\left(-\ln(x)+\frac{\ln(cx+1)}{2}+\frac{\ln(cx-1)}{2}\right)}{d}-\frac{b\left(\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)-\right.}{}$
derivativedivides	$-\frac{a\left(-\ln(cx)+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)\right.}{}$
default	$-\frac{a\left(-\ln(cx)+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)\right.}{}$

input
```
int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d*(-ln(x)+1/2*ln(c*x+1)+1/2*ln(c*x-1))-b/d*(arccosh(c*x)*ln(1+c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))+polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-arccos
h(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)+arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))+polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x\,(d - c^2dx^2)}\,dx = \int -\frac{b\,\text{arcosh}\,(cx) + a}{(c^2dx^2 - d)x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x\,(d - c^2dx^2)}\,dx = -\frac{\int \frac{a}{c^2x^3-x}\,dx + \int \frac{b\,\text{acosh}\,(cx)}{c^2x^3-x}\,dx}{d}$$

input
```
integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**3 - x), x) + Integral(b*acosh(c*x)/(c**2*x**3 - x),
x))/d
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x\,(d - c^2dx^2)}\,dx = \int -\frac{b\,\text{arcosh}\,(cx) + a}{(c^2dx^2 - d)x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - b*integrate(log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^3 - d*x), x)
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x\left(d - c^2dx^2\right)}\,dx = \int -\frac{b\operatorname{arcosh}\left(cx\right) + a}{(c^2dx^2 - d)x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x\left(d - c^2dx^2\right)}\,dx = \int \frac{a + b\operatorname{acosh}(c\,x)}{x\,\left(d - c^2\,d\,x^2\right)}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x\left(d - c^2dx^2\right)}\,dx = \frac{-2\left(\int \frac{acosh(cx)}{c^2x^3-x}dx\right)b - \log(c^2x - c)\,a - \log(c^2x + c)\,a + 2\log(x)\,a}{2d}$$

input
```
int((a+b*acosh(c*x))/x/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acosh(c*x)/(c**2*x**3 - x),x)*b - log(c**2*x - c)*a - log(c**2*x
 + c)*a + 2*log(x)*a)/(2*d)
```

3.34 $\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)}\,dx = -\frac{a+b\text{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{2c(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{d} + \frac{bc\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{d}-\frac{bc\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{d}$$

output

```
-(a+b*arccosh(c*x))/d/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d+2*c*(a+b
*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d+b*c*polylog(2,-c
*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d-b*c*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx$$
$$=\frac{-\frac{a+b\text{arccosh}(cx)}{x}+\frac{bc\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}-c(a+b\text{arccosh}(cx))\log\left(1-e^{\text{arccosh}(cx)}\right)+c(a+b\text{arccos}}{d}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

output
```
(-((a + b*ArcCosh[c*x])/x) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*
x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - c*(a + b*ArcCosh[c*x])*Log[1 - E^A
rcCosh[c*x]] + c*(a + b*ArcCosh[c*x])*Log[1 + E^ArcCosh[c*x]] + b*c*PolyLo
g[2, -E^ArcCosh[c*x]] - b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6347, 27, 103, 218, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx$$
$$\Big\downarrow \; 6347$$
$$c^2\int \frac{a+b\text{arccosh}(cx)}{d\left(1-c^2x^2\right)}dx+\frac{bc\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{d}-\frac{a+b\text{arccosh}(cx)}{dx}$$
$$\Big\downarrow \; 27$$
$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{d}+\frac{bc\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{d}-\frac{a+b\text{arccosh}(cx)}{dx}$$

$$\Big\downarrow \ 103$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{d}+\frac{bc^2\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}-\frac{a+b\text{arccosh}(cx)}{dx}$$

$$\Big\downarrow \ 218$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{d}-\frac{a+b\text{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

$$\Big\downarrow \ 6318$$

$$-\frac{c\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d}-\frac{a+b\text{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$-\frac{c\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d}-\frac{a+b\text{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

$$\Big\downarrow \ 26$$

$$-\frac{ic\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d}-\frac{a+b\text{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

$$\Big\downarrow \ 4670$$

$$-\frac{ic\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a-}{d}$$
$$\frac{a+b\text{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

$$\Big\downarrow \ 2715$$

$$-\frac{ic\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arc}}{d}$$
$$\frac{a+b\text{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

$$\Big\downarrow \ 2838$$

$$-\frac{ic\left(2i\operatorname{arctanh}\left(e^{\operatorname{arccosh}(cx)}\right)(a+b\operatorname{arccosh}(cx))+ib\operatorname{PolyLog}\left(2,-e^{\operatorname{arccosh}(cx)}\right)-ib\operatorname{PolyLog}\left(2,e^{\operatorname{arccosh}(cx)}\right)\right)}{d}-\frac{a+b\operatorname{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

output
```
-((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
/d - (I*c*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLo
g[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.51

method	result
parts	$-\frac{a\left(-\frac{c\ln(cx+1)}{2}+\frac{c\ln(cx-1)}{2}+\frac{1}{x}\right)}{d}-\frac{bc\left(\frac{\operatorname{arccosh}(cx)}{cx}-2\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-\operatorname{dilog}\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-\mathrm{dil}\right.}{d}$
derivativedivides	$c\left(-\frac{a\left(\frac{1}{cx}+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{\operatorname{arccosh}(cx)}{cx}-2\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-\operatorname{dilog}\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-\mathrm{dilo}\right.}{d}\right.$
default	$c\left(-\frac{a\left(\frac{1}{cx}+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{\operatorname{arccosh}(cx)}{cx}-2\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-\operatorname{dilog}\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-\mathrm{dilo}\right.}{d}\right.$

input
```
int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d*(-1/2*c*ln(c*x+1)+1/2*c*ln(c*x-1)+1/x)-b/d*c*(arccosh(c*x)/c/x-2*arct
an(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
-dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-arccosh(c*x)*ln(1+c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx=\int -\frac{b\operatorname{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arccosh}(cx)}{x^2 (d - c^2dx^2)}\,dx = -\frac{\int \frac{a}{c^2x^4 - x^2}\,dx + \int \frac{b\operatorname{acosh}(cx)}{c^2x^4 - x^2}\,dx}{d}$$

input
```
integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**4 - x**2), x) + Integral(b*acosh(c*x)/(c**2*x**4 - x
**2), x))/d
```

Maxima [F]

$$\int \frac{a + b\operatorname{arccosh}(cx)}{x^2 (d - c^2dx^2)}\,dx = \int -\frac{b\operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/8*(24*c^3*integrate(1/4*x*log(c*x - 1)/(c^2*d*x^2 - d), x) - 4*c^2*(log(
c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 8*c^2*integrate(1/4*log(c*x - 1)/(c
^2*d*x^2 - d), x) - (c*x*log(c*x + 1)^2 + 2*c*x*log(c*x + 1)*log(c*x - 1)
- 4*(c*x*log(c*x + 1) - c*x*log(c*x - 1) - 2)*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)))/(d*x) + 8*integrate(1/2*(c^2*x*log(c*x + 1) - c^2*x*log(c*x -
1) - 2*c)/(c^3*d*x^4 - c*d*x^2 + (c^2*d*x^3 - d*x)*sqrt(c*x + 1)*sqrt(c*x
- 1)), x))*b + 1/2*a*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x))
```

Giac [F]

$$\int \frac{a + b\operatorname{arccosh}(cx)}{x^2 (d - c^2dx^2)}\,dx = \int -\frac{b\operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2dx^2)} \, dx = \int \frac{a + b \operatorname{acosh}(c\, x)}{x^2 \, (d - c^2 \, d \, x^2)} \, dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2dx^2)} \, dx = \frac{-2\left(\int \frac{acosh(cx)}{c^2x^4 - x^2} dx\right) bx - \log(c^2x - c) \, acx + \log(c^2x + c) \, acx - 2a}{2dx}$$

input
```
int((a+b*acosh(c*x))/x^2/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acosh(c*x)/(c**2*x**4 - x**2),x)*b*x - log(c**2*x - c)*a*c*x + l
og(c**2*x + c)*a*c*x - 2*a)/(2*d*x)
```

3.35 $\int \frac{a+b\,\textbf{arccosh}(cx)}{x^3(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)}\,dx = \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\text{arccosh}(cx)}{2dx^2} + \frac{2c^2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)}{d} + \frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{2d} - \frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2d}$$

output

```
1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x-1/2*(a+b*arccosh(c*x))/d/x^2+2*c^2
*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*b*c
^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-1/2*b*c^2*polylog(2,(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.80

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)}\,dx =$$
$$-\frac{a-bcx\sqrt{-1+cx}\sqrt{1+cx}+b\text{arccosh}(cx)-2bc^2x^2\text{arccosh}(cx)^2-2bc^2x^2\text{arccosh}(cx)\log\left(1+e^{-2\text{arcco}}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

output
```
-1/2*(a - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*ArcCosh[c*x] - 2*b*c^2*x^
2*ArcCosh[c*x]^2 - 2*b*c^2*x^2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] +
 2*b*c^2*x^2*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 2*b*c^2*x^2*ArcCosh[c*
x]*Log[1 + E^ArcCosh[c*x]] - 2*a*c^2*x^2*Log[x] + a*c^2*x^2*Log[1 - c^2*x^
2] + b*c^2*x^2*PolyLog[2, -E^(-2*ArcCosh[c*x])] + 2*b*c^2*x^2*PolyLog[2, -
E^ArcCosh[c*x]] + 2*b*c^2*x^2*PolyLog[2, E^ArcCosh[c*x]])/(d*x^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6347, 27, 106, 6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)}dx$$

$$\Big\downarrow \text{6347}$$

$$c^2\int \frac{a+b\text{arccosh}(cx)}{dx\left(1-c^2x^2\right)}dx+\frac{bc\int \frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}}dx}{2d}-\frac{a+b\text{arccosh}(cx)}{2dx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d}+\frac{bc\int \frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}}dx}{2d}-\frac{a+b\text{arccosh}(cx)}{2dx^2}$$

$$\Big\downarrow \text{106}$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d}-\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

$$\Big\downarrow \text{6331}$$

$$-\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d}-\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

$$\Big\downarrow \text{5984}$$

$$-\frac{2c^2\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx)}{d}-\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2c^2\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d}-\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2ic^2\int (a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d}-\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{2ic^2\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+i\text{arctanh}\left(e^{2\text{arccosh}(c}\right.\right.}{d}$$
$$\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2ic^2\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(\right.}{d}$$
$$\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2ic^2\left(i\operatorname{arctanh}\left(e^{2\operatorname{arccosh}(cx)}\right)(a+b\operatorname{arccosh}(cx))+\frac{1}{4}ib\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)-\frac{1}{4}ib\operatorname{PolyLog}\left(2,e^{2\operatorname{arccosh}(cx)}\right)\right)}{d}$$
$$\frac{a+b\operatorname{arccosh}(cx)}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

output
```
(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2
) - ((2*I)*c^2*(I*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])] + (I/4)
*b*PolyLog[2, -E^(2*ArcCosh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcCosh[c*x])
]))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.06

method	result
derivativedivides	$c^2\left(-\frac{a\left(\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^2x^2}+\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{}\right.\right.}{}\right.$
default	$c^2\left(-\frac{a\left(\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^2x^2}+\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{}\right.\right.}{}\right.$
parts	$-\frac{a\left(\frac{1}{2x^2}-c^2\ln(x)+\frac{c^2\ln(cx+1)}{2}+\frac{c^2\ln(cx-1)}{2}\right)}{d}-\frac{b\,c^2\left(\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^2x^2}+\operatorname{arccosh}(cx)\ln\left(1+cx+\right.\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-a/d*(1/2/c^2/x^2-ln(c*x)+1/2*ln(c*x-1)+1/2*ln(c*x+1))-b/d*(1/2*(-(c*
x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2+arccosh(c*x))/c^2/x^2+arccosh(c*x)*ln
(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(
1/2))-arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*polylog(2
,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))+polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)}\,dx=\int -\frac{b\operatorname{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)}\,dx = -\frac{\int \frac{a}{c^2x^5-x^3}\,dx + \int \frac{b\,\text{acosh}\,(cx)}{c^2x^5-x^3}\,dx}{d}$$

input
```
integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**5 - x**3), x) + Integral(b*acosh(c*x)/(c**2*x**5 - x
**3), x))/d
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)}\,dx = \int -\frac{b\,\text{arcosh}\,(cx)+a}{(c^2dx^2-d)x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2)
)*a - b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^5 - d*x^
3), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)}\,dx = \int -\frac{b\,\text{arcosh}\,(cx)+a}{(c^2dx^2-d)x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x^3\,\left(d-c^2\,d\,x^2\right)}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)}\,dx$$
$$=\frac{-2\left(\int \frac{acosh(cx)}{c^2x^5-x^3}dx\right)b\,x^2-\log(c^2x-c)\,a\,c^2x^2-\log(c^2x+c)\,a\,c^2x^2+2\log(x)\,a\,c^2x^2-a}{2d\,x^2}$$

input
```
int((a+b*acosh(c*x))/x^3/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acosh(c*x)/(c**2*x**5 - x**3),x)*b*x**2 - log(c**2*x - c)*a*c**2
*x**2 - log(c**2*x + c)*a*c**2*x**2 + 2*log(x)*a*c**2*x**2 - a)/(2*d*x**2)
```

3.36 $\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$
\begin{aligned}
\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)}\,dx &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\text{arccosh}(cx)}{3dx^3} \\
&\quad -\frac{c^2(a+b\text{arccosh}(cx))}{dx}+\frac{7bc^3\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d} \\
&\quad +\frac{2c^3(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{d} \\
&\quad +\frac{bc^3\operatorname{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{d}-\frac{bc^3\operatorname{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{d}
\end{aligned}
$$

output

```
1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2-1/3*(a+b*arccosh(c*x))/d/x^3-c^2
*(a+b*arccosh(c*x))/d/x+7/6*b*c^3*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d+2*
c^3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d+b*c^3*po
lylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d-b*c^3*polylog(2,c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.42

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\,(d-c^2dx^2)}\,dx$$

$$= \frac{-\frac{2a}{x^3}-\frac{6ac^2}{x}+\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{x^2}-\frac{2b\text{arccosh}(cx)}{x^3}-\frac{6bc^2\text{arccosh}(cx)}{x}+\frac{7bc^3\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}-6ac^3\log\left(\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

output
```
((-2*a)/x^3 - (6*a*c^2)/x + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x^2 - (2*b*
ArcCosh[c*x])/x^3 - (6*b*c^2*ArcCosh[c*x])/x + (7*b*c^3*Sqrt[-1 + c^2*x^2]
*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*a*c^3*Log[
1 - E^ArcCosh[c*x]] - 6*b*c^3*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 6*a*c
^3*Log[1 + E^ArcCosh[c*x]] + 6*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]]
+ 6*b*c^3*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c^3*PolyLog[2, E^ArcCosh[c*x]]
)/(6*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6347, 27, 114, 27, 103, 218, 6347, 103, 218, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\,(d-c^2dx^2)}\,dx$$

$$\Big\downarrow \text{6347}$$

$$c^2\int \frac{a+b\text{arccosh}(cx)}{dx^2\,(1-c^2x^2)}dx+\frac{bc\int \frac{1}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx}{3d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)}dx}{d}+\frac{bc\int \frac{1}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx}{3d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}$$

$$\Big\downarrow \text{114}$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)}dx}{d}+\frac{bc\left(\frac{1}{2}\int \frac{c^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)}dx}{d}+\frac{bc\left(\frac{1}{2}c^2\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}$$

$$\Big\downarrow \text{103}$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)}dx}{d}+\frac{bc\left(\frac{1}{2}c^3\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)}dx}{d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{6347}$$

$$\frac{c^2\left(c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{a+b\text{arccosh}(cx)}{x}\right)}{d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{103}$$

$$\frac{c^2\left(c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+bc^2\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{218}$$

$$\frac{c^2\left(c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)}{d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{6318}$$

$$\frac{c^2\left(-c\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)}{d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\left(-c\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)}{d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{c^2\left(-ic\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)}{d}-\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{4670}$$

$$\frac{c^2\left(-ic\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)\right.\right.}{d}$$

$$\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{2715}$$

$$\frac{c^2\left(-ic\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2\right.\right.}{d}$$

$$\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

$$\Big\downarrow \text{2838}$$

$$\frac{c^2\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)-\right.}{d}$$

$$\frac{a+b\text{arccosh}(cx)}{3dx^3}+\frac{bc\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)}{3d}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

output
```
-1/3*(a + b*ArcCosh[c*x])/(d*x^3) + (b*c*((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(
2*x^2) + (c^2*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/2))/(3*d) + (c^2*(-((a
 + b*ArcCosh[c*x])/x) + b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]] - I*c*((2
*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCo
sh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.21

method	result
derivativedivides	$c^{3}\left(-\frac{a\left(\frac{1}{3c^{3}x^{3}}+\frac{1}{cx}+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{6c^{2}x^{2}\operatorname{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+2\operatorname{arccosh}(cx)}{6c^{3}x^{3}}-\frac{7\arctan(cx+\sqrt{cx-}}{3}\right.}{}\right.$
default	$c^{3}\left(-\frac{a\left(\frac{1}{3c^{3}x^{3}}+\frac{1}{cx}+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{6c^{2}x^{2}\operatorname{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+2\operatorname{arccosh}(cx)}{6c^{3}x^{3}}-\frac{7\arctan(cx+\sqrt{cx-}}{3}\right.}{}\right.$
parts	$-\frac{a\left(\frac{1}{3x^{3}}+\frac{c^{2}}{x}-\frac{c^{3}\ln(cx+1)}{2}+\frac{c^{3}\ln(cx-1)}{2}\right)}{d}-\frac{b\,c^{3}\left(\frac{6c^{2}x^{2}\operatorname{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+2\operatorname{arccosh}(cx)}{6c^{3}x^{3}}-\frac{7\arctan(cx+\sqrt{cx-}}{3}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-a/d*(1/3/c^3/x^3+1/c/x+1/2*ln(c*x-1)-1/2*ln(c*x+1))-b/d*(1/6*(6*c^2*
x^2*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+2*arccosh(c*x))/c^3/x^3-7
/3*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-dilog(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))-dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-arccosh(c*x)*ln(1+c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\,(d-c^2dx^2)}\,dx=\int -\frac{b\,\text{arcosh}\,(cx)+a}{(c^2dx^2-d)x^4}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)}\,dx = -\frac{\int \frac{a}{c^2x^6 - x^4}\,dx + \int \frac{b\,\text{acosh}\,(cx)}{c^2x^6 - x^4}\,dx}{d}$$

input
```
integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**6 - x**4), x) + Integral(b*acosh(c*x)/(c**2*x**6 - x
**4), x))/d
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)}\,dx = \int -\frac{b\,\text{arcosh}\,(cx) + a}{(c^2dx^2 - d)x^4}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^
3))*a + 1/24*(216*c^5*integrate(1/12*x^3*log(c*x - 1)/(c^2*d*x^4 - d*x^2),
 x) - 12*c^4*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 72*c^4*integrate(
1/12*x^2*log(c*x - 1)/(c^2*d*x^4 - d*x^2), x) - 4*c^2*(c*log(c*x + 1)/d -
c*log(c*x - 1)/d - 2/(d*x)) - (3*c^3*x^3*log(c*x + 1)^2 + 6*c^3*x^3*log(c*
x + 1)*log(c*x - 1) - 4*(3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(c*x - 1) -
 6*c^2*x^2 - 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(d*x^3) + 24*integ
rate(1/6*(3*c^4*x^3*log(c*x + 1) - 3*c^4*x^3*log(c*x - 1) - 6*c^3*x^2 - 2*
c)/(c^3*d*x^6 - c*d*x^4 + (c^2*d*x^5 - d*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))
, x))*b
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2 dx^2)} dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2 dx^2 - d)x^4} dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2 dx^2)} dx = \int \frac{a + b \operatorname{acosh}(c x)}{x^4 (d - c^2 d x^2)} dx$$

input
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2 dx^2)} dx = \frac{-6\left(\int \frac{a\cosh(cx)}{c^2x^6 - x^4} dx\right) b x^3 - 3\log(c^2x - c) a c^3x^3 + 3\log(c^2x + c) a c^3x^3 - 6a c^2x^2 - 2a}{6d x^3}$$

input
```
int((a+b*acosh(c*x))/x^4/(-c^2*d*x^2+d),x)
```

output
```
( - 6*int(acosh(c*x)/(c**2*x**6 - x**4),x)*b*x**3 - 3*log(c**2*x - c)*a*c*
*3*x**3 + 3*log(c**2*x + c)*a*c**3*x**3 - 6*a*c**2*x**2 - 2*a)/(6*d*x**3)
```

3.37 $\int \frac{x^4(a+b\,\mathrm{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\begin{aligned}\int \frac{x^4(a+b\,\mathrm{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = &-\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5d^2}\\ &+\frac{3x(a+b\,\mathrm{arccosh}(cx))}{2c^4d^2}+\frac{x^3(a+b\,\mathrm{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}\\ &-\frac{3(a+b\,\mathrm{arccosh}(cx))\mathrm{arctanh}\left(e^{\mathrm{arccosh}(cx)}\right)}{c^5d^2}\\ &-\frac{3b\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arccosh}(cx)}\right)}{2c^5d^2}\\ &+\frac{3b\,\mathrm{PolyLog}\left(2,e^{\mathrm{arccosh}(cx)}\right)}{2c^5d^2}\end{aligned}$$

output
```
-1/2*b*x^2/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^5/d^2+3/2*x*(a+b*arccosh(c*x))/c^4/d^2+1/2*x^3*(a+b*arccosh(c*x))
/c^2/d^2/(-c^2*x^2+1)-3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))/c^5/d^2-3/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^
2+3/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.38

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$$

$$= \frac{4acx - 3b\sqrt{\frac{-1+cx}{1+cx}} - 4bcx\sqrt{\frac{-1+cx}{1+cx}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}}{1-cx} + \frac{bcx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx} - \frac{2acx}{-1+c^2x^2} + 4bcx\text{arccosh}(cx) + \frac{b\text{arccosh}(cx)}{1-cx} - \cdots}{\cdots}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(4*a*c*x - 3*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*c*x*Sqrt[(-1 + c*x)/(1 + c
*x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + (b*c*x*Sqrt[(-1 + c*x)/(
1 + c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) + 4*b*c*x*ArcCosh[c*x] + (
b*ArcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 6*b*ArcCosh[c*x]*
Log[1 - E^ArcCosh[c*x]] - 6*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 3*a*L
og[1 - c*x] - 3*a*Log[1 + c*x] - 6*b*PolyLog[2, -E^ArcCosh[c*x]] + 6*b*Pol
yLog[2, E^ArcCosh[c*x]])/(4*c^5*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {6349, 27, 109, 27, 83, 6353, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{6349}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{d(1-c^2x^2)}dx}{2c^2d} + \frac{b\int \frac{x^3}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2cd^2} + \frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\downarrow \text{27}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\int \frac{x^3}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2cd^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\downarrow \text{109}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\left(-\frac{\int -\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c^2}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\downarrow \text{27}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c^2}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\downarrow \text{83}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\downarrow \text{6353}$$

$$-\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2}+\frac{b\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{x(a+b\text{arccosh}(cx))}{c^2}\right)}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\downarrow \text{83}$$

$$-\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\downarrow \text{6318}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$\Big\downarrow$ 26

$$-\frac{3\left(-\frac{i\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$\Big\downarrow$ 4670

$$-\frac{3\left(-\frac{i\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)}{c^3}\right.}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$\Big\downarrow$ 2715

$$-\frac{3\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e^{\text{arccosh}\right.\right.}{c^3}\right.}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$\Big\downarrow$ 2838

$$-\frac{3\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{c^3}-\frac{x(a+b\text{arccosh}(c}{c^2}\right.}{2c^2d^2}+\frac{x^3(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{c^4}-\frac{x^2}{c^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(b*(-(x^2/(c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])/c^4))/(2*c*d^2) + (x^3*(a + b*ArcCosh[c*x]))/(2*c^2*d^2*(1 - c^2*x^
2)) - (3*((b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^3 - (x*(a + b*ArcCosh[c*x]))/
c^2 - (I*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog
[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/c^3))/(2*c^2*d^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{a\left(cx-\frac{1}{4(cx-1)}+\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d^2}+\frac{b\,\mathrm{arccosh}(cx)cx}{d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2\left(c^2x^2-1\right)}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2\left(c^2x^2-1\right)}-}{}$
default	$\frac{\frac{a\left(cx-\frac{1}{4(cx-1)}+\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d^2}+\frac{b\,\mathrm{arccosh}(cx)cx}{d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2\left(c^2x^2-1\right)}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2\left(c^2x^2-1\right)}-}{}$
parts	$\frac{a\left(\frac{x}{c^4}-\frac{1}{4c^5(cx+1)}-\frac{3\ln(cx+1)}{4c^5}-\frac{1}{4c^5(cx-1)}+\frac{3\ln(cx-1)}{4c^5}\right)}{d^2}+\frac{b\,\mathrm{arccosh}(cx)x}{d^2c^4}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^5d^2}-\frac{b\,\mathrm{arccosh}(cx)x}{2d^2c^4(c^2x^2-1)}-$

input
```
int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a/d^2*(c*x-1/4/(c*x-1)+3/4*ln(c*x-1)-1/4/(c*x+1)-3/4*ln(c*x+1))-b/d
^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+b/d^2*arccosh(c*x)*c*x-1/2*b/d^2/(c^2*x^2-1
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*c*x-3/2*b
/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/2*b/d^2*polylog(
2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))+3/2*b/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))
```

Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)x^4}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccosh(c*x) + a*x^4)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
 x)
```

Sympy [F]

$$\int \frac{x^4(a + b\operatorname{arccosh}(cx))}{(d - c^2dx^2)^2} dx = \frac{\int \frac{ax^4}{c^4x^4 - 2c^2x^2 + 1} dx + \int \frac{bx^4 \operatorname{acosh}(cx)}{c^4x^4 - 2c^2x^2 + 1} dx}{d^2}$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**4*acosh
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^4(a + b\operatorname{arccosh}(cx))}{(d - c^2dx^2)^2} dx = \int \frac{(b\operatorname{arcosh}(cx) + a)x^4}{(c^2dx^2 - d)^2} dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/64*(16*c^4*(2*x/(c^10*d^2*x^2 - c^8*d^2) - 4*x/(c^8*d^2) + 3*log(c*x + 1
)/(c^9*d^2) - 3*log(c*x - 1)/(c^9*d^2)) - 576*c^3*integrate(1/8*x^3*log(c*
x - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x) - 24*c^2*(2*x/(c^8*d^2*
x^2 - c^6*d^2) + log(c*x + 1)/(c^7*d^2) - log(c*x - 1)/(c^7*d^2)) + 192*c^
2*integrate(1/8*x^2*log(c*x - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2),
x) - 9*(c*(2/(c^8*d^2*x - c^7*d^2) - log(c*x + 1)/(c^7*d^2) + log(c*x - 1)
/(c^7*d^2)) + 4*log(c*x - 1)/(c^8*d^2*x^2 - c^6*d^2))*c + 4*(3*(c^2*x^2 -
1)*log(c*x + 1)^2 + 6*(c^2*x^2 - 1)*log(c*x + 1)*log(c*x - 1) + 4*(4*c^3*x
^3 - 6*c*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^2*x^2 - c^5*d^2) - 64*integ
rate(-1/4*(4*c^3*x^3 - 6*c*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 -
 1)*log(c*x - 1))/(c^9*d^2*x^5 - 2*c^7*d^2*x^3 + c^5*d^2*x + (c^8*d^2*x^4
- 2*c^6*d^2*x^2 + c^4*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 192*integrat
e(1/8*log(c*x - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x))*b - 1/4*a*
(2*x/(c^6*d^2*x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^5*d^2) -
3*log(c*x - 1)/(c^5*d^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acosh}(c\,x))}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \frac{4\left(\int \frac{acosh(cx)x^4}{c^4x^4-2c^2x^2+1}dx\right)b\,c^7x^2-4\left(\int \frac{acosh(cx)x^4}{c^4x^4-2c^2x^2+1}dx\right)b\,c^5+3\log(c^2x-c)\,a\,c^2x^2-3\log(c^2x-c)\,a-3\log(c}{4c^5d^2\,(c^2x^2-1)}$$

input
```
int(x^4*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int((acosh(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**7*x**2 - 4*
int((acosh(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**5 + 3*log(c**2
*x - c)*a*c**2*x**2 - 3*log(c**2*x - c)*a - 3*log(c**2*x + c)*a*c**2*x**2
+ 3*log(c**2*x + c)*a + 4*a*c**3*x**3 - 6*a*c*x)/(4*c**5*d**2*(c**2*x**2 -
 1))
```

3.38 $\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{bx}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\text{arccosh}(cx)}{2c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{(a+b\text{arccosh}(cx))^2}{2bc^4d^2}+\frac{(a+b\text{arccosh}(cx))\log\left(1-e^{2\text{arccosh}(cx)}\right)}{c^4d^2}+\frac{b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2c^4d^2}$$

output
```
-1/2*b*x/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*arccosh(c*x)/c^4/d^2+1/
2*x^2*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x^2+1)-1/2*(a+b*arccosh(c*x))^2/b/c
^4/d^2+(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^4/d^
2+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^4/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.38

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{-b\sqrt{\frac{-1+cx}{1+cx}}+\frac{b\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}+\frac{bcx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}-\frac{2a}{-1+c^2x^2}+\frac{b\text{arccosh}(cx)}{1-cx}+\frac{b\text{arccosh}(cx)}{1+cx}-2b\text{arccosh}(cx)^2+4b\text{arccos}}{}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(-(b*Sqrt[(-1 + c*x)/(1 + c*x)]) + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x
) + (b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a)/(-1 + c^2*x^2) +
(b*ArcCosh[c*x])/(1 - c*x) + (b*ArcCosh[c*x])/(1 + c*x) - 2*b*ArcCosh[c*x]
^2 + 4*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 4*b*ArcCosh[c*x]*Log[1 + E
^ArcCosh[c*x]] + 2*a*Log[1 - c^2*x^2] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] +
4*b*PolyLog[2, E^ArcCosh[c*x]])/(4*c^4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {6349, 27, 100, 27, 87, 43, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \text{6349}$$
$$-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{d(1-c^2x^2)}dx}{c^2d}+\frac{b\int \frac{x^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2cd^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}$$
$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{b\int \frac{x^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2cd^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \ 100$$

$$-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{b\left(\frac{\int \frac{c^2x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c^3}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{b\left(\frac{\int \frac{x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \ 87$$

$$-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{b\left(\frac{\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \ 43$$

$$-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \ 6328$$

$$\frac{\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{i\int (a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{4199}$$

$$-\frac{i\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccosh}(cx)}{2b}\right.}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a}{}\right.}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4d^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2cd^2}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(x^2*(a + b*ArcCosh[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) + (b*(-(1/(c^3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])) + (-(Sqrt[-1 + c*x]/(c^2*Sqrt[1 + c*x])) + ArcCos
h[c*x]/c^2)/c))/(2*c*d^2) - (I*(((-1/2*I)*(a + b*ArcCosh[c*x])^2)/b - (2*I
)*(-1/2*((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])]) - (b*PolyLog[2,
 E^(2*ArcCosh[c*x])])/4)))/(c^4*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}+\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)^2}{2}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,cx-c^2x^2+\operatorname{arccosh}(cx)+1}{2\left(c^2x^2-1\right)}+\operatorname{arccosh}(cx)\ln(1+c\right.}{}}{c^4}$
default	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}+\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)^2}{2}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,cx-c^2x^2+\operatorname{arccosh}(cx)+1}{2\left(c^2x^2-1\right)}+\operatorname{arccosh}(cx)\ln(1+c\right.}{}}{c^4}$
parts	$\frac{a\left(\frac{1}{4c^4(cx+1)}+\frac{\ln(cx+1)}{2c^4}-\frac{1}{4c^4(cx-1)}+\frac{\ln(cx-1)}{2c^4}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)^2}{2}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,cx-c^2x^2+\operatorname{arccosh}(cx)+1}{2\left(c^2x^2-1\right)}+\operatorname{arccosh}(\right.}{}$

input
```
int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d^2*(-1/4/(c*x-1)+1/2*ln(c*x-1)+1/4/(c*x+1)+1/2*ln(c*x+1))+b/d^2*
(-1/2*arccosh(c*x)^2-1/2*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-c^2*x^2+arccosh(
c*x)+1)/(c^2*x^2-1)+arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+pol
ylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))+polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arccosh(c*x) + a*x^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
 x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{ax^3}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^3\,\text{acosh}\,(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**3*acosh
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/8*b*(((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)*log(c
*x - 1) + (c^2*x^2 - 1)*log(c*x - 1)^2 - 4*((c^2*x^2 - 1)*log(c*x + 1) + (
c^2*x^2 - 1)*log(c*x - 1) - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2)
/(c^6*d^2*x^2 - c^4*d^2) - 8*integrate(1/2*((c^2*x^2 - 1)*log(c*x + 1) + (
c^2*x^2 - 1)*log(c*x - 1) - 1)/(c^8*d^2*x^5 - 2*c^6*d^2*x^3 + c^4*d^2*x +
(c^7*d^2*x^4 - 2*c^5*d^2*x^2 + c^3*d^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x
- 1))), x)) - 1/2*a*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2
))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acosh}(c\,x))}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = \frac{2\left(\int \frac{acosh(cx)x^3}{c^4x^4-2c^2x^2+1}dx\right)b\,c^6x^2-2\left(\int \frac{acosh(cx)x^3}{c^4x^4-2c^2x^2+1}dx\right)b\,c^4+\log(c^2x-c)\,a\,c^2x^2-\log(c^2x-c)\,a+\log(c^2x+}{2c^4d^2\,(c^2x^2-1)}$$

input
```
int(x^3*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int((acosh(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**6*x**2 - 2*
int((acosh(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**4 + log(c**2*x
 - c)*a*c**2*x**2 - log(c**2*x - c)*a + log(c**2*x + c)*a*c**2*x**2 - log(
c**2*x + c)*a - a*c**2*x**2)/(2*c**4*d**2*(c**2*x**2 - 1))
```

3.39 $\int \frac{x^2(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{b}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)} - \frac{(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{c^3d^2} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{2c^3d^2}+\frac{b\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{2c^3d^2}$$

output
```
-1/2*b/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*x*(a+b*arccosh(c*x))/c^2/d^
2/(-c^2*x^2+1)-(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/c^3/d^2-1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2+1/2*b*p
olylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.66

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{b\sqrt{\frac{-1+cx}{1+cx}}+\frac{b\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}+\frac{bcx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}-\frac{2acx}{-1+c^2x^2}+\frac{b\text{arccosh}(cx)}{1-cx}-\frac{b\text{arccosh}(cx)}{1+cx}+2b\text{arccosh}(cx)\log\left(1-e^{\text{arccos}\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(b*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) +
 (b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) +
 (b*ArcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 2*b*ArcCosh[c*x
]*Log[1 - E^ArcCosh[c*x]] - 2*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + a*L
og[1 - c*x] - a*Log[1 + c*x] - 2*b*PolyLog[2, -E^ArcCosh[c*x]] + 2*b*PolyL
og[2, E^ArcCosh[c*x]])/(4*c^3*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {6349, 27, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \text{ 6349}$$
$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{d(1-c^2x^2)}dx}{2c^2d}+\frac{b\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2cd^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}$$
$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2cd^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2c^2d^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{b}{2c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6318}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c^3d^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{b}{2c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c^3d^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{b}{2c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c^3d^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{b}{2c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\right.}{2c^3d^2}$$
$$\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{b}{2c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arcta}\right.}{2c^3d^2}$$
$$\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{b}{2c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c^3d^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2d^2(1-c^2x^2)}-\frac{b}{2c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
-1/2*b/(c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(
2*c^2*d^2*(1 - c^2*x^2)) + ((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^Ar
cCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c
*x]]))/(c^3*d^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\ \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2x^2-1\right)}-\frac{\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{2}-\cdots\right.}{\cdots}}{c^3}$
default	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\ \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2x^2-1\right)}-\frac{\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{2}-\cdots\right.}{\cdots}}{c^3}$
parts	$\frac{a\left(-\frac{1}{4c^3(cx+1)}-\frac{\ln(cx+1)}{4c^3}-\frac{1}{4c^3(cx-1)}+\frac{\ln(cx-1)}{4c^3}\right)}{d^2}+\frac{b\left(-\frac{cx\ \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2x^2-1\right)}-\frac{\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{\cdots}\right.}{2}\cdots\right.}{\cdots}$

input
```
int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d^2*(-1/4/(c*x-1)+1/4*ln(c*x-1)-1/4/(c*x+1)-1/4*ln(c*x+1))+b/d^2*
(-1/2*(c*x*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c^2*x^2-1)-1/2*arcco
sh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*polylog(2,-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))+1/2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))
+1/2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a + b\operatorname{arccosh}(cx))}{(d - c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arcosh}(cx) + a)x^2}{(c^2dx^2 - d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
 x)
```

Sympy [F]

$$\int \frac{x^2(a + b\operatorname{arccosh}(cx))}{(d - c^2dx^2)^2}\,dx = \frac{\int \frac{ax^2}{c^4x^4 - 2c^2x^2 + 1}\,dx + \int \frac{bx^2\operatorname{acosh}(cx)}{c^4x^4 - 2c^2x^2 + 1}\,dx}{d^2}$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**2*acosh
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^2} \, dx = \int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{(c^2dx^2 - d)^2} \, dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2
 + c^2*d^2), x) + 8*c^2*(2*x/(c^6*d^2*x^2 - c^4*d^2) + log(c*x + 1)/(c^5*d
^2) - log(c*x - 1)/(c^5*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^6
*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x) + 3*(c*(2/(c^6*d^2*x - c^5*d^2) -
log(c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 4*log(c*x - 1)/(c^6*d^2
*x^2 - c^4*d^2))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log
(c*x + 1)*log(c*x - 1) - 4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2
- 1)*log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^5*d^2*x^2 -
c^3*d^2) + 64*integrate(1/4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2
 - 1)*log(c*x - 1))/(c^7*d^2*x^5 - 2*c^5*d^2*x^3 + c^3*d^2*x + (c^6*d^2*x^
4 - 2*c^4*d^2*x^2 + c^2*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integra
te(1/8*log(c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x))*b - 1/4*a
*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3
*d^2))
```

Giac [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^2} \, dx = \int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{(c^2dx^2 - d)^2} \, dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{\left(d-c^2dx^2\right)^2}\,dx = \frac{4\left(\int \frac{acosh(cx)x^2}{c^4x^4-2c^2x^2+1}dx\right)b\,c^5x^2-4\left(\int \frac{acosh(cx)x^2}{c^4x^4-2c^2x^2+1}dx\right)b\,c^3+\log(c^2x-c)\,a\,c^2x^2-\log(c^2x-c)\,a-\log(c^2x+}{4c^3d^2\left(c^2x^2-1\right)}$$

input
```
int(x^2*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int((acosh(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**5*x**2 - 4*
int((acosh(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**3 + log(c**2*x
 - c)*a*c**2*x**2 - log(c**2*x - c)*a - log(c**2*x + c)*a*c**2*x**2 + log(
c**2*x + c)*a - 2*a*c*x)/(4*c**3*d**2*(c**2*x**2 - 1))
```

3.40 $\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=-\frac{bx}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\text{arccosh}(cx)}{2c^2d^2(1-c^2x^2)}$$

output
```
-1/2*b*x/c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*(a+b*arccosh(c*x))/c^2/d^2/
(-c^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\frac{a+bcx\sqrt{-1+cx}\sqrt{1+cx}+b\text{arccosh}(cx)}{2c^2d^2-2c^4d^2x^2}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*ArcCosh[c*x])/(2*c^2*d^2 - 2*c
^4*d^2*x^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6329, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}dx$$

$$\Big\downarrow \text{6329}$$

$$\frac{b\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2cd^2}+\frac{a+b\text{arccosh}(cx)}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{41}$$

$$\frac{a+b\text{arccosh}(cx)}{2c^2d^2(1-c^2x^2)}-\frac{bx}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

input

```
Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output

```
-1/2*(b*x)/(c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*
c^2*d^2*(1 - c^2*x^2))
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{a}{2d^2(c^2x^2-1)}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{2(c^2x^2-1)}-\frac{cx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}}{c^2}$
default	$\frac{-\frac{a}{2d^2(c^2x^2-1)}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{2(c^2x^2-1)}-\frac{cx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}}{c^2}$
parts	$-\frac{a}{2d^2c^2(c^2x^2-1)}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{2(c^2x^2-1)}-\frac{cx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2c^2}$
orering	$-\frac{(cx-1)(cx+1)(3c^2x^2+2)(a+b\operatorname{arccosh}(cx))}{2c^2(-c^2dx^2+d)^2}-\frac{(cx-1)^2(cx+1)^2\left(\frac{a+b\operatorname{arccosh}(cx)}{(-c^2dx^2+d)^2}+\frac{xbc}{\sqrt{cx-1}\sqrt{cx+1}(-c^2dx^2+d)^2}+\cdots\right.}{2c^2}$

input
```
int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*a/d^2/(c^2*x^2-1)+b/d^2*(-1/2/(c^2*x^2-1)*arccosh(c*x)-1/2*c/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{\left(d-c^2dx^2\right)^2}\,dx = -\frac{ac^2x^2+\sqrt{c^2x^2-1}bcx+b\log\left(cx+\sqrt{c^2x^2-1}\right)}{2\left(c^4d^2x^2-c^2d^2\right)}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/2*(a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b*c*x + b*log(c*x + sqrt(c^2*x^2 - 1))
)/(c^4*d^2*x^2 - c^2*d^2)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{\left(d-c^2dx^2\right)^2}\,dx = \frac{\int \frac{ax}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{bx\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x*acosh(c*x)/
(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(52) = 104.

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.20

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{\left(d-c^2dx^2\right)^2}\,dx = -\frac{1}{4}\left(\left(\frac{\sqrt{c^2x^2-1}c^2d^2}{c^7d^4x+c^6d^4}+\frac{\sqrt{c^2x^2-1}c^2d^2}{c^7d^4x-c^6d^4}\right)c^2+\frac{2\operatorname{arcosh}(cx)}{c^4d^2x^2-c^2d^2}\right)b-\frac{a}{2\left(c^4d^2x^2-c^2d^2\right)}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*((sqrt(c^2*x^2 - 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(c^2*x^2 - 1)
*c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2 + 2*arccosh(c*x)/(c^4*d^2*x^2 - c^2*d^
2))*b - 1/2*a/(c^4*d^2*x^2 - c^2*d^2)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{x\,(a+b\,\text{acosh}(c\,x))}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\frac{2\left(\int \frac{acosh(cx)x}{c^4x^4-2c^2x^2+1}dx\right)b\,c^2x^2-2\left(\int \frac{acosh(cx)x}{c^4x^4-2c^2x^2+1}dx\right)b-a\,x^2}{2d^2\,(c^2x^2-1)}$$

input
```
int(x*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int((acosh(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**2*x**2 - 2*int
((acosh(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b - a*x**2)/(2*d**2*(c**2
*x**2 - 1))
```

3.41 $\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^2}\,dx = & -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)} \\ & +\frac{(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{cd^2} \\ & +\frac{b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{2cd^2}-\frac{b\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{2cd^2}\end{aligned}$$

output
```
-1/2*b/c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*x*(a+b*arccosh(c*x))/d^2/(-c^
2*x^2+1)+(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2
+1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2-1/2*b*polylog(2,c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.58

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d-c^2dx^2)^2}\,dx$$
$$=\frac{\frac{-2acx-2b\sqrt{\frac{-1+cx}{1+cx}}-2bcx\sqrt{\frac{-1+cx}{1+cx}}-2b\operatorname{arccosh}(cx)\left(cx+(-1+c^2x^2)\log\left(1-e^{\operatorname{arccosh}(cx)}\right)+(1-c^2x^2)\log\left(1+e^{\operatorname{arccosh}(cx)}\right)\right)+(a-ac^2x^2)\log}{-1+c^2x^2}}{4cd^2}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

output
```
((-2*a*c*x - 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*b*c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)] - 2*b*ArcCosh[c*x]*(c*x + (-1 + c^2*x^2)*Log[1 - E^ArcCosh[c*x]] +
(1 - c^2*x^2)*Log[1 + E^ArcCosh[c*x]]) + (a - a*c^2*x^2)*Log[1 - c*x] - a*
Log[1 + c*x] + a*c^2*x^2*Log[1 + c*x])/(-1 + c^2*x^2) + 2*b*PolyLog[2, -E^
ArcCosh[c*x]] - 2*b*PolyLog[2, E^ArcCosh[c*x]])/(4*c*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6316, 27, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \text{ 6316}$$
$$\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{d(1-c^2x^2)}dx}{2d}+\frac{bc\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2}+\frac{x(a+b\operatorname{arccosh}(cx))}{2d^2\,(1-c^2x^2)}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2d^2}+\frac{bc\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2}+\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2d^2}+\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}-\frac{b}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6318}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2cd^2}+\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}-\frac{b}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2cd^2}+\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}-\frac{b}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2cd^2}+\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}-\frac{b}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+\right.}{2cd^2}$$
$$\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}-\frac{b}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arct}\right.}{2cd^2}$$
$$\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}-\frac{b}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2cd^2}+\frac{x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}-\frac{b}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

output
```
-1/2*b/(c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*
d^2*(1 - c^2*x^2)) - ((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[
c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))
/(c*d^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\ \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2x^2-1\right)}-\frac{\operatorname{arccosh}(cx)\ln\left(1-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{2}-\cdots}{\cdots}}{c}$
default	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\ \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2x^2-1\right)}-\frac{\operatorname{arccosh}(cx)\ln\left(1-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{2}-\cdots}{\cdots}}{c}$
parts	$\frac{a\left(-\frac{1}{4c(cx+1)}+\frac{\ln(cx+1)}{4c}-\frac{1}{4c(cx-1)}-\frac{\ln(cx-1)}{4c}\right)}{d^2}+\frac{b\left(-\frac{cx\ \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2x^2-1\right)}-\frac{\operatorname{arccosh}(cx)\ln\left(1-cx-\sqrt{cx-1}\,\sqrt{cx\cdots}\right)}{2}\cdots}{\cdots}$

input
```
int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d^2*(-1/4/(c*x-1)-1/4*ln(c*x-1)-1/4/(c*x+1)+1/4*ln(c*x+1))+b/d^2*(-
1/2*(c*x*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c^2*x^2-1)-1/2*arccosh
(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*polylog(2,c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))+1/2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/
2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^2} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^2} \, dx = \frac{\int \frac{a}{c^4x^4 - 2c^2x^2 + 1} \, dx + \int \frac{b \operatorname{acosh}(cx)}{c^4x^4 - 2c^2x^2 + 1} \, dx}{d^2}$$

input
```
integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*acosh(c*x)/(c**
4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\operatorname{arccosh}(cx)}{(d - c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^2} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2
+ d^2), x) - 8*c^2*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) -
 log(c*x - 1)/(c^3*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^2*
x^4 - 2*c^2*d^2*x^2 + d^2), x) + 3*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x +
 1)/(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^
2*d^2))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)
*log(c*x - 1) + 4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(
c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^3*d^2*x^2 - c*d^2) +
64*integrate(-1/4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(
c*x - 1))/(c^5*d^2*x^5 - 2*c^3*d^2*x^3 + c*d^2*x + (c^4*d^2*x^4 - 2*c^2*d^
2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integrate(1/8*log(c*x -
 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x))*b - 1/4*a*(2*x/(c^2*d^2*x^2 -
 d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2))
```

Giac [F]

$$\int \frac{a + b\operatorname{arccosh}(cx)}{(d - c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^2} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^2}\,dx$$
$$=\frac{4\left(\int \frac{acosh(cx)}{c^4x^4-2c^2x^2+1}dx\right)b\,c^3x^2-4\left(\int \frac{acosh(cx)}{c^4x^4-2c^2x^2+1}dx\right)bc-\log(c^2x-c)\,a\,c^2x^2+\log(c^2x-c)\,a+\log(c^2x+c}{4c\,d^2\,(c^2x^2-1)}$$

input
```
int((a+b*acosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int(acosh(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**3*x**2 - 4*int(aco
sh(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c - log(c**2*x - c)*a*c**2*x**2
 + log(c**2*x - c)*a + log(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a - 2
*a*c*x)/(4*c*d**2*(c**2*x**2 - 1))
```

3.42 $\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^2}\,dx=-\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\text{arccosh}(cx)}{2d^2(1-c^2x^2)}+\frac{2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)}{d^2}+\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2d^2}$$

output
```
-1/2*b*c*x/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*(a+b*arccosh(c*x))/d^2/(-c^
2*x^2+1)+2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
/d^2+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-1/2*b*polyl
og(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 244 vs. $2(116) = 232$.

Time = 0.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.10

$$\int \frac{a + b\text{arccosh}(cx)}{x\,(d - c^2dx^2)^2}\,dx$$
$$= \frac{-b\sqrt{\frac{-1+cx}{1+cx}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}}{1-cx} + \frac{bcx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx} - \frac{2a}{-1+c^2x^2} + \frac{b\text{arccosh}(cx)}{1-cx} + \frac{b\text{arccosh}(cx)}{1+cx} + 4b\text{arccosh}(cx)^2 + 4b\text{arccos}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

output
```
(-(b*Sqrt[(-1 + c*x)/(1 + c*x)]) + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x
) + (b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a)/(-1 + c^2*x^2) +
(b*ArcCosh[c*x])/(1 - c*x) + (b*ArcCosh[c*x])/(1 + c*x) + 4*b*ArcCosh[c*x]
^2 + 4*b*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 4*b*ArcCosh[c*x]*Log[
1 - E^ArcCosh[c*x]] - 4*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 4*a*Log[x
] - 2*a*Log[1 - c^2*x^2] - 2*b*PolyLog[2, -E^(-2*ArcCosh[c*x])] - 4*b*Poly
Log[2, -E^ArcCosh[c*x]] - 4*b*PolyLog[2, E^ArcCosh[c*x]])/(4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6351, 27, 41, 6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arccosh}(cx)}{x\,(d - c^2dx^2)^2}\,dx$$
$$\Big\downarrow \text{6351}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{dx(1-c^2x^2)}dx}{d}+\frac{bc\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2}+\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d^2}+\frac{bc\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2}+\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{ 41}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d^2}+\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6331}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d^2}+\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 5984}$$

$$-\frac{2\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx)}{d^2}+\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d^2}+\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{2i\int (a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d^2}+\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{2i\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)\right.}{d^2}$$
$$\frac{a+b\text{arccosh}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{2i\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx}}{d^2}$$
$$\frac{a+b\text{arccosh}(cx)}{2d^2(1-c^2x^2)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2i\left(i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)}{d^2}+$$
$$\frac{a+b\text{arccosh}(cx)}{2d^2(1-c^2x^2)}-\frac{bcx}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

output
```
-1/2*(b*c*x)/(d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*
d^2*(1 - c^2*x^2)) - ((2*I)*(I*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c
*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcCosh[c*x])] - (I/4)*b*PolyLog[2, E^(2*
ArcCosh[c*x])]))/d^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.19

method	result
parts	$\frac{a\left(\ln(x)+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}\right)}{d^2}+\frac{b\left(-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,cx-c^2x^2+\operatorname{arccosh}(cx)+1}{2(c^2x^2-1)}-\operatorname{arccosh}(cx)\ln(1+cx\right.}{}$
derivativedivides	$\frac{a\left(\ln(cx)-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,cx-c^2x^2+\operatorname{arccosh}(cx)+1}{2(c^2x^2-1)}-\operatorname{arccosh}(cx)\ln(1+cx\right.}{}$
default	$\frac{a\left(\ln(cx)-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,cx-c^2x^2+\operatorname{arccosh}(cx)+1}{2(c^2x^2-1)}-\operatorname{arccosh}(cx)\ln(1+cx\right.}{}$

input
```
int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*(ln(x)+1/4/(c*x+1)-1/2*ln(c*x+1)-1/4/(c*x-1)-1/2*ln(c*x-1))+b/d^2*(-
1/2*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-c^2*x^2+arccosh(c*x)+1)/(c^2*x^2-1)-a
rccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-polylog(2,-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))+arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
+1/2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-arccosh(c*x)*ln(1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))-polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^5-2c^2x^3+x}\,dx+\int \frac{b\,\text{acosh}\,(cx)}{c^4x^5-2c^2x^3+x}\,dx}{d^2}$$

input
```
integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b*acosh(c*x)/(c**
4*x**5 - 2*c**2*x**3 + x), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^2x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*lo
g(x)/d^2) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^4*d^2*x^
5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^2x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x\,\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx = \frac{2\left(\int \frac{acosh(cx)}{c^4x^5-2c^2x^3+x}dx\right)b\,c^2x^2-2\left(\int \frac{acosh(cx)}{c^4x^5-2c^2x^3+x}dx\right)b-\log(c^2x-c)\,a\,c^2x^2+\log(c^2x-c)\,a-\log(c^2x+c}{2d^2\left(c^2x^2-1\right)}$$

input
```
int((a+b*acosh(c*x))/x/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int(acosh(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*b*c**2*x**2 - 2*int(aco
sh(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*b - log(c**2*x - c)*a*c**2*x**2 +
 log(c**2*x - c)*a - log(c**2*x + c)*a*c**2*x**2 + log(c**2*x + c)*a + 2*l
og(x)*a*c**2*x**2 - 2*log(x)*a - a*c**2*x**2)/(2*d**2*(c**2*x**2 - 1))
```

3.43 $\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^2}\,dx = -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\text{arccosh}(cx)}{d^2x} + \frac{c^2x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)} + \frac{bc\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2} + \frac{3c(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{d^2} + \frac{3bc\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{2d^2} - \frac{3bc\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{2d^2}$$

output
```
-1/2*b*c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-(a+b*arccosh(c*x))/d^2/x+1/2*c^2*
x*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/
2))/d^2+3*c*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^
2+3/2*b*c*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2-3/2*b*c*polylog(
2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.79

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx$$
$$=\frac{-\frac{4a}{x}+bc\sqrt{\frac{-1+cx}{1+cx}}+\frac{bc\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}+\frac{bc^2x\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}-\frac{2ac^2x}{-1+c^2x^2}-\frac{4b\operatorname{arccosh}(cx)}{x}+\frac{bc\operatorname{arccosh}(cx)}{1-cx}-\frac{bc\operatorname{arccosh}(cx)}{1+cx}+\frac{4b}{\ldots}}{\ldots}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

output
```
((-4*a)/x + b*c*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*c*Sqrt[(-1 + c*x)/(1 + c*x
)])/(1 - c*x) + (b*c^2*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c^2*
x)/(-1 + c^2*x^2) - (4*b*ArcCosh[c*x])/x + (b*c*ArcCosh[c*x])/(1 - c*x) -
(b*c*ArcCosh[c*x])/(1 + c*x) + (4*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 +
c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*b*c*ArcCosh[c*x]*Log[1 - E^A
rcCosh[c*x]] + 6*b*c*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] - 3*a*c*Log[1 -
c*x] + 3*a*c*Log[1 + c*x] + 6*b*c*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c*Poly
Log[2, E^ArcCosh[c*x]])/(4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.24, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {6347, 27, 115, 27, 103, 218, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx$$

$$\Big\downarrow \; 6347$$

$$3c^2\int \frac{a+b\operatorname{arccosh}(cx)}{d^2\left(1-c^2x^2\right)^2}dx-\frac{bc\int \frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx}{d^2}-\frac{a+b\operatorname{arccosh}(cx)}{d^2x\left(1-c^2x^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{3c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\int \frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x\,(1-c^2x^2)}$$

$$\Big\downarrow \textbf{115}$$

$$\frac{3c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\left(-\frac{\int \frac{c}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x\,(1-c^2x^2)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{3c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\left(-\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x\,(1-c^2x^2)}$$

$$\Big\downarrow \textbf{103}$$

$$\frac{3c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\left(-c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x\,(1-c^2x^2)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{3c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x\,(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \textbf{6316}$$

$$\frac{3c^2\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x\,(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \textbf{83}$$

$$\frac{3c^2\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x\,(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \textbf{6318}$$

$$\frac{3c^2\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3c^2\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{3c^2\left(-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \text{4670}$$

$$\frac{3c^2\left(-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right.}{2c}\right.}{d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3c^2\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right.\right.}{2c}\right.}{d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{3c^2\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}+\frac{x(a+b\text{arccosh}(c}{2(1-c^2x^2)}\right.}{d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{d^2x\left(1-c^2x^2\right)}-\frac{bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

output
```
-((a + b*ArcCosh[c*x])/(d^2*x*(1 - c^2*x^2))) - (b*c*(-(1/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])) - ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))/d^2 + (3*c^2*(-1/
2*b/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*(1 - c^
2*x^2)) - ((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b
*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/c))/d^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.31

method	result
derivativedivides	$c\left(\frac{a\left(-\frac{1}{cx}-\frac{1}{4(cx-1)}-\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{3\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{3c^2x^2\operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}\,cx-2\operatorname{arccosh}(cx)}{2(c^2x^2-1)cx}+2\,\mathrm{a}\right.}{}\right.$
default	$c\left(\frac{a\left(-\frac{1}{cx}-\frac{1}{4(cx-1)}-\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{3\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{3c^2x^2\operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}\,cx-2\operatorname{arccosh}(cx)}{2(c^2x^2-1)cx}+2\,\mathrm{a}\right.}{}\right.$
parts	$\frac{a\left(-\frac{1}{x}-\frac{c}{4(cx+1)}+\frac{3c\ln(cx+1)}{4}-\frac{c}{4(cx-1)}-\frac{3c\ln(cx-1)}{4}\right)}{d^2}+\frac{bc\left(-\frac{3c^2x^2\operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}\,cx-2\operatorname{arccosh}(cx)}{2(c^2x^2-1)cx}+2\,\mathrm{arc}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*(a/d^2*(-1/c/x-1/4/(c*x-1)-3/4*ln(c*x-1)-1/4/(c*x+1)+3/4*ln(c*x+1))+b/d^
2*(-1/2*(3*c^2*x^2*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-2*arccosh(
c*x))/(c^2*x^2-1)/c/x+2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*dilog(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))+3/2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^6-2c^2x^4+x^2}\,dx+\int \frac{b\,\text{acosh}\left(cx\right)}{c^4x^6-2c^2x^4+x^2}\,dx}{d^2}$$

input
```
integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b*acosh(c*x)/(
c**4*x**6 - 2*c**2*x**4 + x**2), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/64*(576*c^5*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2
+ d^2), x) - 24*c^4*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2)
- log(c*x - 1)/(c^3*d^2)) - 192*c^4*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x) + 9*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x
 + 1)/(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 -
c^2*d^2))*c^3 + 16*c^2*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + l
og(c*x - 1)/(c*d^2)) + 192*c^2*integrate(1/8*log(c*x - 1)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x) - 4*(3*(c^3*x^3 - c*x)*log(c*x + 1)^2 + 6*(c^3*x^3
 - c*x)*log(c*x + 1)*log(c*x - 1) + 4*(6*c^2*x^2 - 3*(c^3*x^3 - c*x)*log(c
*x + 1) + 3*(c^3*x^3 - c*x)*log(c*x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)))/(c^2*d^2*x^3 - d^2*x) + 64*integrate(-1/4*(6*c^3*x^2 - 3*(c^4*
x^3 - c^2*x)*log(c*x + 1) + 3*(c^4*x^3 - c^2*x)*log(c*x - 1) - 4*c)/(c^5*d
^2*x^6 - 2*c^3*d^2*x^4 + c*d^2*x^2 + (c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b - 1/4*a*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x
^3 - d^2*x) - 3*c*log(c*x + 1)/d^2 + 3*c*log(c*x - 1)/d^2)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^2\,{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2dx^2)^2} \, dx$$
$$= \frac{4\left(\int \frac{acosh(cx)}{c^4x^6-2c^2x^4+x^2}dx\right) b\,c^2x^3 - 4\left(\int \frac{acosh(cx)}{c^4x^6-2c^2x^4+x^2}dx\right) bx - 3\log(c^2x-c)\,a\,c^3x^3 + 3\log(c^2x-c)\,acx + 3\,\mathrm{lo}}{4d^2x\,(c^2x^2-1)}$$

input
```
int((a+b*acosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int(acosh(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b*c**2*x**3 - 4*int(
acosh(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b*x - 3*log(c**2*x - c)*a*c
**3*x**3 + 3*log(c**2*x - c)*a*c*x + 3*log(c**2*x + c)*a*c**3*x**3 - 3*log
(c**2*x + c)*a*c*x - 6*a*c**2*x**2 + 4*a)/(4*d**2*x*(c**2*x**2 - 1))
```

3.44 $\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^2}\,dx = & -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}} \\ & -\frac{a+b\text{arccosh}(cx)}{2d^2x^2}+\frac{c^2(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)} \\ & +\frac{4c^2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)}{d^2} \\ & +\frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{d^2}-\frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{d^2}\end{aligned}$$

output
```
-1/2*b*c/d^2/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(a+b*arccosh(c*x))/d^2/x^2+
1/2*c^2*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+4*c^2*(a+b*arccosh(c*x))*arcta
nh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2+b*c^2*polylog(2,-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)/d^2-b*c^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(143) = 286.

Time = 1.08 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.02

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^2}\,dx$$
$$= \frac{-\frac{a}{x^2}+\frac{ac^2}{1-c^2x^2}+4ac^2\log(x)-2ac^2\log(1-c^2x^2)+\frac{1}{2}b\left(\frac{2cx\sqrt{-1+cx}\sqrt{1+cx}-2\text{arccosh}(cx)}{x^2}+c^2\left(-\frac{1}{\sqrt{\frac{-1+cx}{1+cx}}}+\text{a}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

output
```
(-(a/x^2) + (a*c^2)/(1 - c^2*x^2) + 4*a*c^2*Log[x] - 2*a*c^2*Log[1 - c^2*x
^2] + (b*((2*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*ArcCosh[c*x])/x^2 + c^2*
(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - c^2*(Sqrt[(-1
 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) + 4*c^2*(ArcCosh[c*x]*(ArcCos
h[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])
]) + 2*c^2*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCosh[c*x]]) - 4*Po
lyLog[2, -E^ArcCosh[c*x]]) + 2*c^2*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 -
 E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]])))/2)/(2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.45, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {6347, 27, 114, 27, 41, 6351, 41, 6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \; 6347$$

$$2c^2\int \frac{a+b\text{arccosh}(cx)}{d^2x\left(1-c^2x^2\right)^2}dx-\frac{bc\int \frac{1}{x^2(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{bc\int \frac{1}{x^2(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{114}$$

$$\frac{2c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{bc\left(\int \frac{2c^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{bc\left(2c^2\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{41}$$

$$\frac{2c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{6351}$$

$$\frac{2c^2\left(\int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)}dx+\frac{1}{2}bc\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{a+b\text{arccosh}(cx)}{2\left(1-c^2x^2\right)}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{41}$$

$$\frac{2c^2\left(\int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)}dx+\frac{a+b\text{arccosh}(cx)}{2\left(1-c^2x^2\right)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{6331}$$

$$\frac{2c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2\left(1-c^2x^2\right)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2c^2\left(-2\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c^2\left(-2\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{2c^2\left(-2i\int (a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{4670}$$

$$\frac{2c^2\left(-2i\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+i\text{arctanh}\left(e^{2\text{arccos}}\right.\right.\right.}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2c^2\left(-2i\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e^{2\text{arccosh}(cx)}\right)de^{2\text{arcco}}\right.\right.}{d^2}-\frac{a+b\text{arccosh}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2c^2\left(-2i\left(i\operatorname{arctanh}\left(e^{2\operatorname{arccosh}(cx)}\right)(a+b\operatorname{arccosh}(cx))+\frac{1}{4}ib\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)-\frac{1}{4}ib\operatorname{PolyLog}\left(2,e^{2\operatorname{arccosh}(c\cdots\right.\right.\right.}{d^2}$$

$$\frac{a+b\operatorname{arccosh}(cx)}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2x}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{2d^2}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

output
```
-1/2*(b*c*(1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*c^2*x)/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])))/d^2 - (a + b*ArcCosh[c*x])/(2*d^2*x^2*(1 - c^2*x^2)) + (2
*c^2*(-1/2*(b*c*x)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(
2*(1 - c^2*x^2)) - (2*I)*(I*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x]
)] + (I/4)*b*PolyLog[2, -E^(2*ArcCosh[c*x])] - (I/4)*b*PolyLog[2, E^(2*Arc
Cosh[c*x])])))/d^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.96

method	result
derivativedivides	$c^2 \left(\frac{a\left(-\frac{1}{2c^2x^2}+2\ln(cx)-\frac{1}{4(cx-1)}-\ln(cx-1)+\frac{1}{4cx+4}-\ln(cx+1)\right)}{d^2} + \frac{b\left(-\frac{2c^2x^2 \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}\,cx-\operatorname{arccos}}{2(c^2x^2-1)c^2x^2}\right.}{} \right.$
default	$c^2 \left(\frac{a\left(-\frac{1}{2c^2x^2}+2\ln(cx)-\frac{1}{4(cx-1)}-\ln(cx-1)+\frac{1}{4cx+4}-\ln(cx+1)\right)}{d^2} + \frac{b\left(-\frac{2c^2x^2 \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}\,cx-\operatorname{arccos}}{2(c^2x^2-1)c^2x^2}\right.}{} \right.$
parts	$\frac{a\left(-\frac{1}{2x^2}+2c^2\ln(x)+\frac{c^2}{4cx+4}-c^2\ln(cx+1)-\frac{c^2}{4(cx-1)}-c^2\ln(cx-1)\right)}{d^2} + \frac{b\,c^2\left(-\frac{2c^2x^2 \operatorname{arccosh}(cx)+\sqrt{cx-1}\,\sqrt{cx+1}\,cx-\operatorname{arccc}}{2(c^2x^2-1)c^2x^2}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/d^2*(-1/2/c^2/x^2+2*ln(c*x)-1/4/(c*x-1)-ln(c*x-1)+1/4/(c*x+1)-ln(c*
x+1))+b/d^2*(-1/2*(2*c^2*x^2*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-
arccosh(c*x))/(c^2*x^2-1)/c^2/x^2-2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))-2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*arccosh(c*x)*
ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+polylog(2,-(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))^2)-2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*pol
ylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^2}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(c^2dx^2-d)^2x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a}{c^4x^7-2c^2x^5+x^3}\,dx+\int \frac{b\,\text{acosh}\,(cx)}{c^4x^7-2c^2x^5+x^3}\,dx}{d^2}$$

input
```
integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b*acosh(c*x)/(
c**4*x**7 - 2*c**2*x**5 + x**3), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^2}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(c^2dx^2-d)^2x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d^2
 + (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + b*integrate(log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x^3\,\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx$$
$$=\frac{2\left(\int \frac{acosh(cx)}{c^4x^7-2c^2x^5+x^3}dx\right)b\,c^2x^4-2\left(\int \frac{acosh(cx)}{c^4x^7-2c^2x^5+x^3}dx\right)b\,x^2-2\log(c^2x-c)\,a\,c^4x^4+2\log(c^2x-c)\,a\,c^2x^2-}{2d^2x^2\left(c^2x^2-\right.}$$

input
```
int((a+b*acosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int(acosh(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b*c**2*x**4 - 2*int(
acosh(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b*x**2 - 2*log(c**2*x - c)*
a*c**4*x**4 + 2*log(c**2*x - c)*a*c**2*x**2 - 2*log(c**2*x + c)*a*c**4*x**
4 + 2*log(c**2*x + c)*a*c**2*x**2 + 4*log(x)*a*c**4*x**4 - 4*log(x)*a*c**2
*x**2 - 2*a*c**4*x**4 + a)/(2*d**2*x**2*(c**2*x**2 - 1))
```

3.45 $\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^2}\,dx = & -\frac{bc^3}{3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\sqrt{1+cx}}\\ & -\frac{a+b\text{arccosh}(cx)}{3d^2x^3}-\frac{2c^2(a+b\text{arccosh}(cx))}{d^2x}\\ & +\frac{c^4x(a+b\text{arccosh}(cx))}{2d^2(1-c^2x^2)}+\frac{13bc^3\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d^2}\\ & +\frac{5c^3(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{d^2}\\ & +\frac{5bc^3\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{2d^2}\\ & -\frac{5bc^3\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{2d^2}\end{aligned}$$

output
```
-1/3*b*c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c/d^2/x^2/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-1/3*(a+b*arccosh(c*x))/d^2/x^3-2*c^2*(a+b*arccosh(c*x))/d^2/x
+1/2*c^4*x*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+13/6*b*c^3*arctan((c*x-1)^(
1/2)*(c*x+1)^(1/2))/d^2+5*c^3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/d^2+5/2*b*c^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/
d^2-5/2*b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.70

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx =$$

$$-\frac{\frac{4a}{x^3}+\frac{24ac^2}{x}-3bc^3\sqrt{\frac{-1+cx}{1+cx}}+\frac{3bc^3\sqrt{\frac{-1+cx}{1+cx}}}{-1+cx}+\frac{3bc^4x\sqrt{\frac{-1+cx}{1+cx}}}{-1+cx}-\frac{2bc^3}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bc}{x^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{6ac^4x}{-1+c^2x^2}+4b\text{a}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

output
```
-1/12*((4*a)/x^3 + (24*a*c^2)/x - 3*b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)] + (3*
b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)])/(-1 + c*x) + (3*b*c^4*x*Sqrt[(-1 + c*x)/
(1 + c*x)])/(-1 + c*x) - (2*b*c^3)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c
)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*a*c^4*x)/(-1 + c^2*x^2) + (4*b*A
rcCosh[c*x])/x^3 + (24*b*c^2*ArcCosh[c*x])/x + (3*b*c^3*ArcCosh[c*x])/(-1
+ c*x) + (3*b*c^3*ArcCosh[c*x])/(1 + c*x) - (26*b*c^3*Sqrt[-1 + c^2*x^2]*A
rcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 30*b*c^3*ArcCo
sh[c*x]*Log[1 - E^ArcCosh[c*x]] - 30*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[
c*x]] + 15*a*c^3*Log[1 - c*x] - 15*a*c^3*Log[1 + c*x] - 30*b*c^3*PolyLog[2
, -E^ArcCosh[c*x]] + 30*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/d^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.41, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {6347, 27, 114, 27, 115, 27, 103, 218, 6347, 115, 27, 103, 218, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx$$

$$\Big\downarrow \text{ 6347}$$

$$\frac{5}{3}c^2\int \frac{a+b\text{arccosh}(cx)}{d^2x^2\left(1-c^2x^2\right)^2}dx-\frac{bc\int \frac{1}{x^3(cx-1)^{3/2}(cx+1)^{3/2}}dx}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\int \frac{1}{x^3(cx-1)^{3/2}(cx+1)^{3/2}}dx}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{114}$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{1}{2}\int \frac{3c^2}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{3}{2}c^2\int \frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{115}$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{3}{2}c^2\left(-\frac{\int \frac{c}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{3}{2}c^2\left(-\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{103}$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{3}{2}c^2\left(-c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{5c^2\int\frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$\Big\downarrow$ 6347

$$\frac{5c^2\left(3c^2\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx-bc\int\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$\Big\downarrow$ 115

$$\frac{5c^2\left(3c^2\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx-bc\left(-\frac{\int\frac{c}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$\Big\downarrow$ 27

$$\frac{5c^2\left(3c^2\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx-bc\left(-\int\frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$\Big\downarrow$ 103

$$\frac{5c^2\left(3c^2\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx-bc\left(-c\int\frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$\Big\downarrow$ 218

$$\frac{5c^2\left(3c^2\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}-bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{3d^2}-\frac{a+b\text{arccosh}(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$\Big\downarrow$ 6316

$$\frac{5c^2\left(3c^2\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}-bc\left(-\arctan\right.\right.}{3d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{5c^2\left(3c^2\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}-bc\left(-\arctan\left(\sqrt{cx-1}\sqrt{\right.\right.\right.}{3d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$$\Big\downarrow \text{6318}$$

$$\frac{5c^2\left(3c^2\left(-\frac{\int\frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}-bc\left(-\arctan\left(\right.\right.\right.}{3d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5c^2\left(3c^2\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}\right.}{3d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{5c^2\left(3c^2\left(-\frac{i\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}\right.}{3d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$$\Big\downarrow \text{4670}$$

$$\frac{5c^2\left(3c^2\left(-\frac{i\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}}{2c}\right.\right.}{3}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$$\Big\downarrow \; 2715$$

$$\frac{5c^2\left(3c^2\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e\right.\right.}{2c}\right.\right.}{}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{5c^2\left(3c^2\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}+\frac{x(a+b\text{arcc}}{2(1-c^2}\right.\right.}{3d^2}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(-\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{2x^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2}$$

input

```
Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

output

```
-1/3*(a + b*ArcCosh[c*x])/(d^2*x^3*(1 - c^2*x^2)) - (b*c*(1/(2*x^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + (3*c^2*(-(1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - Arc
Tan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))/2))/(3*d^2) + (5*c^2*(-((a + b*ArcCosh
[c*x])/(x*(1 - c^2*x^2))) - b*c*(-(1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - Arc
Tan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]) + 3*c^2*(-1/2*b/(c*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*(1 - c^2*x^2)) - ((I/2)*((2*I)*(a
+ b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]
] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/c)))/(3*d^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.13

method	result
derivativedivides	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}-\frac{2}{cx}-\frac{1}{4(cx-1)}-\frac{5\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{5\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{2c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}+15\,\operatorname{arccosh}(cx)c^4x^4+\sqrt{c}}{6c^3x^3}\right.}{}\right.$
default	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}-\frac{2}{cx}-\frac{1}{4(cx-1)}-\frac{5\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{5\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{2c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}+15\,\operatorname{arccosh}(cx)c^4x^4+\sqrt{c}}{6c^3x^3}\right.}{}\right.$
parts	$\frac{a\left(-\frac{1}{3x^3}-\frac{2c^2}{x}-\frac{c^3}{4(cx+1)}+\frac{5c^3\ln(cx+1)}{4}-\frac{c^3}{4(cx-1)}-\frac{5c^3\ln(cx-1)}{4}\right)}{d^2}+\frac{b\,c^3\left(-\frac{2c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}+15\,\operatorname{arccosh}(cx)c^4x^4+\sqrt{c}}{6c^3x^3}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(a/d^2*(-1/3/c^3/x^3-2/c/x-1/4/(c*x-1)-5/4*ln(c*x-1)-1/4/(c*x+1)+5/4*l
n(c*x+1))+b/d^2*(-1/6*(2*c^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+15*arccosh(c*
x)*c^4*x^4+(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-10*c^2*x^2*arccosh(c*x)-2*arcco
sh(c*x))/c^3/x^3/(c^2*x^2-1)+13/3*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+
5/2*dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/2*dilog(1+c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))+5/2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^2 x^4} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^2} \, dx = \frac{\int \frac{a}{c^4x^8 - 2c^2x^6 + x^4} \, dx + \int \frac{b \operatorname{acosh}(cx)}{c^4x^8 - 2c^2x^6 + x^4} \, dx}{d^2}$$

input
```
integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b*acosh(c*x)/(
c**4*x**8 - 2*c**2*x**6 + x**4), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^2 x^4} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 -
10*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a + 1/192*(8640*c^7*integrate(1/2
4*x^5*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x) - 120*c^6*(
2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d
^2)) - 2880*c^6*integrate(1/24*x^4*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x
^4 + d^2*x^2), x) + 45*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1)/(c^3*d^2
) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d^2))*c^5
+ 80*c^4*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c
*d^2)) + 2880*c^4*integrate(1/24*x^2*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2
*x^4 + d^2*x^2), x) + 16*c^2*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*
c*log(c*x + 1)/d^2 + 3*c*log(c*x - 1)/d^2) - 4*(15*(c^5*x^5 - c^3*x^3)*log
(c*x + 1)^2 + 30*(c^5*x^5 - c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(30*c^4
*x^4 - 20*c^2*x^2 - 15*(c^5*x^5 - c^3*x^3)*log(c*x + 1) + 15*(c^5*x^5 - c^
3*x^3)*log(c*x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*d^2*
x^5 - d^2*x^3) + 192*integrate(-1/12*(30*c^5*x^4 - 20*c^3*x^2 - 15*(c^6*x^
5 - c^4*x^3)*log(c*x + 1) + 15*(c^6*x^5 - c^4*x^3)*log(c*x - 1) - 4*c)/(c^
5*d^2*x^8 - 2*c^3*d^2*x^6 + c*d^2*x^4 + (c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2
*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^2 x^4} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^4\,(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^2}\,dx = \frac{12\left(\int \frac{acosh(cx)}{c^4x^8-2c^2x^6+x^4}dx\right)b\,c^2x^5-12\left(\int \frac{acosh(cx)}{c^4x^8-2c^2x^6+x^4}dx\right)b\,x^3-15\log(c^2x-c)\,a\,c^5x^5+15\log(c^2x-c)\,a\,c^3}{12d^2x^3\,(c^2x^2-1)}$$

input
```
int((a+b*acosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x)
```

output
```
(12*int(acosh(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b*c**2*x**5 - 12*in
t(acosh(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b*x**3 - 15*log(c**2*x -
c)*a*c**5*x**5 + 15*log(c**2*x - c)*a*c**3*x**3 + 15*log(c**2*x + c)*a*c**
5*x**5 - 15*log(c**2*x + c)*a*c**3*x**3 - 30*a*c**4*x**4 + 20*a*c**2*x**2
+ 4*a)/(12*d**2*x**3*(c**2*x**2 - 1))
```

3.46 $\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 188

$$\begin{aligned}\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx =& \frac{b}{12c^5d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{5b}{8c^5d^3\sqrt{-1+cx}\sqrt{1+cx}}\\&+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\,(1-c^2x^2)^2}-\frac{3x(a+b\text{arccosh}(cx))}{8c^4d^3\,(1-c^2x^2)}\\&+\frac{3(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{4c^5d^3}\\&+\frac{3b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{8c^5d^3}\\&-\frac{3b\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{8c^5d^3}\end{aligned}$$

output
```
1/12*b/c^5/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+5/8*b/c^5/d^3/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+1/4*x^3*(a+b*arccosh(c*x))/c^2/d^3/(-c^2*x^2+1)^2-3/8*x*(a+b*ar
ccosh(c*x))/c^4/d^3/(-c^2*x^2+1)+3/4*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))/c^5/d^3+3/8*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^
(1/2))/c^5/d^3-3/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.53

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{-\frac{b(-2+cx)\sqrt{1+cx}}{(-1+cx)^{3/2}} + \frac{b\sqrt{-1+cx}(2+cx)}{(1+cx)^{3/2}} + \frac{12acx}{(-1+c^2x^2)^2} + \frac{30acx}{-1+c^2x^2} + \frac{3b\operatorname{arccosh}(cx)}{(-1+cx)^2} - \frac{3b\operatorname{arccosh}(cx)}{(1+cx)^2} - 15b\left(-\frac{1}{\sqrt{\frac{-1+cx}{1+cx}}} + \right.}{}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
 c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (30*a*c*x)/(-1 + c^
2*x^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2
- 15*b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 15*b*(
Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) + (9*b*ArcCosh[c*x]*(
ArcCosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 - (9*b*ArcCosh[c*x]*(ArcCosh[
c*x] - 4*Log[1 + E^ArcCosh[c*x]]))/2 - 9*a*Log[1 - c*x] + 9*a*Log[1 + c*x]
 + 18*b*PolyLog[2, -E^ArcCosh[c*x]] - 18*b*PolyLog[2, E^ArcCosh[c*x]])/(48
*c^5*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {6349, 27, 105, 100, 27, 48, 6349, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$$
$$\Big\downarrow \; 6349$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{d^2(1-c^2x^2)^2}dx}{4c^2d}-\frac{b\int \frac{x^3}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4cd^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}-\frac{b\int \frac{x^3}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4cd^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}-\frac{b\left(\frac{\int \frac{x^2}{(cx-1)^{3/2}(cx+1)^{5/2}}dx}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}-\frac{b\left(\frac{\frac{\int \frac{c\sqrt{cx-1}}{(cx+1)^{5/2}}dx}{c^3}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{cx-1}}{(cx+1)^{5/2}}dx}{c^2}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \text{ 6349}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2c^2}+\frac{b\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \; 83$$

$$-\frac{3\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2c^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \; 6318$$

$$-\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c^3}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c^3}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \; 26$$

$$-\frac{3\left(\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c^3}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{3\left(\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)}{2c^3}\right.}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{3\left(\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e^{\text{arccosh}(c}\right.\right.}{2c^3}\right.}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{3\left(\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c^3}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right.}{4c^2d^3}+\frac{x^3(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\left(\frac{\frac{(cx-1)^{3/2}}{3c^3(cx+1)^{3/2}}-\frac{1}{c^3\sqrt{cx-1}(cx+1)^{3/2}}}{c}-\frac{x^3}{3c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
-1/4*(b*(-1/3*x^3/(c*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (-(1/(c^3*Sqrt[-1
 + c*x]*(1 + c*x)^(3/2))) + (-1 + c*x)^(3/2)/(3*c^3*(1 + c*x)^(3/2)))/c))/
(c*d^3) + (x^3*(a + b*ArcCosh[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (3*(-1/
2*b/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*c^2*(
1 - c^2*x^2)) + ((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]]
 + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/c^3)
)/(4*c^2*d^3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.36

method	result
derivativedivides	$-\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{5}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{15c^3x^3\operatorname{arccosh}(cx)+15\sqrt{cx-1}\sqrt{cx+1}\,c^2}{24\left(c^4x^4-2\right.}\right.}{}$
default	$-\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{5}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{15c^3x^3\operatorname{arccosh}(cx)+15\sqrt{cx-1}\sqrt{cx+1}\,c^2}{24\left(c^4x^4-2\right.}\right.}{}$
parts	$-\frac{a\left(\frac{1}{16c^5(cx+1)^2}-\frac{5}{16c^5(cx+1)}-\frac{3\ln(cx+1)}{16c^5}-\frac{1}{16c^5(cx-1)^2}-\frac{5}{16c^5(cx-1)}+\frac{3\ln(cx-1)}{16c^5}\right)}{d^3}-\frac{b\left(-\frac{15c^3x^3\operatorname{arccosh}(cx)+15\sqrt{cx-}}{}\right.}{}$

input
```
int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-a/d^3*(-1/16/(c*x-1)^2-5/16/(c*x-1)+3/16*ln(c*x-1)+1/16/(c*x+1)^2-
5/16/(c*x+1)-3/16*ln(c*x+1))-b/d^3*(-1/24*(15*c^3*x^3*arccosh(c*x)+15*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-9*c*x*arccosh(c*x)-13*(c*x-1)^(1/2)*(c*x+1
)^(1/2))/(c^4*x^4-2*c^2*x^2+1)-3/8*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))-3/8*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/8*arccosh(c*
x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/8*polylog(2,c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = \int -\frac{(b \operatorname{arcosh}(cx) + a)x^4}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^
2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = -\frac{\int \frac{ax^4}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} \, dx + \int \frac{bx^4 \operatorname{acosh}(cx)}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} \, dx}{d^3}$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b*x**4*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = \int -\frac{(b \operatorname{arcosh}(cx) + a)x^4}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*d^
3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 80*c^4*(2*(5*c^2*x^3 - 3*x)/(c^12*d
^3*x^4 - 2*c^10*d^3*x^2 + c^8*d^3) + 3*log(c*x + 1)/(c^9*d^3) - 3*log(c*x
- 1)/(c^9*d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^10*d^3*x^6 -
 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*
x - 6)/(c^12*d^3*x^3 - c^11*d^3*x^2 - c^10*d^3*x + c^9*d^3) - 5*log(c*x +
1)/(c^9*d^3) + 5*log(c*x - 1)/(c^9*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)
/(c^12*d^3*x^4 - 2*c^10*d^3*x^2 + c^8*d^3))*c^3 - 48*c^2*(2*(c^2*x^3 + x)/
(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3) - log(c*x + 1)/(c^7*d^3) + log(c*
x - 1)/(c^7*d^3)) + 12288*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^10*d^3*x^
6 - 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*
c*x - 2)/(c^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 3*log(c*x +
1)/(c^7*d^3) + 3*log(c*x - 1)/(c^7*d^3)) - 16*log(c*x - 1)/(c^10*d^3*x^4 -
 2*c^8*d^3*x^2 + c^6*d^3))*c - 32*(3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1
)^2 + 6*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)*log(c*x - 1) - 4*(10*c^3*x^
3 - 6*c*x + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*
x^2 + 1)*log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^9*d^3*x^
4 - 2*c^7*d^3*x^2 + c^5*d^3) + 2048*integrate(1/16*(10*c^3*x^3 - 6*c*x + 3
*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(
c*x - 1))/(c^11*d^3*x^7 - 3*c^9*d^3*x^5 + 3*c^7*d^3*x^3 - c^5*d^3*x + (...
```

Giac [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx=\int -\frac{(b\operatorname{arcosh}(cx)+a)x^4}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)*x^4/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx=\int \frac{x^4\,(a+b\,\text{acosh}(c\,x))}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$$
$$=\frac{-16\left(\int \frac{acosh(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^9x^4+32\left(\int \frac{acosh(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^7x^2-16\left(\int \frac{acosh(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c}{}$$

input
```
int(x^4*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int((acosh(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x
)*b*c**9*x**4 + 32*int((acosh(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2
*x**2 - 1),x)*b*c**7*x**2 - 16*int((acosh(c*x)*x**4)/(c**6*x**6 - 3*c**4*x
**4 + 3*c**2*x**2 - 1),x)*b*c**5 - 3*log(c**2*x - c)*a*c**4*x**4 + 6*log(c
**2*x - c)*a*c**2*x**2 - 3*log(c**2*x - c)*a + 3*log(c**2*x + c)*a*c**4*x*
*4 - 6*log(c**2*x + c)*a*c**2*x**2 + 3*log(c**2*x + c)*a + 10*a*c**3*x**3
- 6*a*c*x)/(16*c**5*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.47 $\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\text{arccosh}(cx)}{4c^4d^3}+\frac{x^4(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}$$

output
```
1/12*b*x^3/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*b*x/c^3/d^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)-1/4*b*arccosh(c*x)/c^4/d^3+1/4*x^4*(a+b*arccosh(c*x))/d^3/(
-c^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}(-3+4c^2x^2)+a(-3+6c^2x^2)+3b(-1+2c^2x^2)\,\text{arccosh}(cx)}{12c^4d^3(-1+c^2x^2)^2}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + 4*c^2*x^2) + a*(-3 + 6*c^2*x^2)
+ 3*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x])/(12*c^4*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6332, 109, 27, 100, 27, 87, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{6332}$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}-\frac{bc\int \frac{x^4}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4d^3}$$

$$\Big\downarrow \text{109}$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}-\frac{bc\left(-\frac{\int -\frac{3x^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{3c^2}-\frac{x^3}{3c^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}-\frac{bc\left(\frac{\int \frac{x^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{c^2}-\frac{x^3}{3c^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{100}$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}-\frac{bc\left(\frac{\frac{\int \frac{c^2x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c^3}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}}{c^2}-\frac{x^3}{3c^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(a+b\operatorname{arccosh}(cx))}{4d^3(1-c^2x^2)^2}-\frac{bc\left(\frac{\frac{\int\frac{x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}}{c^2}-\frac{x^3}{3c^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{87}$$

$$\frac{x^4(a+b\operatorname{arccosh}(cx))}{4d^3(1-c^2x^2)^2}-\frac{bc\left(\frac{\frac{\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}}{c^2}-\frac{x^3}{3c^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{43}$$

$$\frac{x^4(a+b\operatorname{arccosh}(cx))}{4d^3(1-c^2x^2)^2}-\frac{bc\left(\frac{\frac{\frac{\operatorname{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}}{c^2}-\frac{x^3}{3c^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) - (b*c*(-1/3*x^3/(c^2*(
-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (-(1/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
) + (-(Sqrt[-1 + c*x]/(c^2*Sqrt[1 + c*x])) + ArcCosh[c*x]/c^2)/c)/c^2))/(4
*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}\right)}{d^3}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{16(cx-1)^2}-\frac{3\operatorname{arccosh}(cx)}{16(cx-1)}-\frac{\operatorname{arccosh}(cx)}{16(cx+1)^2}+\frac{3\operatorname{arccosh}(cx)}{16(cx+1)}-\frac{cx(}{12(cx+}\right.}{d^3}}{c^4}$
default	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}\right)}{d^3}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{16(cx-1)^2}-\frac{3\operatorname{arccosh}(cx)}{16(cx-1)}-\frac{\operatorname{arccosh}(cx)}{16(cx+1)^2}+\frac{3\operatorname{arccosh}(cx)}{16(cx+1)}-\frac{cx(}{12(cx+}\right.}{d^3}}{c^4}$
parts	$-\frac{a\left(-\frac{1}{16c^4(cx+1)^2}+\frac{3}{16c^4(cx+1)}-\frac{1}{16c^4(cx-1)^2}-\frac{3}{16c^4(cx-1)}\right)}{d^3}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{16(cx-1)^2}-\frac{3\operatorname{arccosh}(cx)}{16(cx-1)}-\frac{\operatorname{arccosh}(cx)}{16(cx+1)^2}+\frac{3\operatorname{arccos}}{16(cx}\right.}{d^3c^4}$
orering	$-\frac{(cx-1)(cx+1)(4c^4x^4+3c^2x^2-4)(a+b\operatorname{arccosh}(cx))}{4c^4(-c^2dx^2+d)^3}-\frac{(4c^2x^2-3)(cx-1)^2(cx+1)^2\left(\frac{3x^2(a+b\operatorname{arccosh}(cx))}{(-c^2dx^2+d)^3}+\frac{}{\sqrt{cx-1}}\right.}{12x^2c^4}$

input
```
int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-a/d^3*(-1/16/(c*x-1)^2-3/16/(c*x-1)-1/16/(c*x+1)^2+3/16/(c*x+1))-b
/d^3*(-1/16*arccosh(c*x)/(c*x-1)^2-3/16*arccosh(c*x)/(c*x-1)-1/16*arccosh(
c*x)/(c*x+1)^2+3/16*arccosh(c*x)/(c*x+1)-1/12*c*x*(4*c^2*x^2-3)/(c*x+1)^(3
/2)/(c*x-1)^(3/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$$
$$=\frac{3\,ac^4x^4+3\,(2\,bc^2x^2-b)\log\left(cx+\sqrt{c^2x^2-1}\right)+(4\,bc^3x^3-3\,bcx)\sqrt{c^2x^2-1}}{12\,(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3)}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*x^4 + 3*(2*b*c^2*x^2 - b)*log(c*x + sqrt(c^2*x^2 - 1)) + (4*
b*c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4
*d^3)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{ax^3}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^3\,\text{acosh}\,(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b*x**3*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx)+a)x^3}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1)) - 3)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 16*integrate(1/4*(2*c^2
*x^2 - 1)/(c^10*d^3*x^7 - 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 - c^4*d^3*x + (c^9
*d^3*x^6 - 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3)*e^(1/2*log(c*x + 1) +
1/2*log(c*x - 1))), x)) + 1/4*(2*c^2*x^2 - 1)*a/(c^8*d^3*x^4 - 2*c^6*d^3*x
^2 + c^4*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \int \frac{x^3\,(a+b\operatorname{acosh}(c\,x))}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \frac{-4\left(\int \frac{acosh(cx)x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^4x^4+8\left(\int \frac{acosh(cx)x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^2x^2-4\left(\int \frac{acosh(cx)x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b+a\,x}{4d^3\,(c^4x^4-2c^2x^2+1)}$$

input
```
int(x^3*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int((acosh(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)
*b*c**4*x**4 + 8*int((acosh(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x
**2 - 1),x)*b*c**2*x**2 - 4*int((acosh(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4
 + 3*c**2*x**2 - 1),x)*b + a*x**4)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.48 $\int \frac{x^2(a+b\textbf{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x(a+b\operatorname{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{x(a+b\operatorname{arccosh}(cx))}{8c^2d^3\left(1-c^2x^2\right)} - \frac{(a+b\operatorname{arccosh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arccosh}(cx)}\right)}{4c^3d^3} - \frac{b\operatorname{PolyLog}\left(2,-e^{\operatorname{arccosh}(cx)}\right)}{8c^3d^3}+\frac{b\operatorname{PolyLog}\left(2,e^{\operatorname{arccosh}(cx)}\right)}{8c^3d^3}$$

output
```
1/12*b/c^3/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/8*b/c^3/d^3/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+1/4*x*(a+b*arccosh(c*x))/c^2/d^3/(-c^2*x^2+1)^2-1/8*x*(a+b*arcc
osh(c*x))/c^2/d^3/(-c^2*x^2+1)-1/4*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))/c^3/d^3-1/8*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/c^3/d^3+1/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.54

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3}\,dx$$
$$= \frac{-\frac{b(-2+cx)\sqrt{1+cx}}{(-1+cx)^{3/2}} + \frac{b\sqrt{-1+cx}(2+cx)}{(1+cx)^{3/2}} + \frac{12acx}{(-1+c^2x^2)^2} + \frac{6acx}{-1+c^2x^2} + \frac{3b\text{arccosh}(cx)}{(-1+cx)^2} - \frac{3b\text{arccosh}(cx)}{(1+cx)^2} - 3b\left(-\frac{1}{\sqrt{\frac{-1+cx}{1+cx}}} + \right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
 c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (6*a*c*x)/(-1 + c^2
*x^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2 -
 3*b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 3*b*(Sqr
t[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) - (3*b*ArcCosh[c*x]*(Arc
Cosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 + (3*b*ArcCosh[c*x]*(ArcCosh[c*x
] - 4*Log[1 + E^ArcCosh[c*x]]))/2 + 3*a*Log[1 - c*x] - 3*a*Log[1 + c*x] -
6*b*PolyLog[2, -E^ArcCosh[c*x]] + 6*b*PolyLog[2, E^ArcCosh[c*x]])/(48*c^3*
d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {6349, 27, 83, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3}\,dx$$
$$\Big\downarrow \; 6349$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{d^2(1-c^2x^2)^2}dx}{4c^2d}-\frac{b\int \frac{x}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4cd^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{4c^2d^3}-\frac{b\int \frac{x}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4cd^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{83}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{6316}$$

$$-\frac{\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{83}$$

$$-\frac{\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{6318}$$

$$-\frac{-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$\downarrow$ 4670

$$-\frac{-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)}{2c}+}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$\downarrow$ 2715

$$-\frac{-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right.\right.}{2c}}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}}{4c^2d^3}+\frac{x(a+b\text{arccosh}(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
b/(12*c^3*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (x*(a + b*ArcCosh[c*x]))
/(4*c^2*d^3*(1 - c^2*x^2)^2) - (-1/2*b/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(x*(a + b*ArcCosh[c*x]))/(2*(1 - c^2*x^2)) - ((I/2)*((2*I)*(a + b*ArcCosh[
c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*Poly
Log[2, E^ArcCosh[c*x]]))/c)/(4*c^2*d^3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.38

method	result
derivativedivides	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{1}{16(cx-1)}-\frac{\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{1}{16(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{3c^3x^3\operatorname{arccosh}(cx)+3\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2+3}{24\left(c^4x^4-2c^2x^2\right.}\right.}{}}{}$
default	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{1}{16(cx-1)}-\frac{\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{1}{16(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{3c^3x^3\operatorname{arccosh}(cx)+3\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2+3}{24\left(c^4x^4-2c^2x^2\right.}\right.}{}}{}$
parts	$-\frac{a\left(\frac{1}{16c^3(cx+1)^2}-\frac{1}{16c^3(cx+1)}+\frac{\ln(cx+1)}{16c^3}-\frac{1}{16c^3(cx-1)^2}-\frac{1}{16c^3(cx-1)}-\frac{\ln(cx-1)}{16c^3}\right)}{d^3}-\frac{b\left(-\frac{3c^3x^3\operatorname{arccosh}(cx)+3\sqrt{cx-1}\,\sqrt{c}}{24\left(\right.}\right.}{}$

input
```
int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a/d^3*(-1/16/(c*x-1)^2-1/16/(c*x-1)-1/16*ln(c*x-1)+1/16/(c*x+1)^2-
1/16/(c*x+1)+1/16*ln(c*x+1))-b/d^3*(-1/24*(3*c^3*x^3*arccosh(c*x)+3*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+3*c*x*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/(c^4*x^4-2*c^2*x^2+1)+1/8*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))+1/8*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8*arccosh(c*x)*ln
(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx=\int -\frac{(b\operatorname{arcosh}(cx)+a)x^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^
2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx=-\frac{\int \frac{ax^2}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx^2\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b*x**2*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = \int -\frac{(b \operatorname{arcosh}(cx) + a)x^2}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/2048*(6144*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*d^3
*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) - 16*c^4*(2*(5*c^2*x^3 - 3*x)/(c^10*d^
3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3) + 3*log(c*x + 1)/(c^7*d^3) - 3*log(c*x -
1)/(c^7*d^3)) - 2048*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^8*d^3*x^6 - 3*
c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) + 6*(c*(2*(5*c^2*x^2 + 3*c*x -
6)/(c^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 5*log(c*x + 1)/(c^
7*d^3) + 5*log(c*x - 1)/(c^7*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^10
*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3))*c^3 - 16*c^2*(2*(c^2*x^3 + x)/(c^8*d^
3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) - log(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(
c^5*d^3)) + 4096*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*
d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) + 3*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(
c^8*d^3*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 3*log(c*x + 1)/(c^5*d^3
) + 3*log(c*x - 1)/(c^5*d^3)) - 16*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^6*d^3*x
^2 + c^4*d^3))*c - 32*((c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 2*(c^4*x
^4 - 2*c^2*x^2 + 1)*log(c*x + 1)*log(c*x - 1) + 4*(2*c^3*x^3 + 2*c*x - (c^
4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x -
1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 +
 c^3*d^3) + 2048*integrate(-1/16*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x^2
 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^9*d^3*x^7
- 3*c^7*d^3*x^5 + 3*c^5*d^3*x^3 - c^3*d^3*x + (c^8*d^3*x^6 - 3*c^6*d^3*...
```

Giac [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = \int -\frac{(b \operatorname{arcosh}(cx) + a)x^2}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3}\, dx = \int \frac{x^2\,(a + b\,\text{acosh}(c\,x))}{(d - c^2\,d\,x^2)^3}\, dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3}\, dx = \frac{-16\left(\int \frac{acosh(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right) b\,c^7x^4 + 32\left(\int \frac{acosh(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right) b\,c^5x^2 - 16\left(\int \frac{acosh(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right) b\,c$$

input
```
int(x^2*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int((acosh(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x
)*b*c**7*x**4 + 32*int((acosh(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2
*x**2 - 1),x)*b*c**5*x**2 - 16*int((acosh(c*x)*x**2)/(c**6*x**6 - 3*c**4*x
**4 + 3*c**2*x**2 - 1),x)*b*c**3 + log(c**2*x - c)*a*c**4*x**4 - 2*log(c**
2*x - c)*a*c**2*x**2 + log(c**2*x - c)*a - log(c**2*x + c)*a*c**4*x**4 + 2
*log(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a + 2*a*c**3*x**3 + 2*a*c*x
)/(16*c**3*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.49 $\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3} dx = \frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{bx}{6cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\text{arccosh}(cx)}{4c^2d^3(1-c^2x^2)^2}$$

output
```
1/12*b*x/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-1/6*b*x/c/d^3/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+1/4*(a+b*arccosh(c*x))/c^2/d^3/(-c^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3} dx = \frac{3a+bcx\sqrt{-1+cx}\sqrt{1+cx}(3-2c^2x^2)+3b\text{arccosh}(cx)}{12c^2d^3(-1+c^2x^2)^2}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(3*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - 2*c^2*x^2) + 3*b*ArcCosh[c*
x])/(12*c^2*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6329, 42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{ 6329}$$

$$\frac{a+b\text{arccosh}(cx)}{4c^2d^3(1-c^2x^2)^2}-\frac{b\int \frac{1}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4cd^3}$$

$$\Big\downarrow \text{ 42}$$

$$\frac{a+b\text{arccosh}(cx)}{4c^2d^3(1-c^2x^2)^2}-\frac{b\left(-\frac{2}{3}\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

$$\Big\downarrow \text{ 41}$$

$$\frac{a+b\text{arccosh}(cx)}{4c^2d^3(1-c^2x^2)^2}-\frac{b\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4cd^3}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
-1/4*(b*(-1/3*x/((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (2*x)/(3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])))/(c*d^3) + (a + b*ArcCosh[c*x])/(4*c^2*d^3*(1 - c^2*x^2
)^2)
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{a}{4d^3\left(c^2x^2-1\right)^2}-\frac{b\left(-\frac{\text{arccosh}(cx)}{4\left(c^2x^2-1\right)^2}+\frac{cx\left(2c^2x^2-3\right)}{12\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)}\right)}{d^3}}{c^2}$
default	$\frac{\frac{a}{4d^3\left(c^2x^2-1\right)^2}-\frac{b\left(-\frac{\text{arccosh}(cx)}{4\left(c^2x^2-1\right)^2}+\frac{cx\left(2c^2x^2-3\right)}{12\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)}\right)}{d^3}}{c^2}$
parts	$\frac{a}{4d^3c^2(c^2x^2-1)^2}-\frac{b\left(-\frac{\text{arccosh}(cx)}{4\left(c^2x^2-1\right)^2}+\frac{cx\left(2c^2x^2-3\right)}{12\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)}\right)}{d^3c^2}$
orering	$\frac{(cx-1)(cx+1)\left(10c^4x^4-13c^2x^2-6\right)(a+b\ \text{arccosh}(cx))}{12c^2(-c^2d\,x^2+d)^3}+\frac{\left(2c^2x^2-3\right)(cx-1)^2(cx+1)^2\left(\frac{a+b\ \text{arccosh}(cx)}{\left(-c^2d\,x^2+d\right)^3}+\cdots\right.}{12c^2}$

input
```
int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4*a/d^3/(c^2*x^2-1)^2-b/d^3*(-1/4/(c^2*x^2-1)^2*arccosh(c*x)+1/12
/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c*x*(2*c^2*x^2-3)/(c^2*x^2-1)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{3\,ac^4x^4-6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+(2\,bc^3x^3-3\,bcx)\sqrt{c^2x^2-1}}{12\,(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3)}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/12*(3*a*c^4*x^4 - 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 - 1)) + (2*b
*c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*
d^3)
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{ax}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx\,\text{acosh}\,(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(
b*x*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = \int -\frac{(b \text{arcosh}(cx) + a)x}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*b*((4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 1)/(c^6*d^3*x^4 - 2*c^
4*d^3*x^2 + c^2*d^3) + 16*integrate(1/4/(c^8*d^3*x^7 - 3*c^6*d^3*x^5 + 3*c
^4*d^3*x^3 - c^2*d^3*x + (c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*
d^3)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) + 1/4*a/(c^6*d^3*x^4 -
2*c^4*d^3*x^2 + c^2*d^3)
```

Giac [F]

$$\int \frac{x(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = \int -\frac{(b \text{arcosh}(cx) + a)x}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3} \, dx = \int \frac{x\,(a + b\,\text{acosh}(c\,x))}{(d - c^2\,d\,x^2)^3} \, dx$$

input
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{\left(d-c^2dx^2\right)^3}\,dx$$
$$= \frac{-4\left(\int \frac{acosh(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^6x^4+8\left(\int \frac{acosh(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^4x^2-4\left(\int \frac{acosh(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^2+}{4c^2d^3\left(c^4x^4-2c^2x^2+1\right)}$$

input
```
int(x*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int((acosh(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*
c**6*x**4 + 8*int((acosh(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 -
1),x)*b*c**4*x**2 - 4*int((acosh(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2
*x**2 - 1),x)*b*c**2 + a)/(4*c**2*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.50 $\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 180

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^3}\,dx = \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2} + \frac{3x(a+b\text{arccosh}(cx))}{8d^3(1-c^2x^2)} + \frac{3(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{4cd^3} + \frac{3b\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{8cd^3} - \frac{3b\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{8cd^3}$$

output
```
1/12*b/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-3/8*b/c/d^3/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+1/4*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+3/8*x*(a+b*arccosh(c*x)
)/d^3/(-c^2*x^2+1)+3/4*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/c/d^3+3/8*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3-3/8
*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.76

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^3}\,dx$$

$$= \frac{\frac{4ax}{(-1+c^2x^2)^2} - \frac{6ax}{-1+c^2x^2} + \frac{b\left(\sqrt{-1+cx}\sqrt{1+cx}(2+cx)-3\text{arccosh}(cx)\right)}{3c(1+cx)^2} + \frac{b\left((2-cx)\sqrt{-1+cx}\sqrt{1+cx}+3\text{arccosh}(cx)\right)}{3c(-1+cx)^2} + \frac{3b\left(-\frac{1}{\sqrt{\frac{-1}{1+}}}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```

output
```
((4*a*x)/(-1 + c^2*x^2)^2 - (6*a*x)/(-1 + c^2*x^2) + (b*(Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(3*c*(1 + c*x)^2) + (b*((2 - c*x)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 3*ArcCosh[c*x]))/(3*c*(-1 + c*x)^2) + (3*b
*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)))/c + (3*b*(Sqr
t[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)))/c - (3*a*Log[1 - c*x])/
c + (3*a*Log[1 + c*x])/c - (3*b*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^
ArcCosh[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/(2*c) + (3*b*(ArcCosh[c*x
]*(ArcCosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]
]))/(2*c))/(16*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6316, 27, 83, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^3}\,dx$$

$$\Big\downarrow \text{ 6316}$$

$$\frac{3\int \frac{a+b\text{arccosh}(cx)}{d^2(1-c^2x^2)^2}dx}{4d}-\frac{bc\int \frac{x}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{4d^3}-\frac{bc\int \frac{x}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{3\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{6316}$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{83}$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{6318}$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{3\left(-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{4d^3}+\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \; 4670$$

$$\frac{3\left(-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)}{2c}+\right.}{4d^3}$$
$$\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{3\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e^{\text{arccosh}(c}\right.\right.}{2c}\right.}{4d^3}$$
$$\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{3\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right.}{4d^3}$$
$$\frac{x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```

output
```
b/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (x*(a + b*ArcCosh[c*x]))/(
4*d^3*(1 - c^2*x^2)^2) + (3*(-1/2*b/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*
(a + b*ArcCosh[c*x]))/(2*(1 - c^2*x^2)) - ((I/2)*((2*I)*(a + b*ArcCosh[c*x
])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog
[2, E^ArcCosh[c*x]]))/c))/(4*d^3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.42

method	result
derivativedivides	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\operatorname{arccosh}(cx)+9\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2-}{24c^4x^4-48c^2}\right.}{}}{}$
default	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\operatorname{arccosh}(cx)+9\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2-}{24c^4x^4-48c^2}\right.}{}}{}$
parts	$-\frac{a\left(\frac{1}{16c(cx+1)^2}+\frac{3}{16c(cx+1)}-\frac{3\ln(cx+1)}{16c}-\frac{1}{16c(cx-1)^2}+\frac{3}{16c(cx-1)}+\frac{3\ln(cx-1)}{16c}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\operatorname{arccosh}(cx)+9\sqrt{cx-1}\,\sqrt{cx+1}}{24c^4x}\right.}{}$

input
```
int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-a/d^3*(-1/16/(c*x-1)^2+3/16/(c*x-1)+3/16*ln(c*x-1)+1/16/(c*x+1)^2+3/
16/(c*x+1)-3/16*ln(c*x+1))-b/d^3*(1/24*(9*c^3*x^3*arccosh(c*x)+9*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*c^2*x^2-15*c*x*arccosh(c*x)-11*(c*x-1)^(1/2)*(c*x+1)^(1
/2))/(c^4*x^4-2*c^2*x^2+1)+3/8*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))+3/8*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/8*arccosh(c*x)*ln
(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/8*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^
2 - d^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^3} \, dx = -\frac{\int \frac{a}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} \, dx + \int \frac{b \operatorname{acosh}(cx)}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} \, dx}{d^3}$$

input
```
integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b*
acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3
*x^4 + 3*c^2*d^3*x^2 - d^3), x) - 48*c^4*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4
 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c
^5*d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d
^3*x^4 + 3*c^2*d^3*x^2 - d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^8*
d^3*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3) +
5*log(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4 -
 2*c^6*d^3*x^2 + c^4*d^3))*c^3 + 80*c^2*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*
c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3))
+ 12288*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 +
 3*c^2*d^3*x^2 - d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x^3 -
 c^5*d^3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x
 - 1)/(c^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)
)*c - 32*(3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 6*(c^4*x^4 - 2*c^2*
x^2 + 1)*log(c*x + 1)*log(c*x - 1) + 4*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 -
2*c^2*x^2 + 1)*log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3
) + 2048*integrate(-1/16*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)
*log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^7*d^3*x^7 - 3
*c^5*d^3*x^5 + 3*c^3*d^3*x^3 - c*d^3*x + (c^6*d^3*x^6 - 3*c^4*d^3*x^4 +...
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d - c^2dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d-c^2dx^2)^3}\,dx=\int \frac{a+b\,\operatorname{acosh}(c\,x)}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d-c^2dx^2)^3}\,dx = \frac{-16\left(\int \frac{acosh(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^5x^4+32\left(\int \frac{acosh(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^3x^2-16\left(\int \frac{acosh(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)bc}{}$$

input
```
int((a+b*acosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int(acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**
5*x**4 + 32*int(acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*
b*c**3*x**2 - 16*int(acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1
),x)*b*c - 3*log(c**2*x - c)*a*c**4*x**4 + 6*log(c**2*x - c)*a*c**2*x**2 -
 3*log(c**2*x - c)*a + 3*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a
*c**2*x**2 + 3*log(c**2*x + c)*a - 6*a*c**3*x**3 + 10*a*c*x)/(16*c*d**3*(c
**4*x**4 - 2*c**2*x**2 + 1))
```

3.51 $\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^3}\,dx &= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}}\\ &+\frac{a+b\text{arccosh}(cx)}{4d^3(1-c^2x^2)^2}+\frac{a+b\text{arccosh}(cx)}{2d^3(1-c^2x^2)}\\ &+\frac{2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)}{d^3}\\ &+\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{2d^3}-\frac{b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2d^3}\end{aligned}$$

output
```
1/12*b*c*x/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-2/3*b*c*x/d^3/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+1/4*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+1/2*(a+b*arccosh(c*x)
)/d^3/(-c^2*x^2+1)+2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2)/d^3+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-1
/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.92

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx$$
$$= \frac{\frac{a}{(-1+c^2x^2)^2}-\frac{2a}{-1+c^2x^2}-\frac{b\left(\sqrt{-1+cx}\sqrt{1+cx}(2+cx)-3\operatorname{arccosh}(cx)\right)}{12(1+cx)^2}+\frac{b\left((2-cx)\sqrt{-1+cx}\sqrt{1+cx}+3\operatorname{arccosh}(cx)\right)}{12(-1+cx)^2}+\frac{5}{4}b\left(-\frac{}{\sqrt{}}\right.}{}$$

input

```
Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

output

```
(a/(-1 + c^2*x^2)^2 - (2*a)/(-1 + c^2*x^2) - (b*(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(12*(1 + c*x)^2) + (b*((2 - c*x)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x] + 3*ArcCosh[c*x]))/(12*(-1 + c*x)^2) + (5*b*(-(1/Sqrt
[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)))/4 - (5*b*(Sqrt[(-1 + c*
x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)))/4 + 4*a*Log[x] - 2*a*Log[1 - c^2*
x^2] + 2*b*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) -
 PolyLog[2, -E^(-2*ArcCosh[c*x])]) + b*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log
[1 + E^ArcCosh[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]) + b*(ArcCosh[c*x]*(
ArcCosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))
/(4*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {6351, 27, 42, 41, 6351, 41, 6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx$$

$$\Big\downarrow \; 6351$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{d^2x(1-c^2x^2)^2}dx}{d}-\frac{bc\int \frac{1}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^3}-\frac{bc\int \frac{1}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \; 42$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^3}-\frac{bc\left(-\frac{2}{3}\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \; 41$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 6351$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}}{d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 41$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}}{d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 6331$$

$$\frac{-\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}}{d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 5984$$

$$\frac{-2\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}}{d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{-2\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}}{d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 26$$

$$\frac{-2i\int (a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}}{d^3}+\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 4670$$

$$\frac{-2i\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 2715$$

$$\frac{-2i\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx}\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \; 2838$$

$$\frac{-2i\left(i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{4d^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{4d^3}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

output
```
-1/4*(b*c*(-1/3*x/((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (2*x)/(3*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])))/d^3 + (a + b*ArcCosh[c*x])/(4*d^3*(1 - c^2*x^2)^2) +
 (-1/2*(b*c*x)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*(1
 - c^2*x^2)) - (2*I)*(I*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])] +
 (I/4)*b*PolyLog[2, -E^(2*ArcCosh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcCosh
[c*x])]))/d^3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.89

method	result
parts	$-\frac{a\left(-\ln(x)-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}\right)}{d^3}-\frac{b\left(8c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-8c^4x^4+6c\right.}{}$
derivativedivides	$-\frac{a\left(-\ln(cx)-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b\left(8c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-8c^4x^4+6\right.}{}$
default	$-\frac{a\left(-\ln(cx)-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b\left(8c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-8c^4x^4+6\right.}{}$

input
```
int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-a/d^3*(-ln(x)-1/16/(c*x+1)^2-5/16/(c*x+1)+1/2*ln(c*x+1)-1/16/(c*x-1)^2+5/
16/(c*x-1)+1/2*ln(c*x-1))-b/d^3*(1/12*(8*c^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/
2)-8*c^4*x^4+6*c^2*x^2*arccosh(c*x)-9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+16*c
^2*x^2-9*arccosh(c*x)-8)/(c^4*x^4-2*c^2*x^2+1)+arccosh(c*x)*ln(1+c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))+polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-arccos
h(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)+arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))+polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx=\int -\frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^3x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^
3 - d^3*x), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x (d - c^2 d x^2)^3} \, dx = -\frac{\int \frac{a}{c^6x^7-3c^4x^5+3c^2x^3-x} \, dx + \int \frac{b \operatorname{acosh}(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x} \, dx}{d^3}$$

input
```
integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integral(b*
acosh(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x (d - c^2 d x^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2 d x^2 - d)^3 x} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x +
1)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - b*integrate(log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*
x), x)
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x (d - c^2 d x^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2 d x^2 - d)^3 x} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x\,\left(d-c^2\,d\,x^2\right)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx$$
$$=\frac{-4\left(\int \frac{acosh(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)b\,c^4x^4+8\left(\int \frac{acosh(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)b\,c^2x^2-4\left(\int \frac{acosh(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)b-2}{}$$

input
```
int((a+b*acosh(c*x))/x/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int(acosh(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b*c**4
*x**4 + 8*int(acosh(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b*
c**2*x**2 - 4*int(acosh(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x
)*b - 2*log(c**2*x - c)*a*c**4*x**4 + 4*log(c**2*x - c)*a*c**2*x**2 - 2*lo
g(c**2*x - c)*a - 2*log(c**2*x + c)*a*c**4*x**4 + 4*log(c**2*x + c)*a*c**2
*x**2 - 2*log(c**2*x + c)*a + 4*log(x)*a*c**4*x**4 - 8*log(x)*a*c**2*x**2
+ 4*log(x)*a - a*c**4*x**4 + 2*a)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.52 $\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^3}\,dx = \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\text{arccosh}(cx)}{d^3x} + \frac{c^2x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2} + \frac{7c^2x(a+b\text{arccosh}(cx))}{8d^3(1-c^2x^2)} + \frac{bc\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d^3} + \frac{15c(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{4d^3} + \frac{15bc\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{8d^3} - \frac{15bc\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{8d^3}$$

output

```
1/12*b*c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-7/8*b*c/d^3/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-(a+b*arccosh(c*x))/d^3/x+1/4*c^2*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2
+1)^2+7/8*c^2*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+b*c*arctan((c*x-1)^(1/
2)*(c*x+1)^(1/2))/d^3+15/4*c*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))/d^3+15/8*b*c*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^
3-15/8*b*c*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.66

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx$$

$$= \frac{-\frac{96a}{x}+\frac{24ac^2x}{(-1+c^2x^2)^2}-\frac{84ac^2x}{-1+c^2x^2}-\frac{2bc\left((-2+cx)\sqrt{-1+cx}\sqrt{1+cx}-3\text{arccosh}(cx)\right)}{(-1+cx)^2}+\frac{2bc\left(\sqrt{-1+cx}\sqrt{1+cx}(2+cx)-3\text{arccosh}(cx)\right)}{(1+cx)^2}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

output
```
((-96*a)/x + (24*a*c^2*x)/(-1 + c^2*x^2)^2 - (84*a*c^2*x)/(-1 + c^2*x^2) -
 (2*b*c*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCosh[c*x]))/(-1 +
c*x)^2 + (2*b*c*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))
/(1 + c*x)^2 - (96*b*ArcCosh[c*x])/x + 42*b*c*(-(1/Sqrt[(-1 + c*x)/(1 + c*
x)]) + ArcCosh[c*x]/(1 - c*x)) + 42*b*c*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcC
osh[c*x]/(1 + c*x)) + (96*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]
])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 90*a*c*Log[1 - c*x] + 90*a*c*Log[1 + c
*x] - 45*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCosh[c*x]]) - 4*
PolyLog[2, -E^ArcCosh[c*x]]) + 45*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[
1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(96*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.24, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {6347, 27, 115, 27, 115, 27, 103, 218, 6316, 83, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx$$

$$\Big\downarrow \ 6347$$

$$5c^2\int \frac{a+b\text{arccosh}(cx)}{d^3(1-c^2x^2)^3}dx+\frac{bc\int \frac{1}{x(cx-1)^{5/2}(cx+1)^{5/2}}dx}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx}{d^3}+\frac{bc\int \frac{1}{x(cx-1)^{5/2}(cx+1)^{5/2}}dx}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \ 115$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx}{d^3}+\frac{bc\left(-\frac{\int \frac{3c}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx}{3c}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx}{d^3}+\frac{bc\left(-\int \frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \ 115$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx}{d^3}+\frac{bc\left(\frac{\int \frac{c}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{c}+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx}{d^3}+\frac{bc\left(\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \ 103$$

$$\frac{5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx}{d^3}+\frac{bc\left(c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{5c^2\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \text{ 6316}$$

$$\frac{5c^2\left(\frac{3}{4}\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx-\frac{1}{4}bc\int\frac{x}{(cx-1)^{5/2}(cx+1)^{5/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{5c^2\left(\frac{3}{4}\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \text{ 6316}$$

$$\frac{5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(cx-1)^{3/2}(}\right.}{d^3}$$
$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}-$$
$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \text{ 6318}$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(cx-1)^{3/2}(c}\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \; 26$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \; 4670$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(\right.}{2c}\right.\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \; 2715$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\left(e^{\text{arc}}\right.\right.}{2c}\right.\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

$$\Big\downarrow \; 2838$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}+\frac{x(a+b\text{arccos}}{2(1-c^2x^2}\right.\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{d^3}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

output
```
-((a + b*ArcCosh[c*x])/(d^3*x*(1 - c^2*x^2)^2)) + (b*c*(-1/3*1/((-1 + c*x)
^(3/2)*(1 + c*x)^(3/2)) + 1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ArcTan[Sqrt[-
1 + c*x]*Sqrt[1 + c*x]]))/d^3 + (5*c^2*(b/(12*c*(-1 + c*x)^(3/2)*(1 + c*x)
^(3/2)) + (x*(a + b*ArcCosh[c*x]))/(4*(1 - c^2*x^2)^2) + (3*(-1/2*b/(c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*(1 - c^2*x^2)) -
((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2
, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/c))/4))/d^3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.23

method	result
derivativedivides	$c\left(-\frac{a\left(\frac{1}{cx}-\frac{1}{16(cx-1)^2}+\frac{7}{16(cx-1)}+\frac{15\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{7}{16(cx+1)}-\frac{15\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{45\operatorname{arccosh}(cx)c^4x^4+21c^3x}{}\right.}{}\right.$
default	$c\left(-\frac{a\left(\frac{1}{cx}-\frac{1}{16(cx-1)^2}+\frac{7}{16(cx-1)}+\frac{15\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{7}{16(cx+1)}-\frac{15\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{45\operatorname{arccosh}(cx)c^4x^4+21c^3x}{}\right.}{}\right.$
parts	$-\frac{a\left(\frac{1}{x}+\frac{c}{16(cx+1)^2}+\frac{7c}{16(cx+1)}-\frac{15c\ln(cx+1)}{16}-\frac{c}{16(cx-1)^2}+\frac{7c}{16(cx-1)}+\frac{15c\ln(cx-1)}{16}\right)}{d^3}-\frac{bc\left(\frac{45\operatorname{arccosh}(cx)c^4x^4+21c^3x^3\sqrt{}}{}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-a/d^3*(1/c/x-1/16/(c*x-1)^2+7/16/(c*x-1)+15/16*ln(c*x-1)+1/16/(c*x+1)^
2+7/16/(c*x+1)-15/16*ln(c*x+1))-b/d^3*(1/24*(45*arccosh(c*x)*c^4*x^4+21*c^
3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-75*c^2*x^2*arccosh(c*x)-23*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*c*x+24*arccosh(c*x))/c/x/(c^4*x^4-2*c^2*x^2+1)-2*arctan(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))-15/8*dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-
15/8*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-15/8*arccosh(c*x)*ln(1+c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2 d x^2)^3}\, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2 d x^2 - d)^3 x^2}\, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^
4 - d^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2 d x^2)^3}\, dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^3 x^2} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/2048*(92160*c^7*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3
*x^4 + 3*c^2*d^3*x^2 - d^3), x) - 240*c^6*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^
4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(
c^5*d^3)) - 30720*c^6*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4
*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x) + 90*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^
8*d^3*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3)
+ 5*log(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4
 - 2*c^6*d^3*x^2 + c^4*d^3))*c^5 + 400*c^4*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 -
 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3
)) + 61440*c^4*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^
4 + 3*c^2*d^3*x^2 - d^3), x) + 45*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x
^3 - c^5*d^3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log
(c*x - 1)/(c^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*
d^3))*c^3 + 128*c^2*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^
3) - 3*log(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 30720*c^2*integrat
e(1/32*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x
) - 32*(15*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(c*x + 1)^2 + 30*(c^5*x^5 - 2*c^
3*x^3 + c*x)*log(c*x + 1)*log(c*x - 1) + 4*(30*c^4*x^4 - 50*c^2*x^2 - 15*(
c^5*x^5 - 2*c^3*x^3 + c*x)*log(c*x + 1) + 15*(c^5*x^5 - 2*c^3*x^3 + c*x)*l
og(c*x - 1) + 16)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^4*d^3*x^5 ...
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^3 x^2} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^3}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x^2\,(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^3}\,dx = \frac{-16\left(\int \frac{acosh(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)b\,c^4x^5+32\left(\int \frac{acosh(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)b\,c^2x^3-16\left(\int \frac{acosh(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)}{}$$

input
```
int((a+b*acosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int(acosh(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),x)*b*
c**4*x**5 + 32*int(acosh(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**
2),x)*b*c**2*x**3 - 16*int(acosh(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x*
*4 - x**2),x)*b*x - 15*log(c**2*x - c)*a*c**5*x**5 + 30*log(c**2*x - c)*a*
c**3*x**3 - 15*log(c**2*x - c)*a*c*x + 15*log(c**2*x + c)*a*c**5*x**5 - 30
*log(c**2*x + c)*a*c**3*x**3 + 15*log(c**2*x + c)*a*c*x - 30*a*c**4*x**4 +
 50*a*c**2*x**2 - 16*a)/(16*d**3*x*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.53 $\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx &= \frac{bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc^3x}{6d^3\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} - \frac{a+b\text{arccosh}(cx)}{2d^3x^2} \\ &\quad + \frac{c^2\left(3-2c^2x^2\right)^2(a+b\text{arccosh}(cx))}{4d^3\left(1-c^2x^2\right)^2} \\ &\quad + \frac{6c^2(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)}{d^3} \\ &\quad - \frac{bc^2\sqrt{-1+c^2x^2}\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad + \frac{3bc^2\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{2d^3} \\ &\quad - \frac{3bc^2\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{2d^3}\end{aligned}$$

output
```
1/12*b*c^3*x/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-7/6*b*c^3*x/d^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/x-1/2*(a+b*arccosh(
c*x))/d^3/x^2+1/4*c^2*(-2*c^2*x^2+3)^2*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)
^2+6*c^2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d
^3-b*c^2*(c^2*x^2-1)^(1/2)*arctanh(c*x/(c^2*x^2-1)^(1/2))/d^3/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+3/2*b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/
d^3-3/2*b*c^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.45

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^3}\,dx$$
$$= \frac{-\frac{2a}{x^2}+\frac{ac^2}{(-1+c^2x^2)^2}-\frac{4ac^2}{-1+c^2x^2}-\frac{bc^2\left((-2+cx)\sqrt{-1+cx}\sqrt{1+cx}-3\text{arccosh}(cx)\right)}{12(-1+cx)^2}-\frac{bc^2\left(\sqrt{-1+cx}\sqrt{1+cx}(2+cx)-3\text{arccosh}(cx)\right)}{12(1+cx)^2}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

output
```
((-2*a)/x^2 + (a*c^2)/(-1 + c^2*x^2)^2 - (4*a*c^2)/(-1 + c^2*x^2) - (b*c^2
*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCosh[c*x]))/(12*(-1 + c*x
)^2) - (b*c^2*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(
12*(1 + c*x)^2) + (2*b*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - ArcCosh[c*x]))/
x^2 + (9*b*c^2*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)))
/4 - (9*b*c^2*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)))/4 + 1
2*a*c^2*Log[x] - 6*a*c^2*Log[1 - c^2*x^2] + 6*b*c^2*(ArcCosh[c*x]*(ArcCosh
[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]
) + 3*b*c^2*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCosh[c*x]]) - 4*P
olyLog[2, -E^ArcCosh[c*x]]) + 3*b*c^2*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[
1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(4*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.13, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {6347, 27, 114, 27, 42, 41, 6351, 42, 41, 6351, 41, 6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 \left(d-c^2dx^2\right)^3} \, dx$$

$$\Big\downarrow \text{6347}$$

$$3c^2 \int \frac{a+b\text{arccosh}(cx)}{d^3x \left(1-c^2x^2\right)^3}dx+\frac{bc\int \frac{1}{x^2(cx-1)^{5/2}(cx+1)^{5/2}}dx}{2d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2 \left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3c^2 \int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)^3}dx}{d^3}+\frac{bc\int \frac{1}{x^2(cx-1)^{5/2}(cx+1)^{5/2}}dx}{2d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2 \left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{114}$$

$$\frac{3c^2 \int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)^3}dx}{d^3}+\frac{bc\left(\int \frac{4c^2}{(cx-1)^{5/2}(cx+1)^{5/2}}dx+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2 \left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3c^2 \int \frac{a+b\text{arccosh}(cx)}{x\left(1-c^2x^2\right)^3}dx}{d^3}+\frac{bc\left(4c^2\int \frac{1}{(cx-1)^{5/2}(cx+1)^{5/2}}dx+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2 \left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{42}$$

$$\frac{3c^2\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^3}dx}{d^3}+\frac{bc\left(4c^2\left(-\frac{2}{3}\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \ 41$$

$$\frac{3c^2\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^3}dx}{d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 6351$$

$$\frac{3c^2\left(\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx-\frac{1}{4}bc\int \frac{1}{(cx-1)^{5/2}(cx+1)^{5/2}}dx+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 42$$

$$\frac{3c^2\left(\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx-\frac{1}{4}bc\left(-\frac{2}{3}\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 41$$

$$\frac{3c^2\left(\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)\right)}{d^3}-\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 6351$$

$$\frac{3c^2\left(\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-3(\right.\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \; 41$$

$$\frac{3c^2\left(\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1}\right.\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \; 6331$$

$$\frac{3c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{}{3}\right.\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \; 5984$$

$$\frac{3c^2\left(-2\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3c^2\left(-2\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \; 26$$

$$\frac{3c^2\left(-2i\int (a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}+\frac{a+b\text{arccosh}(cx)}{4(1-c^2x^2)^2}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \; 4670$$

$$\frac{3c^2\left(-2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+i\text{arctanh}\left(e^{2\text{arccos}\right.\right.\right.}{}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{3c^2\left(-2i\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e^{2\text{arccosh}(cx)}\right)de^{2\text{arccc}\right.\right.}{}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{3c^2\left(-2i\left(i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arccosh}(c}\right.\right.\right.}{d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{x}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{2d^3}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

output
```
(b*c*(1/(x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + 4*c^2*(-1/3*x/((-1 + c*x)^(
3/2)*(1 + c*x)^(3/2)) + (2*x)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))))/(2*d^3)
- (a + b*ArcCosh[c*x])/(2*d^3*x^2*(1 - c^2*x^2)^2) + (3*c^2*(-1/2*(b*c*x)/
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(-1/3*x/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)) + (2*x)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/4 + (a + b*ArcCosh[c*x])
/(4*(1 - c^2*x^2)^2) + (a + b*ArcCosh[c*x])/(2*(1 - c^2*x^2)) - (2*I)*(I*(
a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])] + (I/4)*b*PolyLog[2, -E^(2
*ArcCosh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcCosh[c*x])])))/d^3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.41

method	result
derivativedivides	$c^2\left(-\frac{a\left(\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{16(cx-1)^2}+\frac{9}{16(cx-1)}+\frac{3\ln(cx-1)}{2}-\frac{1}{16(cx+1)^2}-\frac{9}{16(cx+1)}+\frac{3\ln(cx+1)}{2}\right)}{d^3}-\frac{b\left(\frac{8c^5x^5\sqrt{cx-1}}{\cdots}\right.}{\cdots}\right.$
default	$c^2\left(-\frac{a\left(\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{16(cx-1)^2}+\frac{9}{16(cx-1)}+\frac{3\ln(cx-1)}{2}-\frac{1}{16(cx+1)^2}-\frac{9}{16(cx+1)}+\frac{3\ln(cx+1)}{2}\right)}{d^3}-\frac{b\left(\frac{8c^5x^5\sqrt{cx-1}}{\cdots}\right.}{\cdots}\right.$
parts	$-\frac{a\left(\frac{1}{2x^2}-3c^2\ln(x)-\frac{c^2}{16(cx+1)^2}-\frac{9c^2}{16(cx+1)}+\frac{3c^2\ln(cx+1)}{2}-\frac{c^2}{16(cx-1)^2}+\frac{9c^2}{16(cx-1)}+\frac{3c^2\ln(cx-1)}{2}\right)}{d^3}-\frac{b\,c^2\left(\frac{8c^5x^5\sqrt{cx-}}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-a/d^3*(1/2/c^2/x^2-3*ln(c*x)-1/16/(c*x-1)^2+9/16/(c*x-1)+3/2*ln(c*x-
1)-1/16/(c*x+1)^2-9/16/(c*x+1)+3/2*ln(c*x+1))-b/d^3*(1/12/(c^4*x^4-2*c^2*x
^2+1)/c^2/x^2*(8*c^5*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-8*c^6*x^6+18*arccosh(
c*x)*c^4*x^4-3*c^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+16*c^4*x^4-27*c^2*x^2*a
rccosh(c*x)-6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-8*c^2*x^2+6*arccosh(c*x))+3*
arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3*polylog(2,-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))-3*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2)-3/2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+3*arccosh(c*x)*l
n(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d - c^2 dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2 dx^2 - d)^3 x^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^
5 - d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d - c^2 dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\mathrm{arccosh}(cx)}{x^3\,(d-c^2dx^2)^3}\,dx = \int -\frac{b\,\mathrm{arcosh}\,(cx)+a}{(c^2dx^2-d)^3x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x^2
) + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3) -
 b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^6*d^3*x^9 - 3*c^4*d
^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Giac [F]

$$\int \frac{a+b\mathrm{arccosh}(cx)}{x^3\,(d-c^2dx^2)^3}\,dx = \int -\frac{b\,\mathrm{arcosh}\,(cx)+a}{(c^2dx^2-d)^3x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\mathrm{arccosh}(cx)}{x^3\,(d-c^2dx^2)^3}\,dx = \int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^3\,(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx$$
$$= \frac{-4\left(\int \frac{acosh(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)b\,c^4x^6+8\left(\int \frac{acosh(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)b\,c^2x^4-4\left(\int \frac{acosh(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)b\,x^2}{}$$

input
```
int((a+b*acosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int(acosh(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x)*b*c
**4*x**6 + 8*int(acosh(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3)
,x)*b*c**2*x**4 - 4*int(acosh(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5
- x**3),x)*b*x**2 - 6*log(c**2*x - c)*a*c**6*x**6 + 12*log(c**2*x - c)*a*c
**4*x**4 - 6*log(c**2*x - c)*a*c**2*x**2 - 6*log(c**2*x + c)*a*c**6*x**6 +
 12*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a*c**2*x**2 + 12*log(x
)*a*c**6*x**6 - 24*log(x)*a*c**4*x**4 + 12*log(x)*a*c**2*x**2 - 3*a*c**6*x
**6 + 6*a*c**2*x**2 - 2*a)/(4*d**3*x**2*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.54 $\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 284

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^3}\,dx &= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} \\ &\quad+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad-\frac{a+b\text{arccosh}(cx)}{3d^3x^3}-\frac{3c^2(a+b\text{arccosh}(cx))}{d^3x} \\ &\quad+\frac{c^4x(a+b\text{arccosh}(cx))}{4d^3(1-c^2x^2)^2}+\frac{11c^4x(a+b\text{arccosh}(cx))}{8d^3(1-c^2x^2)} \\ &\quad+\frac{19bc^3\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d^3} \\ &\quad+\frac{35c^3(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{4d^3} \\ &\quad+\frac{35bc^3\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{8d^3} \\ &\quad-\frac{35bc^3\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{8d^3}\end{aligned}$$

output
```
-1/12*b*c^3/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/6*b*c/d^3/x^2/(c*x-1)^(3/2)/
(c*x+1)^(3/2)-29/24*b*c^3/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(a+b*arccosh
(c*x))/d^3/x^3-3*c^2*(a+b*arccosh(c*x))/d^3/x+1/4*c^4*x*(a+b*arccosh(c*x))
/d^3/(-c^2*x^2+1)^2+11/8*c^4*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+19/6*b*
c^3*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+35/4*c^3*(a+b*arccosh(c*x))*ar
ctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+35/8*b*c^3*polylog(2,-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))/d^3-35/8*b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.66

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4(d-c^2dx^2)^3}\,dx$$

$$= \frac{-\frac{16a}{x^3}-\frac{144ac^2}{x}+\frac{12ac^4x}{(-1+c^2x^2)^2}-\frac{66ac^4x}{-1+c^2x^2}-\frac{bc^3\left((-2+cx)\sqrt{-1+cx}\sqrt{1+cx}-3\operatorname{arccosh}(cx)\right)}{(-1+cx)^2}+\frac{bc^3\left(\sqrt{-1+cx}\sqrt{1+cx}(2+cx)-3\operatorname{arc}\right.}{(1+cx)^2}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

output
```
((-16*a)/x^3 - (144*a*c^2)/x + (12*a*c^4*x)/(-1 + c^2*x^2)^2 - (66*a*c^4*x
)/(-1 + c^2*x^2) - (b*c^3*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*Arc
Cosh[c*x]))/(-1 + c*x)^2 + (b*c^3*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x)
- 3*ArcCosh[c*x]))/(1 + c*x)^2 + 33*b*c^3*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)])
 + ArcCosh[c*x]/(1 - c*x)) + 33*b*c^3*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCos
h[c*x]/(1 + c*x)) + 144*b*c^2*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^2]*A
rcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (8*b*(-2*ArcC
osh[c*x] + (c*x*(-1 + c^2*x^2 + c^2*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1
+ c^2*x^2]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/x^3 - 105*a*c^3*Log[1 - c*x
] + 105*a*c^3*Log[1 + c*x] - (105*b*c^3*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Lo
g[1 + E^ArcCosh[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/2 + (105*b*c^3*(A
rcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^Ar
cCosh[c*x]]))/2)/(48*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.43, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.080, Rules used = {6347, 27, 114, 27, 115, 27, 115, 27, 103, 218, 6347, 115, 27, 115, 27, 103, 218, 6316, 83, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4 \left(d-c^2dx^2\right)^3} \, dx$$

$$\Big\downarrow \text{6347}$$

$$\frac{7}{3}c^2\int \frac{a+b\text{arccosh}(cx)}{d^3x^2\left(1-c^2x^2\right)^3}dx+\frac{bc\int \frac{1}{x^3(cx-1)^{5/2}(cx+1)^{5/2}}dx}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2\left(1-c^2x^2\right)^3}dx}{3d^3}+\frac{bc\int \frac{1}{x^3(cx-1)^{5/2}(cx+1)^{5/2}}dx}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{114}$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2\left(1-c^2x^2\right)^3}dx}{3d^3}+\frac{bc\left(\frac{1}{2}\int \frac{5c^2}{x(cx-1)^{5/2}(cx+1)^{5/2}}dx+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2\left(1-c^2x^2\right)^3}dx}{3d^3}+\frac{bc\left(\frac{5}{2}c^2\int \frac{1}{x(cx-1)^{5/2}(cx+1)^{5/2}}dx+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{115}$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}+\frac{bc\left(\frac{5}{2}c^2\left(-\frac{\int \frac{3c}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx}{3c}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}+\frac{bc\left(\frac{5}{2}c^2\left(-\int \frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 115$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}+\frac{bc\left(\frac{5}{2}c^2\left(\frac{\int \frac{c}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{c}+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}+\frac{bc\left(\frac{5}{2}c^2\left(\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 103$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}+\frac{bc\left(\frac{5}{2}c^2\left(c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{7c^2\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow \; 6347$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx+bc\int \frac{1}{x(cx-1)^{5/2}(cx+1)^{5/2}}dx-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow \; 115$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx+bc\left(-\frac{\int \frac{3c}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx}{3c}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow \; 27$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx+bc\left(-\int \frac{1}{x(cx-1)^{3/2}(cx+1)^{3/2}}dx-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow \; 115$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx+bc\left(\frac{\int \frac{c}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{c}+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 27$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx+bc\left(\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 103$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx+bc\left(c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)-\right.}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 218$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^3}dx-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right.\right.}{3d^3}-\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 6316$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx-\frac{1}{4}bc\int\frac{x}{(cx-1)^{5/2}(cx+1)^{5/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}+bc\left(\arctan\left(\sqrt{\right.\right.}{3d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 83$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\int\frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(cx-1)^{3/2}(cx+1)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}+bc\left(\arctan\left(\sqrt{cx-\right.\right.}{3d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 6316$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{}{12c(cx-}\right.\right.}{3d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 83$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(cx-1)^{3/2}(cx+1)^{3/}}\right.\right.}{3d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow 6318$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccosh}(cx))}{4(1-c^2x^2)^2}+\frac{}{12c(cx-}\right.\right.}{3d^3}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$\downarrow$ 3042

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccos}}{4(1-c^2x^2)}\right.\right.$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$\downarrow$ 26

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{i\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{x(a+b\text{arccos}}{4(1-c^2x^2)}\right.\right.$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$\downarrow$ 4670

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{i\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arcc}\right.}{2c}\right.\right.\right.$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$\downarrow$ 2715

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctan}}{2c}\right.\right.\right.}{}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

$$\Big\downarrow \text{2838}$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}+\frac{x(a+ba}{2(}\right.\right.\right.}{}$$

$$\frac{a+b\text{arccosh}(cx)}{3d^3x^3(1-c^2x^2)^2}+$$

$$\frac{bc\left(\frac{5}{2}c^2\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{3(cx-1)^{3/2}(cx+1)^{3/2}}\right)+\frac{1}{2x^2(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{3d^3}$$

input

```
Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

output

```
-1/3*(a + b*ArcCosh[c*x])/(d^3*x^3*(1 - c^2*x^2)^2) + (b*c*(1/(2*x^2*(-1 +
 c*x)^(3/2)*(1 + c*x)^(3/2)) + (5*c^2*(-1/3*1/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)) + 1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]]))/2))/(3*d^3) + (7*c^2*(-((a + b*ArcCosh[c*x])/(x*(1 - c^2*x^2)^2))
 + b*c*(-1/3*1/((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + 1/(Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]) + 5*c^2*(b/(12*c*(-1 +
c*x)^(3/2)*(1 + c*x)^(3/2)) + (x*(a + b*ArcCosh[c*x]))/(4*(1 - c^2*x^2)^2)
 + (3*(-1/2*b/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/
(2*(1 - c^2*x^2)) - ((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c
*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/
c))/4)))/(3*d^3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.11

method	result
derivativedivides	$c^3\left(-\frac{a\left(\frac{1}{3c^3x^3}+\frac{3}{cx}-\frac{1}{16(cx-1)^2}+\frac{11}{16(cx-1)}+\frac{35\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{11}{16(cx+1)}-\frac{35\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{105\,\mathrm{arccosh}(cx)c}{}\right.}{}\right.$
default	$c^3\left(-\frac{a\left(\frac{1}{3c^3x^3}+\frac{3}{cx}-\frac{1}{16(cx-1)^2}+\frac{11}{16(cx-1)}+\frac{35\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{11}{16(cx+1)}-\frac{35\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{105\,\mathrm{arccosh}(cx)c}{}\right.}{}\right.$
parts	$-\frac{a\left(\frac{1}{3x^3}+\frac{3c^2}{x}+\frac{c^3}{16(cx+1)^2}+\frac{11c^3}{16(cx+1)}-\frac{35c^3\ln(cx+1)}{16}-\frac{c^3}{16(cx-1)^2}+\frac{11c^3}{16(cx-1)}+\frac{35c^3\ln(cx-1)}{16}\right)}{d^3}-\frac{b\,c^3\left(\frac{105\,\mathrm{arccosh}(cx)}{}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-a/d^3*(1/3/c^3/x^3+3/c/x-1/16/(c*x-1)^2+11/16/(c*x-1)+35/16*ln(c*x-1
)+1/16/(c*x+1)^2+11/16/(c*x+1)-35/16*ln(c*x+1))-b/d^3*(1/24*(105*arccosh(c
*x)*c^6*x^6+29*c^5*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-175*arccosh(c*x)*c^4*x^
4-27*c^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+56*c^2*x^2*arccosh(c*x)-4*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*c*x+8*arccosh(c*x))/(c^4*x^4-2*c^2*x^2+1)/c^3/x^3-19/
3*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-35/8*dilog(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))-35/8*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-35/8*arccosh(c*x
)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^3 x^4} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x
^6 - d^3*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^3} \, dx = \int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)^3 x^4} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/6144*(1935360*c^9*integrate(1/96*x^7*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d
^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) - 1680*c^8*(2*(5*c^2*x^3 - 3*x)/(c^8
*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x
 - 1)/(c^5*d^3)) - 645120*c^8*integrate(1/96*x^6*log(c*x - 1)/(c^6*d^3*x^8
 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 630*(c*(2*(5*c^2*x^2 + 3
*c*x - 6)/(c^8*d^3*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x +
1)/(c^5*d^3) + 5*log(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)
/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3))*c^7 + 2800*c^6*(2*(c^2*x^3 + x)/
(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x
 - 1)/(c^3*d^3)) + 1290240*c^6*integrate(1/96*x^4*log(c*x - 1)/(c^6*d^3*x^
8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 315*(c*(2*(3*c^2*x^2 -
3*c*x - 2)/(c^6*d^3*x^3 - c^5*d^3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x +
 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 -
 2*c^4*d^3*x^2 + c^2*d^3))*c^5 + 896*c^4*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4
 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3))
 - 645120*c^4*integrate(1/96*x^2*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6
 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 128*c^2*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)
/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(
c*x - 1)/d^3) - 32*(105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1)^2 + 2
10*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(210*c...
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx=\int -\frac{b\,\text{arcosh}\left(cx\right)+a}{\left(c^2dx^2-d\right)^3x^4}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^4\,\left(d-c^2\,d\,x^2\right)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx = \frac{-48\left(\int \frac{acosh(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right)b\,c^4x^7+96\left(\int \frac{acosh(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right)b\,c^2x^5-48\left(\int \frac{acosh(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right.}{}$$

input
```
int((a+b*acosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 48*int(acosh(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*b
*c**4*x**7 + 96*int(acosh(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x
**4),x)*b*c**2*x**5 - 48*int(acosh(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2
*x**6 - x**4),x)*b*x**3 - 105*log(c**2*x - c)*a*c**7*x**7 + 210*log(c**2*x
 - c)*a*c**5*x**5 - 105*log(c**2*x - c)*a*c**3*x**3 + 105*log(c**2*x + c)*
a*c**7*x**7 - 210*log(c**2*x + c)*a*c**5*x**5 + 105*log(c**2*x + c)*a*c**3
*x**3 - 210*a*c**6*x**6 + 350*a*c**4*x**4 - 112*a*c**2*x**2 - 16*a)/(48*d*
*3*x**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.55 $\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 278

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \frac{bx^2\sqrt{d-c^2dx^2}}{32c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{16c^4} - \frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{24c^2} + \frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{32bc^5\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/32*b*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/96*b*x^4
*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/36*b*c*x^6*(-c^2*d*x
^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*x*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))/c^4-1/24*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^2+1/6*
x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-1/32*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))^2/b/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.71

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$

$$= \frac{48acx\sqrt{d-c^2dx^2}(-3-2c^2x^2+8c^4x^4)-144a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{b\sqrt{d-c^2dx^2}\left(-72\text{arccosh}(cx)^2+18\,\text{co}\right.}{}}{}$$

input
```
Integrate[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(48*a*c*x*Sqrt[d - c^2*d*x^2]*(-3 - 2*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt[d]
*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d -
c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCos
h[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]
] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 + c*x)))/(2304*c^5)
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6341, 15, 6354, 15, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6341}$$

$$-\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^5dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6354}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6354}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6308}$$

$$\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}\left(\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}+\frac{3\left(\frac{(a+b\text{arccosh}(cx))^2}{4bc^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/36*(b*c*x^6*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^5*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (Sqrt[d - c^2*d*x^2]*(-1/16*
(b*x^4)/c + (x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(4*c^2
) + (3*(-1/4*(b*x^2)/c + (x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*
x]))/(2*c^2) + (a + b*ArcCosh[c*x])^2/(4*b*c^3)))/(4*c^2)))/(6*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(234) = 468$.

Time = 0.40 (sec) , antiderivative size = 878, normalized size of antiderivative = 3.16

method	result
default	$-\frac{a\,x^{3}\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{6 c^{2} d}-\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{8 c^{4} d}+\frac{a x \sqrt{-c^{2} d\,x^{2}+d}}{16 c^{4}}+\frac{a d \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{16 c^{4} \sqrt{c^{2} d}}+b\left(-\frac{\sqrt{-d\left(c^{2} x^{2}-1\right)}\,\operatorname{arccosh}(\right.}{32 \sqrt{c x -1}\,\sqrt{c x +1}\,c^{5}}$
parts	$-\frac{a\,x^{3}\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{6 c^{2} d}-\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{8 c^{4} d}+\frac{a x \sqrt{-c^{2} d\,x^{2}+d}}{16 c^{4}}+\frac{a d \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{16 c^{4} \sqrt{c^{2} d}}+b\left(-\frac{\sqrt{-d\left(c^{2} x^{2}-1\right)}\,\operatorname{arccosh}(\right.}{32 \sqrt{c x -1}\,\sqrt{c x +1}\,c^{5}}$

input
```
int(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*x^3*(-c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^4*x*(-c^2*d*x^2+d)^(3/2)/d+1
/16*a/c^4*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^4*d/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)/c^5*arccosh(c*x)^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*
x^7-64*c^5*x^5+32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+38*c^3*x^3-48*c^4*x^
4*(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x+18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2
-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)+1/51
2*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*
x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*(-1+4*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)-1/256*(-d*(c^2*x^2-1))^(
1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*
(c*x+1)^(1/2))*(-1+2*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)-1/256*(-d*(c^2*x^2-
1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*
(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)+1/512*(-d*(c^2
*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)
*(1+4*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32
*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+32*c^7*x^7+48*c^4*x^4*(c*x-1)^(1/2)*(
c*x+1)^(1/2)-64*c^5*x^5-18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+38*c^3*x^3+
(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))/(c*x+1)/c^5/(c*x-...
```

Fricas [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)x^4\,dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int x^4\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))\,\,dx$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)x^4\,dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/48*(8*(-c^2*d*x^2 + d)^(3/2)*x^3/(c^2*d) - 3*sqrt(-c^2*d*x^2 + d)*x/c^4
 + 6*(-c^2*d*x^2 + d)^(3/2)*x/(c^4*d) - 3*sqrt(d)*arcsin(c*x)/c^5)*a + b*i
ntegrate(sqrt(-c^2*d*x^2 + d)*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)),
x)
```

Giac [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)x^4\,dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int x^4\,(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{\sqrt{d}\,\big(3asin(cx)\,a+8\sqrt{-c^2x^2+1}\,a\,c^5x^5-2\sqrt{-c^2x^2+1}\,a\,c^3x^3-3\sqrt{-c^2x^2+1}\,acx+48\big(\int\sqrt{-c^2x^2+1}\,a}{48c^5}$$

input
```
int(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*(3*asin(c*x)*a + 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 - 2*sqrt( -
 c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt( - c**2*x**2 + 1)*a*c*x + 48*int(sqrt
( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*b*c**5))/(48*c**5)
```

3.56 $\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 201

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\frac{bx^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/16*b*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*b*c*x^4
*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*x*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccosh(c*x))/c^2+1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-
1/16*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/c^3/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx =$$

$$-\frac{-16acx(-1+2c^2x^2)\sqrt{d-c^2dx^2}+16a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{b\sqrt{d-c^2dx^2}\left(8\text{arccosh}(cx)^2+\cosh(4\text{arccos}\cdots\right.}{\sqrt{\frac{-1+\cdots}{1+c\cdots}}\cdots}}{128c^3}$$

input
```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/128*(-16*a*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 16*a*Sqrt[d]*ArcT
an[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^2*d
*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*Arc
Cosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/c^3
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6341, 15, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6341}$$

$$-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^3dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6354$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 15$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6308$$

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))^2}{4bc^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/16*(b*c*x^4*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (Sqrt[d - c^2*d*x^2]*(-1/4*(
b*x^2)/c + (x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c^2) +
 (a + b*ArcCosh[c*x])^2/(4*b*c^3)))/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(169) = 338$.

Time = 0.25 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.83

method	result
default	$-\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}+\frac{ax\sqrt{-c^2dx^2+d}}{8c^2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2}{16\sqrt{cx-1}\,\sqrt{cx+1}\,c^3}+\frac{\sqrt{-d(c^2x^2-1)}}{}\right.$
parts	$-\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}+\frac{ax\sqrt{-c^2dx^2+d}}{8c^2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2}{16\sqrt{cx-1}\,\sqrt{cx+1}\,c^3}+\frac{\sqrt{-d(c^2x^2-1)}}{}\right.$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*a
/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/16
*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2+1/2
56*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c
*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(-1+4*arccosh(c*x))/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^
(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arc
cosh(c*x))/(c*x+1)/c^3/(c*x-1))
```

Fricas [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))\;dx$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
1/8*a*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + s
qrt(d)*arcsin(c*x)/c^3) + b*integrate(sqrt(-c^2*d*x^2 + d)*x^2*log(c*x + s
qrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int x^2\,(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(asin(cx)\,a+2\sqrt{-c^2x^2+1}\,a\,c^3x^3-\sqrt{-c^2x^2+1}\,acx+8\left(\int\sqrt{-c^2x^2+1}\,acosh(cx)\,x^2dx\right)b\,c^3\right)}{8c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*(asin(c*x)*a + 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - sqrt( - c**
2*x**2 + 1)*a*c*x + 8*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*b*c**3
))/(8*c**3)
```

3.57 $\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = -\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4bc\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/4*b*c*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*x*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-1/4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*
x))^2/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \frac{1}{8}\left(4ax\sqrt{d-c^2dx^2} - \frac{4a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{c} - \frac{b\sqrt{d-c^2dx^2}\left(2\text{arccosh}(cx)^2+\cosh(2\text{arccosh}(cx))-2\text{arccosh}(cx)\sinh(2\text{arccosh}(cx))\right)}{c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(4*a*x*Sqrt[d - c^2*d*x^2] - (4*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])
/(Sqrt[d]*(-1 + c^2*x^2))])/c - (b*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 +
 Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)))/8
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6310, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6310}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6308}$$

$$\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/4*(b*c*x^2*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(104) = 208$.

Time = 0.00 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.24

method	result
default	$\frac{ax\sqrt{-c^2d\,x^2+d}}{2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2}{4\sqrt{cx-1}\,\sqrt{cx+1}\,c}+\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-2cx+2\sqrt{cx-1}\,\sqrt{}}{16(c}\right.$
parts	$\frac{ax\sqrt{-c^2d\,x^2+d}}{2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2}{4\sqrt{cx-1}\,\sqrt{cx+1}\,c}+\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-2cx+2\sqrt{cx-1}\,\sqrt{}}{16(c}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/
(-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)/c*arccosh(c*x)^2+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccos
h(c*x))/(c*x-1)/(c*x+1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arcc
osh(c*x))/(c*x-1)/(c*x+1)/c)
```

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int \sqrt{-d\,(cx-1)\,(cx+1)}\left(a+b\,\text{acosh}\,(cx)\right)\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\,dx=\int \sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a + b*integrate(sqrt(
-c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\,dx=\int (a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{\sqrt{d}\left(asin(cx)\,a+\sqrt{-c^2x^2+1}\,acx+2\left(\int\sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)bc\right)}{2c}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*(asin(c*x)*a + sqrt( - c**2*x**2 + 1)*a*c*x + 2*int(sqrt( - c**2*
x**2 + 1)*acosh(c*x),x)*b*c))/(2*c)
```

3.58 $\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x} + \frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc\sqrt{d-c^2dx^2}\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x+1/2*c*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))^2/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*(-c^2*d*x^2+d)^(1/2)*ln
(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{a\sqrt{d-c^2dx^2}}{x} + ac\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + \frac{1}{2}bc\sqrt{d-c^2dx^2}\left(-\frac{2\text{arccosh}(cx)}{cx} + \frac{\text{arccosh}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((a*Sqrt[d - c^2*d*x^2])/x) + a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2]
)/(Sqrt[d]*(-1 + c^2*x^2))] + (b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/
(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
))))/2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6339, 14, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{6339}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \; 14$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6308$$

$$\frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d -
c^2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(102) = 204$.

Time = 0.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.42

method	result
default	$-\frac{a(-c^2 d x^2+d)^{\frac{3}{2}}}{dx} - a c^2 x\sqrt{-c^2 d x^2+d} - \frac{a c^2 d \arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d x^2+d}}\right)}{\sqrt{c^2 d}} + \frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2 c}{2\sqrt{cx-1}\sqrt{cx+1}} - \frac{b\sqrt{-d(c \cdots}}{\cdots}$
parts	$-\frac{a(-c^2 d x^2+d)^{\frac{3}{2}}}{dx} - a c^2 x\sqrt{-c^2 d x^2+d} - \frac{a c^2 d \arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d x^2+d}}\right)}{\sqrt{c^2 d}} + \frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2 c}{2\sqrt{cx-1}\sqrt{cx+1}} - \frac{b\sqrt{-d(c \cdots}}{\cdots}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(3/2)-a*c^2*x*(-c^2*d*x^2+d)^(1/2)-a*c^2*d/(c^2*d)^(
1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c-b*(-d*(c^2*x^2-1))^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c-b*(-d*(c^2*x^2-1))^(1/2)*arcco
sh(c*x)/(c*x-1)/(c*x+1)*x*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-1
)/(c*x+1)/x+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))/x**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima
")
```

output
```
-(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a + b*integrate(sqrt(-c^
2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{\sqrt{d}\left(-asin(cx)\,acx-\sqrt{-c^2x^2+1}\,a+\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^2}dx\right)bx\right)}{x}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))/x^2,x)
```

output
```
(sqrt(d)*( - asin(c*x)*a*c*x - sqrt( - c**2*x**2 + 1)*a + int((sqrt( - c**
2*x**2 + 1)*acosh(c*x))/x**2,x)*b*x))/x
```

3.59 $\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^4}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3dx^3} - \frac{bc^3\sqrt{d-c^2dx^2}\log(x)}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*d*
x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^3-1/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*ln(x)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\left(\frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\text{arccosh}(cx))}{3x^3} - \frac{1}{3}bc\left(\frac{1}{2x^2}+c^2\log(x)\right)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x
]))/(3*x^3) - (b*c*(1/(2*x^2) + c^2*Log[x]))/3))/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6332, 25, 82, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6332}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{82}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{244}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^3}-\frac{c^2}{x}\right)dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3dx^3}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(d*x^3) + (b*c*Sqrt[d -
c^2*d*x^2]*(-1/2*1/x^2 - c^2*Log[x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.29

method	result
default	$-\frac{a(-c^2 d\,x^2+d)^{\frac{3}{2}}}{3d\,x^3}+\frac{b\sqrt{-d(c^2x^2-1)}\,\left(2\,\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2+2c^3x^3\,\operatorname{arccosh}(cx)-2\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)x}{6\sqrt{cx-1}\,\sqrt{cx+1}\,x^3}$
parts	$-\frac{a(-c^2 d\,x^2+d)^{\frac{3}{2}}}{3d\,x^3}+\frac{b\sqrt{-d(c^2x^2-1)}\,\left(2\,\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2+2c^3x^3\,\operatorname{arccosh}(cx)-2\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)x}{6\sqrt{cx-1}\,\sqrt{cx+1}\,x^3}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(3/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)*(2*arccosh(
c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+2*c^3*x^3*arccosh(c*x)-2*ln(1+(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)-c*x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.89

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^4}\,dx$$
$$= \left[\frac{2\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^5x^5-bc^3x^3\right)\sqrt{-d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+}{}\right.}{6\,(c}\right.$$
$$-\frac{2\left(bc^5x^5-bc^3x^3\right)\sqrt{d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}(x^2-1)\sqrt{d}}{c^2dx^4+(c^2-1)dx^2-d}\right)-2\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\log\left(cx\right.}{6\left(c^2x^5-\right.}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas
")
```

output
```
[1/6*(2*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) + (b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^
2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)
/(c^2*x^4 - x^2)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 -
1) + 2*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3)
, -1/6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - 2*(b*
c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1
)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(a*c^4*x
^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{d - c^2dx^2}(a + b\operatorname{arccosh}(cx))}{x^4}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a + b\operatorname{acosh}(cx))}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))/x**4,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{d - c^2dx^2}(a + b\operatorname{arccosh}(cx))}{x^4}\,dx$$
$$= \frac{\left(c^4d^2\sqrt{-\frac{1}{c^4d}}\log\left(x^2 - \frac{1}{c^2}\right) + i\,(-1)^{-2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(-2\,c^2d + \frac{2\,d}{x^2}\right) + \frac{\sqrt{-c^4dx^4+2\,c^2dx^2-dd}}{x^2}\right)bc}{6\,d}$$
$$- \frac{(-c^2dx^2 + d)^{\frac{3}{2}}b\operatorname{arcosh}(cx)}{3\,dx^3} - \frac{(-c^2dx^2 + d)^{\frac{3}{2}}a}{3\,dx^3}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima
")
```

output
```
1/6*(c^4*d^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*
d)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d
)*d/x^2)*b*c/d - 1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccosh(c*x)/(d*x^3) - 1/3*(
-c^2*d*x^2 + d)^(3/2)*a/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^4}\,dx$$
$$=\frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a+3\left(\int\frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^4}dx\right)b\,x^3\right)}{3x^3}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))/x^4,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a +
3*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**4,x)*b*x**3))/(3*x**3)
```

3.60 $\int \frac{\sqrt{d-c^2dx^2}(a+b\,\mathbf{arccosh}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{5dx^5} - \frac{2c^2(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{15dx^3} - \frac{2bc^5\sqrt{d-c^2dx^2}\log(x)}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/20*b*c*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/30*b*c^3*
(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(3
/2)*(a+b*arccosh(c*x))/d/x^5-2/15*c^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*
x))/d/x^3-2/15*b*c^5*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx$$
$$= \frac{\sqrt{d-c^2dx^2}\left(12(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\operatorname{arccosh}(cx))+8c^2x^2(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\operatorname{arccosh}\right.}{60x^5\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c
*x]) + 8*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) - b
*c*x*(3 - 2*c^2*x^2 + 8*c^4*x^4*Log[x])))/(60*x^5*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6337, 27, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx$$

$$\Big\downarrow \; 6337$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-2c^4x^4-c^2x^2+3}{15x^5}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{15dx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\sqrt{d-c^2dx^2}\int\frac{-2c^4x^4-c^2x^2+3}{x^5}dx}{15\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{15dx^3}$$

$$\Big\downarrow \text{1433}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int\left(-\frac{2c^4}{x}-\frac{c^2}{x^3}+\frac{3}{x^5}\right)dx}{15\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{15dx^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{15dx^3}+\frac{bc\sqrt{d-c^2dx^2}\left(-2c^4\log(x)+\frac{c^2}{2x^2}-\frac{3}{4x^4}\right)}{15\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(d*x^5) - (2*c^2*(d - c^
2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(15*d*x^3) + (b*c*Sqrt[d - c^2*d*x^2]
*(-3/(4*x^4) + c^2/(2*x^2) - 2*c^4*Log[x]))/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(167) = 334$.

Time = 0.39 (sec) , antiderivative size = 1742, normalized size of antiderivative = 8.75

method	result	size
default	Expression too large to display	1742
parts	Expression too large to display	1742

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
2/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^7*c^12
-2/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5*c^1
0-3/10*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3*c^
8+3/10*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x*c^6+
a*(-1/5/d/x^5*(-c^2*d*x^2+d)^(3/2)-2/15*c^2/d/x^3*(-c^2*d*x^2+d)^(3/2))-1/
4*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)*c^5-2/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^5+4/15*b*(-d*(c^2*x^2-1))
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^5+3/10*b*(-d*(c^2*x^2-1)
)^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x/(c*x-1)/(c*x+1)*c^6+9/5*b*(-
d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^5/(c*x-1)/(c*x+
1)*arccosh(c*x)+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*
x^2+9)*x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^9-6/5*b*(-d*(c^2*x^2-1))^(1/2)/(1
5*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)
*c^5-11/12*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^
2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^7+21/20*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x
^6-5*c^4*x^4-15*c^2*x^2+9)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^3+12/5*b*(-d*
(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x/(c*x-1)/(c*x+1)*a
rccosh(c*x)*c^4-27/5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2
*x^2+9)/x^3/(c*x-1)/(c*x+1)*arccosh(c*x)*c^2+2*b*(-d*(c^2*x^2-1))^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.76

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx$$
$$= \left[\frac{4\left(2\,bc^6x^6-bc^4x^4-4\,bc^2x^2+3\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+4\left(bc^7x^7-bc^5x^5\right)\sqrt{-d}\log\left(\frac{c^2d}{}\right.}{}\right.$$
$$-\frac{8\left(bc^7x^7-bc^5x^5\right)\sqrt{d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}(x^2-1)\sqrt{d}}{c^2dx^4+(c^2-1)dx^2-d}\right)-4\left(2\,bc^6x^6-bc^4x^4-4\,bc^2x^2+3\,b\right)\sqrt{-c^2dx^2}}{}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas
")
```

output
```
[1/60*(4*(2*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(b*c^7*x^7 - b*c^5*x^5)*sqrt(-d)*log((c
^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4
 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^
5 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(2*a*c^6*x^6 - a*c
^4*x^4 - 4*a*c^2*x^2 + 3*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5), -1/60*(
8*(b*c^7*x^7 - b*c^5*x^5)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2
 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - 4*(2*b*c^6*x^
6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 - 1)) - (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqrt(-c^2*d*x
^2 + d)*sqrt(c^2*x^2 - 1) - 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 3*a
)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^6}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))}{x^6}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))/x**6,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.73

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^6}\,dx \\
&= -\frac{1}{60}\left(8\,c^4\sqrt{-d}\log\left(x\right)-\frac{2\,c^2\sqrt{-d}x^2-3\,\sqrt{-d}}{x^4}\right)bc \\
&\quad -\frac{1}{15}\,b\left(\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}c^2}{dx^3}+\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}}{dx^5}\right)\text{arcosh}\left(cx\right) \\
&\quad -\frac{1}{15}\,a\left(\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}c^2}{dx^3}+\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}}{dx^5}\right)
\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima
")
```

output
```
-1/60*(8*c^4*sqrt(-d)*log(x) - (2*c^2*sqrt(-d)*x^2 - 3*sqrt(-d))/x^4)*b*c
- 1/15*b*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(3/2)/
(d*x^5))*arccosh(c*x) - 1/15*a*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(
-c^2*d*x^2 + d)^(3/2)/(d*x^5))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^6}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^6}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^6,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx$$
$$=\frac{\sqrt{d}\left(2\sqrt{-c^2x^2+1}\,a\,c^4x^4+\sqrt{-c^2x^2+1}\,a\,c^2x^2-3\sqrt{-c^2x^2+1}\,a+15\left(\int\frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^6}dx\right)b\,x^5\right)}{15x^5}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))/x^6,x)
```

output
```
(sqrt(d)*(2*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + sqrt( - c**2*x**2 + 1)*a*
c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*a + 15*int((sqrt( - c**2*x**2 + 1)*ac
osh(c*x))/x**6,x)*b*x**5))/(15*x**5)
```

3.61 $\int \frac{\sqrt{d-c^2dx^2}(a+b\,\mathbf{arccosh}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 279

$$\begin{aligned}\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx = & -\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\sqrt{1+cx}} \\ & +\frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{-1+cx}\sqrt{1+cx}} \\ & +\frac{2bc^5\sqrt{d-c^2dx^2}}{105x^2\sqrt{-1+cx}\sqrt{1+cx}} \\ & -\frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{7dx^7} \\ & -\frac{4c^2(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{35dx^5} \\ & -\frac{8c^4(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{105dx^3} \\ & -\frac{8bc^7\sqrt{d-c^2dx^2}\log(x)}{105\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/42*b*c*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/140*b*c^3
*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/105*b*c^5*(-c^2*d*
x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(3/2)*(a+b
*arccosh(c*x))/d/x^7-4/35*c^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^
5-8/105*c^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^3-8/105*b*c^7*(-c^
2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(60(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\operatorname{arccosh}(cx))+16c^2x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(3+2c^2x^2\right)\right.}{420x^7\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(60*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c
*x]) + 16*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(3 + 2*c^2*x^2)*(a + b*
ArcCosh[c*x]) - b*c*x*(10 - 3*c^2*x^2 - 8*c^4*x^4 + 32*c^6*x^6*Log[x])))/(
420*x^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6337, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx$$
$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-8c^6x^6-4c^4x^4-3c^2x^2+15}{105x^7}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{105dx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{-8c^6x^6-4c^4x^4-3c^2x^2+15}{x^7}dx}{105\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{105dx^3}$$

$$\Big\downarrow \; 2010$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \left(-\frac{8c^6}{x}-\frac{4c^4}{x^3}-\frac{3c^2}{x^5}+\frac{15}{x^7}\right)dx}{105\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{105dx^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{105dx^3}+\frac{bc\sqrt{d-c^2dx^2}\left(-8c^6\log(x)+\frac{2c^4}{x^2}+\frac{3c^2}{4x^4}-\frac{5}{2x^6}\right)}{105\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

output
```
-1/7*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(d*x^7) - (4*c^2*(d - c^
2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(35*d*x^5) - (8*c^4*(d - c^2*d*x^2)^(
3/2)*(a + b*ArcCosh[c*x]))/(105*d*x^3) + (b*c*Sqrt[d - c^2*d*x^2]*(-5/(2*x
^6) + (3*c^2)/(4*x^4) + (2*c^4)/x^2 - 8*c^6*Log[x]))/(105*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2536 vs. $2(235) = 470$.

Time = 0.42 (sec) , antiderivative size = 2537, normalized size of antiderivative = 9.09

method	result	size
default	Expression too large to display	2537
parts	Expression too large to display	2537

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
-120/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^
2*x^2+225)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^7-128/105*b*(-d*(c^2
*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^13/(
c*x-1)/(c*x+1)*c^20+16/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x
^6-21*c^4*x^4-315*c^2*x^2+225)*x^11/(c*x-1)/(c*x+1)*c^18+40/21*b*(-d*(c^2*
x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^9/(c*
x-1)/(c*x+1)*c^16+214/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^
6-21*c^4*x^4-315*c^2*x^2+225)*x^7/(c*x-1)/(c*x+1)*c^14-152/105*b*(-d*(c^2*
x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^5/(c*
x-1)/(c*x+1)*c^12-30/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-2
1*c^4*x^4-315*c^2*x^2+225)*x^3/(c*x-1)/(c*x+1)*c^10+20/7*b*(-d*(c^2*x^2-1)
)^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x/(c*x-1)/(c*
x+1)*c^8+225/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^
4-315*c^2*x^2+225)/x^7/(c*x-1)/(c*x+1)*arccosh(c*x)+16/3*b*(-d*(c^2*x^2-1)
)^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^6/(c*x-1)^(
1/2)/(c*x+1)^(1/2)*c^13-469/60*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c
^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^9+71/
28*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^
2+225)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^5+255/28*b*(-d*(c^2*x^2-1))^(1/2)
/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^4/(c*x-1)^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^8}\,dx$$
$$= \left[\frac{4\left(8\,bc^8x^8-4\,bc^6x^6-bc^4x^4-18\,bc^2x^2+15\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+16\left(bc^9x^9-bc^7x^7\right)\right.$$
$$-\frac{32\left(bc^9x^9-bc^7x^7\right)\sqrt{d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}(x^2-1)\sqrt{d}}{c^2dx^4+(c^2-1)dx^2-d}\right)-4\left(8\,bc^8x^8-4\,bc^6x^6-bc^4x^4-18\,bc^2x^2+\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas
")
```

output
```
[1/420*(4*(8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4*x^4 - 18*b*c^2*x^2 + 15*b)*sq
rt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 16*(b*c^9*x^9 - b*c^7*x^
7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (8*b*c^5*x^5 - (8
*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b*c*x)*sqrt(-c^2*d*x^2 +
 d)*sqrt(c^2*x^2 - 1) + 4*(8*a*c^8*x^8 - 4*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^
2*x^2 + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/420*(32*(b*c^9*x^9
 - b*c^7*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 -
 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - 4*(8*b*c^8*x^8 - 4*b*c^6*
x^6 - b*c^4*x^4 - 18*b*c^2*x^2 + 15*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 - 1)) - (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3
*x^3 - 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(8*a*c^8*x^8 -
 4*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2 + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2
*x^9 - x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))/x**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^8}\,dx$$
$$= -\frac{1}{420}\left(32\,c^6\sqrt{-d}\log\left(x\right)-\frac{8\,c^4\sqrt{-d}x^4+3\,c^2\sqrt{-d}x^2-10\,\sqrt{-d}}{x^6}\right)bc$$
$$-\frac{1}{105}\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4}{dx^3}+\frac{12\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{dx^5}+\frac{15\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^7}\right)b\,\text{arcosh}\left(cx\right)$$
$$-\frac{1}{105}\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4}{dx^3}+\frac{12\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{dx^5}+\frac{15\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^7}\right)a$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima
")
```

output
```
-1/420*(32*c^6*sqrt(-d)*log(x) - (8*c^4*sqrt(-d)*x^4 + 3*c^2*sqrt(-d)*x^2
- 10*sqrt(-d))/x^6)*b*c - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12
*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*b
*arccosh(c*x) - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(-c^2*d*x
^2 + d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^8}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^8}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^8,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \frac{\sqrt{d}\left(8\sqrt{-c^2x^2+1}\,a\,c^6x^6+4\sqrt{-c^2x^2+1}\,a\,c^4x^4+3\sqrt{-c^2x^2+1}\,a\,c^2x^2-15\sqrt{-c^2x^2+1}\,a+105\left(\int \frac{\sqrt{-c^2}}{}\right.\right.}{105x^7}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))/x^8,x)
```

output
```
(sqrt(d)*(8*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 + 4*sqrt( - c**2*x**2 + 1)*
a*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 15*sqrt( - c**2*x**2
+ 1)*a + 105*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**8,x)*b*x**7))/(105
*x**7)
```

3.62 $\int x^5\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 272

$$\int x^5\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))\,dx = \frac{8bx\sqrt{d-c^2dx^2}}{105c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4bx^3\sqrt{d-c^2dx^2}}{315c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx^5\sqrt{d-c^2dx^2}}{175c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^7\sqrt{d-c^2dx^2}}{49\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))}{3c^6d} + \frac{2(d-c^2dx^2)^{5/2}(a+b\,\mathrm{arccosh}(cx))}{5c^6d^2} - \frac{(d-c^2dx^2)^{7/2}(a+b\,\mathrm{arccosh}(cx))}{7c^6d^3}$$

output
```
8/105*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/315*b*x^3
*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/175*b*x^5*(-c^2*d*
x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/49*b*c*x^7*(-c^2*d*x^2+d)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))
/c^6/d+2/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^6/d^2-1/7*(-c^2*d*x^2
+d)^(7/2)*(a+b*arccosh(c*x))/c^6/d^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.62

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(bcx(840+140c^2x^2+63c^4x^4-225c^6x^6)+105a\sqrt{-1+cx}\sqrt{1+cx}(-8-4c^2x^2-3c^4x^4+1\right.}{11025c^6\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x*(840 + 140*c^2*x^2 + 63*c^4*x^4 - 225*c^6*x^6)
 + 105*a*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-8 - 4*c^2*x^2 - 3*c^4*x^4 + 15*c^6
*x^6) + 105*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-8 - 4*c^2*x^2 - 3*c^4*x^4 + 1
5*c^6*x^6)*ArcCosh[c*x]))/(11025*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6337, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-15c^6x^6+3c^4x^4+4c^2x^2+8}{105c^6}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{d-c^2dx^2}\int \left(-15c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{105c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^6d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^6d}+\frac{b\left(-\frac{15}{7}c^6x^7+\frac{3c^4x^5}{5}+\frac{4c^2x^3}{3}+8x\right)\sqrt{d-c^2dx^2}}{105c^5\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)
/7))/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a +
b*ArcCosh[c*x]))/(3*c^6*d) + (2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])
)/(5*c^6*d^2) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^6*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.72

method	result
orering	$\frac{(2925c^8x^8-3393c^6x^6-630c^4x^4-4760c^2x^2+5040)\sqrt{-c^2d\,x^2+d}\,(a+b\,\mathrm{arccosh}(cx))}{11025c^6(c^2x^2-1)} - \frac{(225c^6x^6-63c^4x^4-140c^2x^2-840)\left(5x^4\sqrt{-\right.}{}$
default	$a\left(-\frac{x^4(-c^2d\,x^2+d)^{\frac{3}{2}}}{7c^2d}+\frac{-\frac{4x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{35c^2d}-\frac{8\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{105d\,c^4}}{c^2}\right)+b\Big(\frac{\sqrt{-d(c^2x^2-1)}\,(64c^8x^8-144c^6x^6+64c^7x^7\sqrt{cx-1}\,\sqrt{}}{}$
parts	$a\left(-\frac{x^4(-c^2d\,x^2+d)^{\frac{3}{2}}}{7c^2d}+\frac{-\frac{4x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{35c^2d}-\frac{8\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{105d\,c^4}}{c^2}\right)+b\Big(\frac{\sqrt{-d(c^2x^2-1)}\,(64c^8x^8-144c^6x^6+64c^7x^7\sqrt{cx-1}\,\sqrt{}}{}$

input
```
int(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/11025*(2925*c^8*x^8-3393*c^6*x^6-630*c^4*x^4-4760*c^2*x^2+5040)/c^6/(c^2
*x^2-1)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-1/11025/x^4*(225*c^6*x^6-6
3*c^4*x^4-140*c^2*x^2-840)/c^6*(5*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*
x))-x^6/(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*c^2*d+b*c*x^5*(-c^2*d*x^2+
d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \frac{105\,(15\,bc^8x^8-18\,bc^6x^6-bc^4x^4-4\,bc^2x^2+8\,b)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-(225\,bc^7x^7-63\,b\ldots}{11\ldots}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
1/11025*(105*(15*b*c^8*x^8 - 18*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 8*b)
*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (225*b*c^7*x^7 - 63*b
*c^5*x^5 - 140*b*c^3*x^3 - 840*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
1) + 105*(15*a*c^8*x^8 - 18*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 8*a)*sqr
t(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F]

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \int x^5\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))\;dx$$

input
```
integrate(x**5*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**5*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$= -\frac{1}{105}\left(\frac{15\,(-c^2dx^2+d)^{\frac{3}{2}}x^4}{c^2d}+\frac{12\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^6d}\right)b\,\text{arcosh}\,(cx)$$
$$-\frac{1}{105}\left(\frac{15\,(-c^2dx^2+d)^{\frac{3}{2}}x^4}{c^2d}+\frac{12\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^6d}\right)a$$
$$-\frac{\left(225\,c^6\sqrt{-d}x^7-63\,c^4\sqrt{-d}x^5-140\,c^2\sqrt{-d}x^3-840\,\sqrt{-d}x\right)b}{11025\,c^5}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/105*(15*(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*
x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*b*arccosh(c*x) - 1/105*(15
*(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^4*d
) + 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*a - 1/11025*(225*c^6*sqrt(-d)*x^7 -
63*c^4*sqrt(-d)*x^5 - 140*c^2*sqrt(-d)*x^3 - 840*sqrt(-d)*x)*b/c^5
```

Giac [F(-2)]

Exception generated.

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int x^5\,(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{\sqrt{d}\left(15\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4-4\sqrt{-c^2x^2+1}\,a\,c^2x^2-8\sqrt{-c^2x^2+1}\,a+105\left(\int\sqrt{-c\right.\right.}{105c^6}$$

input
```
int(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*(15*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)
*a*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt( - c**2*x**2
+ 1)*a + 105*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**5,x)*b*c**6))/(105*c
**6)
```

3.63 $\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 195

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \frac{2bx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3c^4d} + \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d^2}$$

output
```
2/15*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/45*b*x^3*(
-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/25*b*c*x^5*(-c^2*d*x^2
+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos
h(c*x))/c^4/d+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.65

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \frac{\sqrt{d-c^2dx^2}\left(bc(30x+5c^2x^3-9c^4x^5)+30(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\text{arccosh}(cx))+45c^2x^2(-1+cx\right.}{225c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*(30*x + 5*c^2*x^3 - 9*c^4*x^5) + 30*(-1 + c*x)^(
3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + 45*c^2*x^2*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2)*(a + b*ArcCosh[c*x])))/(225*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6337, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-3c^4x^4+c^2x^2+2}{15c^4}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^4d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\sqrt{d-c^2dx^2}\int \left(-3c^4x^4+c^2x^2+2\right)dx}{15c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^4d}+\frac{b\left(-\frac{3}{5}c^4x^5+\frac{c^2x^3}{3}+2x\right)\sqrt{d-c^2dx^2}}{15c^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - (3*c^4*x^5)/5))/(15*c^3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(3*
c^4*d) + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(5*c^4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

method	result
orering	$\frac{(81c^6x^6-107c^4x^4-120c^2x^2+120)\sqrt{-c^2d\,x^2+d}\,(a+b\text{ arccosh}(cx))}{225c^4(c^2x^2-1)}-\frac{(9c^4x^4-5c^2x^2-30)\left(3x^2\sqrt{-c^2d\,x^2+d}\,(a+b\text{ arccosh}(cx))-\right.}{225x^2c^4}$
default	$a\left(-\frac{x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}{5c^2d}-\frac{2(-c^2d\,x^2+d)^{\frac{3}{2}}}{15d\,c^4}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,(16c^6x^6-28c^4x^4+16c^5x^5\sqrt{cx-1}\,\sqrt{cx+1}+13c^2x^2-20c^3x^3}{800(cx+1)c^4(cx-}\right.$
parts	$a\left(-\frac{x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}{5c^2d}-\frac{2(-c^2d\,x^2+d)^{\frac{3}{2}}}{15d\,c^4}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,(16c^6x^6-28c^4x^4+16c^5x^5\sqrt{cx-1}\,\sqrt{cx+1}+13c^2x^2-20c^3x^3}{800(cx+1)c^4(cx-}\right.$

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/225*(81*c^6*x^6-107*c^4*x^4-120*c^2*x^2+120)/c^4/(c^2*x^2-1)*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccosh(c*x))-1/225/x^2*(9*c^4*x^4-5*c^2*x^2-30)/c^4*(3*x^2
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-x^4/(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cosh(c*x))*c^2*d+b*c*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{15\,(3\,bc^6x^6-4\,bc^4x^4-bc^2x^2+2\,b)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-(9\,bc^5x^5-5\,bc^3x^3-30\,bcx)\sqrt{-}}{225\,(c^6x^2-c^4)}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
1/225*(15*(3*b*c^6*x^6 - 4*b*c^4*x^4 - b*c^2*x^2 + 2*b)*sqrt(-c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^5*x^5 - 5*b*c^3*x^3 - 30*b*c*x)*s
qrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(3*a*c^6*x^6 - 4*a*c^4*x^4 - a*
c^2*x^2 + 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))\;dx$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.74

$$\begin{aligned}&\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx\\&=-\frac{1}{15}\,b\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\text{arcosh}\,(cx)\\&\quad-\frac{1}{15}\,a\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\\&\quad-\frac{(9\,c^4\sqrt{-d}x^5-5\,c^2\sqrt{-d}x^3-30\,\sqrt{-d}x)\,b}{225\,c^3}\end{aligned}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/15*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(
c^4*d))*arccosh(c*x) - 1/15*a*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-
c^2*d*x^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(-d)*x^5 - 5*c^2*sqrt(-d)
*x^3 - 30*sqrt(-d)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \int x^3\,(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$= \frac{\sqrt{d}\,\left(3\sqrt{-c^2x^2+1}\,a\,c^4x^4-\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+15\left(\int\sqrt{-c^2x^2+1}\,acosh(cx)\,x^3dx\right)b\,c}{15c^4}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x)),x)
```

output

```
(sqrt(d)*(3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - sqrt( - c**2*x**2 + 1)*a*
c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 15*int(sqrt( - c**2*x**2 + 1)*aco
sh(c*x)*x**3,x)*b*c**4))/(15*c**4)
```

3.64 $\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3c^2d}$$

output
```
1/3*b*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*c*x^3*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+
b*arccosh(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{-1+cx}\sqrt{1+cx}(3-c^2x^2)+3a(-1+c^2x^2)^2+3b(-1+c^2x^2)^2\text{arccosh}(cx)\right)}{9c^2(-1+c^2x^2)}$$

input
```
Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3
*a*(-1 + c^2*x^2)^2 + 3*b*(-1 + c^2*x^2)^2*ArcCosh[c*x]))/(9*c^2*(-1 + c^2
*x^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6329, 25, 39, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{b\sqrt{d-c^2dx^2}\int -((1-cx)(cx+1))dx}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{d-c^2dx^2}\int (1-cx)(cx+1)dx}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{39}$$

$$\frac{b\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)dx}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3c^2d}$$

input
```
Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(3*c^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.31

method	result
orering	$\frac{(5c^4x^4-13c^2x^2+6)\sqrt{-c^2dx^2+d}\,(a+b\,\mathrm{arccosh}(cx))}{9c^2(c^2x^2-1)}-\frac{(c^2x^2-3)\left(\sqrt{-c^2dx^2+d}\,(a+b\,\mathrm{arccosh}(cx))-\frac{x^2(a+b\,\mathrm{arccosh}(cx))c^2d}{\sqrt{-c^2dx^2+d}}+\frac{bcx}{\sqrt{c}}\right.}{9c^2}$
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx+1)(-1+3\,\mathrm{arccosh}(cx))}{72(cx+1)c^2(cx-1)}-\right.$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx+1)(-1+3\,\mathrm{arccosh}(cx))}{72(cx+1)c^2(cx-1)}-\right.$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/9*(5*c^4*x^4-13*c^2*x^2+6)/c^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cosh(c*x))-1/9*(c^2*x^2-3)/c^2*((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-x^
2/(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*c^2*d+b*c*x*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.20

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{3\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^3x^3-3\,bcx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}+3}{9\left(c^4x^2-c^2\right)}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 - 1)) - (b*c^3*x^3 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1
) + 3*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int x\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = -\frac{(-c^2dx^2+d)^{\frac{3}{2}}b\,\text{arcosh}\,(cx)}{3\,c^2d} - \frac{\left(c^2\sqrt{-d}dx^3-3\,\sqrt{-d}dx\right)b}{9\,cd} - \frac{(-c^2dx^2+d)^{\frac{3}{2}}a}{3\,c^2d}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccosh(c*x)/(c^2*d) - 1/9*(c^2*sqrt(-d)*d*x
^3 - 3*sqrt(-d)*d*x)*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \int x\,(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = \frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a+3\left(\int \sqrt{-c^2x^2+1}\,acosh(cx)\,xdx\right)b\,c^2\right)}{3c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a +
3*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*b*c**2))/(3*c**2)
```

3.65 $\int \frac{\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 213

$$
\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx \\
&= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) \\
&\quad - \frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}
$$

output

```
-b*c*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+(-c^2*d*x^2+d)^(1/
2)*(a+b*arccosh(c*x))-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b*(-c^2*d*x^2+
d)^(1/2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx$$
$$= a\sqrt{d-c^2dx^2} + a\sqrt{d}\log(x) - a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)$$
$$+ \frac{b\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)+i\text{arccosh}(cx)\log\left(1-ie^{-\text{arccosh}(c\ldots}\right.\right.}{\sqrt{\frac{-1+cx}{1+cx}}\ldots}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
a*Sqrt[d - c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[
d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*
x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh
[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]]
 + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6341, 24, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx$$
$$\Big\downarrow \text{ 6341}$$
$$-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}} - \frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{cx-1}\sqrt{cx+1}} + \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))$$
$$\Big\downarrow \text{ 24}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6362

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 3042

$$-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+ b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 4668

$$-\frac{\sqrt{d-c^2dx^2}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\left(e^{\text{arc}\cdots}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2715

$$-\frac{\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)de^{\text{arcc}\cdots}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2838

$$-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arccosh}(\cdots}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
-((b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*
x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*x]] + I*b*Pol
yLog[2, I*E^ArcCosh[c*x]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.85

method	result
default	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)a+\sqrt{-c^2d\,x^2+d}\,a+\frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)x^2c^2}{(cx-1)(cx+1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,cx}{\sqrt{cx-1}\,\sqrt{cx+1}}-$
parts	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)a+\sqrt{-c^2d\,x^2+d}\,a+\frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)x^2c^2}{(cx-1)(cx+1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,cx}{\sqrt{cx-1}\,\sqrt{cx+1}}-$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*a+(-c^2*d*x^2+d)^(1/2)
*a+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)/(c*x+1)*arccosh(c*x)*x^2*c^2-b*(-d*(c^
2*x^2-1))^(1/2)*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)*x-b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)/(c*x+1)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c
^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
)))
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))/x,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
-(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-
c^2*d*x^2 + d))*a + b*integrate(sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\,dx = \sqrt{d}\left(\sqrt{-c^2x^2+1}\,a + \left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x}dx\right)b + \log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a - a\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))/x,x)
```

output
```
sqrt(d)*(sqrt( - c**2*x**2 + 1)*a + int((sqrt( - c**2*x**2 + 1)*acosh(c*x)
)/x,x)*b + log(tan(asin(c*x)/2))*a - a)
```

3.66 $\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 235

$$\begin{aligned}\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx &= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}\\&\quad+\frac{c^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\\&\quad-\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}\\&\quad+\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/2*b*c*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccosh(c*x))/x^2+c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*
x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/
2*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(
2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$=\frac{1}{2}\left(-\frac{a\sqrt{d-c^2dx^2}}{x^2}-ac^2\sqrt{d}\log(x)+ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)\right.$$
$$+\frac{bd(1+cx)\left(cx\sqrt{\frac{-1+cx}{1+cx}}-\text{arccosh}(cx)+cx\text{arccosh}(cx)+ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)\log\left(1-ie^{-\text{arccosh}(}\right.\right.}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
(-((a*Sqrt[d - c^2*d*x^2])/x^2) - a*c^2*Sqrt[d]*Log[x] + a*c^2*Sqrt[d]*Log
[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1
 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)
/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.70, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6339, 15, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{6339}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \; 15$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6362$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 4668$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\left(e^{\text{arc}\cdots}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2715$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)de^{\text{arcc}\cdots}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2838$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arccosh}(\cdots}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*Sqrt[d - c^2*d*x^2])/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*(2*(
a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh
[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.86

method	result
default	$a\left(-\frac{\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}-\frac{c^{2}\left(\sqrt{-c^{2}d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)\right)}{2}\right)-\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\operatorname{arccosh}\left(cx\right)c^{2}}{2\left(cx-1\right)\left(cx+1\right)}-\frac{bc\sqrt{-d(c^{2}}}{2x\sqrt{cx-1}}$
parts	$a\left(-\frac{\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}-\frac{c^{2}\left(\sqrt{-c^{2}d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)\right)}{2}\right)-\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\operatorname{arccosh}\left(cx\right)c^{2}}{2\left(cx-1\right)\left(cx+1\right)}-\frac{bc\sqrt{-d(c^{2}}}{2x\sqrt{cx-1}}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(3/2)-1/2*c^2*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*l
n((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)))-1/2*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)/(c*x+1)*arccosh(c*x)*c^2-1/2*b*c*(-d*(c^2*x^2-1))^(1/2)/x/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+1/2*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-1)/(c*x+
1)/x^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(
c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2-1)
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2)))*c^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))*c^2
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{\sqrt{-d(cx-1)(cx+1)}(a+b\operatorname{acosh}(cx))}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))/x**3,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima
")
```

output
```
1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a + b*integrat
e(sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}\,dx = \frac{\sqrt{d}\left(-\sqrt{-c^2x^2+1}\,a+2\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^3}dx\right)b\,x^2-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a\,c^2x^2\right)}{2x^2}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))/x^3,x)
```

output
```
(sqrt(d)*( - sqrt( - c**2*x**2 + 1)*a + 2*int((sqrt( - c**2*x**2 + 1)*acos
h(c*x))/x**3,x)*b*x**2 - log(tan(asin(c*x)/2))*a*c**2*x**2))/(2*x**2)
```

3.67 $\int \frac{\sqrt{d-c^2dx^2}(a+b\,\mathbf{arccosh}(cx))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 315

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^5}\,dx \\
&= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad -\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{4x^4}+\frac{c^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{8x^2} \\
&\quad +\frac{c^4\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{4\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad -\frac{ibc^4\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(cx)}\right)}{8\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad +\frac{ibc^4\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(cx)}\right)}{8\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
-1/12*b*c*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*c^3*(
-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccosh(c*x))/x^4+1/8*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^
2+1/4*c^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*I*b*c^4*(-c^2*d*x^2+d)^(1/
2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+1/8*I*b*c^4*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^5}\,dx$$
$$= \frac{1}{24}\left(\frac{3a(-2+c^2x^2)\sqrt{d-c^2dx^2}}{x^4} - 3ac^4\sqrt{d}\log(x) + 3ac^4\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)\right.$$
$$+ \frac{b\sqrt{d-c^2dx^2}\left(-2cx+3c^3x^3-6\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{arccosh}(cx)+3c^2x^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{arccosh}(cx)-3\right.}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```

output
```
((3*a*(-2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^4 - 3*a*c^4*Sqrt[d]*Log[x] + 3
*a*c^4*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^
2]*(-2*c*x + 3*c^3*x^3 - 6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*
x] + 3*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - (3*I)*c
^4*x^4*ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]
) - (3*I)*c^4*x^4*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCos
h[c*x]])))/(x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6339, 15, 6348, 15, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^5}dx$$

$$\Big\downarrow \text{6339}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x^4}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{15}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6348}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{2}bc\int \frac{1}{x^2}dx+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2x^2}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bc}{2x}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6362}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bc}{2x}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int(a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bc}{2x}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 4668

$$\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\left(-ib\int\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int\log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\right.\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 2715

$$\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)d\right.\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 2838

$$\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{ar}}\right.\right.\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*Sqrt[d - c^2*d*x^2])/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*x^4) + (c^2*Sqrt[d - c^2*d*x^2]*(
(b*c)/(2*x) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*x^2)
+ (c^2*(2*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I
)*E^ArcCosh[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/2))/(4*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6348
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp[b*c*(n/(f
*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]
   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.72

method	result
default	$-\frac{a(-c^2 d x^2+d)^{\frac{3}{2}}}{4d x^4}-\frac{a c^2(-c^2 d x^2+d)^{\frac{3}{2}}}{8d x^2}+\frac{a c^4\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2 d x^2+d}}{x}\right)}{8}-\frac{a c^4\sqrt{-c^2 d x^2+d}}{8}+\frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccos}}{8(cx+1)(cx-1)}$
parts	$-\frac{a(-c^2 d x^2+d)^{\frac{3}{2}}}{4d x^4}-\frac{a c^2(-c^2 d x^2+d)^{\frac{3}{2}}}{8d x^2}+\frac{a c^4\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2 d x^2+d}}{x}\right)}{8}-\frac{a c^4\sqrt{-c^2 d x^2+d}}{8}+\frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccos}}{8(cx+1)(cx-1)}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/x^4*(-c^2*d*x^2+d)^(3/2)-1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/8
*a*c^4*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/8*a*c^4*(-c^2*
d*x^2+d)^(1/2)+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^4+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-3/8*b*(-
d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*c*(-d*(c^
2*x^2-1))^(1/2)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/4*b*(-d*(c^2*x^2-1))^(1/
2)/(c*x+1)/x^4/(c*x-1)*arccosh(c*x)-1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)*c^4+1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c
*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4-1/8*I*b*(-d*(c^2*x^2-1))
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*c^4+1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog
(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \int \frac{\sqrt{-d(cx-1)(cx+1)}(a+b\operatorname{acosh}(cx))}{x^5}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))/x**5,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^5}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima
")
```

output
```
1/8*(c^4*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 sqrt(-c^2*d*x^2 + d)*c^4 - (-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^2) - 2*(-c^2*d
*x^2 + d)^(3/2)/(d*x^4))*a + b*integrate(sqrt(-c^2*d*x^2 + d)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^5}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^5}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^5}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^5,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x^5}\,dx = \frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+8\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^5}dx\right)b\,x^4-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a\,c^4x^4\right)}{8x^4}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))/x^5,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a
+ 8*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**5,x)*b*x**4 - log(tan(asin(
c*x)/2))*a*c**4*x**4))/(8*x**4)
```

3.68 $\int x^4(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 360

$$\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx=\frac{3bdx^2\sqrt{d-c^2dx^2}}{256c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3dx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{128c^4}$$
$$-\frac{dx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{64c^2}+\frac{1}{16}dx^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))$$
$$+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{3d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{256bc^5\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
3/256*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/256*b
*d*x^4*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*b*c*d*x^6*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/64*b*c^3*d*x^8*(-c^2*d*x
^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/128*d*x*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccosh(c*x))/c^4-1/64*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^2
+1/16*d*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))+1/8*x^5*(-c^2*d*x^2+d)
^(3/2)*(a+b*arccosh(c*x))-3/256*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^
2/b/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.52 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.94

$$\int x^4 \left(d-c^2 d x^2\right)^{3/2} (a + b \operatorname{arccosh}(cx)) \, dx = \frac{d\left(-576acx\sqrt{d-c^2dx^2}(3+2c^2x^2-24c^4x^4+16c^6x^6)-1728a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right.\right.}{}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*(-576*a*c*x*Sqrt[d - c^2*d*x^2]*(3 + 2*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^
6) - 1728*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^
2))] + (32*b*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c
*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(
-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))
/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^2]*(1440*Arc
Cosh[c*x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cos
h[6*ArcCosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] - 24*ArcCosh[c*x]*(-48*Sinh[2*A
rcCosh[c*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[
8*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(73728*c^5)
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6345, 25, 82, 244, 2009, 6341, 15, 6354, 15, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d-c^2 d x^2\right)^{3/2} (a + b \operatorname{arccosh}(cx)) \, dx$$

$$\Big\downarrow \text{ 6345}$$

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int -x^5(1-cx)(cx+1)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^5(1-cx)(cx+1)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{82}$$

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^5\left(1-c^2x^2\right)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(x^5-c^2x^7\right)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6341}$$

$$\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^5dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}\right)+$$
$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\right.$$
$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arcc}\right.$$
$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}-\right.}{6\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}-\frac{bx^4}{16c}\right.}{6\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6308

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))+$$

$$\frac{3}{8}d\left(\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}\left(\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}+\frac{3\left(\frac{(a+b\text{arccosh}(cx))^2}{4bc^3}+\frac{x\sqrt{c\ldots}}{}\right.}{}\right.}{6\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/8*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^6/6 - (c^2*x^8)/8))/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/8 + (3*d*(-
1/36*(b*c*x^6*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^5*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (Sqrt[d - c^2*d*x^2]*(-1/16*(
b*x^4)/c + (x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(4*c^2)
 + (3*(-1/4*(b*x^2)/c + (x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
]))/(2*c^2) + (a + b*ArcCosh[c*x])^2/(4*b*c^3)))/(4*c^2)))/(6*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])))/8
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(304) = 608$.

Time = 0.35 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.18

method	result
default	$-\frac{a\,x^3\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{8c^2 d}-\frac{ax\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{16c^4 d}+\frac{ax\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{64c^4}+\frac{3adx\sqrt{-c^2 d\,x^2+d}}{128c^4}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{128c^4\sqrt{c^2 d}}+b\Bigg($
parts	$-\frac{a\,x^3\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{8c^2 d}-\frac{ax\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{16c^4 d}+\frac{ax\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{64c^4}+\frac{3adx\sqrt{-c^2 d\,x^2+d}}{128c^4}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{128c^4\sqrt{c^2 d}}+b\Bigg($

input
```
int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-1/8*a*x^3*(-c^2*d*x^2+d)^(5/2)/c^2/d-1/16*a/c^4*x*(-c^2*d*x^2+d)^(5/2)/d+
1/64*a/c^4*x*(-c^2*d*x^2+d)^(3/2)+3/128*a/c^4*d*x*(-c^2*d*x^2+d)^(1/2)+3/1
28*a/c^4*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*
(-3/256*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^5*arccosh(c*x
)^2*d-1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*c^8*x^8+272*c^5*x^5-256*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^
(1/2)-88*c^3*x^3+160*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x-32*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+8*arccosh(c*x
))*d/(c*x+1)/c^5/(c*x-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x
^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d/(c*x+1)/c^5/
(c*x-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1
/2)+8*c^5*x^5+8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/
2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh(c*x))*d/(c*x+1)/c^5/(c*x-1)-1/16384*(
-d*(c^2*x^2-1))^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^8*x^8+128*c^9*x^
9+256*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)-320*c^7*x^7-160*c^4*x^4*(c*x-1)^
(1/2)*(c*x+1)^(1/2)+272*c^5*x^5+32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-88*
c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x)*(1+8*arccosh(c*x))*d/(c*x+1)/c^
5/(c*x-1))
```

Fricas [F]

$$\int x^4 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (b\,\text{arcosh}\,(cx)+a)x^4\,dx$$

input

```
integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output

```
integral(-(a*c^2*d*x^6 - a*d*x^4 + (b*c^2*d*x^6 - b*d*x^4)*arccosh(c*x))*s
qrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \left(d-c^2 d x^2\right)^{3/2} (a+b \text{arccosh}(c x)) \, dx = \text{Timed out}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 \left(d-c^2 d x^2\right)^{3/2} (a+b \text{arccosh}(c x)) \, dx = \int \left(-c^2 d x^2+d\right)^{\frac{3}{2}} (b \text{arcosh}\left(c x\right)+a) x^4 \, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/128*(16*(-c^2*d*x^2 + d)^(5/2)*x^3/(c^2*d) - 2*(-c^2*d*x^2 + d)^(3/2)*x
/c^4 + 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^4*d) - 3*sqrt(-c^2*d*x^2 + d)*d*x/c^4
 - 3*d^(3/2)*arcsin(c*x)/c^5)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*x^4*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F]

$$\int x^4 \left(d-c^2 d x^2\right)^{3/2} (a+b \text{arccosh}(c x)) \, dx = \int \left(-c^2 d x^2+d\right)^{\frac{3}{2}} (b \text{arcosh}\left(c x\right)+a) x^4 \, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx=\int x^4\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^4 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx=\frac{\sqrt{d}\,d\left(3asin(cx)\,a-16\sqrt{-c^2x^2+1}\,a\,c^7x^7+24\sqrt{-c^2x^2+1}\,a\,c^5x^5-2\sqrt{-c^2x^2+1}\,a\,c^3\right.}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a - 16*sqrt( - c**2*x**2 + 1)*a*c**7*x**7 + 24*sqr
t( - c**2*x**2 + 1)*a*c**5*x**5 - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - 3
*sqrt( - c**2*x**2 + 1)*a*c*x - 128*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*
x**6,x)*b*c**7 + 128*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*b*c**5)
)/(128*c**5)
```

3.69 $\int x^2(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 281

$$\begin{aligned}\int x^2(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx &= \frac{bdx^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\sqrt{1+cx}}\\ &- \frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}\\ &- \frac{dx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{16c^2} + \frac{1}{8}dx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\\ &+ \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx)) - \frac{d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
1/32*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/96*b*c*d
*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*c^3*d*x^6*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*d*x*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccosh(c*x))/c^2+1/8*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))
+1/6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/32*d*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccosh(c*x))^2/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.47 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b \text{arccosh}(cx)) \, dx = \frac{d\left(-48acx\sqrt{d - c^2dx^2}(3 - 14c^2x^2 + 8c^4x^4) - 144a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right) - \frac{18b\sqrt{d-}}{}\right.}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*(-48*a*c*x*Sqrt[d - c^2*d*x^2]*(3 - 14*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqr
t[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - (18*b*Sq
rt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x
]*Sinh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[
d - c^2*d*x^2]*(72*ArcCosh[c*x]^2 - 18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*Arc
Cosh[c*x]] + 2*Cosh[6*ArcCosh[c*x]] - 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c
*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(
1 + c*x)]*(1 + c*x))))/(2304*c^3)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6345, 25, 82, 244, 2009, 6341, 15, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b \text{arccosh}(cx)) \, dx$$

$$\Big\downarrow \text{ 6345}$$

$$\frac{1}{2} d \int x^2 \sqrt{d - c^2 d x^2} (a + b \text{arccosh}(cx)) dx + \frac{bcd\sqrt{d - c^2dx^2} \int -x^3(1 - cx)(cx + 1)dx}{6\sqrt{cx - 1}\sqrt{cx + 1}} + \frac{1}{6} x^3 (d - c^2 d x^2)^{3/2} (a + b \text{arccosh}(cx))$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^3(1-cx)(cx+1)dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 82}$$

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^3\left(1-c^2x^2\right)dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 244}$$

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(x^3-c^2x^5\right)dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6341}$$

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^3dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6354}$$

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccos}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(c\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6308}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))+$$

$$\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))^2}{4bc^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/6*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^4/4 - (c^2*x^6)/6))/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/6 + (d*(-1/
16*(b*c*x^4*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (Sqrt[d - c^2*d*x^2]*(-1/4*(b*x
^2)/c + (x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c^2) + (a
 + b*ArcCosh[c*x])^2/(4*b*c^3)))/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(237) = 474$.

Time = 0.33 (sec) , antiderivative size = 883, normalized size of antiderivative = 3.14

method	result
default	$-\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6c^2d}+\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{24c^2}+\frac{adx\sqrt{-c^2dx^2+d}}{16c^2}+\frac{a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{16c^2\sqrt{c^2d}}+b\Big(-\frac{\sqrt{-d(c^2x^2-1)}\,\text{arccosh}}{32\sqrt{cx-1}\,\sqrt{cx+1}\,c}$
parts	$-\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6c^2d}+\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{24c^2}+\frac{adx\sqrt{-c^2dx^2+d}}{16c^2}+\frac{a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{16c^2\sqrt{c^2d}}+b\Big(-\frac{\sqrt{-d(c^2x^2-1)}\,\text{arccosh}}{32\sqrt{cx-1}\,\sqrt{cx+1}\,c}$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/16
*a/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d
)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c
^7*x^7-64*c^5*x^5+32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+38*c^3*x^3-48*c^4
*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x+18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*
x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)
+1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2
)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^
2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*
(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-
1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/5
12*(-d*(c^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^
5+8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(
1/2)+4*c*x)*(1+4*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1
))^(1/2)*(-32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+32*c^7*x^7+48*c^4*x^4*(c
*x-1)^(1/2)*(c*x+1)^(1/2)-64*c^5*x^5-18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^
2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d/(c...
```

Fricas [F]

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (b\operatorname{arcosh}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral(-(a*c^2*d*x^4 - a*d*x^2 + (b*c^2*d*x^4 - b*d*x^2)*arccosh(c*x))*s
qrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b\text{arccosh}(cx)) \, dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b\text{arccosh}(cx)) \, dx = \int (-c^2 d x^2 + d)^{\frac{3}{2}} (b \operatorname{arcosh}(cx) + a) x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
1/48*a*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d
) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + b*integr
ate((-c^2*d*x^2 + d)^(3/2)*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F]

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b\text{arccosh}(cx)) \, dx = \int (-c^2 d x^2 + d)^{\frac{3}{2}} (b \operatorname{arcosh}(cx) + a) x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2dx^2)^{3/2} (a + b\text{arccosh}(cx)) \, dx = \int x^2 (a + b\,\text{acosh}(c\,x)) \left(d - c^2\,d\,x^2\right)^{3/2} dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2 (d - c^2dx^2)^{3/2} (a + b\text{arccosh}(cx)) \, dx = \frac{\sqrt{d}\,d\left(3asin(cx)\,a - 8\sqrt{-c^2x^2+1}\,a\,c^5x^5 + 14\sqrt{-c^2x^2+1}\,a\,c^3x^3 - 3\sqrt{-c^2x^2+1}\,acx\right.}{48c}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a - 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 + 14*sqrt
( - c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt( - c**2*x**2 + 1)*a*c*x - 48*int(s
qrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*b*c**5 + 48*int(sqrt( - c**2*x**2
 + 1)*acosh(c*x)*x**2,x)*b*c**3))/(48*c**3)
```

3.70 $\int (d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 197

$$\int (d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx = -\frac{3bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd(-1+cx)^{3/2}(1+cx)^{3/2}\sqrt{d-c^2dx^2}}{16c}+\frac{3}{8}dx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))+\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))-\frac{3d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx$$

output

```
-3/16*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*d*
(c*x-1)^(3/2)*(c*x+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccosh(c*x))+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-3/1
6*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/c/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.19

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx =$$

$$-\frac{1}{8}adx\left(-5+2c^2x^2\right)\sqrt{d-c^2dx^2}-\frac{3ad^{3/2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{8c}$$
$$-\frac{bd\sqrt{d-c^2dx^2}(2\text{arccosh}(cx)^2+\cosh(2\text{arccosh}(cx))-2\text{arccosh}(cx)\sinh(2\text{arccosh}(cx)))}{8c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$
$$+\frac{bd\sqrt{d-c^2dx^2}(8\text{arccosh}(cx)^2+\cosh(4\text{arccosh}(cx))-4\text{arccosh}(cx)\sinh(4\text{arccosh}(cx)))}{128c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/8*(a*d*x*(-5 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (3*a*d^(3/2)*ArcTan[(c
*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/(8*c) - (b*d*Sqrt[d - c
^2*d*x^2]*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2
*ArcCosh[c*x]]))/(8*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d*Sqrt[d
- c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sin
h[4*ArcCosh[c*x]]))/(128*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6312, 25, 82, 244, 2009, 6310, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6312}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int -x(1-cx)(cx+1)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{82}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(x-c^2x^3\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6310}$$

$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6308}$$

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/4*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/4 + (3*d*(-1/
4*(b*c*x^2*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6312
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p
)]   Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(165) = 330$.

Time = 0.00 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.77

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4} + \frac{3adx\sqrt{-c^2dx^2+d}}{8} + \frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}} + b\left(-\frac{3\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2d}{16\sqrt{cx-1}\,\sqrt{cx+1}\,c} - \frac{\sqrt{-d(c^2x^2}{}\right.$
parts	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4} + \frac{3adx\sqrt{-c^2dx^2+d}}{8} + \frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}} + b\left(-\frac{3\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2d}{16\sqrt{cx-1}\,\sqrt{cx+1}\,c} - \frac{\sqrt{-d(c^2x^2}{}\right.$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*d^2/(c^2
*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/16*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d-1/256*(-d*(c^2
*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)
+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
(-1+4*arccosh(c*x))*d/(c*x-1)/(c*x+1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3
*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/
2))*(-1+2*arccosh(c*x))*d/(c*x-1)/(c*x+1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-
2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2
)-2*c*x)*(1+2*arccosh(c*x))*d/(c*x-1)/(c*x+1)/c-1/256*(-d*(c^2*x^2-1))^(1/
2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-1)^(1/2)*(c*x+
1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccos
h(c*x))*d/(c*x-1)/(c*x+1)/c)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2
*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d - c^2 d x^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))\, dx = \int \left(-d(cx - 1)\,(cx + 1)\right)^{\frac{3}{2}} (a + b\operatorname{acosh}(cx))\, dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \left(d - c^2 d x^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))\, dx = \int \left(-c^2 d x^2 + d\right)^{\frac{3}{2}} (b\operatorname{arcosh}(cx) + a)\, dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*a
rcsin(c*x)/c)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d - c^2 d x^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx = \int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx = \frac{\sqrt{d}\,d\left(3asin(cx)\,a-2\sqrt{-c^2x^2+1}\,a\,c^3x^3+5\sqrt{-c^2x^2+1}\,acx-8\left(\int\sqrt{-c^2x^2+1}\,acosh\right.\right.}{8c}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 5*sqrt(
 - c**2*x**2 + 1)*a*c*x - 8*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*
b*c**3 + 8*int(sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*b*c))/(8*c)
```

3.71 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 197

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{bc^3dx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) - \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x} + \frac{3cd\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bcd\sqrt{d-c^2dx^2}\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/4*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*c^2*d
*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-(-c^2*d*x^2+d)^(3/2)*(a+b*arcco
sh(c*x))/x+3/4*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+b*c*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{1}{8}\left(-\frac{4ad(2+c^2x^2)\sqrt{d-c^2dx^2}}{x}\right.$$
$$+12acd^{3/2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+4bcd\sqrt{d-c^2dx^2}\left(-\frac{2\text{arccosh}(cx)}{cx}+\frac{\text{arccosh}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)+\frac{bc}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
((-4*a*d*(2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x + 12*a*c*d^(3/2)*ArcTan[(c*x
*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 4*b*c*d*Sqrt[d - c^2*d*x
^2]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x))) + (b*c*d*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2
+ Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)))/8
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6343, 25, 82, 244, 2009, 6310, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx$$
$$\Big\downarrow \; 6343$$
$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \ 25$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \ 82$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \ 244$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x}-c^2x\right)dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \ 2009$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 6310$$

$$-3c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 15$$

$$-3c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6308}$$

$$-3c^2d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x) - 3*c^2*d*(-1/4*(b*c*x^2
*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)
/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/2*
(c^2*x^2) + Log[x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.24

method	result
default	$-\frac{a\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{d x}-a\,c^{2} x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}-\frac{3 a\,c^{2} d x\sqrt{-c^{2} d\,x^{2}+d}}{2}-\frac{3 a\,c^{2} d^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{2\sqrt{c^{2} d}}+\frac{b\sqrt{-d\left(c^{2} x^{2}-\right.}}{}$
parts	$-\frac{a\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{d x}-a\,c^{2} x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}-\frac{3 a\,c^{2} d x\sqrt{-c^{2} d\,x^{2}+d}}{2}-\frac{3 a\,c^{2} d^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{2\sqrt{c^{2} d}}+\frac{b\sqrt{-d\left(c^{2} x^{2}-\right.}}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(5/2)-a*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d*x*(-c
^2*d*x^2+d)^(1/2)-3/2*a*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2
*d*x^2+d)^(1/2))+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/
x*(-4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+2*c^3*x^3+6*arccosh
(c*x)^2*c*x-8*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-8*c*x*arccosh(c*x)+
8*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c-c*x)*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas
")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2
*d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\text{acosh}\,(cx))}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**2, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima
")
```

output
```
-1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)/x)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{\sqrt{d}\,d\Big(-3asin(cx)\,acx-\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^2,x)
```

output
```
(sqrt(d)*d*( - 3*asin(c*x)*a*c*x - sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*
sqrt( - c**2*x**2 + 1)*a + 2*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**2,
x)*b*x - 2*int(sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*b*c**2*x))/(2*x)
```

3.72 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\textbf{arccosh}(cx))}{x^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^4}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x} - \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3x^3} - \frac{c^3d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{4bc^3d\sqrt{d-c^2dx^2}\log(x)}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/6*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+c^2*d*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos
h(c*x))/x^3-1/2*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-4/3*b*c^3*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.28

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \frac{-2bd^2\sqrt{\frac{-1+cx}{1+cx}}(1-5c^2x^2+4c^4x^4)\,\text{arccosh}(cx)+3bc^3d^2x^3(-1+$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
(-2*b*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4)*ArcCosh[c
*x] + 3*b*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 6*a*c^3*d^(3/2)*x^3*Sqrt
[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2]
)/(Sqrt[d]*(-1 + c^2*x^2))] + d^2*(b*c*x*(-1 + c*x) - 2*a*Sqrt[(-1 + c*x)/
(1 + c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4) + 8*b*c^3*x^3*(-1 + c*x)*Log[c*x]))
/(6*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6343, 25, 82, 244, 2009, 6339, 14, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6343}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 82

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 244

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^3}-\frac{c^2}{x}\right)dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$\Big\downarrow$ 2009

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6339

$$c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 14

$$c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6308

$$c^2(-d)\left(\frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3 + (b*c*d*Sqrt[d - c^
2*d*x^2]*(-1/2*1/x^2 - c^2*Log[x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - c^2
*d*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x) + (c*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[
d - c^2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.33

method	result
default	$-\frac{a\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d\,x^{3}}+\frac{2 a\,c^{2} \left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d x}+\frac{2 a\,c^{4} x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+a\,c^{4} d x \sqrt{-c^{2} d\,x^{2}+d}+\frac{a\,c^{4} d^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,\ldots}{\sqrt{-c^{2} d\,x^{2}\ldots}}\right.}{\sqrt{c^{2} d}}$
parts	$-\frac{a\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d\,x^{3}}+\frac{2 a\,c^{2} \left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d x}+\frac{2 a\,c^{4} x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+a\,c^{4} d x \sqrt{-c^{2} d\,x^{2}+d}+\frac{a\,c^{4} d^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,\ldots}{\sqrt{-c^{2} d\,x^{2}\ldots}}\right.}{\sqrt{c^{2} d}}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+2/3*a
*c^4*x*(-c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a*c^4*d^2/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/6*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x^3*(3*arccosh(c*x)^2*c^3*x^3-8*arcco
sh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2-8*c^3*x^3*arccosh(c*x)+8*ln(1+
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3+2*arccosh(c*x)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)+c*x)*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas
")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2
*d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**4,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**4, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\mathrm{arccosh}(cx))}{x^4}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima
")
```

output
```
1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*
d*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a + b*integrate((
-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\mathrm{arccosh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\mathrm{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^4}\,dx = \frac{\sqrt{d}\,d\Big(3asin(cx)\,a\,c^3x^3+4\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a+$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^4,x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a*c**3*x**3 + 4*sqrt( - c**2*x**2 + 1)*a*c**2*x**2
 - sqrt( - c**2*x**2 + 1)*a + 3*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x*
*4,x)*b*x**3 - 3*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**2,x)*b*c**2*x*
*3))/(3*x**3)
```

3.73 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\mathbf{arccosh}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{5dx^5}+\frac{bc^5d\sqrt{d-c^2dx^2}\log(x)}{5\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/20*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/5*b*c^3
*d*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/5*(-c^2*d*x^2+d)
^(5/2)*(a+b*arccosh(c*x))/d/x^5+1/5*b*c^5*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.57

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx = -\frac{d\sqrt{d-c^2dx^2}\left(\frac{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^5}-bc\left(-\frac{1}{4x^4}+\frac{c^2}{x^2}+c^4\log(x)\right)\right)}{5\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

output
```
-1/5*(d*Sqrt[d - c^2*d*x^2]*(((-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcC
osh[c*x]))/x^5 - b*c*(-1/4*1/x^4 + c^2/x^2 + c^4*Log[x])))/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6332, 82, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{6332}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2}{x^5}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{5dx^5}$$

$$\Big\downarrow \text{82}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2}{x^5}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{5dx^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2}{x^6}dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{5dx^5}$$

$$\Big\downarrow \text{49}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{c^4}{x^2}-\frac{2c^2}{x^4}+\frac{1}{x^6}\right)dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{5dx^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{5dx^5}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(d*x^5) + (b*c*d*Sqrt[d
- c^2*d*x^2]*(-1/2*1/x^4 + (2*c^2)/x^2 + c^4*Log[x^2]))/(10*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2170 vs. $2(138) = 276$.

Time = 0.45 (sec) , antiderivative size = 2171, normalized size of antiderivative = 13.08

method	result	size
default	Expression too large to display	2171
parts	Expression too large to display	2171

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-11*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)*x^5/(c*x-1)/(c*x+1)*arccosh(c*x)*c^10+14*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*
c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3/(c*x-1)/(c*x+1)*arccosh(c*x
)*c^8-56/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c
^2*x^2+1)*x/(c*x-1)/(c*x+1)*arccosh(c*x)*c^6+28/5*b*(-d*(c^2*x^2-1))^(1/2)
*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x/(c*x-1)/(c*x+1)*arccosh
(c*x)*c^4-8/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-
5*c^2*x^2+1)/x^3/(c*x-1)/(c*x+1)*arccosh(c*x)*c^2-b*(-d*(c^2*x^2-1))^(1/2)
*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^9/(c*x-1)/(c*x+1)*arcco
sh(c*x)*c^14-b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5
*c^2*x^2+1)*x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^7+b*(-d*(c^2*x^
2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^8/(c*x-1)^(1
/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^13-2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^
8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcco
sh(c*x)*c^11+2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4
-5*c^2*x^2+1)*x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^9+5*b*(-d*(c^
2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^7/(c*x-1
)/(c*x+1)*arccosh(c*x)*c^12-1/5*a/d/x^5*(-c^2*d*x^2+d)^(5/2)-3/2*b*(-d*(c^
2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)*c^5+1/5*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.45

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^6}\,dx = \left[-\frac{4\left(bc^6dx^6-3\,bc^4dx^4+3\,bc^2dx^2-bd\right)\sqrt{-c^2dx^2+d}\log\left(cx+\right.}{}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas
")
```

output
```
[-1/20*(4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*
x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt
(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^
2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) - (4*b*c^3*d*x^3 - (4*b*c^
3 - b*c)*d*x^5 - b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(a*c^
6*d*x^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*
x^7 - x^5), 1/20*(4*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(d)*arctan(sqrt(-c^2*d
*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2
 - d)) - 4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d
*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + (4*b*c^3*d*x^3 - (4*b*c^3 - b*c)*
d*x^5 - b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(a*c^6*d*x^6 -
 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**6,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^6}\,dx = -\frac{\left(2\,c^6d^3\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)+2i\,(-1)^{-2\,c^2dx^2+2\,d}\,c^4d^{\frac{5}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+\frac{3\sqrt{-c^4dx^4+2\,c^2dx^2-d}c^2d^2}{x^2}-\frac{\sqrt{-c^4dx}\cdots}{}\right.}{20\,d} - \frac{(-c^2dx^2+d)^{\frac{5}{2}}b\,\text{arcosh}\,(cx)}{5\,dx^5} - \frac{(-c^2dx^2+d)^{\frac{5}{2}}a}{5\,dx^5}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima
")
```

output
```
-1/20*(2*c^6*d^3*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + 2*I*(-1)^(-2*c^2*d*x^
2 + 2*d)*c^4*d^(5/2)*log(-2*c^2*d + 2*d/x^2) + 3*sqrt(-c^4*d*x^4 + 2*c^2*d
*x^2 - d)*c^2*d^2/x^2 - sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*d^2/x^4)*b*c/d
- 1/5*(-c^2*d*x^2 + d)^(5/2)*b*arccosh(c*x)/(d*x^5) - 1/5*(-c^2*d*x^2 + d)
^(5/2)*a/(d*x^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^6}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^6}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^6,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^6}\,dx = \frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a\,c^4x^4+2\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^6,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 2*sqrt( - c**2*x**2 +
1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a + 5*int((sqrt( - c**2*x**2 + 1)*
acosh(c*x))/x**6,x)*b*x**5 - 5*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**
4,x)*b*c**2*x**5))/(5*x**5)
```

3.74 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 247

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^8}\,dx =&-\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{-1+cx}\sqrt{1+cx}}\\&-\frac{bc^5d\sqrt{d-c^2dx^2}}{70x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{7dx^7}\\&-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{35dx^5}+\frac{2bc^7d\sqrt{d-c^2dx^2}\log(x)}{35\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/42*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/35*b*c^
3*d*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/70*b*c^5*d*(-c^
2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(5/2)*
(a+b*arccosh(c*x))/d/x^7-2/35*c^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/
d/x^5+2/35*b*c^7*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.55

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^8}\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\left(30(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\text{arccosh}(cx))+12c^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\text{arccos}\right.}{210x^7\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

output
```
-1/210*(d*Sqrt[d - c^2*d*x^2]*(30*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*
ArcCosh[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh
[c*x]) + b*c*x*(5 - 12*c^2*x^2 + 3*c^4*x^4 - 12*c^6*x^6*Log[x])))/(x^7*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6337, 27, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^8}\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(2c^2x^2+5)}{35x^7}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{7dx^7}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{35dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2\left(2c^2x^2+5\right)}{x^7}dx}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{7dx^7}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{35dx^5}$$

$$\Big\downarrow \ 354$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2\left(2c^2x^2+5\right)}{x^8}dx^2}{70\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{7dx^7}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{35dx^5}$$

$$\Big\downarrow \ 85$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{2c^6}{x^2}+\frac{c^4}{x^4}-\frac{8c^2}{x^6}+\frac{5}{x^8}\right)dx^2}{70\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{7dx^7}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{35dx^5}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{7dx^7}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{35dx^5}+\frac{bcd\sqrt{d-c^2dx^2}\left(2c^6\log\left(x^2\right)-\frac{c^4}{x^2}+\frac{4c^2}{x^4}-\frac{5}{3x^6}\right)}{70\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

output
```
-1/7*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(d*x^7) - (2*c^2*(d - c^
2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(35*d*x^5) + (b*c*d*Sqrt[d - c^2*d*x^
2]*(-5/(3*x^6) + (4*c^2)/x^4 - c^4/x^2 + 2*c^6*Log[x^2]))/(70*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3144 vs. $2(207) = 414$.

Time = 0.48 (sec) , antiderivative size = 3145, normalized size of antiderivative = 12.73

method	result	size
default	Expression too large to display	3145
parts	Expression too large to display	3145

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
-116/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+15
4*c^4*x^4-105*c^2*x^2+25)*x^5*c^12-5/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^1
0*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x*c^8-2/35*b*(-d*
(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c
^2*x^2+25)*x^11*c^18+20/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8
*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3*c^10+1/5*b*(-d*(c^2*x^2-1)
)^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*
x^9*c^16+26/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6
*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^7*c^14-2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35
*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^11/(c*x-1)/
(c*x+1)*arccosh(c*x)*c^18+3*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^
8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c*x-1)/(c*x+1)*arccosh(c
*x)*c^16+12*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6
+154*c^4*x^4-105*c^2*x^2+25)*x^7/(c*x-1)/(c*x+1)*arccosh(c*x)*c^14-164/5*b
*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-
105*c^2*x^2+25)*x^5/(c*x-1)/(c*x+1)*arccosh(c*x)*c^12+52/5*b*(-d*(c^2*x^2-
1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25
)*x^3/(c*x-1)/(c*x+1)*arccosh(c*x)*c^10+1966/35*b*(-d*(c^2*x^2-1))^(1/2)*d
/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x/(c*x-1)
/(c*x+1)*arccosh(c*x)*c^8-3272/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x...
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.63

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \Bigg[-\frac{6\left(2\,bc^8dx^8-bc^6dx^6-9\,bc^4dx^4+13\,bc^2dx^2-5\,bd\right)\sqrt{-c^2dx^2}}{}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas
")
```

output
```
[-1/210*(6*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 -
 5*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(b*c^9*d*x^9
 - b*c^7*d*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*
x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (3*b
*c^5*d*x^5 - (3*b*c^5 - 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d
*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 6*(2*a*c^8*d*x^8 - a*c^6*d*x^
6 - 9*a*c^4*d*x^4 + 13*a*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9
 - x^7), 1/210*(12*(b*c^9*d*x^9 - b*c^7*d*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*
x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2
- d)) - 6*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 -
5*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5
- (3*b*c^5 - 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4
*d*x^4 + 13*a*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}{x^8}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.66

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}{x^8}\,dx=\frac{1}{210}\left(12\,c^6\sqrt{-d}d\log\left(x\right)-\frac{3\,c^4\sqrt{-d}dx^4-12\,c^2\sqrt{-d}dx^2+5\,\sqrt{-}}{x^6}\right.$$
$$-\frac{1}{35}\,b\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{dx^5}+\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{dx^7}\right)\text{arcosh}\left(cx\right)$$
$$-\frac{1}{35}\,a\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{dx^5}+\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{dx^7}\right)$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima
")
```

output
```
1/210*(12*c^6*sqrt(-d)*d*log(x) - (3*c^4*sqrt(-d)*d*x^4 - 12*c^2*sqrt(-d)*
d*x^2 + 5*sqrt(-d)*d)/x^6)*b*c - 1/35*b*(2*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x
^5) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))*arccosh(c*x) - 1/35*a*(2*(-c^2*d*x
^2 + d)^(5/2)*c^2/(d*x^5) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^8}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^8,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(d - c^2dx^2)^{3/2}\,(a + b\text{arccosh}(cx))}{x^8}\,dx = \frac{\sqrt{d}\,d\Big(-2\sqrt{-c^2x^2+1}\,a\,c^6x^6 - \sqrt{-c^2x^2+1}\,a\,c^4x^4 + 8\sqrt{-c^2x^2+}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^8,x)
```

output
```
(sqrt(d)*d*( - 2*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - sqrt( - c**2*x**2 +
1)*a*c**4*x**4 + 8*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 5*sqrt( - c**2*x**
2 + 1)*a + 35*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**8,x)*b*x**7 - 35*
int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**6,x)*b*c**2*x**7))/(35*x**7)
```

3.75 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))}{x^{10}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 328

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))}{x^{10}}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{420x^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bc^7d\sqrt{d-c^2dx^2}}{315x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathrm{arccosh}(cx))}{9dx^9} - \frac{4c^2(d-c^2dx^2)^{5/2}(a+b\,\mathrm{arccosh}(cx))}{63dx^7} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\,\mathrm{arccosh}(cx))}{315dx^5} + \frac{8bc^9d\sqrt{d-c^2dx^2}\log(x)}{315\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/72*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/189*b*c
^3*d*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/420*b*c^5*d*(-
c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/315*b*c^7*d*(-c^2*d*x
^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*(-c^2*d*x^2+d)^(5/2)*(a+b*
arccosh(c*x))/d/x^9-4/63*c^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^7
-8/315*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^5+8/315*b*c^9*d*(-c
^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.47

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\left(840(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\text{arccosh}(cx))+96c^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(5+2c^2x^2\right.\right.}{7560x^9\sqrt{-1+cx}\sqrt{1+c}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

output
```
-1/7560*(d*Sqrt[d - c^2*d*x^2]*(840*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a +
b*ArcCosh[c*x]) + 96*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(5 + 2*c^2*x
^2)*(a + b*ArcCosh[c*x]) + b*c*x*(105 - 200*c^2*x^2 + 18*c^4*x^4 + 48*c^6*
x^6 - 192*c^8*x^8*Log[x])))/(x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6337, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{315x^9}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{315dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{x^9}dx}{315\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{315dx^5}$$

$$\Big\downarrow \; 1578$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{x^{10}}dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{315dx^5}$$

$$\Big\downarrow \; 1195$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{8c^8}{x^2}+\frac{4c^6}{x^4}+\frac{3c^4}{x^6}-\frac{50c^2}{x^8}+\frac{35}{x^{10}}\right)dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{315dx^5}$$

$$\Big\downarrow \; 2009$$

$$-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{315dx^5}+\frac{bcd\sqrt{d-c^2dx^2}\left(8c^8\log(x^2)-\frac{4c^6}{x^2}-\frac{3c^4}{2x^4}+\frac{50c^2}{3x^6}-\frac{35}{4x^8}\right)}{630\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

output
```
-1/9*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(d*x^9) - (4*c^2*(d - c^
2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(63*d*x^7) - (8*c^4*(d - c^2*d*x^2)^(
5/2)*(a + b*ArcCosh[c*x]))/(315*d*x^5) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-35/(
4*x^8) + (50*c^2)/(3*x^6) - (3*c^4)/(2*x^4) - (4*c^6)/x^2 + 8*c^8*Log[x^2]
))/(630*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4261 vs. $2(276) = 552$.

Time = 0.54 (sec) , antiderivative size = 4262, normalized size of antiderivative = 12.99

method	result	size
default	Expression too large to display	4262
parts	Expression too large to display	4262

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x,method=_RETURNVERBOSE)
```

output
```
-24*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-27
30*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^10/(c*x-1)^(1/2)/(c*x+1)^(1/2
)*arccosh(c*x)*c^19+24/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^1
0*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^8/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^17-208/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(
840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2
*x^2+1225)*x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^15+1104/7*b*(-d*
(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6
+6210*c^4*x^4-4725*c^2*x^2+1225)*x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c
*x)*c^13-120*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c
^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^2/(c*x-1)^(1/2)/(c*x
+1)^(1/2)*arccosh(c*x)*c^11+113594/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12
*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+122
5)/x/(c*x-1)/(c*x+1)*arccosh(c*x)*c^8-174520/63*b*(-d*(c^2*x^2-1))^(1/2)*d
/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c
^2*x^2+1225)/x^3/(c*x-1)/(c*x+1)*arccosh(c*x)*c^6+19540/9*b*(-d*(c^2*x^2-1
))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*
x^4-4725*c^2*x^2+1225)/x^5/(c*x-1)/(c*x+1)*arccosh(c*x)*c^4+59884/105*b*(-
d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x
^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^3/(c*x-1)/(c*x+1)*arccosh(c*x)*c^1...
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.20

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \left[-\frac{24\left(8\,bc^{10}dx^{10}-4\,bc^8dx^8-bc^6dx^6-53\,bc^4dx^4+85\,bc^2dx^2-\right.}{}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="frica
s")
```

output
```
[-1/7560*(24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x
^4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2
- 1)) - 96*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x
^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d
)/(c^2*x^4 - x^2)) + (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c
^5 - 200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2
*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^
6*d*x^6 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/
(c^2*x^11 - x^9), 1/7560*(192*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(d)*arctan
(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^
2 - 1)*d*x^2 - d)) - 24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 5
3*b*c^4*d*x^4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 - 1)) - (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5
 - 200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d
*x^2 + d)*sqrt(c^2*x^2 - 1) - 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*
d*x^6 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/(c
^2*x^11 - x^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.69

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \frac{1}{7560}\left(192\,c^8\sqrt{-d}d\log(x) - \frac{48\,c^6\sqrt{-d}dx^6+18\,c^4\sqrt{-d}dx^4-2\cdots}{x^8}\right.$$
$$-\frac{1}{315}b\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^4}{dx^5}+\frac{20\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{dx^7}+\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{dx^9}\right)\text{arcosh}(cx)$$
$$-\frac{1}{315}a\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^4}{dx^5}+\frac{20\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{dx^7}+\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{dx^9}\right)$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxim
a")
```

output
```
1/7560*(192*c^8*sqrt(-d)*d*log(x) - (48*c^6*sqrt(-d)*d*x^6 + 18*c^4*sqrt(-
d)*d*x^4 - 200*c^2*sqrt(-d)*d*x^2 + 105*sqrt(-d)*d)/x^8)*b*c - 1/315*b*(8*
(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7)
 + 35*(-c^2*d*x^2 + d)^(5/2)/(d*x^9))*arccosh(c*x) - 1/315*a*(8*(-c^2*d*x^
2 + d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) + 35*(-c^
2*d*x^2 + d)^(5/2)/(d*x^9))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^{10}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^10,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \frac{\sqrt{d}\,d\Big(-8\sqrt{-c^2x^2+1}\,a\,c^8x^8-4\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^10,x)
```

output
```
(sqrt(d)*d*( - 8*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 - 4*sqrt( - c**2*x**2
+ 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 50*sqrt( - c**2*
x**2 + 1)*a*c**2*x**2 - 35*sqrt( - c**2*x**2 + 1)*a + 315*int((sqrt( - c**
2*x**2 + 1)*acosh(c*x))/x**10,x)*b*x**9 - 315*int((sqrt( - c**2*x**2 + 1)*
acosh(c*x))/x**8,x)*b*c**2*x**9))/(315*x**9)
```

3.76 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\text{arccosh}(cx))}{x^{12}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 409

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{66x^8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{1386x^6\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^7d\sqrt{d-c^2dx^2}}{770x^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{4bc^9d\sqrt{d-c^2dx^2}}{1155x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{11dx^{11}} - \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{33dx^9}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{231dx^7} - \frac{16c^6(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{1155dx^5}+\frac{16bc^{11}d\sqrt{d-c^2dx^2}\log(x)}{1155\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/110*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^10/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/66*b*
c^3*d*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/1386*b*c^5*d*
(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/770*b*c^7*d*(-c^2*d
*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/1155*b*c^9*d*(-c^2*d*x^2+d
)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/11*(-c^2*d*x^2+d)^(5/2)*(a+b*arc
cosh(c*x))/d/x^11-2/33*c^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^9-8
/231*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^7-16/1155*c^6*(-c^2*d
*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^5+16/1155*b*c^11*d*(-c^2*d*x^2+d)^(1/
2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.42

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\left(630(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\text{arccosh}(cx))+12c^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(35+20c^2\right.\right.}{6930x^{11}\sqrt{-1+c}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

output
```
-1/6930*(d*Sqrt[d - c^2*d*x^2]*(630*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a +
b*ArcCosh[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(35 + 20*c^2
*x^2 + 8*c^4*x^4)*(a + b*ArcCosh[c*x]) + b*c*x*(63 - 105*c^2*x^2 + 5*c^4*x
^4 + 9*c^6*x^6 + 24*c^8*x^8 - 96*c^10*x^10*Log[x])))/(x^11*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6337, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^{12}}dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d\left(1-c^2x^2\right)^2\left(16c^6x^6+40c^4x^4+70c^2x^2+105\right)}{1155x^{11}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{33dx^9}-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{1155dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{231dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2\left(16c^6x^6+40c^4x^4+70c^2x^2+105\right)}{x^{11}}dx}{1155\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{33dx^9}-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{1155dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{231dx^7}$$

$$\Big\downarrow \text{2331}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2\left(16c^6x^6+40c^4x^4+70c^2x^2+105\right)}{x^{12}}dx^2}{2310\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{33dx^9}-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{1155dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{231dx^7}$$

$$\Big\downarrow \text{2123}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{16c^{10}}{x^2}+\frac{8c^8}{x^4}+\frac{6c^6}{x^6}+\frac{5c^4}{x^8}-\frac{140c^2}{x^{10}}+\frac{105}{x^{12}}\right)dx^2}{2310\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{33dx^9}-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{1155dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{231dx^7}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{33dx^9}-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{1155dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{231dx^7}+\frac{bcd\sqrt{d-c^2dx^2}\left(16c^{10}\log\left(x^2\right)-\frac{8c^8}{x^2}-\frac{3c^6}{x^4}-\frac{5c^4}{3x^6}+\frac{35c^2}{x^8}-\frac{21}{x^{10}}\right)}{2310\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

output
```
-1/11*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(d*x^11) - (2*c^2*(d -
c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(33*d*x^9) - (8*c^4*(d - c^2*d*x^2)
^(5/2)*(a + b*ArcCosh[c*x]))/(231*d*x^7) - (16*c^6*(d - c^2*d*x^2)^(5/2)*(
a + b*ArcCosh[c*x]))/(1155*d*x^5) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-21/x^10 +
 (35*c^2)/x^8 - (5*c^4)/(3*x^6) - (3*c^6)/x^4 - (8*c^8)/x^2 + 16*c^10*Log[
x^2]))/(2310*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5522 vs. $2(345) = 690$.

Time = 0.62 (sec) , antiderivative size = 5523, normalized size of antiderivative = 13.50

method	result	size
default	Expression too large to display	5523
parts	Expression too large to display	5523

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.94

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="frica
s")
```

output
```
[-1/6930*(6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*
x^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 + d)*lo
g(c*x + sqrt(c^2*x^2 - 1)) - 48*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(-d)*l
og((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (24*b*c^9*d*x^9 + 9*b*c^7*d*x^
7 - (24*b*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d
*x^5 - 105*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1
) + 6*(16*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 -
145*a*c^4*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^
13 - x^11), 1/6930*(96*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(d)*arctan(sqrt
(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1
)*d*x^2 - d)) - 6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*
c^6*d*x^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 +
 d)*log(c*x + sqrt(c^2*x^2 - 1)) - (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (24*b
*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 - 10
5*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 6*(16
*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^4
*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)^{3/2} (a + b\text{arccosh}(cx))}{x^{12}} \, dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**12,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.70

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^{12}}\,dx = \frac{1}{6930}\left(96\,c^{10}\sqrt{-d}d\log(x) - \frac{24\,c^8\sqrt{-d}dx^8 + 9\,c^6\sqrt{-d}dx^6 + 5\,c^4\sqrt{-d}dx^4 - 105\,c^2\sqrt{-d}dx^2 + 63\,\sqrt{-d}d}{x^{10}}\right)bc - \frac{1}{1155}\left(\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}c^6}{dx^5} + \frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}c^4}{dx^7} + \frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^9} + \frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^{11}}\right)b\operatorname{arccosh}(cx) - \frac{1}{1155}\left(\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}c^6}{dx^5} + \frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}c^4}{dx^7} + \frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^9} + \frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^{11}}\right)a$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxim
a")
```

output
```
1/6930*(96*c^10*sqrt(-d)*d*log(x) - (24*c^8*sqrt(-d)*d*x^8 + 9*c^6*sqrt(-d
)*d*x^6 + 5*c^4*sqrt(-d)*d*x^4 - 105*c^2*sqrt(-d)*d*x^2 + 63*sqrt(-d)*d)/x
^10)*b*c - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*c^6/(d*x^5) + 40*(-c^2*d*x^2
+ d)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^9) + 105*(-c^2
*d*x^2 + d)^(5/2)/(d*x^11))*b*arccosh(c*x) - 1/1155*(16*(-c^2*d*x^2 + d)^(
5/2)*c^6/(d*x^5) + 40*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2
+ d)^(5/2)*c^2/(d*x^9) + 105*(-c^2*d*x^2 + d)^(5/2)/(d*x^11))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^{12}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^{12}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^12,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx=\frac{\sqrt{d}\,d\Big(-16\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}-8\sqrt{-c^2x^2+1}\,a\,c^8x^8-6\sqrt{-c^2}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^12,x)
```

output
```
(sqrt(d)*d*( - 16*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 - 8*sqrt( - c**2*x*
*2 + 1)*a*c**8*x**8 - 6*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 5*sqrt( - c**
2*x**2 + 1)*a*c**4*x**4 + 140*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 105*sqr
t( - c**2*x**2 + 1)*a + 1155*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**12
,x)*b*x**11 - 1155*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**10,x)*b*c**2
*x**11))/(1155*x**11)
```

3.77 $\int x^7(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 399

$$\int x^7\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx=\frac{16bdx\sqrt{d-c^2dx^2}}{1155c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{8bdx^3\sqrt{d-c^2dx^2}}{3465c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bdx^5\sqrt{d-c^2dx^2}}{1925c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdx^7\sqrt{d-c^2dx^2}}{1617c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdx^9\sqrt{d-c^2dx^2}}{297\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^{11}\sqrt{d-c^2dx^2}}{121\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^8d}+\frac{3\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d^2}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^3}+\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^4}$$

output
```
16/1155*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/3465*
b*d*x^3*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/1925*b*d*x^
5*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1617*b*d*x^7*(-c^
2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/297*b*c*d*x^9*(-c^2*d*x^2
+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/121*b*c^3*d*x^11*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)
)/c^8/d+3/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^8/d^2-1/3*(-c^2*d*x^
2+d)^(9/2)*(a+b*arccosh(c*x))/c^8/d^3+1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arcc
osh(c*x))/c^8/d^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.52

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\operatorname{arccosh}(cx))\,dx =$$

$$-\frac{d\sqrt{d-c^2dx^2}\Big(3465a\sqrt{-1+cx}\sqrt{1+cx}(-1+c^2x^2)^2\left(16+40c^2x^2+70c^4x^4+105c^6x^6\right)-bcx(55440+9\cdots}{\cdots}$$

input
```
Integrate[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/4002075*(d*Sqrt[d - c^2*d*x^2]*(3465*a*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1
 + c^2*x^2)^2*(16 + 40*c^2*x^2 + 70*c^4*x^4 + 105*c^6*x^6) - b*c*x*(55440
+ 9240*c^2*x^2 + 4158*c^4*x^4 + 2475*c^6*x^6 - 53900*c^8*x^8 + 33075*c^10*
x^10) + 3465*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)^2*(16 + 40*c^2*
x^2 + 70*c^4*x^4 + 105*c^6*x^6)*ArcCosh[c*x]))/(c^8*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6337, 27, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \; 6337$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(105c^6x^6+70c^4x^4+40c^2x^2+16)}{1155c^8}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\operatorname{arccosh}(cx))}{11c^8d^4}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\operatorname{arccosh}(cx))}{3c^8d^3}+\frac{3\left(d-c^2dx^2\right)^{7/2}(a+b\operatorname{arccosh}(cx))}{7c^8d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))}{5c^8d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2 \left(105c^6x^6+70c^4x^4+40c^2x^2+16\right)dx}{1155c^7\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^4}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^3}+\frac{3\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^8d}$$

$$\Big\downarrow \text{2341}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int \left(105c^{10}x^{10}-140c^8x^8+5c^6x^6+6c^4x^4+8c^2x^2+16\right)dx}{1155c^7\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^4}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^3}+\frac{3\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^8d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^4}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^3}+\frac{3\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^8d}+\frac{bd\left(\frac{105c^{10}x^{11}}{11}-\frac{140c^8x^9}{9}+\frac{5c^6x^7}{7}+\frac{6c^4x^5}{5}+\frac{8c^2x^3}{3}+16x\right)\sqrt{d-c^2dx^2}}{1155c^7\sqrt{cx-1}\sqrt{cx+1}}$$

input

```
Int[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output

```
(b*d*Sqrt[d - c^2*d*x^2]*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (5*c^6*x^
7)/7 - (140*c^8*x^9)/9 + (105*c^10*x^11)/11))/(1155*c^7*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(5*c^8*d) + (3*
(d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^8*d^2) - ((d - c^2*d*x^2)
^(9/2)*(a + b*ArcCosh[c*x]))/(3*c^8*d^3) + ((d - c^2*d*x^2)^(11/2)*(a + b*
ArcCosh[c*x]))/(11*c^8*d^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.64

method	result
orering	$\frac{(694575x^{12}c^{12}-1619450c^{10}x^{10}+904475c^8x^8+27720c^6x^6+70224c^4x^4+517440c^2x^2-443520)\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\,\mathrm{arccosh}(cx))}{4002075c^8(cx-1)(cx+1)(c^2x^2-1)}$ –
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4002075*(694575*c^12*x^12-1619450*c^10*x^10+904475*c^8*x^8+27720*c^6*x^6
+70224*c^4*x^4+517440*c^2*x^2-443520)/c^8/(c*x-1)/(c*x+1)/(c^2*x^2-1)*(-c^
2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/4002075/x^6*(33075*c^10*x^10-53900*c
^8*x^8+2475*c^6*x^6+4158*c^4*x^4+9240*c^2*x^2+55440)/c^8/(c*x-1)/(c*x+1)*(
7*x^6*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-3*x^8*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccosh(c*x))*c^2*d+x^7*(-c^2*d*x^2+d)^(3/2)*b*c/(c*x-1)^(1/2)/(c*x+1)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.69

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx =$$

$$-\frac{3465\,(105\,bc^{12}dx^{12}-245\,bc^{10}dx^{10}+145\,bc^8dx^8+bc^6dx^6+2\,bc^4dx^4+8\,bc^2dx^2-16\,bd)\sqrt{-c^2dx^2+d}\log}{}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
-1/4002075*(3465*(105*b*c^12*d*x^12 - 245*b*c^10*d*x^10 + 145*b*c^8*d*x^8
+ b*c^6*d*x^6 + 2*b*c^4*d*x^4 + 8*b*c^2*d*x^2 - 16*b*d)*sqrt(-c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 - 1)) - (33075*b*c^11*d*x^11 - 53900*b*c^9*d*x^9
 + 2475*b*c^7*d*x^7 + 4158*b*c^5*d*x^5 + 9240*b*c^3*d*x^3 + 55440*b*c*d*x)
*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(105*a*c^12*d*x^12 - 245*a*
c^10*d*x^10 + 145*a*c^8*d*x^8 + a*c^6*d*x^6 + 2*a*c^4*d*x^4 + 8*a*c^2*d*x^
2 - 16*a*d)*sqrt(-c^2*d*x^2 + d))/(c^10*x^2 - c^8)
```

Sympy [F(-1)]

Timed out.

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**7*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.71

$$\int x^7 (d-c^2dx^2)^{3/2} (a+b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{1}{1155}\left(\frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}x^6}{c^2d}+\frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}x^4}{c^4d}+\frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^6d}+\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^8d}\right)b\operatorname{arccos}$$
$$-\frac{1}{1155}\left(\frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}x^6}{c^2d}+\frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}x^4}{c^4d}+\frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^6d}+\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^8d}\right)a$$
$$+\frac{(33075\,c^{10}\sqrt{-d}dx^{11}-53900\,c^8\sqrt{-d}dx^9+2475\,c^6\sqrt{-d}dx^7+4158\,c^4\sqrt{-d}dx^5+9240\,c^2\sqrt{-d}dx^3+55440}{4002075\,c^7}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2*d) + 70*(-c^2*d*x^2 + d)^(5/2
)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d
)^(5/2)/(c^8*d))*b*arccosh(c*x) - 1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(
c^2*d) + 70*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)
*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^(5/2)/(c^8*d))*a + 1/4002075*(33075*c^1
0*sqrt(-d)*d*x^11 - 53900*c^8*sqrt(-d)*d*x^9 + 2475*c^6*sqrt(-d)*d*x^7 + 4
158*c^4*sqrt(-d)*d*x^5 + 9240*c^2*sqrt(-d)*d*x^3 + 55440*sqrt(-d)*d*x)*b/c
^7
```

Giac [F(-2)]

Exception generated.

$$\int x^7 (d-c^2dx^2)^{3/2} (a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx=\int x^7\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx=\frac{\sqrt{d}\,d\left(-105\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}+140\sqrt{-c^2x^2+1}\,a\,c^8x^8-5\sqrt{-c^2x^2+1}\,a\,c^6x^6-6\sqrt{}\right.}{}$$

input
```
int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d*( - 105*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 + 140*sqrt( - c**2
*x**2 + 1)*a*c**8*x**8 - 5*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 6*sqrt( -
c**2*x**2 + 1)*a*c**4*x**4 - 8*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 16*sqr
t( - c**2*x**2 + 1)*a - 1155*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**9,x)
*b*c**10 + 1155*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**7,x)*b*c**8))/(11
55*c**8)
```

3.78 $\int x^5(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 321

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx=\frac{8bdx\sqrt{d-c^2dx^2}}{315c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bdx^3\sqrt{d-c^2dx^2}}{945c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdx^5\sqrt{d-c^2dx^2}}{525c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{10bcdx^7\sqrt{d-c^2dx^2}}{441\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^9\sqrt{d-c^2dx^2}}{81\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{9c^6d^3}$$

output
```
8/315*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/945*b*d
*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/525*b*d*x^5*(-
c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-10/441*b*c*d*x^7*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/81*b*c^3*d*x^9*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x
))/c^6/d+2/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^6/d^2-1/9*(-c^2*d*x
^2+d)^(9/2)*(a+b*arccosh(c*x))/c^6/d^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.48

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx=$$
$$-\frac{d\sqrt{d-c^2dx^2}\left(-bcx(2520+420c^2x^2+189c^4x^4-2250c^6x^6+1225c^8x^8)+11025c^4x^4(-1+cx)^{5/2}(1+cx\right.}{99225c^6\sqrt{-1+cx}\sqrt{1}}$$

input
```
Integrate[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/99225*(d*Sqrt[d - c^2*d*x^2]*(-(b*c*x*(2520 + 420*c^2*x^2 + 189*c^4*x^4
 - 2250*c^6*x^6 + 1225*c^8*x^8)) + 11025*c^4*x^4*(-1 + c*x)^(5/2)*(1 + c*x
)^(5/2)*(a + b*ArcCosh[c*x]) + 1260*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(2 +
5*c^2*x^2)*(a + b*ArcCosh[c*x])))/(c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6337, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \ 6337$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(35c^4x^4+20c^2x^2+8)}{315c^6}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d}$$

$$\Big\downarrow \ 27$$

$$\frac{bd\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^2\left(35c^4x^4+20c^2x^2+8\right)dx}{315c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d}$$

$$\Big\downarrow \text{1467}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int\left(35c^8x^8-50c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{315c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^6d}+\frac{bd\left(\frac{35c^8x^9}{9}-\frac{50c^6x^7}{7}+\frac{3c^4x^5}{5}+\frac{4c^2x^3}{3}+8x\right)\sqrt{d-c^2dx^2}}{315c^5\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (50*c^6*x^
7)/7 + (35*c^8*x^9)/9))/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2
*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(5*c^6*d) + (2*(d - c^2*d*x^2)^(7/2)*(
a + b*ArcCosh[c*x]))/(7*c^6*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCosh[c
*x]))/(9*c^6*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75

method	result
orering	$\frac{(20825c^{10}x^{10}-50900c^8x^8+29457c^6x^6+2730c^4x^4+19320c^2x^2-15120)(-c^2dx^2+d)^{\frac{3}{2}}(a+b\,\mathrm{arccosh}(cx))}{99225c^6(cx-1)(cx+1)(c^2x^2-1)} - \frac{(1225c^8x^8-2250c^6x^6}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/99225*(20825*c^10*x^10-50900*c^8*x^8+29457*c^6*x^6+2730*c^4*x^4+19320*c^
2*x^2-15120)/c^6/(c*x-1)/(c*x+1)/(c^2*x^2-1)*(-c^2*d*x^2+d)^(3/2)*(a+b*arc
cosh(c*x))-1/99225/x^4*(1225*c^8*x^8-2250*c^6*x^6+189*c^4*x^4+420*c^2*x^2+
2520)/c^6/(c*x-1)/(c*x+1)*(5*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-3
*x^6*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*c^2*d+x^5*(-c^2*d*x^2+d)^(3/2
)*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.76

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx =$$

$$-\frac{315\left(35\,bc^{10}dx^{10}-85\,bc^8dx^8+53\,bc^6dx^6+bc^4dx^4+4\,bc^2dx^2-8\,bd\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)}{}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
-1/99225*(315*(35*b*c^10*d*x^10 - 85*b*c^8*d*x^8 + 53*b*c^6*d*x^6 + b*c^4*
d*x^4 + 4*b*c^2*d*x^2 - 8*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2
 - 1)) - (1225*b*c^9*d*x^9 - 2250*b*c^7*d*x^7 + 189*b*c^5*d*x^5 + 420*b*c^
3*d*x^3 + 2520*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 315*(35*a
*c^10*d*x^10 - 85*a*c^8*d*x^8 + 53*a*c^6*d*x^6 + a*c^4*d*x^4 + 4*a*c^2*d*x
^2 - 8*a*d)*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F(-1)]

Timed out.

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**5*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.69

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a+b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{1}{315}\left(\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^2d}+\frac{20\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^4d}+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^6d}\right)b\operatorname{arcosh}(cx)$$
$$-\frac{1}{315}\left(\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^2d}+\frac{20\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^4d}+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^6d}\right)a$$
$$+\frac{\left(1225\,c^8\sqrt{-d}dx^9-2250\,c^6\sqrt{-d}dx^7+189\,c^4\sqrt{-d}dx^5+420\,c^2\sqrt{-d}dx^3+2520\,\sqrt{-d}dx\right)b}{99225\,c^5}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/315*(35*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*
x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*b*arccosh(c*x) - 1/315*(35
*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^4*d
) + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*a + 1/99225*(1225*c^8*sqrt(-d)*d*x^9
 - 2250*c^6*sqrt(-d)*d*x^7 + 189*c^4*sqrt(-d)*d*x^5 + 420*c^2*sqrt(-d)*d*x
^3 + 2520*sqrt(-d)*d*x)*b/c^5
```

Giac [F(-2)]

Exception generated.

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx = \int x^5\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a +b\text{arccosh}(cx))\,dx = \frac{\sqrt{d}\,d\big(-35\sqrt{-c^2x^2+1}\,a\,c^8x^8+50\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4-4\sqrt{-c^2}}{}$$

input
```
int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d*( - 35*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 + 50*sqrt( - c**2*x**
2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt( - c**2
*x**2 + 1)*a*c**2*x**2 - 8*sqrt( - c**2*x**2 + 1)*a - 315*int(sqrt( - c**2
*x**2 + 1)*acosh(c*x)*x**7,x)*b*c**8 + 315*int(sqrt( - c**2*x**2 + 1)*acos
h(c*x)*x**5,x)*b*c**6))/(315*c**6)
```

3.79 $\int x^3(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 243

$$\int x^3(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx=\frac{2bdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{8bcdx^5\sqrt{d-c^2dx^2}}{175\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^7\sqrt{d-c^2dx^2}}{49\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d}+\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{7c^4d^2}$$

output
```
2/35*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/105*b*d*
x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-8/175*b*c*d*x^5*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/49*b*c^3*d*x^7*(-c^2*d*x^2
+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos
h(c*x))/c^4/d+1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.56

$$\int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \operatorname{arccosh}(c x)) \, dx = -\frac{d \sqrt{d-c^2 d x^2} \left(-b c x \left(210+35 c^2 x^2-168 c^4 x^4+75 c^6 x^6\right)+210 (-1+c x)^{5/2} (1+c x)^{5/2} (a+b \operatorname{arccosh}(c x))\right.}{3675 c^4 \sqrt{-1+c x} \sqrt{1+c x}}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/3675*(d*Sqrt[d - c^2*d*x^2]*(-(b*c*x*(210 + 35*c^2*x^2 - 168*c^4*x^4 +
75*c^6*x^6)) + 210*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) +
 525*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x])))/(c^4*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6337, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \operatorname{arccosh}(c x)) \, dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{b c \sqrt{d-c^2 d x^2} \int -\frac{d \left(1-c^2 x^2\right)^2 \left(5 c^2 x^2+2\right)}{35 c^4} dx}{\sqrt{c x-1} \sqrt{c x+1}}+\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \operatorname{arccosh}(c x))}{7 c^4 d^2}-\frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \operatorname{arccosh}(c x))}{5 c^4 d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2\left(5c^2x^2+2\right)dx}{35c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d}$$

$$\Big\downarrow \text{ 290}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int \left(5c^6x^6-8c^4x^4+c^2x^2+2\right)dx}{35c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^4d}+\frac{bd\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)\sqrt{d-c^2dx^2}}{35c^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - (8*c^4*x^5)/5 + (5*c^6*x^7)/
7))/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/2)*(a + b*
ArcCosh[c*x]))/(5*c^4*d) + ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7
*c^4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93

method	result
orering	$\frac{(325c^8x^8-866c^6x^6+553c^4x^4+420c^2x^2-280)\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\operatorname{arccosh}(cx))}{1225c^4(cx-1)(cx+1)(c^2x^2-1)} - \frac{(75c^6x^6-168c^4x^4+35c^2x^2+210)\left(3x^2(-c^2d\right.}{}$
default	$a\left(-\frac{x^2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{7c^2d}-\frac{2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{35d\,c^4}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(64c^8x^8-144c^6x^6+64c^7x^7\sqrt{cx-1}\,\sqrt{cx+1}+104c^4x^4-11}{}\right.$
parts	$a\left(-\frac{x^2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{7c^2d}-\frac{2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{35d\,c^4}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(64c^8x^8-144c^6x^6+64c^7x^7\sqrt{cx-1}\,\sqrt{cx+1}+104c^4x^4-11}{}\right.$

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/1225*(325*c^8*x^8-866*c^6*x^6+553*c^4*x^4+420*c^2*x^2-280)/c^4/(c*x-1)/(
c*x+1)/(c^2*x^2-1)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/3675/x^2*(75*
c^6*x^6-168*c^4*x^4+35*c^2*x^2+210)/c^4/(c*x-1)/(c*x+1)*(3*x^2*(-c^2*d*x^2
+d)^(3/2)*(a+b*arccosh(c*x))-3*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))
*c^2*d+x^3*(-c^2*d*x^2+d)^(3/2)*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx =$$

$$-\frac{105\,(5\,bc^8dx^8-13\,bc^6dx^6+9\,bc^4dx^4+bc^2dx^2-2\,bd)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-(75\,bc^7dx^7-}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
-1/3675*(105*(5*b*c^8*d*x^8 - 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 + b*c^2*d*x^2
 - 2*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^7*d*
x^7 - 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 + 210*b*c*d*x)*sqrt(-c^2*d*x^2 + d)
*sqrt(c^2*x^2 - 1) + 105*(5*a*c^8*d*x^8 - 13*a*c^6*d*x^6 + 9*a*c^4*d*x^4 +
 a*c^2*d*x^2 - 2*a*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.66

$$\begin{aligned}\int x^3 \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx = & -\frac{1}{35}\left(\frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)b\,\text{arcosh}\,(cx) \\ & -\frac{1}{35}\left(\frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)a \\ & +\frac{\left(75\,c^6\sqrt{-d}dx^7-168\,c^4\sqrt{-d}dx^5+35\,c^2\sqrt{-d}dx^3+210\,\sqrt{-d}dx\right)b}{3675\,c^3}\end{aligned}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^
4*d))*b*arccosh(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^
2*d*x^2 + d)^(5/2)/(c^4*d))*a + 1/3675*(75*c^6*sqrt(-d)*d*x^7 - 168*c^4*sq
rt(-d)*d*x^5 + 35*c^2*sqrt(-d)*d*x^3 + 210*sqrt(-d)*d*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d - c^2dx^2)^{3/2} (a + b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2dx^2)^{3/2} (a + b\text{arccosh}(cx))\,dx = \int x^3\,(a + b\,\text{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\mathrm{arccosh}(cx))\,dx=\frac{\sqrt{d}\,d\left(-5\sqrt{-c^2x^2+1}\,a\,c^6x^6+8\sqrt{-c^2x^2+1}\,a\,c^4x^4-\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+}\right.}{3}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d*( - 5*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 + 8*sqrt( - c**2*x**2
+ 1)*a*c**4*x**4 - sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**
2 + 1)*a - 35*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**5,x)*b*c**6 + 35*in
t(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**3,x)*b*c**4))/(35*c**4)
```

3.80 $\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx=\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^2d}$$

output
```
1/5*b*d*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/15*b*c*d*x^
3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/25*b*c^3*d*x^5*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*a
rccosh(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx=$$

$$-\frac{d\sqrt{d-c^2dx^2}\Big(15a(-1+c^2x^2)^3+bcx\sqrt{-1+cx}\sqrt{1+cx}(-15+10c^2x^2-3c^4x^4)+15b(-1+c^2x^2)^3\,\text{arcc}}{75c^2\left(-1+c^2x^2\right)}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/75*(d*Sqrt[d - c^2*d*x^2]*(15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(-15 + 10*c^2*x^2 - 3*c^4*x^4) + 15*b*(-1 + c^2*x^2)^3*ArcC
osh[c*x]))/(c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6329, 39, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6329}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int(1-cx)^2(cx+1)^2dx}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^2d}$$

$$\Big\downarrow \text{ 39}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^2dx}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^2d}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{bd\sqrt{d-c^2dx^2}\int\left(c^4x^4-2c^2x^2+1\right)dx}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bd\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5c^2d}$$

input
```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d*Sqrt[d - c^2*d*x^2]*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(5*c*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(5*c^2
*d)
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.21

method	result
orering	$\frac{(27c^6x^6-88c^4x^4+115c^2x^2-30)(-c^2dx^2+d)^{\frac{3}{2}}(a+b\,\text{arccosh}(cx))}{75c^2(cx-1)(cx+1)(c^2x^2-1)} - \frac{(3c^4x^4-10c^2x^2+15)\left((-c^2dx^2+d)^{\frac{3}{2}}(a+b\,\text{arccosh}(cx))-3\right.}{75c^2(cx-}$
default	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(16c^6x^6-28c^4x^4+16c^5x^5\sqrt{cx-1}\,\sqrt{cx+1}+13c^2x^2-20c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}+5\sqrt{cx-1}\,\right.}{800(cx+1)c^2(cx-1)}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(16c^6x^6-28c^4x^4+16c^5x^5\sqrt{cx-1}\,\sqrt{cx+1}+13c^2x^2-20c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}+5\sqrt{cx-1}\,\right.}{800(cx+1)c^2(cx-1)}$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/75*(27*c^6*x^6-88*c^4*x^4+115*c^2*x^2-30)/c^2/(c*x-1)/(c*x+1)/(c^2*x^2-1
)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/75*(3*c^4*x^4-10*c^2*x^2+15)/c
^2/(c*x-1)/(c*x+1)*((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-3*x^2*(-c^2*d*
x^2+d)^(1/2)*(a+b*arccosh(c*x))*c^2*d+x*(-c^2*d*x^2+d)^(3/2)*b*c/(c*x-1)^(
1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.12

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\mathrm{arccosh}(cx))\,dx = -\frac{15\,(bc^6dx^6-3\,bc^4dx^4+3\,bc^2dx^2-bd)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-(3\,bc^5dx^5-10\,bc^3dx^3+15\,b}{75\,(c^4x^2-c^2)}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-1/75*(15*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*
x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5 - 10*b*c^3*d*x^3 +
15*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(a*c^6*d*x^6 - 3*a
*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\mathrm{arccosh}(cx))\,dx = \int x(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))\,\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.62

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx = -\frac{(-c^2dx^2+d)^{\frac{5}{2}}b\,\text{arcosh}\,(cx)}{5\,c^2d} - \frac{(-c^2dx^2+d)^{\frac{5}{2}}a}{5\,c^2d} + \frac{(3\,c^4\sqrt{-d}d^2x^5-10\,c^2\sqrt{-d}d^2x^3+15\,\sqrt{-d}d^2x)b}{75\,cd}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/5*(-c^2*d*x^2 + d)^(5/2)*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^
(5/2)*a/(c^2*d) + 1/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 +
 15*sqrt(-d)*d^2*x)*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx=\int x\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))\,dx=\frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a\,c^4x^4+2\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a-5\left(\int\sqrt{-c^2x^2+1}\right.\right.}{5c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 2*sqrt( - c**2*x**2 +
1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a - 5*int(sqrt( - c**2*x**2 + 1)*a
cosh(c*x)*x**3,x)*b*c**4 + 5*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*b*
c**2))/(5*c**2)
```

3.81 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 292

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}\,dx =& -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}\\ &+d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))\\ &-\frac{2d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\\ &+\frac{ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-4/3*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/9*b*c^3*d*
x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccosh(c*x))+1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-2*d*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*d*(-c^2*d*x^
2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c
*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.15

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}\,dx = -\frac{1}{3}ad(-4+c^2x^2)\sqrt{d-c^2dx^2} - \frac{bd\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\text{arccosh}(cx)-\cosh(3\text{arccosh}(cx))\right)}{36\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} + ad^{3/2}\log(x) - ad^{3/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{bd\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\text{arc}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
-1/3*(a*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (b*d*Sqrt[d - c^2*d*x^2]*(
9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*
ArcCosh[c*x]]))/(36*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a*d^(3/2)*Log[
x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*Sqrt[d - c^2*d*
x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*
ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]]
 - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6345, 25, 39, 2009, 6341, 24, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}dx$$

$$\Big\downarrow \ 6345$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int -((1-cx)(cx+1))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 25$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int (1-cx)(cx+1)dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 39$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 2009$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 6341$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{24}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)+$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6362}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)+$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{3042}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\right.$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{4668}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{2715}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)de^{\text{a}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{2838}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcc}}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
-1/3*(b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/3 + d*(-((b*c*x*Sqr
t[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*x])*ArcTan[E
^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*x]] + I*b*PolyLog[2, I*E^
ArcCosh[c*x]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.71

method	result
default	$\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} a}{3}-a\,d^{\frac{3}{2}} \ln \left(\frac{2 d+2 \sqrt{d}\, \sqrt{-c^{2} d\,x^{2}+d}}{x}\right)+a d \sqrt{-c^{2} d\,x^{2}+d}-\frac{b \sqrt{-d \left(c^{2} x^{2}-1\right)}\, d \,\mathrm{arccosh}\left(c x \right) x^{4} c^{4}}{3 \left(c x -1\right) \left(c x +1\right)}+\frac{5 b}{}$
parts	$\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} a}{3}-a\,d^{\frac{3}{2}} \ln \left(\frac{2 d+2 \sqrt{d}\, \sqrt{-c^{2} d\,x^{2}+d}}{x}\right)+a d \sqrt{-c^{2} d\,x^{2}+d}-\frac{b \sqrt{-d \left(c^{2} x^{2}-1\right)}\, d \,\mathrm{arccosh}\left(c x \right) x^{4} c^{4}}{3 \left(c x -1\right) \left(c x +1\right)}+\frac{5 b}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2
))/x)+a*d*(-c^2*d*x^2+d)^(1/2)-1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x
+1)*arccosh(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x+1)*ar
ccosh(c*x)*x^2*c^2+1/9*b*c^3*d*x^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-d*(c^2*x^
2-1))^(1/2)-4/3*b*c*d*x/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)
-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)))*d-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x+1
)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcc
osh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d-I*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2)))*d+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*di
log(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2
*d*x^2 + d)/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\text{acosh}\,(cx))}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
-1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
(-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a + b*integrate((-c^2*d
*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x}\,dx = \frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a\,c^2x^2+4\sqrt{-c^2x^2+1}\,a+3\left(\int \frac{\sqrt{-c^2x^2+1}\,aco}{x}\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + 4*sqrt( - c**2*x**2 +
1)*a + 3*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x,x)*b - 3*int(sqrt( - c*
*2*x**2 + 1)*acosh(c*x)*x,x)*b*c**2 + 3*log(tan(asin(c*x)/2))*a - 4*a))/3
```

3.82 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 311

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^3}\,dx = &-\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}\\ &+\frac{3c^2d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}\\ &+\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/2*b*c*d*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c^3*d*x*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*c^2*d*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccosh(c*x))-1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2+3*c
^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*
polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+3/2*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.31 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.61

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^3}\,dx = \frac{1}{2}\left(-\frac{ad(1+2c^2x^2)\sqrt{d-c^2dx^2}}{x^2}\right.$$

$$-3ac^2d^{3/2}\log(x)+3ac^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)-\frac{2bc^2d\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\operatorname{arccosh}(cx)+cx\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
(-((a*d*(1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^2) - 3*a*c^2*d^(3/2)*Log[x]
 + 3*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*d*Sqrt[
d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqr
t[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[
c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^Arc
Cosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)) + (b*d^2*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*
x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*
Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c
*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyL
og[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[
2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.77, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6343, 25, 82, 244, 2009, 6341, 24, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^3}dx$$

$$\Big\downarrow \text{6343}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \text{82}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \text{244}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^2}-c^2\right)dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6341}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6362}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx\right.$$
$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\,\text{arc}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
(b*c*d*(-x^(-1) - c^2*x)*Sqrt[d - c^2*d*x^2])/(2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(2*x^2) - (3*c^2*d*(-(
(b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*x]
)*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*x]] + I*b*PolyL
og[2, I*E^ArcCosh[c*x]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.73

method	result
default	$a\left(-\frac{\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{2d\,x^2}-\frac{3c^2\left(\frac{\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^2 d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)\right)\right)}{2}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,c^4 d\ \text{a}}{(cx-1)(cx+}$
parts	$a\left(-\frac{\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{2d\,x^2}-\frac{3c^2\left(\frac{\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^2 d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)\right)\right)}{2}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,c^4 d\ \text{a}}{(cx-1)(cx+}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(5/2)-3/2*c^2*(1/3*(-c^2*d*x^2+d)^(3/2)+d*((-
c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))))-b
*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x-1)/(c*x+1)*arccosh(c*x)*x^2+b*(-d*(c^2*
x^2-1))^(1/2)*c^3*d/(c*x-1)^(1/2)/(c*x+1)^(1/2)*x+1/2*b*(-d*(c^2*x^2-1))^(
1/2)*c^2*d/(c*x-1)/(c*x+1)*arccosh(c*x)-1/2*b*(-d*(c^2*x^2-1))^(1/2)*d/x/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)*d/x^2/(c*x-1)/(c
*x+1)*arccosh(c*x)-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+3/2*I*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+3
/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas
")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2
*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**3,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\operatorname{arccosh}(cx))}{x^3}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima
")
```

output
```
1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2
 + d)^(5/2)/(d*x^2))*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\operatorname{arccosh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\operatorname{arccosh}(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\mathrm{arccosh}(cx))}{x^3}\,dx = \frac{\sqrt{d}\,d\left(-8\sqrt{-c^2x^2+1}\,a\,c^2x^2-4\sqrt{-c^2x^2+1}\,a+8\left(\int \frac{\sqrt{-c^2x^2+1}\,a}{x^3}\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^3,x)
```

output
```
(sqrt(d)*d*( - 8*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 4*sqrt( - c**2*x**2
+ 1)*a + 8*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**3,x)*b*x**2 - 8*int(
(sqrt( - c**2*x**2 + 1)*acosh(c*x))/x,x)*b*c**2*x**2 - 12*log(tan(asin(c*x
)/2))*a*c**2*x**2 + 9*a*c**2*x**2))/(8*x**2)
```

3.83 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\text{arccosh}(cx))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 321

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\text{arccosh}(cx))}{x^5}\,dx = &-\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}}\\&+\frac{3c^2d\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx))}{8x^2}-\frac{(d-c^2dx^2)^{3/2}(a+b\,\text{arccosh}(cx))}{4x^4}\\&-\frac{3c^4d\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{4\sqrt{-1+cx}\sqrt{1+cx}}\\&+\frac{3ibc^4d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{8\sqrt{-1+cx}\sqrt{1+cx}}\\&-\frac{3ibc^4d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{8\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/12*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/8*b*c^3
*d*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/8*c^2*d*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccosh(c*x))/x^2-1/4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*
x))/x^4-3/4*c^4*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/8*I*b*c^4*d*(-c^2*d*
x^2+d)^(1/2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)
/(c*x+1)^(1/2)-3/8*I*b*c^4*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.79

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))}{x^5}\,dx = \frac{-2bcd^2x+2bc^2d^2x^2+15bc^3d^2x^3-15bc^4d^2x^4-6ad^2\sqrt{\frac{-1+cx}{1+cx}}+}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

output
```
(-2*b*c*d^2*x + 2*b*c^2*d^2*x^2 + 15*b*c^3*d^2*x^3 - 15*b*c^4*d^2*x^4 - 6*
a*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 21*a*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c
*x)] - 15*a*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^2*Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x] + 21*b*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*A
rcCosh[c*x] - 15*b*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (
9*I)*b*c^4*d^2*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^5*d^
2*x^5*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^4*d^2*x^4*ArcCosh
[c*x]*Log[1 + I/E^ArcCosh[c*x]] + (9*I)*b*c^5*d^2*x^5*ArcCosh[c*x]*Log[1 +
 I/E^ArcCosh[c*x]] + 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d
 - c^2*d*x^2]*Log[x] - 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt
[d - c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (9*I)*b*c^4*d^2*x^4
*(-1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (9*I)*b*c^4*d^2*x^4*(-1 + c*
x)*PolyLog[2, I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d
 - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6343, 25, 82, 244, 2009, 6339, 15, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^5}dx$$

$$\Big\downarrow \text{ 6343}$$

$$-\frac{3}{4}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x^4}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3}{4}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x^4}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{ 82}$$

$$-\frac{3}{4}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^4}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{3}{4}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^4}-\frac{c^2}{x^2}\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3}{4}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^3}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 6339$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 15$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 6362$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx}{2x^2}\right.-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 4668$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\left(-ib\int\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int\log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\,\text{ar}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2715$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(c}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)\left(a+b\text{arccosh}(cx)\right)-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

output
```
(b*c*d*(-1/3*1/x^3 + c^2/x)*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(4*x^4) - (3*c^2*d*
(-1/2*(b*c*Sqrt[d - c^2*d*x^2])/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*(2*
(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCos
h[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.78

method	result
default	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{4 d \,x^{4}}+\frac{a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8 d \,x^{2}}+\frac{a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{3 a \,c^{4} d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}+d}}{x}\right)}{8}+\frac{3 a \,c^{4} d \sqrt{-c^{2} d \,x^{2}+d}}{8}$
parts	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{4 d \,x^{4}}+\frac{a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8 d \,x^{2}}+\frac{a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{3 a \,c^{4} d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}+d}}{x}\right)}{8}+\frac{3 a \,c^{4} d \sqrt{-c^{2} d \,x^{2}+d}}{8}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/x^4*(-c^2*d*x^2+d)^(5/2)+1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(5/2)+1/8
*a*c^4*(-c^2*d*x^2+d)^(3/2)-3/8*a*c^4*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^
2+d)^(1/2))/x)+3/8*a*c^4*d*(-c^2*d*x^2+d)^(1/2)+5/8*b*d*(-d*(c^2*x^2-1))^(
1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+5/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+
1)^(1/2)/x/(c*x-1)^(1/2)*c^3-7/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c
*x-1)*arccosh(c*x)*c^2-1/12*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x^3/(
c*x-1)^(1/2)*c+1/4*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^4/(c*x-1)*arccosh(
c*x)+3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*
x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d-3/8*I*b*(-d*(c^2*x^2-1)
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2)))*c^4*d+3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d-3/8*I*b*(-d*(c^
2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2)))*c^4*d
```

Fricas [F]

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^5}\,dx=\int \frac{{\left(-c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}}{x^{5}}\,{d x}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas
")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2
*d*x^2 + d)/x^5, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^5}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))}{x^5}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**5,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**5, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^5}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima
")
```

output
```
-1/8*(3*c^4*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)
) - (-c^2*d*x^2 + d)^(3/2)*c^4 - 3*sqrt(-c^2*d*x^2 + d)*c^4*d - (-c^2*d*x^
2 + d)^(5/2)*c^2/(d*x^2) + 2*(-c^2*d*x^2 + d)^(5/2)/(d*x^4))*a + b*integra
te((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^5}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^5}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^5,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^5, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))}{x^5}\,dx = \frac{\sqrt{d}\,d\left(5\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+8\left(\int \frac{\sqrt{-c^2x^2+1}\,acos}{x^5}\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))/x^5,x)
```

output
```
(sqrt(d)*d*(5*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1
)*a + 8*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**5,x)*b*x**4 - 8*int((sq
rt( - c**2*x**2 + 1)*acosh(c*x))/x**3,x)*b*c**2*x**4 + 3*log(tan(asin(c*x)
/2))*a*c**4*x**4))/(8*x**4)
```

3.84 $\int x^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 454

$$\begin{aligned}
\int x^4(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx &= \frac{3bd^2x^2\sqrt{d-c^2dx^2}}{512c^3\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{-1+cx}\sqrt{1+cx}} + \frac{21bc^3d^2x^8\sqrt{d-c^2dx^2}}{640\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{bc^5d^2x^{10}\sqrt{d-c^2dx^2}}{100\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3d^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{256c^4} \\
&- \frac{d^2x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{128c^2} + \frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) \\
&+ \frac{1}{16}dx^5(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx)) \\
&+ \frac{1}{10}x^5(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx)) - \frac{3d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{512bc^5\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
3/512*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/512
*b*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-31/960*b*c*d
^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+21/640*b*c^3*d^2*x
^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/100*b*c^5*d^2*x^10*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/256*d^2*x*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccosh(c*x))/c^4-1/128*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
osh(c*x))/c^2+1/32*d^2*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))+1/16*d*
x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/10*x^5*(-c^2*d*x^2+d)^(5/2)*
(a+b*arccosh(c*x))-3/512*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/c
^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.47 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.10

$$\int x^4\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx=\frac{2880acd^2x\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}(-15-10c^2x^2+248c^4x^4-336c^6x^6+128c^8x^8}{}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(2880*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(
-15 - 10*c^2*x^2 + 248*c^4*x^4 - 336*c^6*x^6 + 128*c^8*x^8) - 43200*a*d^(5
/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/
(Sqrt[d]*(-1 + c^2*x^2))] + 1600*b*d^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*
x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh
[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]]
 + Sinh[6*ArcCosh[c*x]])) + 100*b*d^2*Sqrt[d - c^2*d*x^2]*(1440*ArcCosh[c*
x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*Arc
Cosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] - 24*ArcCosh[c*x]*(-48*Sinh[2*ArcCosh[
c*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8*ArcCo
sh[c*x]])) + b*d^2*Sqrt[d - c^2*d*x^2]*(-50400*ArcCosh[c*x]^2 + 25200*Cosh
[2*ArcCosh[c*x]] - 3600*Cosh[4*ArcCosh[c*x]] - 2600*Cosh[6*ArcCosh[c*x]] -
 675*Cosh[8*ArcCosh[c*x]] - 72*Cosh[10*ArcCosh[c*x]] + 120*ArcCosh[c*x]*(-
420*Sinh[2*ArcCosh[c*x]] + 120*Sinh[4*ArcCosh[c*x]] + 130*Sinh[6*ArcCosh[c
*x]] + 45*Sinh[8*ArcCosh[c*x]] + 6*Sinh[10*ArcCosh[c*x]])))/(3686400*c^5*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {6345, 82, 243, 49, 2009, 6345, 25, 82, 244, 2009, 6341, 15, 6354, 15, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx)) \, dx$$

$$\Big\downarrow \text{ 6345}$$

$$\frac{1}{2} d \int x^4 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))dx-\frac{b c d^2 \sqrt{d-c^2 d x^2} \int x^5 (1-c x)^2 (c x+1)^2dx}{10 \sqrt{c x-1} \sqrt{c x+1}}+\frac{1}{10} x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 82}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int x^5\left(1-c^2x^2\right)^2dx}{10\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int x^4\left(1-c^2x^2\right)^2dx^2}{20\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int\left(c^4x^8-2c^2x^6+x^4\right)dx^2}{20\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6345}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int -x^5(1-cx)(cx+1)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^5(1-cx)(cx+1)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{82}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^5\left(1-c^2x^2\right)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+ba\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 244

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^5-c^2x^7\right)dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+bar\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2009

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-}}{8\sqrt{cx-1}\sqrt{cx+}}\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6341

$$\frac{1}{2}d\left(\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^5dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)-\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{1}{2}d\left(\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}\right)+\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{1}{2}d\left(\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a\right.\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{1}{2}d\left(\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+\right.\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{1}{2}d\left(\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx}{4c^2}\right.}{6\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{1}{2}d\left(\frac{3}{8}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{4c^2}\right.}{6\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6308

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+$$

$$\frac{1}{2}d\left(\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))+\frac{3}{8}d\left(\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}\left(\frac{x^3\sqrt{cx-1}\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.\right.$$

$$\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/20*(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x^6/3 - (c^2*x^8)/2 + (c^4*x^10)/5))/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[
c*x]))/10 + (d*(-1/8*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^6/6 - (c^2*x^8)/8))/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*
x]))/8 + (3*d*(-1/36*(b*c*x^6*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (Sqrt[d - c^2
*d*x^2]*(-1/16*(b*x^4)/c + (x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCos
h[c*x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c + (x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(2*c^2) + (a + b*ArcCosh[c*x])^2/(4*b*c^3)))/(4*c^2)))
/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/8))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(386) = 772$.

Time = 0.44 (sec) , antiderivative size = 1743, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	1743
parts	Expression too large to display	1743

input
```
int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/10*a*x^3*(-c^2*d*x^2+d)^(7/2)/c^2/d-3/80*a/c^4*x*(-c^2*d*x^2+d)^(7/2)/d
+1/160*a/c^4*x*(-c^2*d*x^2+d)^(5/2)+1/128*a/c^4*d*x*(-c^2*d*x^2+d)^(3/2)+3
/256*a/c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+3/256*a/c^4*d^3/(c^2*d)^(1/2)*arctan
((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/512*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)/c^5*arccosh(c*x)^2*d^2+1/102400*(-d*(c^2*x^2-1))
^(1/2)*(-10*c*x-1536*c^9*x^9+512*c^11*x^11+512*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^10*c^10+170*c^3*x^3+50*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-400*c^4*x^4*
(c*x-1)^(1/2)*(c*x+1)^(1/2)+1120*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1696*
c^7*x^7-1280*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^8*x^8-832*c^5*x^5-(c*x-1)^(1/2)
*(c*x+1)^(1/2))*(-1+10*arccosh(c*x))*d^2/(c*x+1)/c^5/(c*x-1)-1/32768*(-d*(
c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*c^8*x^8+272*c^5*x^5-256*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)-88*c^3*x^3+16
0*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x-32*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+8*arccosh(c*x))*d^2/(c*x+1)/c^5/
(c*x-1)-1/12288*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+32*c^6*x^6*(
c*x-1)^(1/2)*(c*x+1)^(1/2)+38*c^3*x^3-48*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/
2)-6*c*x+18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2
))*(-1+6*arccosh(c*x))*d^2/(c*x+1)/c^5/(c*x-1)+1/2048*(-d*(c^2*x^2-1))^(1/
2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arc...
```

Fricas [F]

$$\int x^4 \left(d - c^2 d x^2\right)^{5/2} (a + b\text{arccosh}(cx))\, dx = \int \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a) x^4\, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^8 - 2*a*c^2*d^2*x^6 + a*d^2*x^4 + (b*c^4*d^2*x^8 - 2
*b*c^2*d^2*x^6 + b*d^2*x^4)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\, dx = \text{Timed out}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\, dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arcosh}\,(cx)+a)x^4\, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/1280*(128*(-c^2*d*x^2 + d)^(7/2)*x^3/(c^2*d) - 8*(-c^2*d*x^2 + d)^(5/2)
*x/c^4 + 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^4*d) - 10*(-c^2*d*x^2 + d)^(3/2)*d
*x/c^4 - 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^4 - 15*d^(5/2)*arcsin(c*x)/c^5)*a
 + b*integrate((-c^2*d*x^2 + d)^(5/2)*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)), x)
```

Giac [F]

$$\int x^4 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\, dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arcosh}\,(cx)+a)x^4\, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d-c^2dx^2)^{5/2} (a+b\operatorname{arccosh}(cx))\,dx = \int x^4\,(a+b\,\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^4 (d-c^2dx^2)^{5/2} (a +b\operatorname{arccosh}(cx))\,dx = \frac{\sqrt{d}\,d^2\left(15asin(cx)\,a+128\sqrt{-c^2x^2+1}\,a\,c^9x^9-336\sqrt{-c^2x^2+1}\,a\,c^7x^7+248\sqrt{-c^2x^2}\right.}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a + 128*sqrt( - c**2*x**2 + 1)*a*c**9*x**9 - 3
36*sqrt( - c**2*x**2 + 1)*a*c**7*x**7 + 248*sqrt( - c**2*x**2 + 1)*a*c**5*
x**5 - 10*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - 15*sqrt( - c**2*x**2 + 1)*a
*c*x + 1280*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**8,x)*b*c**9 - 2560*in
t(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**6,x)*b*c**7 + 1280*int(sqrt( - c**2
*x**2 + 1)*acosh(c*x)*x**4,x)*b*c**5))/(1280*c**5)
```

3.85 $\int x^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 371

$$\begin{aligned}\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx &= \frac{5bd^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\sqrt{1+cx}} \\ &- \frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\sqrt{1+cx}} + \frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\sqrt{1+cx}} \\ &- \frac{5d^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{128c^2} + \frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) \\ &+ \frac{5}{48}dx^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx)) \\ &+ \frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx)) - \frac{5d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{256bc^3\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
5/256*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/768*
b*c*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/288*b*c^3*
d^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/64*b*c^5*d^2*x^
8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/128*d^2*x*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccosh(c*x))/c^2+5/64*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccosh(c*x))+5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/8*x^3*(-c
^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-5/256*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))^2/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.55 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.12

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx = \frac{192acd^2x\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}(-15+118c^2x^2-136c^4x^4+48c^6x^6)-2880ad^5}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(192*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-
15 + 118*c^2*x^2 - 136*c^4*x^4 + 48*c^6*x^6) - 2880*a*d^(5/2)*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c
^2*x^2))] - 576*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCo
sh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 64*b*d^2*Sqrt[d - c^2*d*
x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]
] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*
Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])) + b*d^2*Sqrt[d - c^2*d*x^2]*
(-1440*ArcCosh[c*x]^2 + 576*Cosh[2*ArcCosh[c*x]] - 144*Cosh[4*ArcCosh[c*x]
] - 64*Cosh[6*ArcCosh[c*x]] - 9*Cosh[8*ArcCosh[c*x]] + 24*ArcCosh[c*x]*(-4
8*Sinh[2*ArcCosh[c*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]]
 + 3*Sinh[8*ArcCosh[c*x]])))/(73728*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x))
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6345, 82, 243, 49, 2009, 6345, 25, 82, 244, 2009, 6341, 15, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6345}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int x^3(1-cx)^2(cx+1)^2dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{82}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int x^3\left(1-c^2x^2\right)^2dx}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{243}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int x^2\left(1-c^2x^2\right)^2dx^2}{16\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{49}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int\left(c^4x^6-2c^2x^4+x^2\right)dx^2}{16\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6345}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int -x^3(1-cx)(cx+1)dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^3(1-cx)(cx+1)dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 82

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^3\left(1-c^2x^2\right)dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+ba\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 244

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^3-c^2x^5\right)dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+bar\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2009

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-}}{6\sqrt{cx-1}\sqrt{cx+}}\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6341

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^3dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}\right)+\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+ba\right.\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+\text{barcc}\right.\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6308

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+$$
$$\frac{5}{8}d\left(\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))+\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccos}}{4bc^3}\right.}{}\right.\right.$$
$$\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/16*(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x^4/2 - (2*c^2*x^6)/3 + (c^4*x^8)/4))/
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh
[c*x]))/8 + (5*d*(-1/6*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^4/4 - (c^2*x^6)/6))/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[
c*x]))/6 + (d*(-1/16*(b*c*x^4*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (Sqrt[d - c^2
*d*x^2]*(-1/4*(b*x^2)/c + (x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c
*x]))/(2*c^2) + (a + b*ArcCosh[c*x])^2/(4*b*c^3)))/(4*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])))/2))/8
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(315) = 630$.

Time = 0.37 (sec) , antiderivative size = 1289, normalized size of antiderivative = 3.47

method	result	size
default	Expression too large to display	1289
parts	Expression too large to display	1289

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*a*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/19
2*a/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/
128*a/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b
*(-5/256*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*
x)^2*d^2+1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*c^8*x^8+272*c^5*x^5-256*c^6*x^6*(c*x-1)^(1/2)*(c*x+
1)^(1/2)-88*c^3*x^3+160*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x-32*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+8*arccosh(
c*x))*d^2/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64
*c^5*x^5+32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+38*c^3*x^3-48*c^4*x^4*(c*x
-1)^(1/2)*(c*x+1)^(1/2)-6*c*x+18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-
1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/1024
*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x
+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*(-1+4*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1)
)^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/
2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c
^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)
^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/1
024*(-d*(c^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^...
```

Fricas [F]

$$\int x^2 \left(d - c^2 d x^2\right)^{5/2} \left(a + b \text{arccosh}(cx)\right) dx = \int \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} \left(b \operatorname{arcosh}\left(cx\right) + a\right) x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^6 - 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 - 2
*b*c^2*d^2*x^4 + b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d - c^2 d x^2\right)^{5/2} \left(a + b \text{arccosh}(cx)\right) dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(d - c^2 d x^2\right)^{5/2} \left(a + b \text{arccosh}(cx)\right) dx = \int \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} \left(b \operatorname{arcosh}\left(cx\right) + a\right) x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d
) + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2
+ 15*d^(5/2)*arcsin(c*x)/c^3)*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*x^2*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F]

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx)) \, dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arcosh}\,(cx)+a)x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx)) \, dx = \int x^2 \,(a+b\,\text{acosh}(c\,x))\, \left(d-c^2\,d\,x^2\right)^{5/2} \, dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx)) \, dx = \frac{\sqrt{d}\,d^2\left(15asin(cx)\,a+48\sqrt{-c^2x^2+1}\,a\,c^7x^7-136\sqrt{-c^2x^2+1}\,a\,c^5x^5+118\sqrt{-c^2x^2+}\right.}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a + 48*sqrt( - c**2*x**2 + 1)*a*c**7*x**7 - 13
6*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 + 118*sqrt( - c**2*x**2 + 1)*a*c**3*x
**3 - 15*sqrt( - c**2*x**2 + 1)*a*c*x + 384*int(sqrt( - c**2*x**2 + 1)*aco
sh(c*x)*x**6,x)*b*c**7 - 768*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)
*b*c**5 + 384*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*b*c**3))/(384*
c**3)
```

3.86 $\int \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\int \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx = -\frac{5bcd^2x^2\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bd^2(-1+cx)^{3/2}(1+cx)^{3/2}\sqrt{d-c^2dx^2}}{96c} - \frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}\sqrt{d-c^2dx^2}}{36c} + \frac{5}{16}d^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) + \frac{5}{24}dx\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx)) + \frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arcc}$$

output

```
-5/32*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/96*b*
d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c-1/36*b*d^2*(c*x-1)^
(5/2)*(c*x+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c+5/16*d^2*x*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccosh(c*x))+5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/6*x
*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-5/32*d^2*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccosh(c*x))^2/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.69 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.25

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx = \frac{48acd^2x\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}(33-26c^2x^2+8c^4x^4)-720ad^{5/2}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(48*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(33
 - 26*c^2*x^2 + 8*c^4*x^4) - 720*a*d^(5/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 288*b*d
^2*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x
] - Sinh[2*ArcCosh[c*x]])) + 36*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^
2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b*d^2*Sq
rt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4
*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCo
sh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(2304*c*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6312, 82, 241, 6312, 25, 82, 244, 2009, 6310, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \; 6312$$

$$\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 82}$$

$$\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2dx}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 241}$$

$$\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6312}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int -x(1-cx)(cx+1)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 82}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arcco}\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\int\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int\left(x-c^2x^3\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccos}\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2009

$$\frac{5}{6}d\left(\frac{3}{4}d\int\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2d}}{4\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6310

$$\frac{5}{6}d\left(\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+\frac{1}{4}x(\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$\frac{5}{6}d\left(\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{4}x(d\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6308

$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))+$$
$$\frac{5}{6}d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}}{4bc\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$
$$\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/6 + (5*d*(-1/4*(b*c
*d*Sqrt[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/4 + (3*d*(-1/4*(b*c*x^
2*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2
)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/4))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6312
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p
)]   Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(234) = 468$.

Time = 0.00 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.18

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6}+\frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{-c^2dx^2+d}}{16}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{16\sqrt{c^2d}}+b\Big(-\frac{5\sqrt{-d(c^2x^2-1)}\,\text{arc}}{32\sqrt{cx-1}\,\sqrt{c}}$
parts	$\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6}+\frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{-c^2dx^2+d}}{16}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{16\sqrt{c^2d}}+b\Big(-\frac{5\sqrt{-d(c^2x^2-1)}\,\text{arc}}{32\sqrt{cx-1}\,\sqrt{c}}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*
(-c^2*d*x^2+d)^(1/2)+5/16*a*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2
*d*x^2+d)^(1/2))+b*(-5/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)/c*arccosh(c*x)^2*d^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x
^5+32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+38*c^3*x^3-48*c^4*x^4*(c*x-1)^(1
/2)*(c*x+1)^(1/2)-6*c*x+18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/
2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))*d^2/(c*x-1)/(c*x+1)/c-3/512*(-d*(c^2
*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)
+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
(-1+4*arccosh(c*x))*d^2/(c*x-1)/(c*x+1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(2
*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)
^(1/2))*(-1+2*arccosh(c*x))*d^2/(c*x-1)/(c*x+1)/c+15/256*(-d*(c^2*x^2-1))^
(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x
+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d^2/(c*x-1)/(c*x+1)/c-3/512*(-d*(c^2*x
^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(
1+4*arccosh(c*x))*d^2/(c*x-1)/(c*x+1)/c+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32
*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+32*c^7*x^7+48*c^4*x^4*(c*x-1)^(1/2)*(
c*x+1)^(1/2)-64*c^5*x^5-18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+38*c^3*x^3+
(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d^2/(c*x-1)/(c*x+...
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt
(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a + b*integrate((-c^2*d
*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx=\int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx=\frac{\sqrt{d}\,d^2\left(15asin(cx)\,a+8\sqrt{-c^2x^2+1}\,a\,c^5x^5-26\sqrt{-c^2x^2+1}\,a\,c^3x^3+33\sqrt{-c^2x^2+1}\,a}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a + 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 - 26*
sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 33*sqrt( - c**2*x**2 + 1)*a*c*x + 48*
int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*b*c**5 - 96*int(sqrt( - c**2
*x**2 + 1)*acosh(c*x)*x**2,x)*b*c**3 + 48*int(sqrt( - c**2*x**2 + 1)*acosh
(c*x),x)*b*c))/(48*c)
```

3.87 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 326

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{bc^3d^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{bc^5d^2x^4\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5}{16}bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}\sqrt{d-c^2dx^2}$$
$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{5}{4}c^2dx\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{(d-c^2dx^2)^{5/2}(a+\text{barc}}{x}$$

output
```
-1/16*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/4*b
*c^5*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/16*b*c*d^2
*(c*x-1)^(3/2)*(c*x+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)-15/8*c^2*d^2*x*(-c^2*d*x
^2+d)^(1/2)*(a+b*arccosh(c*x))-5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcco
sh(c*x))-(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x+15/16*c*d^2*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccosh(c*x))^2/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d^2*(-c^2
*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.47 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.94

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{1}{128}d^2\left(\frac{16a\sqrt{d-c^2dx^2}(-8-9c^2x^2+2c^4x^4)}{x}\right.$$
$$+240ac\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)$$
$$+64bc\sqrt{d-c^2dx^2}\left(-\frac{2\text{arccosh}(cx)}{cx}+\frac{\text{arccosh}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)$$
$$+\frac{32bc\sqrt{d-c^2dx^2}(2\text{arccosh}(cx)^2+\cosh(2\text{arccosh}(cx))-2\text{arccosh}(cx)\sinh(2\text{arccosh}(cx)))}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$
$$\left.-\frac{bc\sqrt{d-c^2dx^2}(8\text{arccosh}(cx)^2+\cosh(4\text{arccosh}(cx))-4\text{arccosh}(cx)\sinh(4\text{arccosh}(cx)))}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
(d^2*((16*a*Sqrt[d - c^2*d*x^2]*(-8 - 9*c^2*x^2 + 2*c^4*x^4))/x + 240*a*c*
Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 64*b*
c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c
*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (32*b*c*Sqrt[d - c^2*d*x^2]
*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[
c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*c*Sqrt[d - c^2*d*x^2]*
(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c
*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/128
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6343, 82, 243, 49, 2009, 6312, 25, 82, 244, 2009, 6310, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{x^2} \, dx$$

$$\Big\downarrow \; 6343$$

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \; 82$$

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \; 243$$

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^2}dx^2}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \; 49$$

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(x^2c^4-2c^2+\frac{1}{x^2}\right)dx^2}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \; 2009$$

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6312

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int -x(1-cx)(cx+1)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 25

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 82

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+bar\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 244

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(x-c^2x^3\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+barc\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 2009

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-}}{4\sqrt{cx-1}\sqrt{cx+}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6310

$$-5c^2d\left(\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$-5c^2d\left(\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{4}x\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6308

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x}-$$
$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccos}}{4bc\sqrt{cx-1}\sqrt{cx+}}\right.\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

input

```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x) - 5*c^2*d*(-1/4*(b*c*d*S
qrt[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 (x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sq
rt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4
*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/4) + (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-2
*c^2*x^2 + (c^4*x^4)/2 + Log[x^2]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6312
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p
)]   Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && GtQ[p, 0]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.93

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-ac^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5ac^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15ac^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15ac^2d^3\arctan\left(\frac{}{\sqrt{}}\right.}{8\sqrt{c^2d}}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-ac^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5ac^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15ac^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15ac^2d^3\arctan\left(\frac{}{\sqrt{}}\right.}{8\sqrt{c^2d}}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(7/2)-a*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a*c^2*d*x*(-c
^2*d*x^2+d)^(3/2)-15/8*a*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a*c^2*d^3/(c^
2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/128*b*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x*(32*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*arccosh(c*x)*x^4*c^4-8*c^5*x^5-144*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*c^2*x^2+72*c^3*x^3+120*arccosh(c*x)^2*c*x-128*arccosh(c*x)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)-128*c*x*arccosh(c*x)+128*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)*x*c-33*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima
")
```

output
```
-1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*
x + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a + b*integrate
((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^2}\,dx=\int \frac{\left(a+b\operatorname{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^2}\,dx=\frac{\sqrt{d}\,d^2\Big(-15asin(cx)\,acx+2\sqrt{-c^2x^2+1}\,a\,c^4x^4-9\sqrt{-c^2x^2+1}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^2,x)
```

output
```
(sqrt(d)*d**2*( - 15*asin(c*x)*a*c*x + 2*sqrt( - c**2*x**2 + 1)*a*c**4*x**
4 - 9*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt( - c**2*x**2 + 1)*a + 8*
int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**2,x)*b*x + 8*int(sqrt( - c**2*x
**2 + 1)*acosh(c*x)*x**2,x)*b*c**4*x - 16*int(sqrt( - c**2*x**2 + 1)*acosh
(c*x),x)*b*c**2*x))/(8*x)
```

3.88 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 293

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx = -\frac{bcd^2\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)) + \frac{5c^2d(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3x} - \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{3x^3} - \frac{5c^3d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{7bc^3d^2\sqrt{d-c^2dx^2}\log(x)}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/6*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c^
5*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/2*c^4*d^2*x*(
-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))+5/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b
*arccosh(c*x))/x-1/3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3-5/4*c^3*d
^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-7/3*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.26 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.09

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \frac{30bc^3d^3x^3(-1+cx)\text{arccosh}(cx)^2-60ac^3d^{5/2}x^3\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2}}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
(30*b*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 60*a*c^3*d^(5/2)*x^3*Sqrt[(-
1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))] + 3*b*c^3*d^3*x^3*(-1 + c*x)*Cosh[2*ArcCosh[c*x]]
 - 4*d^3*(b*c*x*(1 - c*x) + a*Sqrt[(-1 + c*x)/(1 + c*x)]*(2 - 16*c^2*x^2 +
 11*c^4*x^4 + 3*c^6*x^6) - 14*b*c^3*x^3*(-1 + c*x)*Log[c*x]) - 2*b*d^3*(-1
 + c*x)*ArcCosh[c*x]*(4*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 - c*x + 7*c^2*x^2 +
 7*c^3*x^3) + 3*c^3*x^3*Sinh[2*ArcCosh[c*x]]))/(24*x^3*Sqrt[(-1 + c*x)/(1
+ c*x)]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6343, 82, 243, 49, 2009, 6343, 25, 82, 244, 2009, 6310, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6343}$$

$$-\frac{5}{3}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$$\Big\downarrow \text{82}$$

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$\downarrow$ 243

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^4}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$\downarrow$ 49

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(c^4-\frac{2c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}$$

$\downarrow$ 2009

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6343

$$-\frac{5}{3}c^2d\left(-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b}{x}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 25

$$-\frac{5}{3}c^2d\left(-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+ba}{x}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 82

$$-\frac{5}{3}c^2d\left(-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccos}}{x}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 244

$$-\frac{5}{3}c^2d\left(-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x}-c^2x\right)dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arc}}{x}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2009

$$-\frac{5}{3}c^2d\left(-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}+\frac{bcd\sqrt{d-c^2dx^2}\left(\log(\right.}{\sqrt{cx-1}\sqrt{cx}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6310

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 15

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)-\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6308

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+\text{barccosh}(cx))-\frac{\sqrt{d-c^2dx^2}(a+\text{barccosh}(cx))^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}\right)-\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))}{3x^3}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3 - (5*c^2*d*(-(((d -
c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x) - 3*c^2*d*(-1/4*(b*c*x^2*Sqrt[d
 - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/2*(c^2*x^2
) + Log[x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/3 + (b*c*d^2*Sqrt[d - c^2*d*
x^2]*(-x^(-2) + c^4*x^2 - 2*c^2*Log[x^2]))/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.16

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{3dx^3}+\frac{4ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{3dx}+\frac{4ac^4x(-c^2dx^2+d)^{\frac{5}{2}}}{3}+\frac{5ac^4dx(-c^2dx^2+d)^{\frac{3}{2}}}{3}+\frac{5ac^4d^2x\sqrt{-c^2dx^2+d}}{2}+$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{3dx^3}+\frac{4ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{3dx}+\frac{4ac^4x(-c^2dx^2+d)^{\frac{5}{2}}}{3}+\frac{5ac^4dx(-c^2dx^2+d)^{\frac{3}{2}}}{3}+\frac{5ac^4d^2x\sqrt{-c^2dx^2+d}}{2}+$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(7/2)+4/3*a*c^2/d/x*(-c^2*d*x^2+d)^(7/2)+4/3*a
*c^4*x*(-c^2*d*x^2+d)^(5/2)+5/3*a*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d
^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))-1/24*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)/x^3*(-12*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4+6*c^5*x
^5+30*arccosh(c*x)^2*c^3*x^3-56*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c
^2*x^2-56*c^3*x^3*arccosh(c*x)+56*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)*x^3*c^3-3*c^3*x^3+8*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*d^2
```

Fricas [F]

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima
")
```

output
```
1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
 + 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d
*x^2 + d)^(7/2)/(d*x^3))*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^4}\,dx=\frac{\sqrt{d}\,d^2\Big(15asin(cx)\,a\,c^3x^3+3\sqrt{-c^2x^2+1}\,a\,c^4x^4+14\sqrt{-c^2x^2+}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^4,x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a*c**3*x**3 + 3*sqrt( - c**2*x**2 + 1)*a*c**4*
x**4 + 14*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a
+ 6*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**4,x)*b*x**3 - 12*int((sqrt(
 - c**2*x**2 + 1)*acosh(c*x))/x**2,x)*b*c**2*x**3 + 6*int(sqrt( - c**2*x**
2 + 1)*acosh(c*x),x)*b*c**4*x**3))/(6*x**3)
```

3.89 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 293

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx = -\frac{bcd^2\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x} + \frac{c^2d(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3x^3} - \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{5x^5} + \frac{c^5d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{23bc^5d^2\sqrt{d-c^2dx^2}\log(x)}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/20*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/30*b
*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-c^4*d^2*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x+1/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*
arccosh(c*x))/x^3-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5+1/2*c^5*
d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+23/15*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.69 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.37

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx = \frac{d^2\left(8ad\sqrt{\frac{-1+cx}{1+cx}}(-1+c^2x^2)(3-11c^2x^2+23c^4x^4)+120ac^5\sqrt{dx}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

output
```
(d^2*(8*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + c^2*x^2)*(3 - 11*c^2*x^2 + 23
*c^4*x^4) + 120*a*c^5*Sqrt[d]*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*
d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 40*b*c
^2*d*x^2*(1 - c*x)*(c*x - 2*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCo
sh[c*x] + 2*c^3*x^3*Log[c*x]) - 60*b*c^4*d*x^4*(1 - c*x)*(2*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) -
 b*d*(1 - c*x)*(20*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + Cos
h[5*ArcCosh[c*x]]*Log[c*x] + Cosh[3*ArcCosh[c*x]]*(-1 + 5*Log[c*x]) + c*x*
(3 + 10*Log[c*x]) - 5*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Sin
h[5*ArcCosh[c*x]])))/(120*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^
2])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6343, 82, 243, 49, 2009, 6343, 25, 82, 244, 2009, 6339, 14, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{6343}$$

$$-c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^4}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2}{x^5}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}$$

$$\Big\downarrow \text{82}$$

$$-c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^4}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^5}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$-c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^4}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^6}dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}$$

$$\Big\downarrow \text{49}$$

$$-c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^4}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(\frac{c^4}{x^2}-\frac{2c^2}{x^4}+\frac{1}{x^6}\right)dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$-c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^4}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6343}$$

$$-c^2d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+ba}{3x^3}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{25}$$

$$-c^2d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arc}}{3x^3}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 82

$$-c^2d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}}{3x^3}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 244

$$-c^2d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^3}-\frac{c^2}{x}\right)dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arc}}{3x^3}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 2009

$$-c^2d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{3x^3}+\frac{bcd\sqrt{d-c^2dx^2}(c^2(-}{3\sqrt{cx-1}\sqrt{c}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 6339

$$-c^2d\left(c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}\right)-\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 14

$$-c^2d\left(c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6308

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{5x^5}-$$

$$c^2d\left(c^2(-d)\left(\frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{(d\ldots}{}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5 - c^2*d*(-1/3*((d -
c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3 + (b*c*d*Sqrt[d - c^2*d*x^2]*(-
1/2*1/x^2 - c^2*Log[x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - c^2*d*(-((Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^2*d*x
^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))) + (b*c*d^2*Sqrt[d - c^2*d*x^2
]*(-1/2*1/x^4 + (2*c^2)/x^2 + c^4*Log[x^2]))/(10*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2428 vs. $2(251) = 502$.

Time = 0.47 (sec) , antiderivative size = 2429, normalized size of antiderivative = 8.29

method	result	size
default	Expression too large to display	2429
parts	Expression too large to display	2429

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
1495/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-
75*c^2*x^2+9)*x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^9-115*b*(-d*(
c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*
x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^7+1587*b*(-d*(c^2*x^2-1))^(
1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^8/(c*x-1)^(
1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^13+2/15*a*c^2/d/x^3*(-c^2*d*x^2+d)^(7/2)
+5318/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4
-75*c^2*x^2+9)*x^3/(c*x-1)/(c*x+1)*arccosh(c*x)*c^8-9602/15*b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c*x-
1)/(c*x+1)*arccosh(c*x)*c^6+777/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x
^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x/(c*x-1)/(c*x+1)*arccosh(c*x)*c^
4-117/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4
-75*c^2*x^2+9)/x^3/(c*x-1)/(c*x+1)*arccosh(c*x)*c^2+3519*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c*x-1
)/(c*x+1)*arccosh(c*x)*c^12-1173*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^
8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ar
ccosh(c*x)*c^11-1587*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^
6+325*c^4*x^4-75*c^2*x^2+9)*x^9/(c*x-1)/(c*x+1)*arccosh(c*x)*c^14-9595/3*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x
^2+9)*x^5/(c*x-1)/(c*x+1)*arccosh(c*x)*c^10-1/5*a/d/x^5*(-c^2*d*x^2+d)^...
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima
")
```

output
```
-1/15*(10*(-c^2*d*x^2 + d)^(3/2)*c^6*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^6*d^2
*x + 15*c^5*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^4/x - 2*(-c^2
*d*x^2 + d)^(7/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^5))*a + b*in
tegrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^6,
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^6}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))}{x^6}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^6}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^6,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))}{x^6}\,dx=\frac{\sqrt{d}\,d^2\Big(-15asin(cx)\,a\,c^5x^5-23\sqrt{-c^2x^2+1}\,a\,c^4x^4+11\sqrt{-c^2x^2}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^6,x)
```

output
```
(sqrt(d)*d**2*( - 15*asin(c*x)*a*c**5*x**5 - 23*sqrt( - c**2*x**2 + 1)*a*c
**4*x**4 + 11*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1
)*a + 15*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**6,x)*b*x**5 - 30*int((
sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**4,x)*b*c**2*x**5 + 15*int((sqrt( - c
**2*x**2 + 1)*acosh(c*x))/x**2,x)*b*c**4*x**5))/(15*x**5)
```

3.90 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx = -\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{7/2}(a+b\operatorname{arccosh}(cx))}{7dx^7} - \frac{bc^7d^2\sqrt{d-c^2dx^2}\log(x)}{7\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/42*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/28*b*
c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/14*b*c^5*d^
2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^
(7/2)*(a+b*arccosh(c*x))/d/x^7-1/7*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.48

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx = \frac{d^2\sqrt{d-c^2dx^2}\left(12(-1+cx)^{7/2}(1+cx)^{7/2}(a+b\operatorname{arccosh}(cx))-b\right.}{84x^7\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCo
sh[c*x]) - b*c*x*(2 - 9*c^2*x^2 + 18*c^4*x^4 + 12*c^6*x^6*Log[x])))/(84*x^
7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6332, 25, 82, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx$$

$$\Big\downarrow \ 6332$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int -\frac{(1-cx)^3(cx+1)^3}{x^7}dx}{7\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\operatorname{arccosh}(cx))}{7dx^7}$$

$$\Big\downarrow \ 25$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^3(cx+1)^3}{x^7}dx}{7\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\operatorname{arccosh}(cx))}{7dx^7}$$

$$\Big\downarrow \ 82$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3}{x^7}dx}{7\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\operatorname{arccosh}(cx))}{7dx^7}$$

$$\Big\downarrow \text{243}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3}{x^8}dx^2}{14\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}\,(a+b\text{arccosh}(cx))}{7dx^7}$$

$$\Big\downarrow \text{49}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(-\frac{c^6}{x^2}+\frac{3c^4}{x^4}-\frac{3c^2}{x^6}+\frac{1}{x^8}\right)dx^2}{14\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}\,(a+b\text{arccosh}(cx))}{7dx^7}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^6\left(-\log\left(x^2\right)\right)-\frac{3c^4}{x^2}+\frac{3c^2}{2x^4}-\frac{1}{3x^6}\right)}{14\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}\,(a+b\text{arccosh}(cx))}{7dx^7}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

output
```
-1/7*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(d*x^7) + (b*c*d^2*Sqrt[
d - c^2*d*x^2]*(-1/3*1/x^6 + (3*c^2)/(2*x^4) - (3*c^4)/x^2 - c^6*Log[x^2])
)/(14*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3774 vs. $2(183) = 366$.

Time = 0.48 (sec) , antiderivative size = 3775, normalized size of antiderivative = 17.24

method	result	size
default	Expression too large to display	3775
parts	Expression too large to display	3775

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
55/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35
*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^7+2/7*b*(-d
*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d^2*c^7-1/7*b
*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)*d^2*c^7-1/7*a/d/x^7*(-c^2*d*x^2+d)^(7/2)-3/2*b*(-d*(c^
2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4
*x^4-7*c^2*x^2+1)*x^10/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^17+73/42*b*(-d*(c^2*x
^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^
4-7*c^2*x^2+1)*x^9/(c*x-1)/(c*x+1)*c^16-67/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x
^7/(c*x-1)/(c*x+1)*c^14+11/14*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21
*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5/(c*x-1)/(c*x+
1)*c^12-17/84*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^
8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c*x-1)/(c*x+1)*c^10+1/42*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+
21*c^4*x^4-7*c^2*x^2+1)*x/(c*x-1)/(c*x+1)*c^8+47/4*b*(-d*(c^2*x^2-1))^(1/2
)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2
+1)*x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^11-119/12*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)
*x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c^13+23/84*b*(-d*(c^2*x^2-1))^(1/2)*d^...
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 704, normalized size of antiderivative = 3.21

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^8}\,dx = \left[\frac{12\left(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,bc^2d^2x^2+bd^2\right)\sqrt{-c^2dx}}{}\right.$$

$$-\frac{12\left(bc^9d^2x^9-bc^7d^2x^7\right)\sqrt{d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}(x^2-1)\sqrt{d}}{c^2dx^4+(c^2-1)dx^2-d}\right)-12\left(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,b\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas
")
```

output
```
[1/84*(12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2
*x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 6*(b*c^9
*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sq
rt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^
2)) - (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*c^3*d
^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^8*d
^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt
(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/84*(12*(b*c^9*d^2*x^9 - b*c^7*d^2*x^
7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)
/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - 12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6
+ 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x
+ sqrt(c^2*x^2 - 1)) + (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^
2*x^7 - 9*b*c^3*d^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
 1) - 12*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*
x^2 + a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\,\mathrm{arccosh}(cx))}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**8,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.02

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\,\mathrm{arccosh}(cx))}{x^8}\,dx = \frac{\left(6\,c^8d^4\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)+6i\,(-1)^{-2\,c^2dx^2+2\,d}\,c^6d^{\frac{7}{2}}\log\left(-2\,c^2\right.\right.}{} - \frac{(-c^2dx^2+d)^{\frac{7}{2}}b\,\mathrm{arcosh}\,(cx)}{7\,dx^7} - \frac{(-c^2dx^2+d)^{\frac{7}{2}}a}{7\,dx^7}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima
")
```

output
```
1/84*(6*c^8*d^4*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + 6*I*(-1)^(-2*c^2*d*x^2
 + 2*d)*c^6*d^(7/2)*log(-2*c^2*d + 2*d/x^2) + 11*sqrt(-c^4*d*x^4 + 2*c^2*d
*x^2 - d)*c^4*d^3/x^2 - 7*sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*c^2*d^3/x^4 +
 2*sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*d^3/x^6)*b*c/d - 1/7*(-c^2*d*x^2 + d
)^(7/2)*b*arccosh(c*x)/(d*x^7) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a/(d*x^7)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^8}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^8,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^8}\,dx = \frac{\sqrt{d}\,d^2\left(\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4+3\sqrt{-c^2x^2+\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^8,x)
```

output
```
(sqrt(d)*d**2*(sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 +
1)*a*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*a + 7*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**8,x)*b*x**7 - 14*int
((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**6,x)*b*c**2*x**7 + 7*int((sqrt( -
c**2*x**2 + 1)*acosh(c*x))/x**4,x)*b*c**4*x**7))/(7*x**7)
```

3.91 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\textbf{arccosh}(cx))}{x^{10}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 302

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = -\frac{bc^3d^2\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^7d^2\sqrt{d-c^2dx^2}}{21x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2(-1+cx)^{7/2}(1+cx)^{7/2}\sqrt{d-c^2dx^2}}{72x^8} - \frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{9dx^9} - \frac{2c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{63dx^7} - \frac{2bc^9d^2\sqrt{d-c^2dx^2}\log(x)}{63\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/189*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/42
*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/21*b*c^7
*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/72*b*c*d^2*(c*
x-1)^(7/2)*(c*x+1)^(7/2)*(-c^2*d*x^2+d)^(1/2)/x^8-1/9*(-c^2*d*x^2+d)^(7/2)
*(a+b*arccosh(c*x))/d/x^9-2/63*c^2*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))
/d/x^7-2/63*b*c^9*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.49

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \frac{d^2\sqrt{d-c^2dx^2}\left(168(-1+cx)^{7/2}(1+cx)^{7/2}(a+b\text{arccosh}(cx))+\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(168*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcC
osh[c*x]) + 48*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x
]) - b*c*x*(21 - 76*c^2*x^2 + 90*c^4*x^4 - 12*c^6*x^6 + 48*c^8*x^8*Log[x])
))/(1512*x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6337, 27, 354, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2(1-c^2x^2)^3(2c^2x^2+7)}{63x^9}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{63dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(2c^2x^2+7)}{x^9}dx}{63\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{63dx^7}$$

$$\downarrow \text{354}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(2c^2x^2+7)}{x^{10}}dx^2}{126\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{63dx^7}$$

$$\downarrow \text{87}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(2c^2\int \frac{(1-c^2x^2)^3}{x^8}dx^2-\frac{7(1-c^2x^2)^4}{4x^8}\right)}{126\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{63dx^7}$$

$$\downarrow \text{49}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(2c^2\int \left(-\frac{c^6}{x^2}+\frac{3c^4}{x^4}-\frac{3c^2}{x^6}+\frac{1}{x^8}\right)dx^2-\frac{7(1-c^2x^2)^4}{4x^8}\right)}{126\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{63dx^7}$$

$$\downarrow \text{2009}$$

$$-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{63dx^7}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(2c^2\left(c^6(-\log(x^2))-\frac{3c^4}{x^2}+\frac{3c^2}{2x^4}-\frac{1}{3x^6}\right)-\frac{7(1-c^2x^2)^4}{4x^8}\right)}{126\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

output
```
-1/9*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(d*x^9) - (2*c^2*(d - c^
2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(63*d*x^7) + (b*c*d^2*Sqrt[d - c^2*d*
x^2]*((-7*(1 - c^2*x^2)^4)/(4*x^8) + 2*c^2*(-1/3*1/x^6 + (3*c^2)/(2*x^4) -
 (3*c^4)/x^2 - c^6*Log[x^2])))/(126*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5006 vs. $2(254) = 508$.

Time = 0.56 (sec) , antiderivative size = 5007, normalized size of antiderivative = 16.58

method	result	size
default	Expression too large to display	5007
parts	Expression too large to display	5007

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 796, normalized size of antiderivative = 2.64

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))}{x^{10}}\,dx = \left[\frac{24\,(2\,bc^{10}d^2x^{10} - bc^8d^2x^8 - 16\,bc^6d^2x^6 + 34\,bc^4d^2x^4 - 26\,bc^2d^2}{}\right.$$

$$-\frac{48\,(bc^{11}d^2x^{11} - bc^9d^2x^9)\sqrt{d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,(x^2-1)\sqrt{d}}{c^2dx^4+(c^2-1)dx^2-d}\right) - 24\,(2\,bc^{10}d^2x^{10} - bc^8d^2x^8 - 16\,bc^6d^2x^6}{}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="frica
s")
```

output
```
[1/1512*(24*(2*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c
^4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 - 1)) + 24*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(-d)*log((c^2*
d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 -
1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 -
 (12*b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21
*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 24*(2*a*c^10*d^2*x^10
 - a*c^8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2
+ 7*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9), -1/1512*(48*(b*c^11*d^2
*x^11 - b*c^9*d^2*x^9)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - 24*(2*b*c^10*d^2
*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4 - 26*b*c^2*d^2
*x^2 + 7*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (12*b*
c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)
*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*
x^2 - 1) - 24*(2*a*c^10*d^2*x^10 - a*c^8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a
*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11
 - x^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))}{x^{10}}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.62

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx =$$
$$-\frac{1}{1512}\left(48\,c^8\sqrt{-d}d^2\log(x)-\frac{12\,c^6\sqrt{-d}d^2x^6-90\,c^4\sqrt{-d}d^2x^4+76\,c^2\sqrt{-d}d^2x^2-21\,\sqrt{-d}d^2}{x^8}\right)bc$$
$$-\frac{1}{63}\,b\left(\frac{2\,(-c^2dx^2+d)^{\frac{7}{2}}c^2}{dx^7}+\frac{7\,(-c^2dx^2+d)^{\frac{7}{2}}}{dx^9}\right)\text{arcosh}(cx)$$
$$-\frac{1}{63}\,a\left(\frac{2\,(-c^2dx^2+d)^{\frac{7}{2}}c^2}{dx^7}+\frac{7\,(-c^2dx^2+d)^{\frac{7}{2}}}{dx^9}\right)$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxim
a")
```

output
```
-1/1512*(48*c^8*sqrt(-d)*d^2*log(x) - (12*c^6*sqrt(-d)*d^2*x^6 - 90*c^4*sq
rt(-d)*d^2*x^4 + 76*c^2*sqrt(-d)*d^2*x^2 - 21*sqrt(-d)*d^2)/x^8)*b*c - 1/6
3*b*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^2 + d)^(7/2)/(d*x^
9))*arccosh(c*x) - 1/63*a*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*
d*x^2 + d)^(7/2)/(d*x^9))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^{10}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^10,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{10}}\,dx = \frac{\sqrt{d}\,d^2\left(2\sqrt{-c^2x^2+1}\,a\,c^8x^8+\sqrt{-c^2x^2+1}\,a\,c^6x^6-15\sqrt{-c^2x^2+\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^10,x)
```

output
```
(sqrt(d)*d**2*(2*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 + sqrt( - c**2*x**2 +
1)*a*c**6*x**6 - 15*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 19*sqrt( - c**2*x
**2 + 1)*a*c**2*x**2 - 7*sqrt( - c**2*x**2 + 1)*a + 63*int((sqrt( - c**2*x
**2 + 1)*acosh(c*x))/x**10,x)*b*x**9 - 126*int((sqrt( - c**2*x**2 + 1)*aco
sh(c*x))/x**8,x)*b*c**2*x**9 + 63*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/
x**6,x)*b*c**4*x**9))/(63*x**9)
```

3.92 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\textbf{arccosh}(cx))}{x^{12}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 385

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx = -\frac{bcd^2\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{23bc^3d^2\sqrt{d-c^2dx^2}}{792x^8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{113bc^5d^2\sqrt{d-c^2dx^2}}{4158x^6\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^7d^2\sqrt{d-c^2dx^2}}{924x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^9d^2\sqrt{d-c^2dx^2}}{693x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{693dx^7} - \frac{8bc^{11}d^2\sqrt{d-c^2dx^2}\log(x)}{693\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/110*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^10/(c*x-1)^(1/2)/(c*x+1)^(1/2)+23/79
2*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)-113/4158*
b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/924*b*c^7
*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/693*b*c^9*d^2*
(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/11*(-c^2*d*x^2+d)^(
7/2)*(a+b*arccosh(c*x))/d/x^11-4/99*c^2*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(
c*x))/d/x^9-8/693*c^4*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/d/x^7-8/693*
b*c^11*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.43

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx = \frac{d^2\sqrt{d-c^2dx^2}\left(7560(-1+cx)^{7/2}(1+cx)^{7/2}(a+b\text{arccosh}(cx))\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(7560*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*Arc
Cosh[c*x]) + 480*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(7 + 2*c^2*x^2)*
(a + b*ArcCosh[c*x]) - b*c*x*(756 - 2415*c^2*x^2 + 2260*c^4*x^4 - 90*c^6*x
^6 - 240*c^8*x^8 + 960*c^10*x^10*Log[x])))/(83160*x^11*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6337, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{12}}dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{693x^{11}}dx}{\sqrt{cx-1}\sqrt{cx+1}} - \frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{693dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{x^{11}}dx}{693\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{693dx^7}$$

$$\Big\downarrow \ 1578$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{x^{12}}dx^2}{1386\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{693dx^7}$$

$$\Big\downarrow \ 1195$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(-\frac{8c^{10}}{x^2}-\frac{4c^8}{x^4}-\frac{3c^6}{x^6}+\frac{113c^4}{x^8}-\frac{161c^2}{x^{10}}+\frac{63}{x^{12}}\right)dx^2}{1386\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{693dx^7}$$

$$\Big\downarrow \ 2009$$

$$-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{693dx^7}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(-8c^{10}\log(x^2)+\frac{4c^8}{x^2}+\frac{3c^6}{2x^4}-\frac{113c^4}{3x^6}+\frac{161c^2}{4x^8}-\frac{63}{5x^{10}}\right)}{1386\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

output
```
-1/11*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(d*x^11) - (4*c^2*(d -
c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(99*d*x^9) - (8*c^4*(d - c^2*d*x^2)
^(7/2)*(a + b*ArcCosh[c*x]))/(693*d*x^7) + (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-
63/(5*x^10) + (161*c^2)/(4*x^8) - (113*c^4)/(3*x^6) + (3*c^6)/(2*x^4) + (4
*c^8)/x^2 - 8*c^10*Log[x^2]))/(1386*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6381 vs. $2(325) = 650$.

Time = 0.63 (sec) , antiderivative size = 6382, normalized size of antiderivative = 16.58

method	result	size
default	Expression too large to display	6382
parts	Expression too large to display	6382

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.29

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="frica
s")
```

output
```
[1/83160*(120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116
*b*c^6*d^2*x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(-c
^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 480*(b*c^13*d^2*x^13 - b*c^11
*d^2*x^11)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 +
 d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (240*b*c^
9*d^2*x^9 + 90*b*c^7*d^2*x^7 - (240*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b
*c^3 - 756*b*c)*d^2*x^11 - 2260*b*c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b
*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 120*(8*a*c^12*d^2*x^12
- 4*a*c^10*d^2*x^10 - a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^
4 - 224*a*c^2*d^2*x^2 + 63*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11),
 -1/83160*(960*(b*c^13*d^2*x^13 - b*c^11*d^2*x^11)*sqrt(d)*arctan(sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*
x^2 - d)) - 120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 1
16*b*c^6*d^2*x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(
-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (240*b*c^9*d^2*x^9 + 90*b*c
^7*d^2*x^7 - (240*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^
2*x^11 - 2260*b*c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^1
0 - a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2*
x^2 + 63*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)^{5/2}\,(a + b\mathrm{arccosh}(cx))}{x^{12}}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**12,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.65

$$\int \frac{(d - c^2dx^2)^{5/2}\,(a + b\mathrm{arccosh}(cx))}{x^{12}}\,dx =$$

$$-\frac{1}{83160}\left(960\,c^{10}\sqrt{-d}d^2\log(x) - \frac{240\,c^8\sqrt{-d}d^2x^8 + 90\,c^6\sqrt{-d}d^2x^6 - 2260\,c^4\sqrt{-d}d^2x^4 + 2415\,c^2\sqrt{-d}d^2x^2}{x^{10}}\right.$$

$$-\frac{1}{693}b\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^4}{dx^7} + \frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^9} + \frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^{11}}\right)\mathrm{arcosh}(cx)$$

$$-\frac{1}{693}a\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^4}{dx^7} + \frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^9} + \frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^{11}}\right)$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxim
a")
```

output
```
-1/83160*(960*c^10*sqrt(-d)*d^2*log(x) - (240*c^8*sqrt(-d)*d^2*x^8 + 90*c^
6*sqrt(-d)*d^2*x^6 - 2260*c^4*sqrt(-d)*d^2*x^4 + 2415*c^2*sqrt(-d)*d^2*x^2
 - 756*sqrt(-d)*d^2)/x^10)*b*c - 1/693*b*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*
x^7) + 28*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(
d*x^11))*arccosh(c*x) - 1/693*a*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7) + 28
*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^11))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^{12}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^12,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^{12}}\,dx = \frac{\sqrt{d}\,d^2\Big(8\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}+4\sqrt{-c^2x^2+1}\,a\,c^8x^8+3\sqrt{-c^2x^2}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^12,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 + 4*sqrt( - c**2*x**
2 + 1)*a*c**8*x**8 + 3*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 113*sqrt( - c*
*2*x**2 + 1)*a*c**4*x**4 + 161*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 63*sqr
t( - c**2*x**2 + 1)*a + 693*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**12,
x)*b*x**11 - 1386*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**10,x)*b*c**2*
x**11 + 693*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**8,x)*b*c**4*x**11))
/(693*x**11)
```

3.93 $\int x^7(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 458

$$\int x^7(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))\,dx = \frac{16bd^2x\sqrt{d-c^2dx^2}}{3003c^7\sqrt{-1+cx}\sqrt{1+cx}}
+\frac{8bd^2x^3\sqrt{d-c^2dx^2}}{9009c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^2x^5\sqrt{d-c^2dx^2}}{5005c^3\sqrt{-1+cx}\sqrt{1+cx}}
+\frac{5bd^2x^7\sqrt{d-c^2dx^2}}{21021c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{53bcd^2x^9\sqrt{d-c^2dx^2}}{3861\sqrt{-1+cx}\sqrt{1+cx}}
+\frac{27bc^3d^2x^{11}\sqrt{d-c^2dx^2}}{1573\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^5d^2x^{13}\sqrt{d-c^2dx^2}}{169\sqrt{-1+cx}\sqrt{1+cx}}
-\frac{(d-c^2dx^2)^{7/2}\,(a+b\text{arccosh}(cx))}{7c^8d}+\frac{(d-c^2dx^2)^{9/2}\,(a+b\text{arccosh}(cx))}{3c^8d^2}
-\frac{3(d-c^2dx^2)^{11/2}\,(a+b\text{arccosh}(cx))}{11c^8d^3}+\frac{(d-c^2dx^2)^{13/2}\,(a+b\text{arccosh}(cx))}{13c^8d^4}$$

output
```
16/3003*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/900
9*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/5005*b*
d^2*x^5*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/21021*b*d^2
*x^7*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-53/3861*b*c*d^2*x^
9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+27/1573*b*c^3*d^2*x^11*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/169*b*c^5*d^2*x^13*(-c^
2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b
*arccosh(c*x))/c^8/d+1/3*(-c^2*d*x^2+d)^(9/2)*(a+b*arccosh(c*x))/c^8/d^2-3
/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c^8/d^3+1/13*(-c^2*d*x^2+d)^(
13/2)*(a+b*arccosh(c*x))/c^8/d^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.39

$$\int x^7 (d - c^2dx^2)^{5/2} (a + b\operatorname{arccosh}(cx)) \, dx = \frac{d^2\sqrt{d - c^2dx^2}(bc(720720x + 120120c^2x^3 + 54054c^4x^5 + 32175c^6x^7 - 1856855c^8x^9 +$$

input
```
Integrate[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(b*c*(720720*x + 120120*c^2*x^3 + 54054*c^4*x^5 +
 32175*c^6*x^7 - 1856855*c^8*x^9 + 2321865*c^10*x^11 - 800415*c^12*x^13) +
 10405395*c^6*x^6*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) +
90090*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(8 + 28*c^2*x^2 + 63*c^4*x^4)*(a +
b*ArcCosh[c*x])))/(135270135*c^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6337, 27, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2(1-c^2x^2)^3(231c^6x^6+126c^4x^4+56c^2x^2+16)}{3003c^8}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{13/2}(a+b\text{arccosh}(cx))}{13c^8d^4}-\frac{3\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^3}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^3\left(231c^6x^6+126c^4x^4+56c^2x^2+16\right)dx}{3003c^7\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{13/2}(a+b\text{arccosh}(cx))}{13c^8d^4}-\frac{3\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^3}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d}$$

$$\Big\downarrow \text{2341}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int\left(-231c^{12}x^{12}+567c^{10}x^{10}-371c^8x^8+5c^6x^6+6c^4x^4+8c^2x^2+16\right)dx}{3003c^7\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{13/2}(a+b\text{arccosh}(cx))}{13c^8d^4}-\frac{3\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^3}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d-c^2dx^2\right)^{13/2}(a+b\text{arccosh}(cx))}{13c^8d^4}-\frac{3\left(d-c^2dx^2\right)^{11/2}(a+b\text{arccosh}(cx))}{11c^8d^3}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{3c^8d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^8d}+\frac{bd^2\left(-\frac{231}{13}c^{12}x^{13}+\frac{567c^{10}x^{11}}{11}-\frac{371c^8x^9}{9}+\frac{5c^6x^7}{7}+\frac{6c^4x^5}{5}+\frac{8c^2x^3}{3}+16x\right)\sqrt{d-c^2dx^2}}{3003c^7\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d^2*Sqrt[d - c^2*d*x^2]*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (5*c^6*
x^7)/7 - (371*c^8*x^9)/9 + (567*c^10*x^11)/11 - (231*c^12*x^13)/13))/(3003
*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh
[c*x]))/(7*c^8*d) + ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCosh[c*x]))/(3*c^8*d^
2) - (3*(d - c^2*d*x^2)^(11/2)*(a + b*ArcCosh[c*x]))/(11*c^8*d^3) + ((d -
c^2*d*x^2)^(13/2)*(a + b*ArcCosh[c*x]))/(13*c^8*d^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.60

method	result
orering	$\frac{\left(20010375x^{14}c^{14}-69411195x^{12}c^{12}+80787525c^{10}x^{10}-30321005c^{8}x^{8}-468468c^{6}x^{6}-1153152c^{4}x^{4}-8168160c^{2}x^{2}+5765760\right)\left(-c^{2}d\right.}{135270135c^{8}\left(cx-1\right)^{2}\left(cx+1\right)^{2}\left(c^{2}x^{2}-1\right)}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/135270135*(20010375*c^14*x^14-69411195*c^12*x^12+80787525*c^10*x^10-3032
1005*c^8*x^8-468468*c^6*x^6-1153152*c^4*x^4-8168160*c^2*x^2+5765760)/c^8/(
c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-1/1
35270135/x^6*(800415*c^12*x^12-2321865*c^10*x^10+1856855*c^8*x^8-32175*c^6
*x^6-54054*c^4*x^4-120120*c^2*x^2-720720)/c^8/(c*x-1)^2/(c*x+1)^2*(7*x^6*(
-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-5*x^8*(-c^2*d*x^2+d)^(3/2)*(a+b*arc
cosh(c*x))*c^2*d+x^7*(-c^2*d*x^2+d)^(5/2)*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.77

$$\int x^7\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx = \frac{45045\,(231\,bc^{14}d^2x^{14}-798\,bc^{12}d^2x^{12}+938\,bc^{10}d^2x^{10}-376\,bc^8d^2x^8-bc^6d^2x^6-2\,bc^4d}{}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
1/135270135*(45045*(231*b*c^14*d^2*x^14 - 798*b*c^12*d^2*x^12 + 938*b*c^10
*d^2*x^10 - 376*b*c^8*d^2*x^8 - b*c^6*d^2*x^6 - 2*b*c^4*d^2*x^4 - 8*b*c^2*
d^2*x^2 + 16*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (8
00415*b*c^13*d^2*x^13 - 2321865*b*c^11*d^2*x^11 + 1856855*b*c^9*d^2*x^9 -
32175*b*c^7*d^2*x^7 - 54054*b*c^5*d^2*x^5 - 120120*b*c^3*d^2*x^3 - 720720*
b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 45045*(231*a*c^14*d^2*
x^14 - 798*a*c^12*d^2*x^12 + 938*a*c^10*d^2*x^10 - 376*a*c^8*d^2*x^8 - a*c
^6*d^2*x^6 - 2*a*c^4*d^2*x^4 - 8*a*c^2*d^2*x^2 + 16*a*d^2)*sqrt(-c^2*d*x^2
 + d))/(c^10*x^2 - c^8)
```

Sympy [F(-1)]

Timed out.

$$\int x^7 \left(d - c^2 d x^2\right)^{5/2} \left(a + b \operatorname{arccosh}(cx)\right) dx = \text{Timed out}$$

input
```
integrate(x**7*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.68

$$\int x^7 \left(d - c^2 d x^2\right)^{5/2} \left(a + b \operatorname{arccosh}(cx)\right) dx = -\frac{1}{3003} \left(\frac{231 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}} x^6}{c^2 d} + \frac{126 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}} x^4}{c^4 d} + \frac{56 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}} x^2}{c^6 d} + \frac{16 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}}}{c^8 d}\right) b \operatorname{arcc} - \frac{1}{3003} \left(\frac{231 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}} x^6}{c^2 d} + \frac{126 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}} x^4}{c^4 d} + \frac{56 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}} x^2}{c^6 d} + \frac{16 \left(-c^2 d x^2 + d\right)^{\frac{7}{2}}}{c^8 d}\right) a - \frac{\left(800415 c^{12} \sqrt{-d} d^2 x^{13} - 2321865 c^{10} \sqrt{-d} d^2 x^{11} + 1856855 c^8 \sqrt{-d} d^2 x^9 - 32175 c^6 \sqrt{-d} d^2 x^7 - 54054 c^4 \sqrt{\right.}{135270135 c^7}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/3003*(231*(-c^2*d*x^2 + d)^(7/2)*x^6/(c^2*d) + 126*(-c^2*d*x^2 + d)^(7/
2)*x^4/(c^4*d) + 56*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 +
d)^(7/2)/(c^8*d))*b*arccosh(c*x) - 1/3003*(231*(-c^2*d*x^2 + d)^(7/2)*x^6/
(c^2*d) + 126*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^4*d) + 56*(-c^2*d*x^2 + d)^(7/
2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^(7/2)/(c^8*d))*a - 1/135270135*(80041
5*c^12*sqrt(-d)*d^2*x^13 - 2321865*c^10*sqrt(-d)*d^2*x^11 + 1856855*c^8*sq
rt(-d)*d^2*x^9 - 32175*c^6*sqrt(-d)*d^2*x^7 - 54054*c^4*sqrt(-d)*d^2*x^5 -
 120120*c^2*sqrt(-d)*d^2*x^3 - 720720*sqrt(-d)*d^2*x)*b/c^7
```

Giac [F(-2)]

Exception generated.

$$\int x^7 (d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^7 (d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))\,dx = \int x^7\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^7 (d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))\,dx = \frac{\sqrt{d}\,d^2\left(231\sqrt{-c^2x^2+1}\,a\,c^{12}x^{12}-567\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}+371\sqrt{-c^2x^2+1}\,a\,c^8x^8-\right.}{}$$

input
```
int(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(231*sqrt( - c**2*x**2 + 1)*a*c**12*x**12 - 567*sqrt( - c**2
*x**2 + 1)*a*c**10*x**10 + 371*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 - 5*sqrt
( - c**2*x**2 + 1)*a*c**6*x**6 - 6*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 8*
sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 16*sqrt( - c**2*x**2 + 1)*a + 3003*in
t(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**11,x)*b*c**12 - 6006*int(sqrt( - c*
*2*x**2 + 1)*acosh(c*x)*x**9,x)*b*c**10 + 3003*int(sqrt( - c**2*x**2 + 1)*
acosh(c*x)*x**7,x)*b*c**8))/(3003*c**8)
```

3.94 $\int x^5(d-c^2dx^2)^{5/2}(a+b\mathrm{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 378

$$\int x^5\left(d-c^2dx^2\right)^{5/2}(a+b\mathrm{arccosh}(cx))\,dx=\frac{8bd^2x\sqrt{d-c^2dx^2}}{693c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bd^2x^3\sqrt{d-c^2dx^2}}{2079c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2x^5\sqrt{d-c^2dx^2}}{1155c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{113bcd^2x^7\sqrt{d-c^2dx^2}}{4851\sqrt{-1+cx}\sqrt{1+cx}}+\frac{23bc^3d^2x^9\sqrt{d-c^2dx^2}}{891\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^5d^2x^{11}\sqrt{d-c^2dx^2}}{121\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\mathrm{arccosh}(cx))}{7c^6d}+\frac{2\left(d-c^2dx^2\right)^{9/2}(a+b\mathrm{arccosh}(cx))}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\mathrm{arccosh}(cx))}{11c^6d^3}$$

output

```
8/693*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/2079*
b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1155*b*d^
2*x^5*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-113/4851*b*c*d^2*
x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+23/891*b*c^3*d^2*x^9*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/121*b*c^5*d^2*x^11*(-c^
2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b
*arccosh(c*x))/c^6/d+2/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arccosh(c*x))/c^6/d^2-1
/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c^6/d^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.43

$$\int x^5\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx=\frac{d^2\sqrt{d-c^2dx^2}\left(bc(27720x+4620c^2x^3+2079c^4x^5-55935c^6x^7+61985c^8x^9-19845c\right.}{}$$

input

```
Integrate[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output

```
(d^2*Sqrt[d - c^2*d*x^2]*(b*c*(27720*x + 4620*c^2*x^3 + 2079*c^4*x^5 - 559
35*c^6*x^7 + 61985*c^8*x^9 - 19845*c^10*x^11) + 218295*c^4*x^4*(-1 + c*x)^
(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) + 13860*(-1 + c*x)^(7/2)*(1 + c
*x)^(7/2)*(2 + 7*c^2*x^2)*(a + b*ArcCosh[c*x])))/(2401245*c^6*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6337, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2\left(1-c^2x^2\right)^3\left(63c^4x^4+28c^2x^2+8\right)}{693c^6}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\operatorname{arccosh}(cx))}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}(a+b\operatorname{arccosh}(cx))}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\operatorname{arccosh}(cx))}{7c^6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^3\left(63c^4x^4+28c^2x^2+8\right)dx}{693c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{11/2}\left(a+b\text{arccosh}(cx)\right)}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}\left(a+b\text{arccosh}(cx)\right)}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\text{arccosh}(cx)\right)}{7c^6d}$$

$$\Big\downarrow \text{1467}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int\left(-63c^{10}x^{10}+161c^8x^8-113c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{693c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{11/2}\left(a+b\text{arccosh}(cx)\right)}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}\left(a+b\text{arccosh}(cx)\right)}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\text{arccosh}(cx)\right)}{7c^6d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{11/2}\left(a+b\text{arccosh}(cx)\right)}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}\left(a+b\text{arccosh}(cx)\right)}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\text{arccosh}(cx)\right)}{7c^6d}+\frac{bd^2\left(-\frac{63}{11}c^{10}x^{11}+\frac{161c^8x^9}{9}-\frac{113c^6x^7}{7}+\frac{3c^4x^5}{5}+\frac{4c^2x^3}{3}+8x\right)\sqrt{d-c^2dx^2}}{693c^5\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d^2*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (113*c^6
*x^7)/7 + (161*c^8*x^9)/9 - (63*c^10*x^11)/11))/(693*c^5*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^6*d) + (2
*(d - c^2*d*x^2)^(9/2)*(a + b*ArcCosh[c*x]))/(9*c^6*d^2) - ((d - c^2*d*x^2
)^(11/2)*(a + b*ArcCosh[c*x]))/(11*c^6*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.68

method	result
orering	$\frac{(83349x^{12}c^{12}-299047c^{10}x^{10}+363737c^{8}x^{8}-140481c^{6}x^{6}-7854c^{4}x^{4}-53592c^{2}x^{2}+33264)\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}(a+b\operatorname{arccosh}(cx))}{480249c^{6}(cx-1)^{2}(cx+1)^{2}(c^{2}x^{2}-1)}-\frac{(19}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/480249*(83349*c^12*x^12-299047*c^10*x^10+363737*c^8*x^8-140481*c^6*x^6-7
854*c^4*x^4-53592*c^2*x^2+33264)/c^6/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(-c^2
*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-1/2401245/x^4*(19845*c^10*x^10-61985*c^
8*x^8+55935*c^6*x^6-2079*c^4*x^4-4620*c^2*x^2-27720)/c^6/(c*x-1)^2/(c*x+1)
^2*(5*x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-5*x^6*(-c^2*d*x^2+d)^(3/
2)*(a+b*arccosh(c*x))*c^2*d+x^5*(-c^2*d*x^2+d)^(5/2)*b*c/(c*x-1)^(1/2)/(c*
x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.84

$$\int x^5 (d - c^2dx^2)^{5/2} (a + b\text{arccosh}(cx))\, dx = \frac{3465\,(63\,bc^{12}d^2x^{12} - 224\,bc^{10}d^2x^{10} + 274\,bc^8d^2x^8 - 116\,bc^6d^2x^6 - bc^4d^2x^4 - 4\,bc^2d^2x^2}{}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
1/2401245*(3465*(63*b*c^12*d^2*x^12 - 224*b*c^10*d^2*x^10 + 274*b*c^8*d^2*
x^8 - 116*b*c^6*d^2*x^6 - b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 8*b*d^2)*sqrt(
-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (19845*b*c^11*d^2*x^11 - 61
985*b*c^9*d^2*x^9 + 55935*b*c^7*d^2*x^7 - 2079*b*c^5*d^2*x^5 - 4620*b*c^3*
d^2*x^3 - 27720*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(
63*a*c^12*d^2*x^12 - 224*a*c^10*d^2*x^10 + 274*a*c^8*d^2*x^8 - 116*a*c^6*d
^2*x^6 - a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + 8*a*d^2)*sqrt(-c^2*d*x^2 + d))/
(c^8*x^2 - c^6)
```

Sympy [F(-1)]

Timed out.

$$\int x^5 \left(d - c^2 d x^2\right)^{5/2} (a + b \operatorname{arccosh}(cx))\, dx = \text{Timed out}$$

input
```
integrate(x**5*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.66

$$\int x^5 \left(d - c^2 d x^2\right)^{5/2} (a + b \operatorname{arccosh}(cx))\, dx =$$
$$-\frac{1}{693}\left(\frac{63\,(-c^2dx^2+d)^{\frac{7}{2}}x^4}{c^2d}+\frac{28\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^6d}\right)b\operatorname{arcosh}(cx)$$
$$-\frac{1}{693}\left(\frac{63\,(-c^2dx^2+d)^{\frac{7}{2}}x^4}{c^2d}+\frac{28\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^6d}\right)a$$
$$-\frac{(19845\,c^{10}\sqrt{-d}d^2x^{11}-61985\,c^8\sqrt{-d}d^2x^9+55935\,c^6\sqrt{-d}d^2x^7-2079\,c^4\sqrt{-d}d^2x^5-4620\,c^2\sqrt{-d}d^2x^3-}{2401245\,c^5}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/693*(63*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*
x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*b*arccosh(c*x) - 1/693*(63
*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^4*d
) + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*a - 1/2401245*(19845*c^10*sqrt(-d)*d
^2*x^11 - 61985*c^8*sqrt(-d)*d^2*x^9 + 55935*c^6*sqrt(-d)*d^2*x^7 - 2079*c
^4*sqrt(-d)*d^2*x^5 - 4620*c^2*sqrt(-d)*d^2*x^3 - 27720*sqrt(-d)*d^2*x)*b/
c^5
```

Giac [F(-2)]

Exception generated.

$$\int x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx=\int x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

input
```
int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx=\frac{\sqrt{d}\,d^2\left(63\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}-161\sqrt{-c^2x^2+1}\,a\,c^8x^8+113\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{}\right.}{}$$

input
```
int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(63*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 - 161*sqrt( - c**2*
x**2 + 1)*a*c**8*x**8 + 113*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( -
 c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 8*sqr
t( - c**2*x**2 + 1)*a + 693*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**9,x)*
b*c**10 - 1386*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**7,x)*b*c**8 + 693*
int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**5,x)*b*c**6))/(693*c**6)
```

3.95 $\int x^3(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 298

$$\int x^3(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx=\frac{2bd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2x^5\sqrt{d-c^2dx^2}}{21\sqrt{-1+cx}\sqrt{1+cx}}+\frac{19bc^3d^2x^7\sqrt{d-c^2dx^2}}{441\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^5d^2x^9\sqrt{d-c^2dx^2}}{81\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{7c^4d}+\frac{(d-c^2dx^2)^{9/2}(a+b\text{arccosh}(cx))}{9c^4d^2}$$

output
```
2/63*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/189*b*
d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/21*b*c*d^2*x^
5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+19/441*b*c^3*d^2*x^7*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/81*b*c^5*d^2*x^9*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arc
cosh(c*x))/c^4/d+1/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arccosh(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.49

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\frac{d^2\sqrt{d-c^2dx^2}\left(bc(126x+21c^2x^3-189c^4x^5+171c^6x^7-49c^8x^9)+126(-1+cx)^{7/2}(1\right.}{3969c^4\sqrt{-1+}}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(b*c*(126*x + 21*c^2*x^3 - 189*c^4*x^5 + 171*c^6*
x^7 - 49*c^8*x^9) + 126*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*
x]) + 441*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x])))/
(3969*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6337, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2\left(1-c^2x^2\right)^3\left(7c^2x^2+2\right)}{63c^4}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\text{arccosh}(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^3\left(7c^2x^2+2\right)dx}{63c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+\text{barccosh}(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+\text{barccosh}(cx))}{7c^4d}$$

$$\Big\downarrow \text{290}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int\left(-7c^8x^8+19c^6x^6-15c^4x^4+c^2x^2+2\right)dx}{63c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+\text{barccosh}(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+\text{barccosh}(cx))}{7c^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d-c^2dx^2\right)^{9/2}(a+\text{barccosh}(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+\text{barccosh}(cx))}{7c^4d}+\frac{bd^2\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}+2x\right)\sqrt{d-c^2dx^2}}{63c^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d^2*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - 3*c^4*x^5 + (19*c^6*x^7)/7
 - (7*c^8*x^9)/9))/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2
)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^4*d) + ((d - c^2*d*x^2)^(9/2)*(a + b*Ar
cCosh[c*x]))/(9*c^4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.81

method	result
orering	$\frac{(833c^{10}x^{10}-3153c^8x^8+4167c^6x^6-1743c^4x^4-1008c^2x^2+504)(-c^2dx^2+d)^{\frac{5}{2}}(a+b\,\mathrm{arccosh}(cx))}{3969c^4(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(49c^8x^8-171c^6x^6+189c^4x^4-}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3969*(833*c^10*x^10-3153*c^8*x^8+4167*c^6*x^6-1743*c^4*x^4-1008*c^2*x^2+
504)/c^4/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh
(c*x))-1/3969/x^2*(49*c^8*x^8-171*c^6*x^6+189*c^4*x^4-21*c^2*x^2-126)/c^4/
(c*x-1)^2/(c*x+1)^2*(3*x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-5*x^4*(
-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))*c^2*d+x^3*(-c^2*d*x^2+d)^(5/2)*b*c/
(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.94

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\,\mathrm{arccosh}(cx))\,dx = \frac{63\left(7\,bc^{10}d^2x^{10}-26\,bc^8d^2x^8+34\,bc^6d^2x^6-16\,bc^4d^2x^4-bc^2d^2x^2+2\,bd^2\right)\sqrt{-c^2dx^2+}}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
1/3969*(63*(7*b*c^10*d^2*x^10 - 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 - 16*b
*c^4*d^2*x^4 - b*c^2*d^2*x^2 + 2*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqr
t(c^2*x^2 - 1)) - (49*b*c^9*d^2*x^9 - 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^
5 - 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
1) + 63*(7*a*c^10*d^2*x^10 - 26*a*c^8*d^2*x^8 + 34*a*c^6*d^2*x^6 - 16*a*c^
4*d^2*x^4 - a*c^2*d^2*x^2 + 2*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.62

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b\,\text{arcosh}\left(cx\right)-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)a-\frac{\left(49\,c^8\sqrt{-d}d^2x^9-171\,c^6\sqrt{-d}d^2x^7+189\,c^4\sqrt{-d}d^2x^5-21\,c^2\sqrt{-d}d^2x^3-126\,\sqrt{-d}d^2x\right)b}{3969\,c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^
4*d))*b*arccosh(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^
2*d*x^2 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*
sqrt(-d)*d^2*x^7 + 189*c^4*sqrt(-d)*d^2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 12
6*sqrt(-d)*d^2*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d - c^2 d x^2\right)^{5/2} (a + b \text{arccosh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d - c^2 d x^2\right)^{5/2} (a + b \text{arccosh}(cx))\, dx = \int x^3 \left(a + b \operatorname{acosh}(c x)\right) \left(d - c^2 d x^2\right)^{5/2} dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^3 (d - c^2 d x^2)^{5/2} (a + b \operatorname{arccosh}(cx))\, dx = \frac{\sqrt{d}\, d^2 \left(7\sqrt{-c^2x^2+1}\, a\, c^8 x^8 - 19\sqrt{-c^2x^2+1}\, a\, c^6 x^6 + 15\sqrt{-c^2x^2+1}\, a\, c^4 x^4 - \sqrt{-c^2x^2}\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(7*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 - 19*sqrt( - c**2*x**2
 + 1)*a*c**6*x**6 + 15*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - sqrt( - c**2*x
**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 63*int(sqrt( - c**2*x*
*2 + 1)*acosh(c*x)*x**7,x)*b*c**8 - 126*int(sqrt( - c**2*x**2 + 1)*acosh(c
*x)*x**5,x)*b*c**6 + 63*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**3,x)*b*c*
*4))/(63*c**4)
```

3.96 $\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))}{7c^2d}$$

output
```
1/7*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*b*c*d^2
*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/35*b*c^3*d^2*x^5*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/49*b*c^5*d^2*x^7*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*ar
ccosh(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.54

$$\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx = \frac{d^2\sqrt{d-c^2dx^2}\left(35a(-1+c^2x^2)^4+bcx\sqrt{-1+cx}\sqrt{1+cx}(35-35c^2x^2+21c^4x^4-5c^6$$
$$245c^2(-1+c^2x^2)$$

input
```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(35*a*(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(35 - 35*c^2*x^2 + 21*c^4*x^4 - 5*c^6*x^6) + 35*b*(-1 + c^2*x^2
)^4*ArcCosh[c*x]))/(245*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6329, 25, 39, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{bd^2\sqrt{d-c^2dx^2}\int -(1-cx)^3(cx+1)^3dx}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{7c^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int (1-cx)^3(cx+1)^3dx}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}(a+b\text{arccosh}(cx))}{7c^2d}$$

$$\Big\downarrow \text{39}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^3dx}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+\text{barccosh}(cx))}{7c^2d}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{bd^2\sqrt{d-c^2dx^2}\int \left(-c^6x^6+3c^4x^4-3c^2x^2+1\right)dx}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+\text{barccosh}(cx))}{7c^2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bd^2\left(-\frac{1}{7}c^6x^7+\frac{3c^4x^5}{5}-c^2x^3+x\right)\sqrt{d-c^2dx^2}}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+\text{barccosh}(cx))}{7c^2d}$$

input
```
Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(b*d^2*Sqrt[d - c^2*d*x^2]*(x - c^2*x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7))/(7
*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c
*x]))/(7*c^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.99

method	result
orering	$\frac{(65c^8x^8-271c^6x^6+441c^4x^4-385c^2x^2+70)\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}(a+b\,\mathrm{arccosh}(cx))}{245c^2(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(5c^6x^6-21c^4x^4+35c^2x^2-35)\left(\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}\right.}{}$
default	$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{7c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,(64c^8x^8-144c^6x^6+64c^7x^7\sqrt{cx-1}\,\sqrt{cx+1}+104c^4x^4-112c^5x^5\sqrt{cx-1}\,\sqrt{cx+1}-25c^2x^2+}{6272(cx+1)c^2(cx-1)}\right.$
parts	$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{7c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,(64c^8x^8-144c^6x^6+64c^7x^7\sqrt{cx-1}\,\sqrt{cx+1}+104c^4x^4-112c^5x^5\sqrt{cx-1}\,\sqrt{cx+1}-25c^2x^2+}{6272(cx+1)c^2(cx-1)}\right.$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/245*(65*c^8*x^8-271*c^6*x^6+441*c^4*x^4-385*c^2*x^2+70)/c^2/(c*x-1)^2/(c
*x+1)^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-1/245*(5*c^6*x
^6-21*c^4*x^4+35*c^2*x^2-35)/c^2/(c*x-1)^2/(c*x+1)^2*((-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))-5*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))*c^2*d+x*
(-c^2*d*x^2+d)^(5/2)*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\,\mathrm{arccosh}(cx))\,dx = \frac{35\left(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,bc^2d^2x^2+bd^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-\right.}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/245*(35*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2
*x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (5*b*c^7
*d^2*x^7 - 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 - 35*b*c*d^2*x)*sqrt(-c^2*d
*x^2 + d)*sqrt(c^2*x^2 - 1) + 35*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^
4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\text{Timed out}$$

input
```
integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.54

$$\begin{aligned}\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx= & -\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b\,\text{arcosh}\,(cx)}{7\,c^2d}-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}a}{7\,c^2d}\\ & -\frac{\left(5\,c^6\sqrt{-d}d^3x^7-21\,c^4\sqrt{-d}d^3x^5+35\,c^2\sqrt{-d}d^3x^3-35\,\sqrt{-d}d^3x\right)b}{245\,cd}\end{aligned}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-1/7*(-c^2*d*x^2 + d)^(7/2)*b*arccosh(c*x)/(c^2*d) - 1/7*(-c^2*d*x^2 + d)^
(7/2)*a/(c^2*d) - 1/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5
+ 35*c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\int x\,(a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx=\frac{\sqrt{d}\,d^2\left(\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4+3\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output

```
(sqrt(d)*d**2*(sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 +
1)*a*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*a + 7*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**5,x)*b*c**6 - 14*int(s
qrt( - c**2*x**2 + 1)*acosh(c*x)*x**3,x)*b*c**4 + 7*int(sqrt( - c**2*x**2
+ 1)*acosh(c*x)*x,x)*b*c**2))/(7*c**2)
```

3.97 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathrm{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 379

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathrm{arccosh}(cx))}{x}\,dx = -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} + d^2\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx)) + \frac{1}{3}d(d-c^2dx^2)^{3/2}(a+b\,\mathrm{arccosh}(cx)) + \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\,\mathrm{arccosh}(cx)) - \frac{2d^2\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))\arctan\left(e^{\mathrm{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{ibd^2\sqrt{d-c^2\cdots}}{\sqrt{\cdots}}$$

output
```
-23/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/45*b*
c^3*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/25*b*c^5*d^
2*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d^2*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccosh(c*x))+1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/5
*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))-2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+I*b*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*d^2*(-c^2*d*x^2+d)^(1/2)*polylog
(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.60 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.24

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x}\,dx = \frac{1}{15}ad^2\sqrt{d-c^2dx^2}(23-11c^2x^2+3c^4x^4) - \frac{bd^2\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\operatorname{arccosh}(cx)-\cosh(3\operatorname{arccosh}(cx))\right)}{18\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} + ad^{5/2}\log(x) - ad^{5/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{bd^2\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\operatorname{arccosh}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\operatorname{ar}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(a*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 - (b*d^2*Sqrt
[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCo
sh[c*x] - Cosh[3*ArcCosh[c*x]]))/(18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
 + a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*
d^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]
+ c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E
^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (
-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)) - (b*d^2*Sqrt[d - c^2*d*x^2]*(25*Cosh[3*ArcCosh[c*x]]
+ 9*(-50*c*x + Cosh[5*ArcCosh[c*x]]) + 15*ArcCosh[c*x]*(30*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x) - 5*Sinh[3*ArcCosh[c*x]] - 3*Sinh[5*ArcCosh[c*x]])))
/(3600*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.86, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6345, 39, 210, 2009, 6345, 25, 39, 2009, 6341, 24, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))}{x}dx$$

$\Big\downarrow$ 6345

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int (1-cx)^2(cx+1)^2dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 39

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 210

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(c^4x^4-2c^2x^2+1\right)dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 2009

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))}{x}dx+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6345

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+\text{barccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int -((1-cx)(cx+1))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+\text{bar}\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 25

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+\text{barccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int (1-cx)(cx+1)dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+\text{barccos}\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+\text{barccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 39$$

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2009$$

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}\right)$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6341$$

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+\frac{1}{3}\left(d-c^2d\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 24$$

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{3}\left(d-c^2dx^2\right)\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6362$$

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{3}\left(\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 3042$$

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 4668

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\,\text{arcta}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2715

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2838

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
-1/5*(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/5
 + d*(-1/3*(b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(Sqrt[-1 + c*x]*S
qrt[1 + c*x]) + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/3 + d*(-((b*c
*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*x])*Ar
cTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*x]] + I*b*PolyLog[2
, I*E^ArcCosh[c*x]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.64

method	result
default	$\frac{(-c^2 d\,x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d(-c^2 d\,x^2+d)^{\frac{3}{2}}}{3}-a\,d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)+a\,d^2\sqrt{-c^2 d\,x^2+d}-\frac{ib\sqrt{-d(c^2x^2-1)}}{}$
parts	$\frac{(-c^2 d\,x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d(-c^2 d\,x^2+d)^{\frac{3}{2}}}{3}-a\,d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)+a\,d^2\sqrt{-c^2 d\,x^2+d}-\frac{ib\sqrt{-d(c^2x^2-1)}}{}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*(-c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(-c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+
2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*d^2*(-c^2*d*x^2+d)^(1/2)-I*b*(-d*(c^2
*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)))*d^2-14/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)*arcco
sh(c*x)*x^4*c^4+34/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)*arccosh
(c*x)*x^2*c^2-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)*arccosh(c
*x)+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)*arccosh(c*x)*x^6*c^6+
I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))*d^2-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2-1/25
*b*c^5*d^2*x^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)+11/45*b*
c^3*d^2*x^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)-23/15*b*(-d
*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c*d^2*x+I*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))*d^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)^{5/2}\,(a + b\text{arccosh}(cx))}{x}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)^{5/2}\,(a + b\text{arccosh}(cx))}{x}\,dx = \int \frac{(-c^2dx^2 + d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx) + a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
-1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
- 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x
^2 + d)*d^2)*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x}\,dx = \frac{\sqrt{d}\,d^2\left(3\sqrt{-c^2x^2+1}\,a\,c^4x^4-11\sqrt{-c^2x^2+1}\,a\,c^2x^2+23\sqrt{-c^2x^2}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x,x)
```

output
```
(sqrt(d)*d**2*(3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 11*sqrt( - c**2*x**2
 + 1)*a*c**2*x**2 + 23*sqrt( - c**2*x**2 + 1)*a + 15*int((sqrt( - c**2*x**
2 + 1)*acosh(c*x))/x,x)*b + 15*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**3,
x)*b*c**4 - 30*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*b*c**2 + 15*log(
tan(asin(c*x)/2))*a - 23*a))/15
```

3.98 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 404

$$\begin{aligned}
&\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^3}\,dx = \\
&-\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}} \\
&-\frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx)) \\
&-\frac{5}{6}c^2d\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))}{2x^2} \\
&+\frac{5c^2d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&-\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} \\
&+\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
-1/2*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+7/3*b*c^3*
d^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*c^5*d^2*x^3*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/2*c^2*d^2*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccosh(c*x))-5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)
)-1/2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2+5*c^2*d^2*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-5/2*I*b*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/2*I*b*c^2*d^2
*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-
1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.07 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.48

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^3}\,dx = \frac{1}{36}d^2\left(\frac{6a\sqrt{d-c^2dx^2}(-3-14c^2x^2+2c^4x^4)}{x^2}\right.$$
$$+\frac{bc^2\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\text{arccosh}(cx)-\cosh(3\text{arccosh}(cx))\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$
$$-90ac^2\sqrt{d}\log(x)+90ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)-\frac{72bc^2\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)+cx\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output

```
(d^2*((6*a*Sqrt[d - c^2*d*x^2]*(-3 - 14*c^2*x^2 + 2*c^4*x^4))/x^2 + (b*c^2
*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*
ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
)) - 90*a*c^2*Sqrt[d]*Log[x] + 90*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c
^2*d*x^2]] - (72*b*c^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 +
c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCo
sh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x
]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (18*b*d*(1 + c*x)*(c*x*Sqrt[(-1 +
 c*x)/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-
1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqr
t[(-1 + c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(
-1 + c*x)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^
2]))/36
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.83, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6343, 82, 244, 2009, 6345, 25, 39, 2009, 6341, 24, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{x^3} \, dx$$

$$\Big\downarrow \text{6343}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \text{82}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \ 244$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(x^2c^4-2c^2+\frac{1}{x^2}\right)dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 6345$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int -((1-cx)(cx+1))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 25$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int (1-cx)(cx+1)dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 39$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccos}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))-\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2d}}{3\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6341}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)+\frac{1}{3}(d\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{1}{3}(d-c\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6362}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int\log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2715

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccos}}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2838

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
(b*c*d^2*Sqrt[d - c^2*d*x^2]*(-x^(-1) - 2*c^2*x + (c^4*x^3)/3))/(2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(2*x
^2) - (5*c^2*d*(-1/3*(b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/3 +
 d*(-((b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a + b*ArcCos
h[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*x]] + I*b
*PolyLog[2, I*E^ArcCosh[c*x]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))))/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.97

method	result
default	$a\left(-\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{2 d\,x^{2}}-\frac{5 c^{2}\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{5}+d\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)\right)\right)\right)}{2}\right)+$
parts	$a\left(-\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{2 d\,x^{2}}-\frac{5 c^{2}\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{5}+d\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)\right)\right)\right)}{2}\right)+$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(7/2)-5/2*c^2*(1/5*(-c^2*d*x^2+d)^(5/2)+d*(1/
3*(-c^2*d*x^2+d)^(3/2)+d*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(
-c^2*d*x^2+d)^(1/2))/x)))))+1/18*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/x^2*(-6*I*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4*c^4+2
*I*x^5*c^5+42*I*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*c^2-42*I*x^3*
c^3+45*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*x^2*c^2-45*a
rccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*x^2*c^2+45*dilog(1-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*x^2*c^2-45*dilog(1+I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))*x^2*c^2+9*I*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+9*I
*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima
")
```

output
```
1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)
) - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqr
t(-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a + b*integr
ate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))}{x^3}\,dx = \frac{\sqrt{d}\,d^2\left(8\sqrt{-c^2x^2+1}\,a\,c^4x^4-56\sqrt{-c^2x^2+1}\,a\,c^2x^2-12\sqrt{-c^2x^2}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^3,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 56*sqrt( - c**2*x**2
 + 1)*a*c**2*x**2 - 12*sqrt( - c**2*x**2 + 1)*a + 24*int((sqrt( - c**2*x**
2 + 1)*acosh(c*x))/x**3,x)*b*x**2 - 48*int((sqrt( - c**2*x**2 + 1)*acosh(c
*x))/x,x)*b*c**2*x**2 + 24*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*b*c*
*4*x**2 - 60*log(tan(asin(c*x)/2))*a*c**2*x**2 + 65*a*c**2*x**2))/(24*x**2
)
```

3.99 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\text{arccosh}(cx))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 407

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\text{arccosh}(cx))}{x^5}\,dx = {} & -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} \\
& -\frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx)) \\
& +\frac{5c^2d(d-c^2dx^2)^{3/2}(a+b\,\text{arccosh}(cx))}{8x^2}-\frac{(d-c^2dx^2)^{5/2}(a+b\,\text{arccosh}(cx))}{4x^4} \\
& -\frac{15c^4d^2\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{4\sqrt{-1+cx}\sqrt{1+cx}} \\
& +\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{8\sqrt{-1+cx}\sqrt{1+cx}} \\
& -\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{8\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
-1/12*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+9/8*b*c
^3*d^2*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d^2*x*(-c^
2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/8*c^4*d^2*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccosh(c*x))+5/8*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/
x^2-1/4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4-15/4*c^4*d^2*(-c^2*d*x
^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+15/8*I*b*c^4*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/8*I*b*
c^4*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.28 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.62

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \frac{-2bcd^3x+2bc^2d^3x^2+27bc^3d^3x^3-27bc^4d^3x^4-24bc^5d^3x^5+24}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

output
```
(-2*b*c*d^3*x + 2*b*c^2*d^3*x^2 + 27*b*c^3*d^3*x^3 - 27*b*c^4*d^3*x^4 - 24
*b*c^5*d^3*x^5 + 24*b*c^6*d^3*x^6 - 6*a*d^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 3
3*a*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*a*c^4*d^3*x^4*Sqrt[(-1 + c*
x)/(1 + c*x)] - 24*a*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^3*Sqrt
[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + 33*b*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x] - 3*b*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[
c*x] - 24*b*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (45*I)*b
*c^4*d^3*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^5*d^3*x^5
*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^4*d^3*x^4*ArcCosh[c*x
]*Log[1 + I/E^ArcCosh[c*x]] + (45*I)*b*c^5*d^3*x^5*ArcCosh[c*x]*Log[1 + I/
E^ArcCosh[c*x]] + 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d -
 c^2*d*x^2]*Log[x] - 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[
d - c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (45*I)*b*c^4*d^3*x^4
*(-1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (45*I)*b*c^4*d^3*x^4*(-1 + c
*x)*PolyLog[2, I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[
d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.84, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {6343, 82, 244, 2009, 6343, 25, 82, 244, 2009, 6341, 24, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{x^5}dx$$

$$\Big\downarrow \text{6343}$$

$$-\frac{5}{4}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^3}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2}{x^4}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{82}$$

$$-\frac{5}{4}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^3}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^4}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{244}$$

$$-\frac{5}{4}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^3}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(c^4-\frac{2c^2}{x^2}+\frac{1}{x^4}\right)dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5}{4}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{x^3}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6343}$$

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+}{2x^2}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 25

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+ba}{2x^2}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 82

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^2}dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccos}}{2x^2}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 244

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^2}-c^2\right)dx}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barc}}{2x^2}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2009

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{2x^2}+\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{}}{2\sqrt{cx-1}\sqrt{cx}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6341

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\right)-\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 24

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}\right)-\frac{(a}{}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6362

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{bcx\sqrt{d-c^2d}}{\sqrt{cx-1}\sqrt{cx}}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 3042

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arc}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 4668

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int\log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2715

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2838

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{Pol}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{4x^4}+\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

output
```
(b*c*d^2*(-1/3*1/x^3 + (2*c^2)/x + c^4*x)*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(4*x^
4) - (5*c^2*d*((b*c*d*(-x^(-1) - c^2*x)*Sqrt[d - c^2*d*x^2])/(2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(2*x^2)
 - (3*c^2*d*(-((b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
 + Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a +
 b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*
x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/
2))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.70

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{4dx^4}+\frac{3a\,c^2(-c^2dx^2+d)^{\frac{7}{2}}}{8dx^2}+\frac{3a\,c^4(-c^2dx^2+d)^{\frac{5}{2}}}{8}+\frac{5a\,c^4d(-c^2dx^2+d)^{\frac{3}{2}}}{8}-\frac{15a\,c^4d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2}}{x}\right.}{8}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{4dx^4}+\frac{3a\,c^2(-c^2dx^2+d)^{\frac{7}{2}}}{8dx^2}+\frac{3a\,c^4(-c^2dx^2+d)^{\frac{5}{2}}}{8}+\frac{5a\,c^4d(-c^2dx^2+d)^{\frac{3}{2}}}{8}-\frac{15a\,c^4d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2}}{x}\right.}{8}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/x^4*(-c^2*d*x^2+d)^(7/2)+3/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(7/2)+3/8
*a*c^4*(-c^2*d*x^2+d)^(5/2)+5/8*a*c^4*d*(-c^2*d*x^2+d)^(3/2)-15/8*a*c^4*d^
(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+15/8*a*c^4*d^2*(-c^2*d*x^
2+d)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x-1)/(c*x+1)*arccosh(c*x)*x
^2-b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*x+1/8*b*(-
d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x-1)/(c*x+1)*arccosh(c*x)+9/8*b*d^2*(-d*(c
^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-11/8*b*d^2*(-d*(c^2*x^2
-1))^(1/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*d^2*(-d*(c^2*x^2-1)
)^(1/2)/(c*x+1)^(1/2)/x^3/(c*x-1)^(1/2)*c+1/4*b*d^2*(-d*(c^2*x^2-1))^(1/2)
/(c*x+1)/x^4/(c*x-1)*arccosh(c*x)-15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d^2-1
5/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln
(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d^2+15/8*I*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2)))*c^4*d^2+15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^5}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima
")
```

output
```
-1/8*(15*c^4*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x
)) - 3*(-c^2*d*x^2 + d)^(5/2)*c^4 - 5*(-c^2*d*x^2 + d)^(3/2)*c^4*d - 15*sq
rt(-c^2*d*x^2 + d)*c^4*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^2) + 2*(-c^
2*d*x^2 + d)^(7/2)/(d*x^4))*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^5}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^5,x)
```

output
```
int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^5, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))}{x^5}\,dx = \frac{\sqrt{d}\,d^2\left(8\sqrt{-c^2x^2+1}\,a\,c^4x^4+9\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))/x^5,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 9*sqrt( - c**2*x**2
+ 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 8*int((sqrt( - c**2*x**2 +
 1)*acosh(c*x))/x**5,x)*b*x**4 - 16*int((sqrt( - c**2*x**2 + 1)*acosh(c*x)
)/x**3,x)*b*c**2*x**4 + 8*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x,x)*b*c
**4*x**4 + 15*log(tan(asin(c*x)/2))*a*c**4*x**4 - 10*a*c**4*x**4))/(8*x**4
)
```

3.100 $\int \frac{x^4(a+b\,\mathrm{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 221

$$\int \frac{x^4(a+b\,\mathrm{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{3bx^2\sqrt{d-c^2dx^2}}{16c^3d\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^4\sqrt{d-c^2dx^2}}{16cd\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3x\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{8c^4d} - \frac{x^3\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{4c^2d} - \frac{3\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2}{16bc^5d\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
3/16*b*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*x
^4*(-c^2*d*x^2+d)^(1/2)/c/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/8*x*(-c^2*d*x^2+
d)^(1/2)*(a+b*arccosh(c*x))/c^4/d-1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))/c^2/d-3/16*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/c^5/d/(c*x-
1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.77

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{-\frac{16acx(3+2c^2x^2)\sqrt{d-c^2dx^2}}{d} - \frac{48a\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(-16\cosh(2\operatorname{arccosh}(cx))-\cosh(4\operatorname{arccosh}(cx))+4\operatorname{ar}}{\sqrt{}}}{128c^5}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
((-16*a*c*x*(3 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/d - (48*a*ArcTan[(c*x*Sqr
t[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/
(1 + c*x)]*(1 + c*x)*(-16*Cosh[2*ArcCosh[c*x]] - Cosh[4*ArcCosh[c*x]] + 4*
ArcCosh[c*x]*(6*ArcCosh[c*x] + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x
]])))/Sqrt[d - c^2*d*x^2])/(128*c^5)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6353, 15, 6353, 15, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \ 6353$$
$$\frac{3\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{4c^2} - \frac{b\sqrt{cx-1}\sqrt{cx+1}\int x^3dx}{4c\sqrt{d-c^2dx^2}} - \frac{x^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{4c^2d}$$
$$\Big\downarrow \ 15$$
$$\frac{3\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{4c^2} - \frac{x^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{4c^2d} - \frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{16c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6353}$$

$$\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int xdx}{2c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4c^2d}-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{16c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4c^2d}-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{16c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6307}$$

$$-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{4c^2d}+\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}\right)}{4c^2}-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{16c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/16*(b*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (x^3*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*c^2*d) + (3*(-1/4*(b*x^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Ar
cCosh[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])))/(4*c^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(189) = 378$.

Time = 0.34 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.57

method	result
default	$-\frac{a\,x^{3}\sqrt{-c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{-c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(-\frac{3\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^{2}}{16d\,c^{5}(c^{2}x^{2}-1)}\right.$
parts	$-\frac{a\,x^{3}\sqrt{-c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{-c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(-\frac{3\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^{2}}{16d\,c^{5}(c^{2}x^{2}-1)}\right.$

input
```
int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(-c^2*d*x^2+d)^(1/2)+3
/8*a/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/
16*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1)*ar
ccosh(c*x)^2-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*
(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-d*
(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-
(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-
d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*
x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/256
*(-d*(c^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+
8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/
2)+4*c*x)*(1+4*arccosh(c*x))/d/c^5/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d
), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^4(a+b\operatorname{acosh}(cx))}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
-1/8*a*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^4
*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + b*integrate(x^4*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^4/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^4\,(a+b\,\text{acosh}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{3asin(cx)\,a-2\sqrt{-c^2x^2+1}\,a\,c^3x^3-3\sqrt{-c^2x^2+1}\,acx+8\left(\int \frac{acosh(cx)x^4}{\sqrt{-c^2x^2+1}}dx\right)b\,c^5}{8\sqrt{d}\,c^5}$$

input
```
int(x^4*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(3*asin(c*x)*a - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt( - c**2*x**
2 + 1)*a*c*x + 8*int((acosh(c*x)*x**4)/sqrt( - c**2*x**2 + 1),x)*b*c**5)/(
8*sqrt(d)*c**5)
```

3.101 $\int \frac{x^3(a+b\,\mathrm{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 162

$$\int \frac{x^3(a+b\,\mathrm{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{2bx\sqrt{d-c^2dx^2}}{3c^3d\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx^3\sqrt{d-c^2dx^2}}{9cd\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{3c^4d} - \frac{x^2\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{3c^2d}$$

output
```
2/3*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/9*b*x^3*(
-c^2*d*x^2+d)^(1/2)/c/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/3*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccosh(c*x))/c^4/d-1/3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)
)/c^2/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{-1+cx}\sqrt{1+cx}(6+c^2x^2)-3a(-2+c^2x^2+c^4x^4)-3b(-2+c^2x^2+c^4x^4)\operatorname{arccosh}(\right.}{9c^4d(-1+cx)(1+cx)}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3
*a*(-2 + c^2*x^2 + c^4*x^4) - 3*b*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]))/
(9*c^4*d*(-1 + c*x)*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6353, 15, 6329, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \ 6353$$
$$\frac{2\int \frac{x(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int x^2dx}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{3c^2d}$$
$$\Big\downarrow \ 15$$
$$\frac{2\int \frac{x(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{3c^2d}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \ 6329$$

$$\frac{2\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int 1dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3c^2d}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 24$$

$$-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^2d}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/9*(b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (x^2*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c^2*d) + (2*(-((b*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/(c^2*d)))/(3*c^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

method	result
orering	$\frac{(5c^4x^4+12c^2x^2-24)(a+b\operatorname{arccosh}(cx))}{9c^4\sqrt{-c^2d\,x^2+d}}-\frac{(c^2x^2+6)(cx-1)(cx+1)\left(\frac{3x^2(a+b\operatorname{arccosh}(cx))}{\sqrt{-c^2d\,x^2+d}}+\frac{x^3bc}{\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-c^2d\,x^2+d}}+\frac{x^4(a+b\operatorname{ar}}{(-c^2c}\right.}{9x^2c^4}$
default	$a\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx+1}{72c^4d(c^2x^2-1)}\right.$
parts	$a\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx+1}{72c^4d(c^2x^2-1)}\right.$

input
```
int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(5*c^4*x^4+12*c^2*x^2-24)/c^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)-
1/9/x^2*(c^2*x^2+6)/c^4*(c*x-1)*(c*x+1)*(3*x^2*(a+b*arccosh(c*x))/(-c^2*d*
x^2+d)^(1/2)+x^3*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+x^4*
(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2)*c^2*d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{3\left(bc^4x^4+bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^3x^3+6\,bcx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}+}{9\left(c^6dx^2-c^4d\right)}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
-1/9*(3*(b*c^4*x^4 + b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) - (b*c^3*x^3 + 6*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
1) + 3*(a*c^4*x^4 + a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^
4*d)
```

Sympy [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x^3(a + b\,\text{acosh}\,(cx))}{\sqrt{-d\,(cx - 1)\,(cx + 1)}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = -\frac{1}{3}\,b\left(\frac{\sqrt{-c^2dx^2 + d}x^2}{c^2d} + \frac{2\,\sqrt{-c^2dx^2 + d}}{c^4d}\right)\text{arcosh}\,(cx) - \frac{1}{3}\,a\left(\frac{\sqrt{-c^2dx^2 + d}x^2}{c^2d} + \frac{2\,\sqrt{-c^2dx^2 + d}}{c^4d}\right) + \frac{\left(c^2\sqrt{-d}x^3 + 6\,\sqrt{-d}x\right)b}{9\,c^3d}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
-1/3*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d))
*arccosh(c*x) - 1/3*a*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^
2 + d)/(c^4*d)) + 1/9*(c^2*sqrt(-d)*x^3 + 6*sqrt(-d)*x)*b/(c^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x^3\,(a + b\,\text{acosh}(c\,x))}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{-\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+3\left(\int \frac{acosh(cx)x^3}{\sqrt{-c^2x^2+1}}dx\right)b\,c^4}{3\sqrt{d}\,c^4}$$

input
```
int(x^3*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 3*in
t((acosh(c*x)*x**3)/sqrt( - c**2*x**2 + 1),x)*b*c**4)/(3*sqrt(d)*c**4)
```

3.102 $\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 138

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{bx^2\sqrt{d-c^2dx^2}}{4cd\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d} - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4bc^3d\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/4*b*x^2*(-c^2*d*x^2+d)^(1/2)/c/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*x*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^2/d-1/4*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cosh(c*x))^2/b/c^3/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.02

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{-\frac{4acx\sqrt{d-c^2dx^2}}{d} - \frac{4a\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(-\cosh(2\text{arccosh}(cx))+2\text{arccosh}(cx)(\text{arccosh}(cx)+\sinh(2\text{arc}}{\sqrt{d-c^2dx^2}}}{8c^3}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
((-4*a*c*x*Sqrt[d - c^2*d*x^2])/d - (4*a*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/
(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[
c*x]])))/Sqrt[d - c^2*d*x^2])/(8*c^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6353, 15, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int xdx}{2c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}$$

$$\Big\downarrow \text{15}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6307}$$

$$-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/4*(b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (x*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(118) = 236$.

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.17

method	result
default	$-\frac{ax\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}+b\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{4d\,c^3(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-$
parts	$-\frac{ax\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}+b\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{4d\,c^3(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-$

input
```
int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a/c^2/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^2-1/16*(-d*(c^2*x^2-1))^(1/
2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c
*x+1)^(1/2))*(-1+2*arccosh(c*x))/d/c^3/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(
1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+
1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))/d/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d
), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^2(a+b\operatorname{acosh}(cx))}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)x^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
-1/2*a*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + b*in
tegrate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x
)
```

Giac [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)x^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x^2\,(a + b\,\text{acosh}(c\,x))}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{asin(cx)\,a-\sqrt{-c^2x^2+1}\,acx+2\left(\int \frac{acosh(cx)x^2}{\sqrt{-c^2x^2+1}}dx\right)b\,c^3}{2\sqrt{d}\,c^3}$$

input
```
int(x^2*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(asin(c*x)*a - sqrt( - c**2*x**2 + 1)*a*c*x + 2*int((acosh(c*x)*x**2)/sqrt
( - c**2*x**2 + 1),x)*b*c**3)/(2*sqrt(d)*c**3)
```

3.103 $\int \frac{x(a+b\textbf{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\frac{bx\sqrt{d-c^2dx^2}}{cd\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{c^2d}$$

output
```
b*x*(-c^2*d*x^2+d)^(1/2)/c/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-(-c^2*d*x^2+d)^(1
/2)*(a+b*arccosh(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx =\frac{\sqrt{d-c^2dx^2}\left(a-ac^2x^2+bcx\sqrt{-1+cx}\sqrt{1+cx}+(b-bc^2x^2)\operatorname{arccosh}(cx)\right)}{c^2d(-1+cx)(1+cx)}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
 (b - b*c^2*x^2)*ArcCosh[c*x]))/(c^2*d*(-1 + c*x)*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6329, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int 1dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^2d}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^2d}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-((b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(66) = 132.

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

method	result
orering	$\frac{(c^2x^2-2)(a+b\,\mathrm{arccosh}(cx))}{c^2\sqrt{-c^2d\,x^2+d}}-\frac{(cx-1)(cx+1)\left(\frac{a+b\,\mathrm{arccosh}(cx)}{\sqrt{-c^2d\,x^2+d}}+\frac{xbc}{\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-c^2d\,x^2+d}}+\frac{x^2(a+b\,\mathrm{arccosh}(cx))c^2d}{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\right)}{c^2}$
default	$-\frac{a\sqrt{-c^2d\,x^2+d}}{c^2d}+b\Big(-\frac{\sqrt{-d(c^2x^2-1)}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)(-1+\mathrm{arccosh}(cx))}{2c^2d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,c}{2c^2d(c^2x^2}$
parts	$-\frac{a\sqrt{-c^2d\,x^2+d}}{c^2d}+b\Big(-\frac{\sqrt{-d(c^2x^2-1)}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)(-1+\mathrm{arccosh}(cx))}{2c^2d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,c}{2c^2d(c^2x^2}$

input
```
int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c^2*x^2-2)/c^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)-1/c^2*(c*x-1)*(c*x
+1)*((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)+x*b*c/(c*x-1)^(1/2)/(c*x+1)^(
1/2)/(-c^2*d*x^2+d)^(1/2)+x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2)*c^2*
d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{x(a+b\mathrm{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx-(bc^2x^2-b)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-(ac^2x^2-a)\sqrt{-c^2dx^2+}}{c^4dx^2-c^2d}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^2*x^2 - b)*sqrt(-c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (a*c^2*x^2 - a)*sqrt(-c^2*d*x^2
+ d))/(c^4*d*x^2 - c^2*d)
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x(a+b\,\text{acosh}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{x(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\frac{b\sqrt{-d}x}{cd}-\frac{\sqrt{-c^2dx^2+d}b\,\text{arcosh}\,(cx)}{c^2d}-\frac{\sqrt{-c^2dx^2+d}a}{c^2d}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b*arccosh(c*x)/(c^2*d) - sqrt(-c
^2*d*x^2 + d)*a/(c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x\,(a + b\,\mathrm{acosh}(c\,x))}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a + \left(\int \frac{acosh(cx)x}{\sqrt{-c^2x^2+1}}dx\right) b\,c^2}{\sqrt{d}\,c^2}$$

input
```
int(x*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a + int((acosh(c*x)*x)/sqrt( - c**2*x**2 + 1),x
)*b*c**2)/(sqrt(d)*c**2)
```

3.104 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{a+b\operatorname{arccosh}(cx)}{\sqrt{d-c^2dx^2}}\,dx=-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{2bcd\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/c/d/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{a+b\operatorname{arccosh}(cx)}{\sqrt{d-c^2dx^2}}\,dx=\frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))^2}{2bc\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*
d*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arccosh}(cx)}{\sqrt{d - c^2dx^2}} \, dx$$

$$\Big\downarrow \text{ 6307}$$

$$\frac{\sqrt{cx - 1}\sqrt{cx + 1}(a + b\text{arccosh}(cx))^2}{2bc\sqrt{d - c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*
d*x^2])
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

method	result	size
default	$\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}-\frac{b\sqrt{-d(cx-1)(cx+1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{2d(c^2x^2-1)c}$	89
parts	$\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}-\frac{b\sqrt{-d(cx-1)(cx+1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{2d(c^2x^2-1)c}$	89

input
```
int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c*
x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)/c*arccosh(c*
x)^2
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\text{acosh}\,(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x
) + a*arcsin(c*x)/(c*sqrt(d))
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{a + b\,\text{acosh}(c\,x)}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{\sqrt{d - c^2dx^2}}\,dx = \frac{asin(cx)\,a + \left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}}dx\right)bc}{\sqrt{d}\,c}$$

input
```
int((a+b*acosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(asin(c*x)*a + int(acosh(c*x)/sqrt( - c**2*x**2 + 1),x)*b*c)/(sqrt(d)*c)
```

3.105 $\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 160

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx = -\frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} + \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,
-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*(-
c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \frac{a\log(x)}{\sqrt{d}} - \frac{a\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{ib\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\text{arccosh}(cx)\left(\log\left(1-ie^{-\text{arccosh}(cx)}\right)-\log\left(1+ie^{-\text{arccosh}(cx)}\right)\right)+\text{PolyLog}\left(2,-ie^{-a\cdots}\right.\right.}{\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (I
*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 - I/E^ArcCosh
[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCosh[c*x]] - Po
lyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6361}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\left(\right.}{\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)de\right.}{\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcc}}\right.\right.}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x
]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]
]))/Sqrt[d - c^2*d*x^2]
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.04

method	result
default	$-\frac{a \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)}{\sqrt{d}}+b\left(\frac{i\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\ln(1+i(cx+\sqrt{cx-1}\,\sqrt{cx+1}))}{d(c^{2}x^{2}-1)}-\frac{i\sqrt{-d(c^{2}x^{2}-}{}\right.$
parts	$-\frac{a \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)}{\sqrt{d}}+b\left(\frac{i\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\ln(1+i(cx+\sqrt{cx-1}\,\sqrt{cx+1}))}{d(c^{2}x^{2}-1)}-\frac{i\sqrt{-d(c^{2}x^{2}-}{}\right.$

input
```
int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+b*(I*(-d*(c^2*x^2-1)
)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))+I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*di
log(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+dx}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\text{acosh}\,(cx)}{x\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+dx}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x
), x) - a*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{a+b\,\text{acosh}(c\,x)}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x}dx\right)b+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a}{\sqrt{d}}$$

input
```
int((a+b*acosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*x),x)*b + log(tan(asin(c*x)/2))*a)
/sqrt(d)
```

3.106 $\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 73

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx=-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{dx}+\frac{bc\sqrt{d-c^2dx^2}\log(x)}{d\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/d/x+b*c*(-c^2*d*x^2+d)^(1/2)*ln(x
)/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx=\frac{\sqrt{-1+cx}\sqrt{1+cx}\left(\frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))}{x}-bc\log(x)\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCos
h[c*x]))/x - b*c*Log[x]))/Sqrt[d - c^2*d*x^2]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6332, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6332}$$

$$-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{dx}$$

$$\Big\downarrow \text{14}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{dx}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(d*x)) - (b*c*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*Log[x])/Sqrt[d - c^2*d*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(65) = 130.

Time = 0.40 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.85

method	result
default	$-\frac{a\sqrt{-c^2d\,x^2+d}}{dx}+b\left(-\frac{2\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)c}{d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\operatorname{arcco}}{x(c^2x^2-1)d}\right.$
parts	$-\frac{a\sqrt{-c^2d\,x^2+d}}{dx}+b\left(-\frac{2\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)c}{d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\operatorname{arcco}}{x(c^2x^2-1)d}\right.$

input
```
int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(1/2)+b*(-2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(-(c*x-1)^(
1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*arccosh(c*x)/x/(c^2*x^2-1)/d+(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)*c)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.64

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx$$
$$=\left[-\frac{bc\sqrt{-d}x\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}(x^4-1)\sqrt{-d}-d}{c^2x^4-x^2}\right)+2\sqrt{-c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2-1}\right)}{2\,dx}\right.$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/2*(b*c*sqrt(-d)*x*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + 2*sqrt(
-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)
/(d*x), (b*c*sqrt(d)*x*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2
- 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - sqrt(-c^2*d*x^2 + d)*b*l
og(c*x + sqrt(c^2*x^2 - 1)) - sqrt(-c^2*d*x^2 + d)*a)/(d*x)]
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2\sqrt{d - c^2dx^2}}\, dx = \int \frac{a + b\,\text{acosh}\,(cx)}{x^2\sqrt{-d\,(cx - 1)\,(cx + 1)}}\, dx$$

input
```
integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \frac{a + b\text{arccosh}(cx)}{x^2\sqrt{d - c^2dx^2}}\, dx = -\frac{\left(c^2d\sqrt{-\frac{1}{c^4d}}\log\left(x^2 - \frac{1}{c^2}\right) + i\,(-1)^{-2\,c^2dx^2 + 2\,d}\sqrt{d}\log\left(-2\,c^2d + \frac{2\,d}{x^2}\right)\right)bc}{2\,d} - \frac{\sqrt{-c^2dx^2 + d}b\,\text{arcosh}\,(cx)}{dx} - \frac{\sqrt{-c^2dx^2 + d}a}{dx}$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
-1/2*(c^2*d*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d
)*sqrt(d)*log(-2*c^2*d + 2*d/x^2))*b*c/d - sqrt(-c^2*d*x^2 + d)*b*arccosh(
c*x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a/(d*x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx=\frac{-\sqrt{-c^2x^2+1}\,a+\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x^2}dx\right)bx}{\sqrt{d}\,x}$$

input
```
int((a+b*acosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

output

```
( - sqrt( - c**2*x**2 + 1)*a + int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*x**2
),x)*b*x)/(sqrt(d)*x)
```

3.107 $\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 251

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{2dx\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2} - \frac{c^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} + \frac{ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{2d\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{2d\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/2*b*c*(-c^2*d*x^2+d)^(1/2)/d/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(-c^2*d*
x^2+d)^(1/2)*(a+b*arccosh(c*x))/d/x^2-c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d/(c*x-1)^(1/2)/(c*x+1)^(1
/2)+1/2*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*
polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.23

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{1}{2}\left(-\frac{a\sqrt{d-c^2dx^2}}{dx^2}+\frac{ac^2\log(x)}{\sqrt{d}}-\frac{ac^2\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}}\right.$$
$$+\frac{b(1+cx)\left(cx\sqrt{\frac{-1+cx}{1+cx}}-\text{arccosh}(cx)+cx\text{arccosh}(cx)-ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)\log\left(1-ie^{-\text{arccosh}(c}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(-((a*Sqrt[d - c^2*d*x^2])/(d*x^2)) + (a*c^2*Log[x])/Sqrt[d] - (a*c^2*Log[
d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] + (b*(1 + c*x)*(c*x*Sqrt[(-1 + c
*x)/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] - I*c^2*x^2*Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1
+ c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[
(-1 + c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]
))/2
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.67, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6347, 15, 6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{ 6347}$$

$$\frac{1}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x^2}dx}{2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2}$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 6361$$

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 4668$$

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\right.}{2\sqrt{d-c^2dx^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 2715$$

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)\right.}{2\sqrt{d-c^2dx^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 2838$$

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{ar}}\right.\right.}{2\sqrt{d-c^2dx^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2dx^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*d*x^2) + (c^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(2*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E
^ArcCosh[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/(2*Sqrt[d - c^2*d*x^2]
)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.72

method	result
default	$-\frac{a\sqrt{-c^{2} d\,x^{2}+d}}{2 d\,x^{2}}-\frac{a\,c^{2} \ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)}{2 \sqrt{d}}+b\left(-\frac{\left(c^{2} x^{2} \operatorname{arccosh}\left(c x\right)+\sqrt{c x-1}\,\sqrt{c x+1}\,c x-\operatorname{arccosh}\left(c x\right)\right) \sqrt{-d\left(c^{2} x^{2}-1\right)}}{2 d\left(c^{2} x^{2}-1\right) x^{2}}\right.$
parts	$-\frac{a\sqrt{-c^{2} d\,x^{2}+d}}{2 d\,x^{2}}-\frac{a\,c^{2} \ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)}{2 \sqrt{d}}+b\left(-\frac{\left(c^{2} x^{2} \operatorname{arccosh}\left(c x\right)+\sqrt{c x-1}\,\sqrt{c x+1}\,c x-\operatorname{arccosh}\left(c x\right)\right) \sqrt{-d\left(c^{2} x^{2}-1\right)}}{2 d\left(c^{2} x^{2}-1\right) x^{2}}\right.$

input
```
int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^
2*d*x^2+d)^(1/2))/x)+b*(-1/2*(c^2*x^2*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(
1/2)*c*x-arccosh(c*x))*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/x^2+1/2*I*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*
ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-1/2*I*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-
1/2*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dil
og(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2)
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+dx^3}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x
)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\text{acosh}\,(cx)}{x^3\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{a + b\,\mathrm{arccosh}(cx)}{x^3\sqrt{d - c^2dx^2}}\,dx = \int \frac{b\,\mathrm{arcosh}\,(cx) + a}{\sqrt{-c^2dx^2 + d}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
-1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)
+ sqrt(-c^2*d*x^2 + d)/(d*x^2))*a + b*integrate(log(c*x + sqrt(c*x + 1)*sq
rt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Giac [F]

$$\int \frac{a + b\,\mathrm{arccosh}(cx)}{x^3\sqrt{d - c^2dx^2}}\,dx = \int \frac{b\,\mathrm{arcosh}\,(cx) + a}{\sqrt{-c^2dx^2 + d}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\,\mathrm{arccosh}(cx)}{x^3\sqrt{d - c^2dx^2}}\,dx = \int \frac{a + b\,\mathrm{acosh}(c\,x)}{x^3\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3\sqrt{d - c^2dx^2}}\,dx$$
$$= \frac{-\sqrt{-c^2x^2 + 1}\,a + 2\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x^3}dx\right) b\,x^2 + \log\left(\tan\left(\frac{asin(cx)}{2}\right)\right) a\,c^2x^2}{2\sqrt{d}\,x^2}$$

input
```
int((a+b*acosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a + 2*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*x*
*3),x)*b*x**2 + log(tan(asin(c*x)/2))*a*c**2*x**2)/(2*sqrt(d)*x**2)
```

3.108 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 161

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{6dx^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{3dx^3} -\frac{2c^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{3dx}+\frac{2bc^3\sqrt{d-c^2dx^2}\log(x)}{3d\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/d/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/d/x^3-2/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccosh(c*x))/d/x+2/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*ln(x)/d/(c*x-1)^(1/2)/(c*
x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.08

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{d-c^2dx^2}\left(bcx+6bc^3x^3+2a\sqrt{-1+cx}\sqrt{1+cx}+4ac^2x^2\sqrt{-1+cx}\sqrt{1+cx}+2b\sqrt{-1+cx}\sqrt{1+c\right.}{6dx^3\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-1/6*(Sqrt[d - c^2*d*x^2]*(b*c*x + 6*b*c^3*x^3 + 2*a*Sqrt[-1 + c*x]*Sqrt[1
 + c*x] + 4*a*c^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(1 + 2*c^2*x^2)*ArcCosh[c*x] - 4*b*c^3*x^3*Log[-1 + c*x] - 4*b
*c^3*x^3*Log[1 + (-1 + c*x)^(-1)]))/(d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6347, 15, 6332, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6347}$$

$$\frac{2}{3}c^2\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x^3}dx}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}c^2\int \frac{a+b\text{arccosh}(cx)}{x^2\sqrt{d-c^2dx^2}}dx-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3dx^3}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6332}$$

$$\frac{2}{3}c^2\left(-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{dx}\right)-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3dx^3}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{dx}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{\sqrt{d-c^2dx^2}}\right)-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3dx^3}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*d*x^3) + (2*c^2*(-((Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(d*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x]
)/Sqrt[d - c^2*d*x^2]))/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.19

method	result
default	$a\left(-\frac{\sqrt{-c^2 d\,x^2+d}}{3d\,x^3}-\frac{2c^2\sqrt{-c^2 d\,x^2+d}}{3dx}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(4\,\mathrm{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2+4c^3x^3\,\mathrm{arccosh}\right.}{6d\,x^3(c^2x}$
parts	$a\left(-\frac{\sqrt{-c^2 d\,x^2+d}}{3d\,x^3}-\frac{2c^2\sqrt{-c^2 d\,x^2+d}}{3dx}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(4\,\mathrm{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2+4c^3x^3\,\mathrm{arccosh}\right.}{6d\,x^3(c^2x}$

input
```
int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*c^2/d/x*(-c^2*d*x^2+d)^(1/2))-1/6*b
*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(4*arccosh(c*x)*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+4*c^3*x^3*arccosh(c*x)-4*ln(1+(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)*x^3*c^3+2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+
c*x)/d/x^3/(c^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.98

$$\int \frac{a+b\mathrm{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$$
$$=\left[-\frac{2\left(2\,bc^4x^4-bc^2x^2-b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+2\left(bc^5x^5-bc^3x^3\right)\sqrt{-d}\log\left(\frac{c^2dx^6+c^2dx^2-d}{\ }\right.}{6\,(}\right.$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/6*(2*(2*b*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 - 1)) + 2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d
*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) -
 d)/(c^2*x^4 - x^2)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2
 - 1) + 2*(2*a*c^4*x^4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 -
 d*x^3), 1/6*(4*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1)*(x^2 - 1)*sqrt(d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) -
 2*(2*b*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 - 1)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(2
*a*c^4*x^4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3)]
```

Sympy [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\operatorname{acosh}(cx)}{x^4\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx=&\ \frac{1}{6}\left(\frac{4\,c^2\sqrt{-d}\log(x)}{d}-\frac{\sqrt{-d}}{dx^2}\right)bc\\&-\frac{1}{3}\,b\left(\frac{2\sqrt{-c^2dx^2+d}c^2}{dx}+\frac{\sqrt{-c^2dx^2+d}}{dx^3}\right)\operatorname{arcosh}(cx)\\&-\frac{1}{3}\,a\left(\frac{2\sqrt{-c^2dx^2+d}c^2}{dx}+\frac{\sqrt{-c^2dx^2+d}}{dx^3}\right)\end{aligned}$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
1/6*(4*c^2*sqrt(-d)*log(x)/d - sqrt(-d)/(d*x^2))*b*c - 1/3*b*(2*sqrt(-c^2*
d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccosh(c*x) - 1/3*a*
(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+dx^4}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x^4\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{-2\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a+3\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x^4}dx\right)b\,x^3}{3\sqrt{d}\,x^3}$$

input
```
int((a+b*acosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a + 3*in
t(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*x**4),x)*b*x**3)/(3*sqrt(d)*x**3)
```

3.109 $\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 232

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{5bx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}}+\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{c^6d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^2}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3c^6d^3}+\frac{b\sqrt{-1+cx}\sqrt{1+cx}\text{arctanh}(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

output
```
5/3*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)+1/9*b*x^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))/c
^6/d/(-c^2*d*x^2+d)^(1/2)+2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^6/d^
2-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^6/d^3+b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*arctanh(c*x)/c^6/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.62

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{24a-12ac^2x^2-3ac^4x^4+15bcx\sqrt{-1+cx}\sqrt{1+cx}+bc^3x^3\sqrt{-1+cx}\sqrt{1+}}{9c^6d\sqrt{d}}$$

input
```
Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(24*a - 12*a*c^2*x^2 - 3*a*c^4*x^4 + 15*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
 + b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*b*(-8 + 4*c^2*x^2 + c^4*x^4)
*ArcCosh[c*x] + 9*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(9*c^6*d*Sq
rt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6337, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6337$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{-c^4x^4-4c^2x^2+8}{3c^6d^2(1-c^2x^2)}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^2}+\frac{a+b\text{arccosh}(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\sqrt{d-c^2dx^2}\int \frac{-c^4x^4-4c^2x^2+8}{1-c^2x^2}dx}{3c^5d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^2}+\frac{a+b\text{arccosh}(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 1467}$$

$$-\frac{b\sqrt{d-c^2dx^2}\int \left(c^2x^2+\frac{3}{1-c^2x^2}+5\right)dx}{3c^5d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{c^6d^2}+\frac{a+b\operatorname{arccosh}(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{c^6d^2}+\frac{a+b\operatorname{arccosh}(cx)}{c^6d\sqrt{d-c^2dx^2}}-\frac{b\left(\frac{3\operatorname{arctanh}(cx)}{c}+\frac{c^2x^3}{3}+5x\right)\sqrt{d-c^2dx^2}}{3c^5d^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCosh[c*x])/(c^6*d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(c^6*d^2) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*
x]))/(3*c^6*d^3) - (b*Sqrt[d - c^2*d*x^2]*(5*x + (c^2*x^3)/3 + (3*ArcTanh[
c*x])/c))/(3*c^5*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.40

method	result
default	$a\left(-\frac{x^4}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{-\frac{4x^2}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{8}{3d\,c^4\sqrt{-c^2d\,x^2+d}}}{c^2}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^2x^2-1)}\,(3\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccos}}{}$
parts	$a\left(-\frac{x^4}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{-\frac{4x^2}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{8}{3d\,c^4\sqrt{-c^2d\,x^2+d}}}{c^2}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^2x^2-1)}\,(3\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccos}}{}$

input
```
int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)+4/3/c^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^
(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2)))+1/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-d
*(c^2*x^2-1))^(1/2)*(3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4-c^
5*x^5+12*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2-14*c^3*x^3-9*ln(
1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2+9*ln((c*x-1)^(1/2)*(c*x+1)^(1/2
)+c*x-1)*x^2*c^2-24*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+15*c*x+9*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-9*ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1))
/(c^2*x^2-1)^2/d^2/c^6
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.11

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \left[\frac{12\,(bc^4x^4+4\,bc^2x^2-8\,b)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-9\,(bc^2x^2-\right.$$

$$-\frac{9\,(bc^2x^2-b)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}c\sqrt{d}x}{c^4dx^4-d}\right)-6\,(bc^4x^4+4\,bc^2x^2-8\,b)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x\right.}{18\,(c^8d^2x^2-c^6}$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
[1/36*(12*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 - 1)) - 9*(b*c^2*x^2 - b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x
^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1
)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(b*c^3*x^3 + 15
*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^4*x^4 + 4*a*c^2*x
^2 - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^2*x^2 - c^6*d^2), -1/18*(9*(b*c^2*x
^2 - b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*
x/(c^4*d*x^4 - d)) - 6*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*x^3 + 15*b*c*x)*sqrt(-c^2*d*x^2
+ d)*sqrt(c^2*x^2 - 1) - 6*(a*c^4*x^4 + 4*a*c^2*x^2 - 8*a)*sqrt(-c^2*d*x^2
 + d))/(c^8*d^2*x^2 - c^6*d^2)]
```

Sympy [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^5(a+b\operatorname{acosh}(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^5}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
-1/3*a*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^4
*d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/9*b*(((c^4*sqrt(d)*x^4 + 16*c^2*
sqrt(d)*x^2 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1)/sqrt(-c*x + 1) - 3*(c
^5*sqrt(d)*x^5 + 4*c^3*sqrt(d)*x^3 - 8*c*sqrt(d)*x + (c^4*sqrt(d)*x^4 + 4*
c^2*sqrt(d)*x^2 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/sqrt(-c*x + 1))/(sqrt(c*x + 1)*c^7*d^2*x + (c*x + 1
)*sqrt(c*x - 1)*c^6*d^2) + 9*integrate(1/9*(3*c^7*sqrt(d)*x^7 + 9*c^5*sqrt
(d)*x^5 - 36*c^3*sqrt(d)*x^3 + 24*c*sqrt(d)*x + (3*c^6*sqrt(d)*x^6 + 8*c^4
*sqrt(d)*x^4 - 52*c^2*sqrt(d)*x^2 + 32*sqrt(d))*e^(1/2*log(c*x + 1) + 1/2*
log(c*x - 1)))/(sqrt(-c*x + 1)*((c^7*d^2*x^2 - c^5*d^2)*e^(3/2*log(c*x + 1
) + log(c*x - 1)) + 2*(c^8*d^2*x^3 - c^6*d^2*x)*e^(log(c*x + 1) + 1/2*log(
c*x - 1)) + (c^9*d^2*x^4 - c^7*d^2*x^2)*sqrt(c*x + 1))), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^5\,(a+b\,\text{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\frac{-3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^5}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^6-a\,c^4x^4-4a\,c^2x^2+8a}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^6d}$$

input
```
int(x^5*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**5)/(sqrt( - c**2*x**2 + 1)
*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**6 - a*c**4*x**4 - 4*a*c**2*x*
*2 + 8*a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**6*d)
```

3.110 $\int \frac{x^4(a+b\,\mathbf{arccosh}(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 226

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\operatorname{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{3x\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{2c^4d^2}-\frac{3\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))^2}{4bc^5d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}\log\left(1-c^2x^2\right)}{2c^5d\sqrt{d-c^2dx^2}}$$

output
```
1/4*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b*
arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))/c^4/d^2-3/4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^2/
b/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-c^2*x^2
+1)/c^5/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-4acdx(-3+c^2x^2)+12a\sqrt{d}\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+bd\left(8cx\text{arc}\right.}{}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(-4*a*c*d*x*(-3 + c^2*x^2) + 12*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(8*c*x*ArcCosh[c*x] -
 Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh[2*ArcCosh[c
*x]] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh[c*x]*Sinh[2
*ArcCosh[c*x]])))/(8*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6349, 25, 82, 243, 49, 2009, 6353, 15, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6349}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{x^3}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 82}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2}{1-c^2x^2}dx^2}{2cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x^2-1)}\right)dx^2}{2cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 6353}$$

$$-\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int xdx}{2c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{3\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 6307}$$

$$\frac{x^3(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (3*(-1/4*(b*x^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])))/(c^2*d) + (b*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(-(x^2/c^2) - Log[1 - c^2*x^2]/c^4))/(2*c*d*Sqrt[d - c^2
*d*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.33

method	result
default	$-\frac{a\,x^{3}}{2c^{2}d\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3ax}{2c^{4}d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2c^{4}d\sqrt{c^{2}d}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^{2}x^{2}-1)}\,\left(4\,\mathrm{arccosh}(cx)\sqrt{cx+\right.}{}$
parts	$-\frac{a\,x^{3}}{2c^{2}d\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3ax}{2c^{4}d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2c^{4}d\sqrt{c^{2}d}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^{2}x^{2}-1)}\,\left(4\,\mathrm{arccosh}(cx)\sqrt{cx+\right.}{}$

input
```
int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(-c^2*d*x^2+d)^(1/2)-3
/2*a/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/8*
b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(4*arccosh(c*x)*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-2*c^4*x^4+6*arccosh(c*x)^2*x^2*c^2-12*arcco
sh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x-8*c^2*x^2*arccosh(c*x)+8*ln((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^2*c^2+3*c^2*x^2-6*arccosh(c*x)^2+8*arcc
osh(c*x)-8*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)-1)/(c^2*x^2-1)^2/d^2/
c^5
```

Fricas [F]

$$\int \frac{x^4(a+b\,\mathrm{arccosh}(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)x^4}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 -
2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{x^4(a + b\,\text{acosh}\,(cx))}{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)x^4}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
-1/2*a*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*d
) + 3*arcsin(c*x)/(c^5*d^(3/2))) + b*integrate(x^4*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-3\sqrt{-c^2x^2+1}\,asin(cx)\,a-2\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^4}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^5}{2\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^5d}$$

input
```
int(x^4*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*asin(c*x)*a - 2*sqrt( - c**2*x**2 + 1)*int((a
cosh(c*x)*x**4)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)
),x)*b*c**5 - a*c**3*x**3 + 3*a*c*x)/(2*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**
5*d)
```

3.111 $\int \frac{x^3(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{a+b\text{arccosh}(cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^4d^2} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\text{arctanh}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

output
```
b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*
x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^4
/d^2+b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arctanh(c*x)/c^4/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{2a-ac^2x^2+bcx\sqrt{-1+cx}\sqrt{1+cx}+b(2-c^2x^2)\,\text{arccosh}(cx)+b\sqrt{-1+cx}}{c^4d\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(2*a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(2 - c^2*x^2)*Ar
cCosh[c*x] + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^4*d*Sqrt[d -
c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6337, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{2-c^2x^2}{c^4d^2(1-c^2x^2)}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^4d^2}+\frac{a+b\text{arccosh}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{d-c^2dx^2}\int \frac{2-c^2x^2}{1-c^2x^2}dx}{c^3d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^4d^2}+\frac{a+b\text{arccosh}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{299}$$

$$-\frac{b\sqrt{d-c^2dx^2}\left(\int \frac{1}{1-c^2x^2}dx+x\right)}{c^3d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^4d^2}+\frac{a+b\text{arccosh}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^4d^2}+\frac{a+b\text{arccosh}(cx)}{c^4d\sqrt{d-c^2dx^2}}-\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)\sqrt{d-c^2dx^2}}{c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCosh[c*x])/(c^4*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(c^4*d^2) - (b*Sqrt[d - c^2*d*x^2]*(x + ArcTanh[c*x]/c
))/(c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

method	result
default	$a\left(-\frac{x^2}{c^2 d\sqrt{-c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2 d\,x^2+d}}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^2x^2-1)}\,(\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2-c^3x^3+\ln(\sqrt{c}}{}$
parts	$a\left(-\frac{x^2}{c^2 d\sqrt{-c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2 d\,x^2+d}}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^2x^2-1)}\,(\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2-c^3x^3+\ln(\sqrt{c}}{}$

input
```
int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arccosh(c*x)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*c^2*x^2-c^3*x^3+ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)*x^2*c^2-
ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2-2*arccosh(c*x)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)+c*x-ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)+ln(1+c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2)))/(c^2*x^2-1)^2/d^2/c^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.90

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx-4\left(bc^2x^2-2b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c\right.}}{}\right.$$

$$-\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx+\left(bc^2x^2-b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}c\sqrt{d}x}{c^4dx^4-d}\right)-2\left(bc^2x^2-2b\right)\sqrt{-c^2dx}}{2\left(c^6d^2x^2-c^4d^2\right)}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/4*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - 4*(b*c^2*x^2 - 2*b
)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*sqrt
(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(
-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c
^2*x^2 - 1)) - 4*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^
4*d^2), -1/2*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + (b*c^2*x^2
- b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(
c^4*d*x^4 - d)) - 2*(b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) - 2*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c
^4*d^2)]
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^3(a+b\,\text{acosh}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx &= \\ &-\frac{1}{2}\,bc\left(\frac{2\,\sqrt{-d}x}{c^4d^2}+\frac{\sqrt{-d}\log\,(cx+1)}{c^5d^2}-\frac{\sqrt{-d}\log\,(cx-1)}{c^5d^2}\right) \\ &-b\left(\frac{x^2}{\sqrt{-c^2dx^2+d}c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}c^4d}\right)\text{arcosh}\,(cx) \\ &-a\left(\frac{x^2}{\sqrt{-c^2dx^2+d}c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}c^4d}\right)\end{aligned}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
-1/2*b*c*(2*sqrt(-d)*x/(c^4*d^2) + sqrt(-d)*log(c*x + 1)/(c^5*d^2) - sqrt(
-d)*log(c*x - 1)/(c^5*d^2)) - b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqr
t(-c^2*d*x^2 + d)*c^4*d))*arccosh(c*x) - a*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*
d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\,\mathrm{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\,\mathrm{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\,\mathrm{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\left(\int \frac{\mathrm{acosh}(cx)x^3}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^4-a\,c^2x^2+2a}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^4d}$$

input
```
int(x^3*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)*c
**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**4 - a*c**2*x**2 + 2*a)/(sqrt(d)
*sqrt( - c**2*x**2 + 1)*c**4*d)
```

3.112 $\int \frac{x^2(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2bc^3d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+1/2*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccosh(c*x))^2/b/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*b*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*ln(-c^2*x^2+1)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.11

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{2acdx + 2a\sqrt{d}\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right) + bd\left(2cx\text{arccosh}(cx) - \sqrt{\cdots}\right.}{2c^3d^2\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(2*a*c*d*x + 2*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1 + c*x)/
(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)])))/(2*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6349, 25, 82, 240, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6349}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{x}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{82}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6307}$$

$$\frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(125) = 250$.

Time = 0.43 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.95

method	result
default	$\frac{ax}{c^2d\sqrt{-c^2d\,x^2+d}}-\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{c^2d\sqrt{c^2d}}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{2d^2c^3(c^2x^2-1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+}}{d^2c^3(c^2x^2-1)}$
parts	$\frac{ax}{c^2d\sqrt{-c^2d\,x^2+d}}-\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{c^2d\sqrt{c^2d}}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{2d^2c^3(c^2x^2-1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+}}{d^2c^3(c^2x^2-1)}$

input
```
int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*
x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(
1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2-1)
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 -
2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{x^2(a + b\operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}(cx) + a)x^2}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
a*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + b*integra
te(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}(cx) + a)x^2}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,asin(cx)\,a-\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^2}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^3+}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^3d}$$

input
```
int(x^2*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*asin(c*x)*a - sqrt( - c**2*x**2 + 1)*int((acosh
(c*x)*x**2)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)
*b*c**3 + a*c*x)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*c**3*d)
```

3.113 $\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{a+b\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\text{arctanh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

output
```
(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+b*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*arctanh(c*x)/c^2/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.79

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{a\sqrt{-d(-1+c^2x^2)}}{c^2d^2(-1+c^2x^2)} - \frac{b\sqrt{-d(-1+c^2x^2)}\text{arccosh}(cx)}{c^2d^2(-1+c^2x^2)} + \frac{b\sqrt{d-c^2dx^2}(\log(-1+cx)-\log(1+cx))}{2c^2d^2\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
-((a*Sqrt[-(d*(-1 + c^2*x^2))])/(c^2*d^2*(-1 + c^2*x^2))) - (b*Sqrt[-(d*(-
1 + c^2*x^2))]*ArcCosh[c*x])/(c^2*d^2*(-1 + c^2*x^2)) + (b*Sqrt[d - c^2*d*
x^2]*(Log[-1 + c*x] - Log[1 + c*x]))/(2*c^2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6329, 25, 39, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6329}$$

$$\frac{a+b\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{1}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{39}$$

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a+b\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCosh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

method	result
default	$\frac{a}{c^2 d\sqrt{-c^2 d\,x^2+d}}-\frac{b\sqrt{-d(c^2x^2-1)}\,\left(\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(1+cx+\sqrt{cx-1}\,\sqrt{cx+1})-\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(\sqrt{cx-1}\,\sqrt{cx+1}+cx-1)+\mathrm{arcc}\right.}{d^2c^2(c^2x^2-1)}$
parts	$\frac{a}{c^2 d\sqrt{-c^2 d\,x^2+d}}-\frac{b\sqrt{-d(c^2x^2-1)}\,\left(\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(1+cx+\sqrt{cx-1}\,\sqrt{cx+1})-\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(\sqrt{cx-1}\,\sqrt{cx+1}+cx-1)+\mathrm{arcc}\right.}{d^2c^2(c^2x^2-1)}$

input
```
int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*((c*x-1)^(1/2)*(c*x+
1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-(c*x-1)^(1/2)*(c*x+1)^(1/2)
*ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)+arccosh(c*x))/d^2/c^2/(c^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.30

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \Bigg[-\frac{4\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+(bc^2x^2-b)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4d\cdots}{\cdots}\right.}{4\left(c^4d^2x^2-c^2\cdots\right)}$$

$$-\frac{(bc^2x^2-b)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+2\sqrt{-c^2dx^2+\cdots}}{2\left(c^4d^2x^2-c^2d^2\right)}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2
- b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c
*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*
x^4 + 3*c^2*x^2 - 1)) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2),
 -1/2*((b*c^2*x^2 - b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2
- 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqr
t(c^2*x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2)]
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\,\text{acosh}\,(cx))}{\left(-d\,(cx-1)\,(cx+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*(((c*sqrt(d)*x + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/sqrt(-c*x + 1) + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d)/
sqrt(-c*x + 1))/(sqrt(c*x + 1)*c^3*d^2*x + (c*x + 1)*sqrt(c*x - 1)*c^2*d^2
) - integrate((c^2*x^3 + c*x^2*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)) - x
)/(sqrt(-c*x + 1)*((c^2*d^(3/2)*x^2 - d^(3/2))*e^(3/2*log(c*x + 1) + log(c
*x - 1)) + 2*(c^3*d^(3/2)*x^3 - c*d^(3/2)*x)*e^(log(c*x + 1) + 1/2*log(c*x
 - 1)) + (c^4*d^(3/2)*x^4 - c^2*d^(3/2)*x^2)*sqrt(c*x + 1))), x)) + a/(sqr
t(-c^2*d*x^2 + d)*c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

input
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-\sqrt{-c^{2} x^{2}+1}\, \left(\int \frac{acosh \left(c x \right) x}{\sqrt{-c^{2} x^{2}+1}\, c^{2} x^{2}-\sqrt{-c^{2} x^{2}+1}}d x \right) b \,c^{2}+a}{\sqrt{d}\, \sqrt{-c^{2} x^{2}+1}\, c^{2} d}$$

input
```
int(x*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c**2
*x**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**2 + a)/(sqrt(d)*sqrt( - c**2*x**2
+ 1)*c**2*d)
```

3.114 $\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{2cd\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*ln(-c^2*x^2+1)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\frac{2acx+2bcx\text{arccosh}(cx)-b\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{2cd\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(2*a*c*x + 2*b*c*x*ArcCosh[c*x] - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c
^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6314, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6314}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6314
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]   Int[x*((a
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(74) = 148$.

Time = 0.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.14

method	result
default	$\frac{ax}{d\sqrt{-c^2 d\,x^2+d}} - \frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)}{d^2c(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)x}{d^2(c^2x^2-1)} + \frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{c}}{d^2c}$
parts	$\frac{ax}{d\sqrt{-c^2 d\,x^2+d}} - \frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)}{d^2c(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)x}{d^2(c^2x^2-1)} + \frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{c}}{d^2c}$

input
```
int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*x/d/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)
/d^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^
2/c/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{a + b\operatorname{arccosh}(cx)}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{b\operatorname{arcosh}(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^
2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\,\text{acosh}\,(cx)}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{bc\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)}{2\,d} + \frac{bx\,\text{arcosh}\,(cx)}{\sqrt{-c^2dx^2+d}d} + \frac{ax}{\sqrt{-c^2dx^2+d}d}$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b*x*arccosh(c*x)/(sqrt(-c^2
*d*x^2 + d)*d) + a*x/(sqrt(-c^2*d*x^2 + d)*d)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\frac{-\sqrt{-c^2x^2+1}\,\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b+ax}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**
2 - sqrt( - c**2*x**2 + 1)),x)*b + a*x)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.115 $\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}\,dx = \frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
osh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arctanh(c*x)/d/(-c^2*d*x^2+d)^(1/2)+
I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d
^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.47

$$\int \frac{a + b\operatorname{arccosh}(cx)}{x (d - c^2dx^2)^{3/2}}\,dx =$$

$$-\frac{\frac{a\sqrt{d-c^2dx^2}}{-1+c^2x^2} - a\sqrt{d}\log(x) + a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{ibd\left(i\operatorname{arccosh}(cx)+\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\operatorname{arccosh}(cx)\log\left(1-\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-(((a*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a*Sqrt[d]*Log[x] + a*Sqrt[d]*L
og[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (I*b*d*(I*ArcCosh[c*x] + Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Cosh[ArcCosh[c*x]/2]] - I*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Sinh[ArcCosh[c*x]/2]] + Sqrt[(-1 + c*x)/(
1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6351, 25, 39, 219, 6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\operatorname{arccosh}(cx)}{x (d - c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6351}$$

$$\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d} - \frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}} + \frac{a+b\operatorname{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{39}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6361}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\left(\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)de^{\right.}}{d\sqrt{d-c^2dx^2}}$$
$$\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcc}}\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(a + b*ArcCosh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(2*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^Ar
cCosh[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/(d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(238) = 476$.

Time = 0.54 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.11

method	result
default	$\frac{a}{d\sqrt{-c^2 d\,x^2+d}} - \frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}} + \frac{b\sqrt{-d(c^2x^2-1)}\,(i\,\mathrm{arccosh}(cx)\ln(1+i(cx+\sqrt{cx-1}\,\sqrt{cx+1}))x^2c^2-i\,\mathrm{arccosh}(cx)}{}$
parts	$\frac{a}{d\sqrt{-c^2 d\,x^2+d}} - \frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}} + \frac{b\sqrt{-d(c^2x^2-1)}\,(i\,\mathrm{arccosh}(cx)\ln(1+i(cx+\sqrt{cx-1}\,\sqrt{cx+1}))x^2c^2-i\,\mathrm{arccosh}(cx)}{}$

input
```
int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/d/(-c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))
/x)+b/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arccosh(c*x)*l
n(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*x^2*c^2-I*arccosh(c*x)*ln(1-I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*x^2*c^2-I*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))*x^2*c^2+I*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*x^2*c^
2+ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)*x^2*c^2-ln(1+c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*x^2*c^2-I*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))+I*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-arccosh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)+I*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-
I*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-ln((c*x-1)^(1/2)*(c*x+1)^(1
/2)+c*x-1)+ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{a+b\mathrm{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = \int \frac{b\,\mathrm{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^
2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x\left(d - c^2dx^2\right)^{3/2}}\,dx = \int \frac{a + b\,\text{acosh}\,(cx)}{x\left(-d\,(cx - 1)\,(cx + 1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x\left(d - c^2dx^2\right)^{3/2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-a*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(s
qrt(-c^2*d*x^2 + d)*d)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
))/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^3-\sqrt{-c^2x^2+1}\,x}dx\right)b+\sqrt{-c^2x^2+1}\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**
3 - sqrt( - c**2*x**2 + 1)*x),x)*b + sqrt( - c**2*x**2 + 1)*log(tan(asin(c
*x)/2))*a - sqrt( - c**2*x**2 + 1)*a + a)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*
d)
```

3.116 $\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 157

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx = \frac{a+b\text{arccosh}(cx)}{dx\sqrt{d-c^2dx^2}} - \frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{d^2x} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\log(x)}{d\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{2d\sqrt{d-c^2dx^2}}$$

output

```
(a+b*arccosh(c*x))/d/x/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccosh(c*x))/d^2/x-b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(x)/d/(-c^2*d*x^2+d)^(
1/2)-1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-c^2*x^2+1)/d/(-c^2*d*x^2+d)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.73

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx = \frac{-2a+4ac^2x^2+2b(-1+2c^2x^2)\,\text{arccosh}(cx)-2bcx\sqrt{-1+cx}\sqrt{1+cx}\log(x)-}{2dx\sqrt{d-c^2dx^2}}$$

input

```
Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(-2*a + 4*a*c^2*x^2 + 2*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x] - 2*b*c*x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*Log[x] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c
^2*x^2])/(2*d*x*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6337, 25, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2 (d-c^2dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{1-2c^2x^2}{d^2x(1-c^2x^2)}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2c^2x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-2c^2x^2}{d^2x(1-c^2x^2)}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2c^2x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-2c^2x^2}{x(1-c^2x^2)}dx}{d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{2c^2x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-2c^2x^2}{x^2(1-c^2x^2)}dx^2}{2d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{2c^2x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{c^2}{c^2x^2-1}+\frac{1}{x^2}\right)dx^2}{2d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{2c^2x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2c^2x(a+b\operatorname{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\operatorname{arccosh}(cx)}{dx\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcCosh[c*x])/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCos
h[c*x]))/(d*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*(Log[x^2] + Lo
g[1 - c^2*x^2]))/(2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.69

method	result
default	$a\left(-\frac{1}{dx\sqrt{-c^2 d\,x^2+d}}+\frac{2c^2 x}{d\sqrt{-c^2 d\,x^2+d}}\right)-\frac{b\left(-2\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^4-1\right)x^3c^3+2\ln\left(\left(cx+\sqrt{cx-1}\,\sqrt{cx+\right.\right.\right.}{}$
parts	$a\left(-\frac{1}{dx\sqrt{-c^2 d\,x^2+d}}+\frac{2c^2 x}{d\sqrt{-c^2 d\,x^2+d}}\right)-\frac{b\left(-2\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^4-1\right)x^3c^3+2\ln\left(\left(cx+\sqrt{cx-1}\,\sqrt{cx+\right.\right.\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))-b*(-2*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*x^3*c^3+2
*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*x^4*c^4+(c*x-1)^(1/2)*(c*x+1)^(
1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*x*c-2*ln((c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^4-1)*x^2*c^2+arccosh(c*x))*(2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
c*x+2*c^2*x^2-1)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)/x
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^
2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2dx^2)^{3/2}}\,dx = \int \frac{a + b\,\text{acosh}\,(cx)}{x^2\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d - c^2dx^2)^{3/2}}\,dx = \frac{1}{2}bc\left(\frac{\sqrt{-d}\log(cx+1)}{d^2} + \frac{\sqrt{-d}\log(cx-1)}{d^2} + \frac{2\sqrt{-d}\log(x)}{d^2}\right) + \left(\frac{2\,c^2x}{\sqrt{-c^2dx^2+d}d} - \frac{1}{\sqrt{-c^2dx^2+d}dx}\right)b\,\text{arcosh}\,(cx) + \left(\frac{2\,c^2x}{\sqrt{-c^2dx^2+d}d} - \frac{1}{\sqrt{-c^2dx^2+d}dx}\right)a$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
1/2*b*c*(sqrt(-d)*log(c*x + 1)/d^2 + sqrt(-d)*log(c*x - 1)/d^2 + 2*sqrt(-d
)*log(x)/d^2) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d
)*d*x))*b*arccosh(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*
d*x^2 + d)*d*x))*a
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x^2\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-\sqrt{-c^2x^2+1}\,\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^4-\sqrt{-c^2x^2+1}\,x^2}dx\right)bx+2a\,c^2x^2-a}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,dx}$$

input
```
int((a+b*acosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**
4 - sqrt( - c**2*x**2 + 1)*x**2),x)*b*x + 2*a*c**2*x**2 - a)/(sqrt(d)*sqrt
( - c**2*x**2 + 1)*d*x)
```

3.117 $\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 329

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx = \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}}+\frac{3c^2(a+b\text{arccosh}(cx))}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}} - \frac{3c^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^2\sqrt{-1+cx}\sqrt{1+cx}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{3ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{2d^2\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(a+b*
arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))/d/x^2/(-c^2*d*
x^2+d)^(1/2)-3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*arctanh(c*x)/d/(-c^2*d*x^2+d)^(1/2)+3/2*I*b*c^2*(-c^2*d*x
^2+d)^(1/2)*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-3/2*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.43 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.33

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx = \frac{1}{2}\left(-\frac{a(-1+3c^2x^2)\sqrt{d-c^2dx^2}}{d^2x^2(-1+c^2x^2)}\right.$$
$$+\frac{3ac^2\log(x)}{d^{3/2}}-\frac{3ac^2\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{d^{3/2}}$$
$$-\frac{bc^2\left(-\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}{cx}+\left(-1+\frac{1}{c^2x^2}\right)\text{arccosh}(cx)-2\text{arccosh}(cx)\cosh^2\left(\frac{1}{2}\text{arccosh}(cx)\right)+3i\sqrt{\frac{-1+cx}{1+cx}}(1+cx\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(-((a*(-1 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^2*x^2*(-1 + c^2*x^2))) + (3
*a*c^2*Log[x])/d^(3/2) - (3*a*c^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^
(3/2) - (b*c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + (-1 + 1/
(c^2*x^2))*ArcCosh[c*x] - 2*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 + (3*I)*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]]
- (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcC
osh[c*x]] - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Cosh[ArcCosh[c*x]/2
]] + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Sinh[ArcCosh[c*x]/2]] + (3
*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] -
 (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] +
 2*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2))/(d*Sqrt[d - c^2*d*x^2]))/2
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.77, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6347, 25, 82, 264, 219, 6351, 25, 39, 219, 6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6347}$$

$$\frac{3}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{1}{x^2(1-cx)(cx+1)}dx}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x^2(1-cx)(cx+1)}dx}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{82}$$

$$\frac{3}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x^2(1-c^2x^2)}dx}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6351}$$

$$\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{39}$$

$$\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 219

$$\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6361

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3042

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\right.$$
$$\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 4668

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\text{ar}\right.}{d\sqrt{d-c^2dx^2}}\right.$$
$$\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(c}\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$
$$\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow 2838$$

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/2*(a + b*ArcCosh[c*x])/(d*x^2*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(-x^(-1) + c*ArcTanh[c*x]))/(2*d*Sqrt[d - c^2*d*x^2]) + (3
*c^2*((a + b*ArcCosh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(2*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I
)*E^ArcCosh[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/(d*Sqrt[d - c^2*d*x
^2])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.79

method	result
default	$a\left(-\frac{1}{2d\,x^{2}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3c^{2}\left(\frac{1}{d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\Big(-\frac{\sqrt{-d(c^{2}x^{2}-1)}\,(3c^{2}x^{2}\operatorname{arccosh}(cx)+\sqrt{}}{2d^{2}(c^{2}x^{2}-}$
parts	$a\left(-\frac{1}{2d\,x^{2}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3c^{2}\left(\frac{1}{d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\Big(-\frac{\sqrt{-d(c^{2}x^{2}-1)}\,(3c^{2}x^{2}\operatorname{arccosh}(cx)+\sqrt{}}{2d^{2}(c^{2}x^{2}-}$

input
```
int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(1/d/(-c^2*d*x^2+d)^(1/2)-1/d^(
3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)))+b*(-1/2*(-d*(c^2*x^2-1))
^(1/2)*(3*c^2*x^2*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-arccosh(c*x
))/d^2/(c^2*x^2-1)/x^2+(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(c^2*x^2-1)/d^2*ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)*c^2-(-d*(c^2*x^2-1))
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*ln(1+c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*c^2-3/2*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c
^2+3/2*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^
2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-3/2*I*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*dilog(1-I*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+3/2*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))*c^2)
```

Fricas [F]

$$\int \frac{a + b\mathrm{arccosh}(cx)}{x^3 (d - c^2dx^2)^{3/2}}\, dx = \int \frac{b\,\mathrm{arcosh}\,(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}}x^3}\, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^
2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\mathrm{arccosh}(cx)}{x^3 (d - c^2dx^2)^{3/2}}\, dx = \int \frac{a + b\,\mathrm{acosh}\,(cx)}{x^3\,(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d - c^2dx^2)^{3/2}}\, dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}} x^3}\, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
-1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2
) - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a + b
*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*
x^3), x)
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d - c^2dx^2)^{3/2}}\, dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}} x^3}\, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^3\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-8\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^5-\sqrt{-c^2x^2+1}\,x^3}dx\right)b\,x^2+12\sqrt{-c^2x^2+1}\log\left(\tan\left(\cdots\right.\right.}{8\sqrt{d}\,\sqrt{-c^2x^2+1}\,d\,x^2}$$

input
```
int((a+b*acosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 8*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x
**5 - sqrt( - c**2*x**2 + 1)*x**3),x)*b*x**2 + 12*sqrt( - c**2*x**2 + 1)*l
og(tan(asin(c*x)/2))*a*c**2*x**2 - 9*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 +
12*a*c**2*x**2 - 4*a)/(8*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**2)
```

3.118 $\int \frac{a+b\,\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 246

$$\int \frac{a+b\,\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx = \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2\sqrt{d-c^2dx^2}}+\frac{a+b\,\text{arccosh}(cx)}{dx^3\sqrt{d-c^2dx^2}} - \frac{4\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx))}{3d^2x^3}-\frac{8c^2\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx))}{3d^2x} - \frac{5bc^3\sqrt{-1+cx}\sqrt{1+cx}\log(x)}{3d\sqrt{d-c^2dx^2}}-\frac{bc^3\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{2d\sqrt{d-c^2dx^2}}$$

output
```
1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2/(-c^2*d*x^2+d)^(1/2)+(a+b*arccos
h(c*x))/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c
*x))/d^2/x^3-8/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/d^2/x-5/3*b*c
^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(x)/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*c^3*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*ln(-c^2*x^2+1)/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.65

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx = \frac{-2a-8ac^2x^2+16ac^4x^4+bcx\sqrt{-1+cx}\sqrt{1+cx}+2b(-1-4c^2x^2+8c^4x^4)\,\text{arc}}{6dx^3}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(-2*a - 8*a*c^2*x^2 + 16*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
2*b*(-1 - 4*c^2*x^2 + 8*c^4*x^4)*ArcCosh[c*x] - 10*b*c^3*x^3*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*Log[x] - 3*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 -
c^2*x^2])/(6*d*x^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6337, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-8c^4x^4+4c^2x^2+1}{3d^2x^3(1-c^2x^2)}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{4c^2(a+b\text{arccosh}(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{-8c^4x^4+4c^2x^2+1}{x^3(1-c^2x^2)}dx}{3d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{4c^2(a+b\text{arccosh}(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{-8c^4x^4+4c^2x^2+1}{x^4(1-c^2x^2)}dx^2}{6d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{4c^2(a+b\text{arccosh}(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{3c^4}{c^2x^2-1}+\frac{5c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{4c^2(a+b\text{arccosh}(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c^2(a+b\text{arccosh}(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(5c^2\log\left(x^2\right)+3c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)}{6d^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcCosh[c*x])/(d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcC
osh[c*x]))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x]))/(3
*d*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*(-x^(-2) + 5*c^2*Log[x^
2] + 3*c^2*Log[1 - c^2*x^2]))/(6*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.49

method	result
default	$a\left(-\frac{1}{3d\,x^{3}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{4c^{2}\left(-\frac{1}{dx\sqrt{-c^{2}d\,x^{2}+d}}+\frac{2c^{2}x}{d\sqrt{-c^{2}d\,x^{2}+d}}\right)}{3}\right)-\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(16\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{ar}\right.}{}$
parts	$a\left(-\frac{1}{3d\,x^{3}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{4c^{2}\left(-\frac{1}{dx\sqrt{-c^{2}d\,x^{2}+d}}+\frac{2c^{2}x}{d\sqrt{-c^{2}d\,x^{2}+d}}\right)}{3}\right)-\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(16\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{ar}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3/(-c^2*d*x^2+d)^(1/2)+4/3*c^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*
c^2/d*x/(-c^2*d*x^2+d)^(1/2)))-1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*(16*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4+16*arcc
osh(c*x)*c^5*x^5-6*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^5*c^5-10*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^5*c^5-8*arccosh(c*x)*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*c^2*x^2-16*c^3*x^3*arccosh(c*x)+6*ln((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2-1)*x^3*c^3+10*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^
3*c^3+c^3*x^3-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c*x)/d^2/(c^4*x^4
-2*c^2*x^2+1)/x^3
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^
2*x^6 + d^2*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^{3/2}} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) -
 1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a + b*integrate(log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^{3/2}} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d - c^2dx^2)^{3/2}} \, dx = \int \frac{a + b \operatorname{acosh}(c\,x)}{x^4 \, (d - c^2\,d\,x^2)^{3/2}} \, dx$$

input
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^6-\sqrt{-c^2x^2+1}\,x^4}dx\right)b\,x^3+8a\,c^4x^4-4a\,c^2x^2-a}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,d\,x^3}$$

input
```
int((a+b*acosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x
**6 - sqrt( - c**2*x**2 + 1)*x**4),x)*b*x**3 + 8*a*c**4*x**4 - 4*a*c**2*x**
*2 - a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**3)
```

3.119 $\int \frac{x^5(a+b\,\mathbf{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 232

$$\begin{aligned}\int \frac{x^5(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx &= -\frac{bx}{6c^5d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} \\ &- \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c^5d^2\sqrt{d-c^2dx^2}} + \frac{a+b\operatorname{arccosh}(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}} - \frac{2(a+b\operatorname{arccosh}(cx))}{c^6d^2\sqrt{d-c^2dx^2}} \\ &- \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{c^6d^3} - \frac{11b\sqrt{-1+cx}\sqrt{1+cx}\operatorname{arctanh}(cx)}{6c^6d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/6*b*x/c^5/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-b*x*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccosh(c*x)
)/c^6/d/(-c^2*d*x^2+d)^(3/2)-2*(a+b*arccosh(c*x))/c^6/d^2/(-c^2*d*x^2+d)^(
1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^6/d^3-11/6*b*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*arctanh(c*x)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.72

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{16a-24ac^2x^2+6ac^4x^4+5bcx\sqrt{-1+cx}\sqrt{1+cx}-6bc^3x^3\sqrt{-1+cx}\sqrt{1+\cdots}}{6c^6d^2\left(-\cdots\right.}$$

input
```
Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(16*a - 24*a*c^2*x^2 + 6*a*c^4*x^4 + 5*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
- 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(8 - 12*c^2*x^2 + 3*c^4*x
^4)*ArcCosh[c*x] - 11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTan
h[c*x])/(6*c^6*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6337, 27, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{3c^4x^4-12c^2x^2+8}{3c^6d^3(1-c^2x^2)^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^3}-\frac{2(a+b\text{arccosh}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{d-c^2dx^2}\int \frac{3c^4x^4-12c^2x^2+8}{(1-c^2x^2)^2}dx}{3c^5d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^3}-\frac{2(a+b\text{arccosh}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1471}$$

$$\frac{b\sqrt{d-c^2dx^2}\left(-\frac{1}{2}\int -\frac{17-6c^2x^2}{1-c^2x^2}dx-\frac{x}{2(1-c^2x^2)}\right)}{3c^5d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^3}-\frac{2(a+b\text{arccosh}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{d-c^2dx^2}\left(\frac{1}{2}\int \frac{17-6c^2x^2}{1-c^2x^2}dx-\frac{x}{2(1-c^2x^2)}\right)}{3c^5d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^3}-\frac{2(a+b\text{arccosh}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\sqrt{d-c^2dx^2}\left(\frac{1}{2}\left(11\int \frac{1}{1-c^2x^2}dx+6x\right)-\frac{x}{2(1-c^2x^2)}\right)}{3c^5d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^3}-\frac{2(a+b\text{arccosh}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{c^6d^3}-\frac{2(a+b\text{arccosh}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}}+\frac{b\left(\frac{1}{2}\left(\frac{11\text{arctanh}(cx)}{c}+6x\right)-\frac{x}{2(1-c^2x^2)}\right)\sqrt{d-c^2dx^2}}{3c^5d^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCosh[c*x])/(3*c^6*d*(d - c^2*d*x^2)^(3/2)) - (2*(a + b*ArcCosh[c
*x]))/(c^6*d^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(c^6*d^3) + (b*Sqrt[d - c^2*d*x^2]*(-1/2*x/(1 - c^2*x^2) + (6*x + (
11*ArcTanh[c*x])/c)/2))/(3*c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.72

method	result
default	$a\left(-\frac{x^4}{c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{4x^2}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}-\frac{8}{3dc^4\left(-c^2dx^2+d\right)^{\frac{3}{2}}}}{c^2}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,(6\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arc}$
parts	$a\left(-\frac{x^4}{c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{4x^2}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}-\frac{8}{3dc^4\left(-c^2dx^2+d\right)^{\frac{3}{2}}}}{c^2}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,(6\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arc}$

input
```
int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4/c^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2
/3/d/c^4/(-c^2*d*x^2+d)^(3/2)))-1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*(6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4-6*c^5*x
^5-11*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^4*x^4+11*ln((c*x-1)^(1/2)*(c
*x+1)^(1/2)+c*x-1)*c^4*x^4-24*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2
*x^2+11*c^3*x^3+22*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2-22*ln((c*
x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)*x^2*c^2+16*arccosh(c*x)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)-5*c*x-11*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+11*ln((c*x-1)^(1/
2)*(c*x+1)^(1/2)+c*x-1))/c^6/(c^6*x^6-3*c^4*x^4+3*c^2*x^2-1)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.28

$$\int \frac{x^5(a+b\operatorname{arccosh}(cx))}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\left[-\frac{8\,(3\,bc^4x^4-12\,bc^2x^2+8\,b)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+11\,(bc^4x}{}\right.$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/24*(8*(3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x
+ sqrt(c^2*x^2 - 1)) + 11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)*log(-(c^6
*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) -
 4*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 8*(3*a
*c^4*x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c^8
*d^3*x^2 + c^6*d^3), 1/12*(11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan
(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 4
*(3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 - 1)) + 2*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2
- 1) - 4*(3*a*c^4*x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^
3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^5}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
-1/9*b*(((9*c^4*sqrt(d)*x^4 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1)/sqrt(
-c*x + 1) - 3*(3*c^5*sqrt(d)*x^5 - 12*c^3*sqrt(d)*x^3 + 8*c*sqrt(d)*x + (3
*c^4*sqrt(d)*x^4 - 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x
- 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c*x + 1))/((c^8*d^3*x^2
 - c^6*d^3)*(c*x + 1)*sqrt(c*x - 1) + (c^9*d^3*x^3 - c^7*d^3*x)*sqrt(c*x +
 1)) + 9*integrate(1/9*(9*c^7*sqrt(d)*x^7 - 45*c^5*sqrt(d)*x^5 + 60*c^3*sq
rt(d)*x^3 - 24*c*sqrt(d)*x + (9*c^6*sqrt(d)*x^6 - 54*c^4*sqrt(d)*x^4 + 60*
c^2*sqrt(d)*x^2 - 16*sqrt(d))*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)))/(sq
rt(-c*x + 1)*((c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)*e^(3/2*log(c*x + 1)
+ log(c*x - 1)) + 2*(c^10*d^3*x^5 - 2*c^8*d^3*x^3 + c^6*d^3*x)*e^(log(c*x
+ 1) + 1/2*log(c*x - 1)) + (c^11*d^3*x^6 - 2*c^9*d^3*x^4 + c^7*d^3*x^2)*sq
rt(c*x + 1))), x)) - 1/3*a*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/
((-c^2*d*x^2 + d)^(3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^5}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^8x^2-3\sqrt{-c^2x^2}}{3\sqrt{d}\,\sqrt{-c^2x^2+}}$$

input
```
int(x^5*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**5)/(sqrt( - c**2*x**2 + 1)*c*
*4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*
b*c**8*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**5)/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*b*c**6 + 3*a*c**4*x**4 - 12*a*c**2*x**2 + 8*a)/(3*sqrt(d)*sqr
t( - c**2*x**2 + 1)*c**6*d**2*(c**2*x**2 - 1))
```

3.120 $\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{b}{6c^5d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\text{arccosh}(cx))}{3c^2d\left(d-c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arccosh}(cx))}{c^4d^2\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2bc^5d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2b\sqrt{-1+cx}\sqrt{1+cx}\log\left(1-c^2x^2\right)}{3c^5d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b/c^5/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a
+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)-x*(a+b*arccosh(c*x))/c^4/d^2/(
-c^2*d*x^2+d)^(1/2)-1/2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/b/c^5/d^
3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-c^2*x^
2+1)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\frac{2acx(-3+4c^2x^2)\sqrt{d-c^2dx^2}}{(-1+c^2x^2)^2} - 6a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right) + \frac{bd\left(-8cx\operatorname{arccosh}(cx) - \sqrt{\cdots}\right.}{\cdots}}{6c^5d^3\cdots}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
((2*a*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 6*a*Sqr
t[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*d*(-8
*c*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[
c*x])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh[c*x
]^2 + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^2])/
(6*c^5*d^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6349, 82, 243, 49, 2009, 6349, 25, 82, 240, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6349}$$

$$-\frac{\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d} + \frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3}{(1-cx)^2(cx+1)^2}dx}{3cd^2\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\operatorname{arccosh}(cx))}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{82}$$

$$-\frac{\int \frac{x^2(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\,\text{arccosh}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 243

$$-\frac{\int \frac{x^2(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2}{(1-c^2x^2)^2}dx^2}{6cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\,\text{arccosh}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 49

$$-\frac{\int \frac{x^2(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \left(\frac{1}{c^2(c^2x^2-1)}+\frac{1}{c^2(c^2x^2-1)^2}\right)dx^2}{6cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\,\text{arccosh}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 2009

$$-\frac{\int \frac{x^2(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{x^3(a+b\,\text{arccosh}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 6349

$$-\frac{-\frac{\int \frac{a+b\,\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{x}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\,\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\,\text{arccosh}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int \frac{a+b\,\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\,\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\,\text{arccosh}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 82

$$-\frac{-\frac{\int \frac{a+b\,\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\,\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\,\text{arccosh}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$-\frac{-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6307}$$

$$\frac{x^3(a+b\text{arccosh}(cx))}{3c^2d(d-c^2dx^2)^{3/2}}-\frac{\frac{x(a+b\text{arccosh}(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCosh[c*x]))/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(1/(c^4*(1 - c^2*x^2)) + Log[1 - c^2*x^2]/c^4))/(6*c*d^
2*Sqrt[d - c^2*d*x^2]) - ((x*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x
^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sq
rt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*
c^3*d*Sqrt[d - c^2*d*x^2]))/(c^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.63

method	result
default	$\frac{a\,x^{3}}{3c^{2}d\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}-\frac{ax}{c^{4}d^{2}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{c^{4}d^{2}\sqrt{c^{2}d}}-\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(3\operatorname{arccosh}\left(cx\right)^{2}x^{4}c^{4}-\right.}{}$
parts	$\frac{a\,x^{3}}{3c^{2}d\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}-\frac{ax}{c^{4}d^{2}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{c^{4}d^{2}\sqrt{c^{2}d}}-\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(3\operatorname{arccosh}\left(cx\right)^{2}x^{4}c^{4}-\right.}{}$

input
```
int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+a/c^
4/d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/6*b*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^6*x^6-3*c^4*x^4+3*c^2*x
^2-1)/d^3/c^5*(3*arccosh(c*x)^2*x^4*c^4-8*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3-8*arccosh(c*x)*c^4*x^4+8*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2-1)*x^4*c^4-6*arccosh(c*x)^2*x^2*c^2+6*arccosh(c*x)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*c*x+16*c^2*x^2*arccosh(c*x)-16*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2-1)*x^2*c^2-c^2*x^2+3*arccosh(c*x)^2-8*arccosh(c*x)+8*ln((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2-1)+1)
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 -
 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^4(a+b\,\text{acosh}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c
^4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a
 + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)
^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^4\,(a+b\,\text{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\frac{3\sqrt{-c^2x^2+1}\,asin(cx)\,a\,c^2x^2-3\sqrt{-c^2x^2+1}\,asin(cx)\,a+3\sqrt{-c^2x^2+1}\left(\int\right.}{}$$

input
```
int(x^4*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*asin(c*x)*a*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)
*asin(c*x)*a + 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**4)/(sqrt( - c**
2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*
x**2 + 1)),x)*b*c**7*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**4)
/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 +
sqrt( - c**2*x**2 + 1)),x)*b*c**5 - 4*a*c**3*x**3 + 3*a*c*x)/(3*sqrt(d)*sq
rt( - c**2*x**2 + 1)*c**5*d**2*(c**2*x**2 - 1))
```

3.121 $\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 158

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{bx}{6c^3d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{a+b\text{arccosh}(cx)}{3c^4d\,(d-c^2dx^2)^{3/2}} - \frac{a+b\text{arccosh}(cx)}{c^4d^2\sqrt{d-c^2dx^2}} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\text{arctanh}(cx)}{6c^4d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b*x/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b
*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(3/2)-(a+b*arccosh(c*x))/c^4/d^2/(-c^2
*d*x^2+d)^(1/2)-5/6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arctanh(c*x)/c^4/d^2/(-c
^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{4a-6ac^2x^2-bcx\sqrt{-1+cx}\sqrt{1+cx}+b(4-6c^2x^2)\,\text{arccosh}(cx)-5b\sqrt{-1+}}{6c^4d^2\,(-1+c^2x^2)\,\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(4 - 6*c^2*x^2
)*ArcCosh[c*x] - 5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTanh[c
*x])/(6*c^4*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6337, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{2-3c^2x^2}{3c^4d^3(1-c^2x^2)^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^4d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{d-c^2dx^2}\int \frac{2-3c^2x^2}{(1-c^2x^2)^2}dx}{3c^3d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^4d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{b\sqrt{d-c^2dx^2}\left(\frac{5}{2}\int \frac{1}{1-c^2x^2}dx-\frac{x}{2(1-c^2x^2)}\right)}{3c^3d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^4d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a+b\text{arccosh}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^4d\,(d-c^2dx^2)^{3/2}}+\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)\sqrt{d-c^2dx^2}}{3c^3d^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCosh[c*x])/(3*c^4*d*(d - c^2*d*x^2)^(3/2)) - (a + b*ArcCosh[c*x]
)/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[d - c^2*d*x^2]*(-1/2*x/(1 - c^2*
x^2) + (5*ArcTanh[c*x])/(2*c)))/(3*c^3*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.63

method	result
default	$a\left(\frac{x^{2}}{c^{2} d \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}-\frac{2}{3 d \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\right)+b\left(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(6 c^{2} x^{2} \operatorname{arccosh}\left(c x \right)+\sqrt{c x -1}\, \sqrt{c x +1}\, c x -4 \,\operatorname{arccosh}\left(c x \right)\right)}{6 \left(c^{2} x^{2}-1\right)^{2} d^{3} c^{4}}\right.$
parts	$a\left(\frac{x^{2}}{c^{2} d \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}-\frac{2}{3 d \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\right)+b\left(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(6 c^{2} x^{2} \operatorname{arccosh}\left(c x \right)+\sqrt{c x -1}\, \sqrt{c x +1}\, c x -4 \,\operatorname{arccosh}\left(c x \right)\right)}{6 \left(c^{2} x^{2}-1\right)^{2} d^{3} c^{4}}\right.$

input
```
int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b*(1/6*(
-d*(c^2*x^2-1))^(1/2)*(6*c^2*x^2*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(1/2)*
c*x-4*arccosh(c*x))/(c^2*x^2-1)^2/d^3/c^4+5/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))-5/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2
*x^2-1)*ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.97

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx+8\left(3\,bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c\right.}}{}\right.$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
[1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*(3*b*c^2*x^2 - 2
*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^4*x^4 - 2*b
*c^2*x^2 + b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^
3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6
 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) + 8*(3*a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d
))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3), 1/12*(2*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1)*b*c*x + 5*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*
sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3
*b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(3
*a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4
*d^3)]
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^3(a+b\,\text{acosh}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{1}{12}\,bc\left(\frac{2\,\sqrt{-d}x}{c^6d^3x^2-c^4d^3}+\frac{5\,\sqrt{-d}\log\,(cx+1)}{c^5d^3}-\frac{5\,\sqrt{-d}\log\,(cx-1)}{c^5d^3}\right) + \frac{1}{3}\,b\left(\frac{3\,x^2}{(-c^2dx^2+d)^{\frac{3}{2}}c^2d}-\frac{2}{(-c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)\text{arcosh}\,(cx) + \frac{1}{3}\,a\left(\frac{3\,x^2}{(-c^2dx^2+d)^{\frac{3}{2}}c^2d}-\frac{2}{(-c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
1/12*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/(
c^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 1/3*b*(3*x^2/((-c^2*d*x^2
+ d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/3*a
*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\operatorname{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^3}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^6x^2-3\sqrt{-c^2x^2}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^4d}$$

input
```
int(x^3*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)*c*
*4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*
b*c**6*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**3)/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*b*c**4 - 3*a*c**2*x**2 + 2*a)/(3*sqrt(d)*sqrt( - c**2*x**2 +
1)*c**4*d**2*(c**2*x**2 - 1))
```

3.122 $\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{b}{6c^3d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{6c^3d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a
+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
ln(-c^2*x^2+1)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.76

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(-\frac{2x^3(a+b\text{arccosh}(cx))}{(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b\left(\frac{1}{1-c^2x^2}+\log(1-c^2x^2)\right)}{c^3}\right)}{6d^2\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((-2*x^3*(a + b*ArcCosh[c*x]))/((-1 + c*x)^(
3/2)*(1 + c*x)^(3/2)) + (b*((1 - c^2*x^2)^(-1) + Log[1 - c^2*x^2]))/c^3))/
(6*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6332, 82, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6332}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{82}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2}{(1-c^2x^2)^2}dx^2}{6d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \left(\frac{1}{c^2(c^2x^2-1)}+\frac{1}{c^2(c^2x^2-1)^2}\right)dx^2}{6d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6d^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (b*c*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(1/(c^4*(1 - c^2*x^2)) + Log[1 - c^2*x^2]/c^4))/(6*d^2*Sq
rt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(113) = 226$.

Time = 0.49 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.44

method	result
default	$a\left(\frac{x}{2c^2 d(-c^2 d\,x^2+d)^{\frac{3}{2}}}-\frac{\frac{x}{3d\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2 d\,x^2+d}}}{2c^2}\right)+$ $b\sqrt{-d(c^2x^2-1)}\,(c^3x^3+\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2-\sqrt{cx-1}\,\sqrt{cx+$
parts	$a\left(\frac{x}{2c^2 d(-c^2 d\,x^2+d)^{\frac{3}{2}}}-\frac{\frac{x}{3d\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2 d\,x^2+d}}}{2c^2}\right)+$ $b\sqrt{-d(c^2x^2-1)}\,(c^3x^3+\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2-\sqrt{cx-1}\,\sqrt{cx+$

input

```
int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
a*(1/2*x/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/2/c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+
2/3/d^2*x/(-c^2*d*x^2+d)^(1/2)))+1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c^3*x^3+(c*
x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^5*c^5+2*ln
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^6*c^6+6*arccosh(c*x)*c^4*x^4+2*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^3*c
^3-6*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^4*c^4+c^3*x^3*(c*x-1)^(1/
2)*(c*x+1)^(1/2)-c^4*x^4-6*c^2*x^2*arccosh(c*x)+6*ln((c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))^2-1)*x^2*c^2+2*c^2*x^2+2*arccosh(c*x)-2*ln((c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2-1)-1)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3/c
^3
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input

```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 -
 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{5/2}}\,dx = \int \frac{x^2(a + b\,\text{acosh}\,(cx))}{(-d\,(cx - 1)\,(cx + 1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.27

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{5/2}}\,dx = \frac{1}{6}\,bc\left(\frac{\sqrt{-d}}{c^6d^3x^2 - c^4d^3} - \frac{\sqrt{-d}\log\,(cx + 1)}{c^4d^3} - \frac{\sqrt{-d}\log\,(cx - 1)}{c^4d^3}\right) - \frac{1}{3}\,b\left(\frac{x}{\sqrt{-c^2dx^2 + d}c^2d^2} - \frac{x}{(-c^2dx^2 + d)^{\frac{3}{2}}c^2d}\right)\text{arcosh}\,(cx) - \frac{1}{3}\,a\left(\frac{x}{\sqrt{-c^2dx^2 + d}c^2d^2} - \frac{x}{(-c^2dx^2 + d)^{\frac{3}{2}}c^2d}\right)$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
1/6*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d^3
) - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 1/3*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*
d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a*(x/(sqrt(-c^
2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{acosh}(c\,x))}{{(d-c^2\,d\,x^2)}^{5/2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^2}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^2x^2-3\sqrt{-c^2x^2}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,d^2\,(c^2x^2}$$

input
```
int(x^2*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**2)/(sqrt( - c**2*x**2 + 1)*c*
*4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*
b*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**2)/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*b - a*x**3)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*(c**2*x**2
 - 1))
```

3.123 $\int \frac{x(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{x(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{bx}{6cd^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{a+b\,\text{arccosh}(cx)}{3c^2d\,(d-c^2dx^2)^{3/2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\,\text{arctanh}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b*x/c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*a
rccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
arctanh(c*x)/c^2/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.28

$$\int \frac{x(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(-2bcx+2bc^3x^3+4a\sqrt{-1+cx}\sqrt{1+cx}+4b\sqrt{-1+cx}\sqrt{1+cx}\right.}{12c}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-2*b*c*x + 2*b*c^3*x^3 + 4*a*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x] + 4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x] + b*(-1 + c^2*x^2)^2
*Log[-1 + c*x] - b*Log[1 + c*x] + 2*b*c^2*x^2*Log[1 + c*x] - b*c^4*x^4*Log
[1 + c*x]))/(12*c^2*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6329, 39, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6329}$$

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)^2(cx+1)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{39}$$

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a+b\text{arccosh}(cx)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCosh[c*x])/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*c*d^2*Sqrt[d -
 c^2*d*x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(107) = 214$.

Time = 0.43 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.73

method	result
default	$\frac{a}{3c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}+b\Bigg(\frac{\sqrt{-d(c^2x^2-1)}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+2\,\mathrm{arccosh}(cx))}{6(c^2x^2-1)^2d^3c^2}+\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(\sqrt{cx-1}\,\sqrt{cx-}}{6d^3c^2(c^2x^2-1)}$
parts	$\frac{a}{3c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}+b\Bigg(\frac{\sqrt{-d(c^2x^2-1)}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+2\,\mathrm{arccosh}(cx))}{6(c^2x^2-1)^2d^3c^2}+\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(\sqrt{cx-1}\,\sqrt{cx-}}{6d^3c^2(c^2x^2-1)}$

input
```
int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/c^2/d/(-c^2*d*x^2+d)^(3/2)+b*(1/6*(-d*(c^2*x^2-1))^(1/2)*((c*x-1)^(1
/2)*(c*x+1)^(1/2)*c*x+2*arccosh(c*x))/(c^2*x^2-1)^2/d^3/c^2+1/6*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln((c*x-1)^(1
/2)*(c*x+1)^(1/2)+c*x-1)-1/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.31

$$\int \frac{x(a+b\,\mathrm{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx+8\sqrt{-c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2-1}\right)-(bc}{2}\right.$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*sqrt(-c^2*d*x^2
+ d)*b*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-
d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2
*x^2 - 1)) + 8*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*
d^3), 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^4*x^4 -
2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c
*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*
x^2 - 1)) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d
^3)]
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x(a+b\,\text{acosh}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2
), x) + 1/3*a/((-c^2*d*x^2 + d)^(3/2)*c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{x\,\left(a+b\,\text{acosh}(c\,x)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

input
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\frac{3\sqrt{-c^2x^2+1}\,\left(\int \frac{acosh(cx)x}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^4x^2-3\sqrt{-c^2x^2}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^2d^2\,(c^2x^2}$$

input
```
int(x*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c**4*
x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b*c
**4*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x)/(sqrt( - c**2*x**2
+ 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 +
1)),x)*b*c**2 - a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**2*d**2*(c**2*x**2
- 1))
```

3.124 $\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{b}{6cd^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{x(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}} + \frac{2x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{3cd^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b/c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*a
rccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arccosh(c*x))/d^2/(-c^2*d*x
^2+d)^(1/2)-1/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-c^2*x^2+1)/c/d^2/(-c^2*d
*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \frac{-6acx+4ac^3x^3-b\sqrt{-1+cx}\sqrt{1+cx}+2bcx(-3+2c^2x^2)\,\text{arccosh}(cx)-2b\sqrt{-}}{6cd^2(-1+c^2x^2)\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(-6*a*c*x + 4*a*c^3*x^3 - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*c*x*(-3 + 2
*c^2*x^2)*ArcCosh[c*x] - 2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*L
og[1 - c^2*x^2])/(6*c*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6316, 82, 241, 6314, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6316}$$

$$\frac{2\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{82}$$

$$\frac{2\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6314}$$

$$\frac{2\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+$$
$$\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2
]) + (x*(a + b*ArcCosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*((x*(a + b*
ArcCosh[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*L
og[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])))/(3*d)
```

Defintions of rubi rules used

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6314
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]   Int[x*((a
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(138) = 276$.

Time = 0.00 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.94

method	result
default	$a\left(\frac{x}{3d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2d\,x^2+d}}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-3cx+2\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2-2\sqrt{cx-1}\,\sqrt{cx+1})\,(8\sqrt{cx-1}}{}$
parts	$a\left(\frac{x}{3d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2d\,x^2+d}}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-3cx+2\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2-2\sqrt{cx-1}\,\sqrt{cx+1})\,(8\sqrt{cx-1}}{}$

input
```
int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))-1/6*b*(-d*
(c^2*x^2-1))^(1/2)*(2*c^3*x^3-3*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-
2*(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^5*c^5-8*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2-1)*x^6*c^6-20*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2-1)*x^3*c^3+24*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^4*c^
4+2*c^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c^4*x^4+6*c^2*x^2*arccosh(c*x)+1
2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x*
c-24*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^2*c^2-3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*c*x+4*c^2*x^2-8*arccosh(c*x)+8*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2-1)-2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d
^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\text{acosh}\,(cx)}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \frac{1}{6}\,bc\left(\frac{\sqrt{-d}}{c^4d^3x^2-c^2d^3}+\frac{2\,\sqrt{-d}\log\,(cx+1)}{c^2d^3}+\frac{2\,\sqrt{-d}\log\,(cx-1)}{c^2d^3}\right) + \frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}d^2}+\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}d}\right)\text{arcosh}\,(cx) + \frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}d^2}+\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}d}\right)$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/6*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2*d
^3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 1/3*b*(2*x/(sqrt(-c^2*d*x^2 + d
)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a*(2*x/(sqrt(-c^
2*d*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^2x^2 - 3\sqrt{-c^2x^2+1}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,d^2\,(c^2x}$$

input
```
int((a+b*acosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b*c**2*
x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b
 + 2*a*c**2*x**3 - 3*a*x)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*(c**2*x**
2 - 1))
```

3.125 $\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 305

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{5/2}}\,dx &= -\frac{bcx}{6d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} \\ &+ \frac{a+b\text{arccosh}(cx)}{3d(d-c^2dx^2)^{3/2}} + \frac{a+b\text{arccosh}(cx)}{d^2\sqrt{d-c^2dx^2}} \\ &- \frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\arctan\left(e^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} \\ &+ \frac{7b\sqrt{-1+cx}\sqrt{1+cx}\text{arctanh}(cx)}{6d^2\sqrt{d-c^2dx^2}} + \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} \\ &- \frac{ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/6*b*c*x/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*a
rccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^
(1/2)-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+7/6*b*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*arctanh(c*x)/d^2/(-c^2*d*x^2+d)^(1/2)+I*b*(-c^2*d*x^2+d)^(1/2)*polyl
og(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d
^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.23

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = -\frac{a(-4+3c^2x^2)\sqrt{d-c^2dx^2}}{3d^3\left(-1+c^2x^2\right)^2} + \frac{a\log(x)}{d^{5/2}} - \frac{a\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{d^{5/2}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(14\operatorname{arccosh}(cx)\coth\left(\frac{1}{2}\operatorname{arccosh}(cx)\right)-\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arccosh}(cx)\right)-\frac{1}{2}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\operatorname{arccos}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(a*(-4 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^3*(-1 + c^2*x^2)^2) + (a*
Log[x])/d^(5/2) - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^(5/2) + (b*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCosh[c*x]*Coth[ArcCosh[c*x]/2] -
 Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*
x]*Csch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x
]] + (24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + 28*Log[Cosh[ArcCosh[c
*x]/2]] - 28*Log[Sinh[ArcCosh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcCosh[
c*x]] + (24*I)*PolyLog[2, I/E^ArcCosh[c*x]] - Sech[ArcCosh[c*x]/2]^2 - (8*
ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*
x)^3) - 14*ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2]))/(24*d^2*Sqrt[d - c^2*d*x^2]
)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6351, 39, 215, 219, 6351, 25, 39, 219, 6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6351}$$

$$\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\operatorname{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 39}$$

$$\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\operatorname{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\operatorname{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{a+b\operatorname{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\operatorname{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 6351}$$

$$\frac{\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\operatorname{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\operatorname{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\operatorname{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{a+b\operatorname{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\operatorname{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\operatorname{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\operatorname{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 39}$$

$$\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6361}$$

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int \log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int \log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2\arctan\left(e^{\text{arc}}\right.\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(c}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

input

```
Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output

```
(a + b*ArcCosh[c*x])/(3*d*(d - c^2*d*x^2)^(3/2)) + (b*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*d^2*Sqrt[d - c^2
*d*x^2]) + ((a + b*ArcCosh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(2*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[
2, (-I)*E^ArcCosh[c*x]] + I*b*PolyLog[2, I*E^ArcCosh[c*x]]))/(d*Sqrt[d - c
^2*d*x^2]))/d
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.84

method	result
default	$\frac{a}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{-c^2dx^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(6c^2x^2\,\mathrm{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx+}}{6(c^2x^2-1)^2d^3}\right.$
parts	$\frac{a}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{-c^2dx^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(6c^2x^2\,\mathrm{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx+}}{6(c^2x^2-1)^2d^3}\right.$

input
```
int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/d/(-c^2*d*x^2+d)^(3/2)+a/d^2/(-c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+
2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+b*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(6*c^2*x
^2*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-8*arccosh(c*x))/(c^2*x^2-1
)^2/d^3+7/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^
2-1)*ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)-7/6*(-d*(c^2*x^2-1))^(1/2)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))-I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*a
rccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2)))+I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^3/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+I*(-d*(c^2*x^2
-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d
^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{acosh}\left(cx\right)}{x\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acosh(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\operatorname{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2)
- 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + b*integra
te(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\operatorname{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^5-2\sqrt{-c^2x^2+1}\,c^2x^3+\sqrt{-c^2x^2+1}\,x}dx\right)b\,c^2x^2-3\sqrt{-c^2x^2+}}{}$$

input
```
int((a+b*acosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**5
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x),x)*b*c**
2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c
**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x),
x)*b + 3*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**2*x**2 - 3*sqrt
( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a - 4*sqrt( - c**2*x**2 + 1)*a*c*
*2*x**2 + 4*sqrt( - c**2*x**2 + 1)*a + 3*a*c**2*x**2 - 4*a)/(3*sqrt(d)*sqr
t( - c**2*x**2 + 1)*d**2*(c**2*x**2 - 1))
```

3.126 $\int \frac{a+b\operatorname{arccosh}(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 237

$$\begin{aligned}\int \frac{a+b\operatorname{arccosh}(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx &= -\frac{bc}{6d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} \\ &\quad+\frac{a+b\operatorname{arccosh}(cx)}{3dx(d-c^2dx^2)^{3/2}}+\frac{4(a+b\operatorname{arccosh}(cx))}{3d^2x\sqrt{d-c^2dx^2}}-\frac{8\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{3d^3x} \\ &\quad-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}\log(x)}{d^2\sqrt{d-c^2dx^2}}-\frac{5bc\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{6d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output

```
-1/6*b*c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arc
cosh(c*x))/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*(a+b*arccosh(c*x))/d^2/x/(-c^2*d*x
^2+d)^(1/2)-8/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/d^3/x-b*c*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*ln(x)/d^2/(-c^2*d*x^2+d)^(1/2)-5/6*b*c*(c*x-1)^(1/2)*(
c*x+1)^(1/2)*ln(-c^2*x^2+1)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx=\frac{\sqrt{-1+cx}\sqrt{1+cx}\left(\frac{a+b\text{arccosh}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{4c^2x(a+b\text{arccosh}(cx))}{3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{8c^2x(a+b\text{arccosh}(cx}{3\sqrt{-1+cx}\sqrt{1+cx}}\right.}{d^2\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2)) - (4*c^2*x*(a + b*ArcCosh[c*x]))/(3*(-1 + c*x)^(3/2)*(1 + c
*x)^(3/2)) + (8*c^2*x*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - b*c*(1/(6*(-1 + c^2*x^2)) + Log[x] + (5*Log[1 - c^2*x^2])/6)))/(d^2*S
qrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6337, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 6337$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{8c^4x^4-12c^2x^2+3}{3d^3x\left(1-c^2x^2\right)^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{8c^2x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{8c^4x^4-12c^2x^2+3}{x(1-c^2x^2)^2}dx}{3d^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{8c^2x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{dx(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{8c^4x^4-12c^2x^2+3}{x^2(1-c^2x^2)^2}dx^2}{6d^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{8c^2x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{dx(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{5c^2}{c^2x^2-1}-\frac{c^2}{(c^2x^2-1)^2}+\frac{3}{x^2}\right)dx^2}{6d^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{8c^2x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{dx(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8c^2x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\text{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{dx(d-c^2dx^2)^{3/2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{c^2x^2-1}+5\log(1-c^2x^2)+3\log(x^2)\right)}{6d^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcCosh[c*x])/(d*x*(d - c^2*d*x^2)^(3/2))) + (4*c^2*x*(a + b*ArcC
osh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcCosh[c*x]))/(3
*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((-1 + c^2*x^2)^(-1)
+ 3*Log[x^2] + 5*Log[1 - c^2*x^2]))/(6*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(203) = 406$.

Time = 0.53 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.83

method	result
default	$a\left(-\frac{1}{dx(-c^2d\,x^2+d)^{\frac{3}{2}}}+4c^2\left(\frac{x}{3d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2d\,x^2+d}}\right)\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(16\sqrt{cx-1}\,\sqrt{c}\right.}{}$
parts	$a\left(-\frac{1}{dx(-c^2d\,x^2+d)^{\frac{3}{2}}}+4c^2\left(\frac{x}{3d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2d\,x^2+d}}\right)\right)-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(16\sqrt{cx-1}\,\sqrt{c}\right.}{}$

input
```
int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x/(-c^2*d*x^2+d)^(3/2)+4*c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2
*x/(-c^2*d*x^2+d)^(1/2)))-1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)*(16*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4+16*arccosh(c
*x)*c^5*x^5-6*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^5*c^5-10*ln((c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^5*c^5-24*arccosh(c*x)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*c^2*x^2-32*c^3*x^3*arccosh(c*x)+12*ln(1+(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))^2)*x^3*c^3+20*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^3*c
^3-c^3*x^3+6*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+16*c*x*arccosh(c*x)-
6*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c-10*ln((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2-1)*x*c+c*x)/d^3/(c^6*x^6-3*c^4*x^4+3*c^2*x^2-1)/x
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d
^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
1/3*a*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/2
)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{a+b\,\text{acosh}(c\,x)}{x^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^6-2\sqrt{-c^2x^2+1}\,c^2x^4+\sqrt{-c^2x^2+1}\,x^2}dx\right)b\,c^2x^3-3\sqrt{-c^2x^2+}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}}$$

input
```
int((a+b*acosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**6
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1)*x**2),x)*b*
c**2*x**3 - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1
)*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1)*
x**2),x)*b*x + 8*a*c**4*x**4 - 12*a*c**2*x**2 + 3*a)/(3*sqrt(d)*sqrt( - c*
*2*x**2 + 1)*d**2*x*(c**2*x**2 - 1))
```

3.127 $\int \frac{a+b\operatorname{arccosh}(cx)}{x^3(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 410

$$\begin{aligned}\int \frac{a+b\operatorname{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx &= -\frac{bc^3x}{6d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} \\ &+ \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x\sqrt{d-c^2dx^2}} + \frac{5c^2(a+b\operatorname{arccosh}(cx))}{6d\left(d-c^2dx^2\right)^{3/2}} - \frac{a+b\operatorname{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}} \\ &+ \frac{5c^2(a+b\operatorname{arccosh}(cx))}{2d^2\sqrt{d-c^2dx^2}} - \frac{5c^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} \\ &+ \frac{13bc^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{arctanh}(cx)}{6d^2\sqrt{d-c^2dx^2}} + \frac{5ibc^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(cx)}\right)}{2d^3\sqrt{-1+cx}\sqrt{1+cx}} \\ &- \frac{5ibc^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(cx)}\right)}{2d^3\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/6*b*c^3*x/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/2*b*c*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x/(-c^2*d*x^2+d)^(1/2)+5/6*c^2*(a+b*arccos
h(c*x))/d/(-c^2*d*x^2+d)^(3/2)-1/2*(a+b*arccosh(c*x))/d/x^2/(-c^2*d*x^2+d)
^(3/2)+5/2*c^2*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)-5*c^2*(-c^2*d*x
^2+d)^(1/2)*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+13/6*b*c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arctan
h(c*x)/d^2/(-c^2*d*x^2+d)^(1/2)+5/2*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(2
,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/2
*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 7.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.22

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{-\frac{2a\sqrt{d-c^2dx^2}\left(3-20c^2x^2+15c^4x^4\right)}{x^2\left(-1+c^2x^2\right)^2}+30ac^2\sqrt{d}\log(x)-30ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
((-2*a*Sqrt[d - c^2*d*x^2]*(3 - 20*c^2*x^2 + 15*c^4*x^4))/(x^2*(-1 + c^2*x
^2)^2) + 30*a*c^2*Sqrt[d]*Log[x] - 30*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 - c^2*d*x^2]] + (b*c^2*d*((6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)
+ 6*(1 - 1/(c^2*x^2))*ArcCosh[c*x] + 26*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^
2 - Coth[ArcCosh[c*x]/2] - ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]^2 - (30*I)*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]]
+ (30*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^Arc
Cosh[c*x]] + 26*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Cosh[ArcCosh[c*x]
/2]] - 26*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Sinh[ArcCosh[c*x]/2]] -
 (30*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x
]] + (30*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*
x]] - 26*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2 - Tanh[ArcCosh[c*x]/2] - ArcC
osh[c*x]*Tanh[ArcCosh[c*x]/2]^2))/Sqrt[d - c^2*d*x^2])/(12*d^3)
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6347, 82, 253, 264, 219, 6351, 39, 215, 219, 6351, 25, 39, 219, 6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 \left(d-c^2dx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{6347}$$

$$\frac{5}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x^2(1-cx)^2(cx+1)^2}dx}{2d^2\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{82}$$

$$\frac{5}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{x^2(1-c^2x^2)^2}dx}{2d^2\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{5}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\int \frac{1}{x^2(1-c^2x^2)}dx+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{2}c^2\int \frac{a+b\text{arccosh}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 6351$$

$$\frac{5}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 39$$

$$\frac{5}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 215$$

$$\frac{5}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{5}{2}c^2\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}\right)-\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 6351$$

$$\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx}}{3}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+}}{3c}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 39$$

$$\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\Big(}{3d^2\sqrt{c}}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arccosh}(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\Big(}{3d^2\sqrt{}}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 6361$$

$$\frac{5}{2}c^2\left(\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{cx}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\text{arccosh}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{2}c^2\left(\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{5}{2}c^2\left(\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+ib\int\log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arc}\right.}{d\sqrt{d-c^2dx^2}}}{d}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{5}{2}c^2\left(\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ib\int e^{-\text{arccosh}(cx)}\log\left(1-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+ib\int e^{-\text{arccosh}(cx)}\log\left(1+ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2\,\text{arct}\right.}{d\sqrt{d-c^2dx^2}}}{d}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{5}{2}c^2\left(\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\text{arc}}{d\sqrt{d-}}}{d}\right.$$

$$\frac{a+b\text{arccosh}(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/2*(a + b*ArcCosh[c*x])/(d*x^2*(d - c^2*d*x^2)^(3/2)) - (b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(1/(2*x*(1 - c^2*x^2)) + (3*(-x^(-1) + c*ArcTanh[c*x]))/
2))/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^2*((a + b*ArcCosh[c*x])/(3*d*(d - c
^2*d*x^2)^(3/2)) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(x/(2*(1 - c^2*x^2))
+ ArcTanh[c*x]/(2*c)))/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((a + b*ArcCosh[c*x])
/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(
d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*(a + b*ArcCosh[c
*x])*ArcTan[E^ArcCosh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c*x]] + I*b*Po
lyLog[2, I*E^ArcCosh[c*x]]))/(d*Sqrt[d - c^2*d*x^2]))/d))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.60

method	result
default	$-\frac{a}{2d\,x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{6d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{2d^2\sqrt{-c^2d\,x^2+d}}-\frac{5a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\Big(-\frac{\sqrt{-d(c^2x^2-1)}\,(1$
parts	$-\frac{a}{2d\,x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{6d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{2d^2\sqrt{-c^2d\,x^2+d}}-\frac{5a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\Big(-\frac{\sqrt{-d(c^2x^2-1)}\,(1$

input
```
int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2*a*c
^2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^
2+d)^(1/2))/x)+b*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(15*arccosh(c*x)*c^4*x^4+2*c
^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-20*c^2*x^2*arccosh(c*x)-3*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*c*x+3*arccosh(c*x))/d^3/(c^4*x^4-2*c^2*x^2+1)/x^2+13/6*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x-1)^
(1/2)*(c*x+1)^(1/2)+c*x-1)*c^2-13/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^2+5/2
*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arcc
osh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-5/2*I*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))*c^2+5/2*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c
^2-5/2*I*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1
)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2)
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\operatorname{arcosh}\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d
^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d-c^2dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d-c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(-c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
-1/6*a*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(
5/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d
) + 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) + b*integrate(log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d-c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(-c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d-c^2dx^2)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

input
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d-c^2dx^2)^{5/2}}\,dx=\frac{24\sqrt{-c^2x^2+1}\,\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^7-2\sqrt{-c^2x^2+1}\,c^2x^5+\sqrt{-c^2x^2+1}\,x^3}dx\right) b\,c^2x^4-24\sqrt{-c^2x^2}}{}$$

input
```
int((a+b*acosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(24*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**
7 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**5 + sqrt( - c**2*x**2 + 1)*x**3),x)*b
*c**2*x**4 - 24*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 +
 1)*c**4*x**7 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**5 + sqrt( - c**2*x**2 + 1
)*x**3),x)*b*x**2 + 60*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**4
*x**4 - 60*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**2*x**2 - 65*s
qrt( - c**2*x**2 + 1)*a*c**4*x**4 + 65*sqrt( - c**2*x**2 + 1)*a*c**2*x**2
+ 60*a*c**4*x**4 - 80*a*c**2*x**2 + 12*a)/(24*sqrt(d)*sqrt( - c**2*x**2 +
1)*d**2*x**2*(c**2*x**2 - 1))
```

3.128 $\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 324

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx = -\frac{bc^3}{6d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6d^2x^2\sqrt{d-c^2dx^2}} + \frac{a+b\text{arccosh}(cx)}{3dx^3(d-c^2dx^2)^{3/2}} + \frac{2(a+b\text{arccosh}(cx))}{d^2x^3\sqrt{d-c^2dx^2}} - \frac{8\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3d^3x^3} - \frac{16c^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3d^3x} - \frac{8bc^3\sqrt{-1+cx}\sqrt{1+cx}\log(x)}{3d^2\sqrt{d-c^2dx^2}} - \frac{4bc^3\sqrt{-1+cx}\sqrt{1+cx}\log(1-c^2x^2)}{3d^2\sqrt{d-c^2dx^2}}$$

output

```
-1/6*b*c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/6*b*c*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x^2/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccosh(c*
x))/d/x^3/(-c^2*d*x^2+d)^(3/2)+2*(a+b*arccosh(c*x))/d^2/x^3/(-c^2*d*x^2+d)
^(1/2)-8/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/d^3/x^3-16/3*c^2*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/d^3/x-8/3*b*c^3*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*ln(x)/d^2/(-c^2*d*x^2+d)^(1/2)-4/3*b*c^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*l
n(-c^2*x^2+1)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.73

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx = \frac{2a+12ac^2x^2-48ac^4x^4+32ac^6x^6-bcx\sqrt{-1+cx}\sqrt{1+cx}+2b(1+6c^2x^2-24$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(2*a + 12*a*c^2*x^2 - 48*a*c^4*x^4 + 32*a*c^6*x^6 - b*c*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x] + 2*b*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6)*ArcCosh[c*x]
- 16*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*Log[x] + 8*b*c^
3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2] - 8*b*c^5*x^5*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(6*d^2*x^3*(-1 + c^2*x^2)*Sqrt[d -
c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6337, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6337}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int -\frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{3d^3x^3(1-c^2x^2)^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2(a+b\operatorname{arccosh}(cx))}{dx(d-c^2dx^2)^{3/2}}-\frac{a+b\operatorname{arccosh}(cx)}{3dx^3(d-c^2dx^2)^{3/2}}+\frac{16c^4x(a+b\operatorname{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\operatorname{arccosh}(cx))}{3d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{x^3(1-c^2x^2)^2}dx}{3d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2(a+b\text{arccosh}(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2331}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{x^4(1-c^2x^2)^2}dx^2}{6d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2(a+b\text{arccosh}(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2123}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{8c^4}{c^2x^2-1}-\frac{c^4}{(c^2x^2-1)^2}+\frac{8c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2(a+b\text{arccosh}(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c^2(a+b\text{arccosh}(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\text{arccosh}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arccosh}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\text{arccosh}(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{c^2}{1-c^2x^2}+8c^2\log\left(x^2\right)+8c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)}{6d^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(a + b*ArcCosh[c*x])/(d*x^3*(d - c^2*d*x^2)^(3/2)) - (2*c^2*(a + b*Ar
cCosh[c*x]))/(d*x*(d - c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcCosh[c*x]))/
(3*d*(d - c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[
d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*(-x^(-2) - c^2/(1 - c^2*x^2) +
8*c^2*Log[x^2] + 8*c^2*Log[1 - c^2*x^2]))/(6*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6337
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCo
sh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c
*x])]   Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)]
 && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.24

method	result
default	$a\left(-\frac{1}{3d\,x^{3}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+2c^{2}\left(-\frac{1}{dx\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+4c^{2}\left(\frac{x}{3d\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^{2}\sqrt{-c^{2}d\,x^{2}+d}}\right)\right)\right)-\frac{b\sqrt{-d(c^{2}}}{}$
parts	$a\left(-\frac{1}{3d\,x^{3}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+2c^{2}\left(-\frac{1}{dx\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+4c^{2}\left(\frac{x}{3d\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^{2}\sqrt{-c^{2}d\,x^{2}+d}}\right)\right)\right)-\frac{b\sqrt{-d(c^{2}}}{}$

input
```
int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3/(-c^2*d*x^2+d)^(3/2)+2*c^2*(-1/d/x/(-c^2*d*x^2+d)^(3/2)+4*c^
2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))))-1/6*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(32*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*arccosh(c*x)*c^6*x^6+32*arccosh(c*x)*c^7*x^7-16*ln((c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^4-1)*x^7*c^7-48*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)
*x^4*c^4-64*arccosh(c*x)*c^5*x^5+32*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4
-1)*x^5*c^5+12*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+32*c^3*x^3
*arccosh(c*x)-16*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*x^3*c^3-c^3*x^3
+2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)/d^3/(c^6*x^6-3*c^4*x^4+3*
c^2*x^2-1)/x^3
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{(-c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*
d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.85

$$\int \frac{a+b\text{arccosh}(cx)}{x^4 (d-c^2dx^2)^{5/2}}\,dx = \frac{1}{6}\,bc\Bigg(\frac{8\,c^2\sqrt{-d}\log{(cx+1)}}{d^3}+\frac{8\,c^2\sqrt{-d}\log{(cx-1)}}{d^3}+\frac{16\,c^2\sqrt{-d}\log{(x)}}{d^3}+\frac{\sqrt{\cdots}}{c^2d^3x\cdots}$$
$$+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}d^2}+\frac{8\,c^4x}{(-c^2dx^2+d)^{\frac{3}{2}}d}-\frac{6\,c^2}{(-c^2dx^2+d)^{\frac{3}{2}}dx}-\frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}dx^3}\right)b\,\text{arcosh}\,(cx)$$
$$+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}d^2}+\frac{8\,c^4x}{(-c^2dx^2+d)^{\frac{3}{2}}d}-\frac{6\,c^2}{(-c^2dx^2+d)^{\frac{3}{2}}dx}-\frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}dx^3}\right)a$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
1/6*b*c*(8*c^2*sqrt(-d)*log(c*x + 1)/d^3 + 8*c^2*sqrt(-d)*log(c*x - 1)/d^3
 + 16*c^2*sqrt(-d)*log(x)/d^3 + sqrt(-d)/(c^2*d^3*x^4 - d^3*x^2)) + 1/3*(1
6*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) -
6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*b*a
rccosh(c*x) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*
x^2 + d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 +
d)^(3/2)*d*x^3))*a
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4 (d-c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(-c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^4\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx=\frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^8-2\sqrt{-c^2x^2+1}\,c^2x^6+\sqrt{-c^2x^2+1}\,x^4}dx\right)b\,c^2x^5-3\sqrt{-c^2x^2+}}{3\sqrt{d}\,\sqrt{-c^2x}}$$

input
```
int((a+b*acosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**8
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**6 + sqrt( - c**2*x**2 + 1)*x**4),x)*b*
c**2*x**5 - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1
)*c**4*x**8 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**6 + sqrt( - c**2*x**2 + 1)*
x**4),x)*b*x**3 + 16*a*c**6*x**6 - 24*a*c**4*x**4 + 6*a*c**2*x**2 + a)/(3*
sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*x**3*(c**2*x**2 - 1))
```

3.129 $\int \frac{x^4 \operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{x^4 \operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}} dx = \frac{3x^2\sqrt{1-ax}}{16a^3\sqrt{-1+ax}} + \frac{x^4\sqrt{1-ax}}{16a\sqrt{-1+ax}} - \frac{3x\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)}{4a^2} - \frac{3\sqrt{1-ax}\operatorname{arccosh}(ax)^2}{16a^5\sqrt{-1+ax}}$$

output
```
3/16*x^2*(-a*x+1)^(1/2)/a^3/(a*x-1)^(1/2)+1/16*x^4*(-a*x+1)^(1/2)/a/(a*x-1
)^(1/2)-3/8*x*(-a^2*x^2+1)^(1/2)*arccosh(a*x)/a^4-1/4*x^3*(-a^2*x^2+1)^(1/
2)*arccosh(a*x)/a^2-3/16*(-a*x+1)^(1/2)*arccosh(a*x)^2/a^5/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.64

$$\int \frac{x^4 \operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}} dx = \frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)(-16\cosh(2\operatorname{arccosh}(ax)) - \cosh(4\operatorname{arccosh}(ax)) + 4\operatorname{arccosh}(ax)(6\operatorname{arccosh}(ax) + 8\sinh}{128a^5\sqrt{-((-1+ax)(1+ax))}}$$

input
```
Integrate[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-16*Cosh[2*ArcCosh[a*x]] - Cosh[4*A
rcCosh[a*x]] + 4*ArcCosh[a*x]*(6*ArcCosh[a*x] + 8*Sinh[2*ArcCosh[a*x]] + S
inh[4*ArcCosh[a*x]])))/(128*a^5*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6353, 15, 6353, 15, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6353}$$

$$\frac{3\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\sqrt{ax-1}\int x^3dx}{4a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)}{4a^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{3\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)}{4a^2}-\frac{x^4\sqrt{ax-1}}{16a\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 6353}$$

$$\frac{3\left(\frac{\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\sqrt{ax-1}\int xdx}{2a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)}{4a^2}-\frac{x^4\sqrt{ax-1}}{16a\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{3\left(\frac{\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2a^2}-\frac{x^2\sqrt{ax-1}}{4a\sqrt{1-ax}}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)}{4a^2}-\frac{x^4\sqrt{ax-1}}{16a\sqrt{1-ax}}$$

$$\Big\downarrow \text{6307}$$

$$-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)}{4a^2}+\frac{3\left(\frac{\sqrt{ax-1}\text{arccosh}(ax)^2}{4a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2a^2}-\frac{x^2\sqrt{ax-1}}{4a\sqrt{1-ax}}\right)}{4a^2}-\frac{x^4\sqrt{ax-1}}{16a\sqrt{1-ax}}$$

input
```
Int[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/16*(x^4*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x]) - (x^3*Sqrt[1 - a^2*x^2]*ArcC
osh[a*x])/(4*a^2) + (3*(-1/4*(x^2*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x]) - (x*S
qrt[1 - a^2*x^2]*ArcCosh[a*x])/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(
4*a^3*Sqrt[1 - a*x])))/(4*a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(119) = 238$.

Time = 0.48 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.14

method	result
default	$-\frac{3\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^2}{16a^5(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\,(8a^5x^5-12a^3x^3+8\sqrt{ax-1}\,\sqrt{ax+1}\,a^4x^4+4ax-8a^2x^2\sqrt{ax-1}\,\sqrt{ax+}}{256a^5(a^2x^2-1)}$

input
```
int(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arcco
sh(a*x)^2-1/256*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x-1)^(1/2)*(
a*x+1)^(1/2)*a^4*x^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(
1/2)*(a*x+1)^(1/2))*(-1+4*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^
(1/2)*(2*a^3*x^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(-1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2
)*(-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2*a^3*x^3+(a*x-1)^(1/2)*(a*x+1)^
(1/2)-2*a*x)*(1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/256*(-a^2*x^2+1)^(1/2)*(
-8*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^4*x^4+8*a^5*x^5+8*a^2*x^2*(a*x-1)^(1/2)*(
a*x+1)^(1/2)-12*a^3*x^3-(a*x-1)^(1/2)*(a*x+1)^(1/2)+4*a*x)*(1+4*arccosh(a*
x))/a^5/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^4\operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^4\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**4*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^4\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^4*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^4*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4 \text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)\,x^4}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^4*acosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)*x**4)/sqrt( - a**2*x**2 + 1),x)
```

3.130 $\int \frac{x^3 \text{arccosh}(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{2x\sqrt{1-ax}}{3a^3\sqrt{-1+ax}}+\frac{x^3\sqrt{1-ax}}{9a\sqrt{-1+ax}} - \frac{2\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3a^2}$$

output
```
2/3*x*(-a*x+1)^(1/2)/a^3/(a*x-1)^(1/2)+1/9*x^3*(-a*x+1)^(1/2)/a/(a*x-1)^(1
/2)-2/3*(-a^2*x^2+1)^(1/2)*arccosh(a*x)/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arc
cosh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{ax\sqrt{-1+ax}\sqrt{1+ax}(6+a^2x^2)-3(-2+a^2x^2+a^4x^4)\,\text{arccosh}(ax)}{9a^4\sqrt{1-a^2x^2}}$$

input
```
Integrate[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/9*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(6 + a^2*x^2) - 3*(-2 + a^2*x^2 + a
^4*x^4)*ArcCosh[a*x])/(a^4*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6353, 15, 6329, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\sqrt{ax-1}\int x^2dx}{3a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3a^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3a^2}-\frac{x^3\sqrt{ax-1}}{9a\sqrt{1-ax}}$$

$$\Big\downarrow \text{6329}$$

$$\frac{2\left(-\frac{\sqrt{ax-1}\int 1dx}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3a^2}-\frac{x^3\sqrt{ax-1}}{9a\sqrt{1-ax}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{a^2}-\frac{x\sqrt{ax-1}}{a\sqrt{1-ax}}\right)}{3a^2}-\frac{x^3\sqrt{ax-1}}{9a\sqrt{1-ax}}$$

input
```
Int[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/9*(x^3*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x]) - (x^2*Sqrt[1 - a^2*x^2]*ArcCo
sh[a*x])/(3*a^2) + (2*(-((x*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x])) - (Sqrt[1 -
 a^2*x^2]*ArcCosh[a*x])/a^2))/(3*a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31

method	result
orering	$\frac{(5a^4x^4+12a^2x^2-24)\,\mathrm{arccosh}(ax)}{9a^4\sqrt{-a^2x^2+1}}-\frac{(a^2x^2+6)(ax-1)(ax+1)\left(\frac{3x^2\,\mathrm{arccosh}(ax)}{\sqrt{-a^2x^2+1}}+\frac{x^3a}{\sqrt{ax-1}\,\sqrt{ax+1}\,\sqrt{-a^2x^2+1}}+\frac{x^4\,\mathrm{arccosh}(ax)a^2}{(-a^2x^2+1)^{\frac{3}{2}}}\right)}{9x^2a^4}$
default	$-\frac{\sqrt{-a^2x^2+1}\,(4a^4x^4-5a^2x^2+4a^3x^3\sqrt{ax-1}\,\sqrt{ax+1}-3\sqrt{ax-1}\,\sqrt{ax+1}\,ax+1)(-1+3\,\mathrm{arccosh}(ax))}{72a^4(a^2x^2-1)}-\frac{3\sqrt{-a^2x^2+1}\,(\sqrt{ax-1}\,\sqrt{a}}{8}$

input
```
int(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(5*a^4*x^4+12*a^2*x^2-24)/a^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2)-1/9/x^2*
(a^2*x^2+6)/a^4*(a*x-1)*(a*x+1)*(3*x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2)+x^3
*a/(a*x-1)^(1/2)/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2)+x^4*arccosh(a*x)/(-a^2*x
^2+1)^(3/2)*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

$$\int \frac{x^3\mathrm{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{3\,(a^4x^4+a^2x^2-2)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)-(a^3x^3+6\,ax)\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}}{9\,(a^6x^2-a^4)}$$

input
```
integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(3*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
- 1)) - (a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 -
 a^4)
```

Sympy [F]

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**3*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{1}{9}a\left(\frac{i\,x^3}{a^2}+\frac{6i\,x}{a^4}\right) - \frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)$$

input
```
integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/9*a*(I*x^3/a^2 + 6*I*x/a^4) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-
a^2*x^2 + 1)/a^4)*arccosh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\,\text{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*acosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.131 $\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\frac{x^2\sqrt{1-ax}}{4a\sqrt{-1+ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2a^2}-\frac{\sqrt{1-ax}\text{arccosh}(ax)^2}{4a^3\sqrt{-1+ax}}$$

output
```
1/4*x^2*(-a*x+1)^(1/2)/a/(a*x-1)^(1/2)-1/2*x*(-a^2*x^2+1)^(1/2)*arccosh(a*
x)/a^2-1/4*(-a*x+1)^(1/2)*arccosh(a*x)^2/a^3/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-((-1+ax)(1+ax))}(-\cosh(2\text{arccosh}(ax))+2\text{arccosh}(ax)(\text{arccosh}(ax)+\sinh(2\text{arccosh}(ax))))}{8a^3\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

input
```
Integrate[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/8*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-Cosh[2*ArcCosh[a*x]] + 2*ArcCosh[a*x
]*(ArcCosh[a*x] + Sinh[2*ArcCosh[a*x]])))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*
(1 + a*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6353, 15, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6353}$$

$$\frac{\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\sqrt{ax-1}\int xdx}{2a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2a^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2a^2}-\frac{x^2\sqrt{ax-1}}{4a\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 6307}$$

$$\frac{\sqrt{ax-1}\text{arccosh}(ax)^2}{4a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2a^2}-\frac{x^2\sqrt{ax-1}}{4a\sqrt{1-ax}}$$

input
```
Int[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*(x^2*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x]) - (x*Sqrt[1 - a^2*x^2]*ArcCosh
[a*x])/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(4*a^3*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(72) = 144$.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.53

method	result
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\,\sqrt{a x -1}\,\sqrt{a x +1}\,\operatorname{arccosh}\left(a x \right)^{2}}{4 a^{3} \left(a^{2} x^{2}-1\right)}-\frac{\sqrt{-a^{2} x^{2}+1}\,\left(2 a^{3} x^{3}-2 a x +2 a^{2} x^{2} \sqrt{a x -1}\,\sqrt{a x +1}-\sqrt{a x -1}\,\sqrt{a x +1}\right) \left(-1+2 \operatorname{arccosh}\right.}{16 a^{3} \left(a^{2} x^{2}-1\right)}$

input
```
int(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccos
h(a*x)^2-1/16*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*
(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(-1+2*arccosh(a*x))/a^3/(a^2*x^
2-1)-1/16*(-a^2*x^2+1)^(1/2)*(-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2*a^3
*x^3+(a*x-1)^(1/2)*(a*x+1)^(1/2)-2*a*x)*(1+2*arccosh(a*x))/a^3/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{acosh}\,(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**2*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*acosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.132 $\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{x\sqrt{1-ax}}{a\sqrt{-1+ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{a^2}$$

output
```
x*(-a*x+1)^(1/2)/a/(a*x-1)^(1/2)-(-a^2*x^2+1)^(1/2)*arccosh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{-ax\sqrt{-1+ax}\sqrt{1+ax}+(-1+a^2x^2)\,\text{arccosh}(ax)}{a^2\sqrt{1-a^2x^2}}$$

input
```
Integrate[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(-(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (-1 + a^2*x^2)*ArcCosh[a*x])/(a^2*S
qrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6329, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6329}$$

$$-\frac{\sqrt{ax-1}\int 1dx}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{a^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{a^2}-\frac{x\sqrt{ax-1}}{a\sqrt{1-ax}}$$

input
```
Int[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-((x*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x])) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x])
/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{\left(a^{2} x^{2} \operatorname{arccosh}\left(a x\right)-\sqrt{a x -1}\, \sqrt{a x +1}\, a x -\operatorname{arccosh}\left(a x\right)\right) \sqrt{-a^{2} x^{2}+1}}{\left(a^{2} x^{2}-1\right) a^{2}}$	65
orering	$\frac{\left(a^{2} x^{2}-2\right) \operatorname{arccosh}\left(a x\right)}{a^{2} \sqrt{-a^{2} x^{2}+1}}-\frac{\left(a x -1\right) \left(a x +1\right) \left(\frac{\operatorname{arccosh}\left(a x\right)}{\sqrt{-a^{2} x^{2}+1}}+\frac{x a}{\sqrt{a x -1}\, \sqrt{a x +1}\, \sqrt{-a^{2} x^{2}+1}}+\frac{x^{2} \operatorname{arccosh}\left(a x\right) a^{2}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}\right)}{a^{2}}$	116

input
```
int(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2*arccosh(a*x)-(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-arccosh(a*x))*(-a^2
*x^2+1)^(1/2)/(a^2*x^2-1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}ax + (-a^2x^2+1)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^4x^2-a^2}$$

input
```
integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x + (-a^2*x^2 + 1)^(3/2)*log(a*x +
 sqrt(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

Sympy [F]

$$\int \frac{x\mathrm{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\mathrm{acosh}\,(ax)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(x*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

$$\int \frac{x\mathrm{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{i\,x}{a} - \frac{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)}{a^2}$$

input
```
integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
I*x/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/a^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{x\mathrm{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{i\,x}{a} - \frac{\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}$$

input
```
integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
I*x/a - sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\mathrm{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)\,x}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*acosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)*x)/sqrt( - a**2*x**2 + 1),x)
```

3.133 $\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-ax}\,\text{arccosh}(ax)^2}{2a\sqrt{-1+ax}}$$

output
```
-1/2*(-a*x+1)^(1/2)*arccosh(a*x)^2/a/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{arccosh}(ax)^2}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6307}$$

$$\frac{\sqrt{ax-1}\text{arccosh}(ax)^2}{2a\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{arccosh}(ax)^2}{2(a^2x^2-1)a}$	51

input
```
int(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)/a*ar
ccosh(a*x)^2
```

Fricas [F]

$$\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{acosh}\,(ax)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int(acosh(a*x)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{acosh(ax)}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(acosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)/sqrt( - a**2*x**2 + 1),x)
```

3.134 $\int \frac{\text{arccosh}(ax)}{x\sqrt{1-a^2x^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{\text{arccosh}(ax)}{x\sqrt{1-a^2x^2}} dx = -\frac{2\sqrt{1-ax}\text{arccosh}(ax)\arctan\left(e^{\text{arccosh}(ax)}\right)}{\sqrt{-1+ax}} + \frac{i\sqrt{1-ax}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)}{\sqrt{-1+ax}} - \frac{i\sqrt{1-ax}\,\text{PolyLog}\left(2,ie^{\text{arccosh}(ax)}\right)}{\sqrt{-1+ax}}$$

output
```
-2*(-a*x+1)^(1/2)*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/(a*
x-1)^(1/2)+I*(-a*x+1)^(1/2)*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
)/(a*x-1)^(1/2)-I*(-a*x+1)^(1/2)*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1
/2)))/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{i\sqrt{-((-1+ax)(1+ax))}\left(\operatorname{arccosh}(ax)\left(\log\left(1-ie^{-\operatorname{arccosh}(ax)}\right)-\log\left(1+ie^{-\operatorname{arccosh}(ax)}\right)\right)+\operatorname{PolyLog}\left(2,\right.\right.}{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

input
```
Integrate[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(I*Sqrt[-((-1 + a*x)*(1 + a*x))]*(ArcCosh[a*x]*(Log[1 - I/E^ArcCosh[a*x]]
- Log[1 + I/E^ArcCosh[a*x]]) + PolyLog[2, (-I)/E^ArcCosh[a*x]] - PolyLog[2
, I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{6361}$$
$$\frac{\sqrt{ax-1}\int \frac{\operatorname{arccosh}(ax)}{ax}d\operatorname{arccosh}(ax)}{\sqrt{1-ax}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{ax-1}\int \operatorname{arccosh}(ax)\csc\left(i\operatorname{arccosh}(ax)+\frac{\pi}{2}\right)d\operatorname{arccosh}(ax)}{\sqrt{1-ax}}$$
$$\Big\downarrow \text{4668}$$

$$\frac{\sqrt{ax-1}\left(-i\int \log\left(1-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+i\int \log\left(1+ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+2\text{arccosh}(ax)\arctan\right.}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\sqrt{ax-1}\left(-i\int e^{-\text{arccosh}(ax)}\log\left(1-ie^{\text{arccosh}(ax)}\right)de^{\text{arccosh}(ax)}+i\int e^{-\text{arccosh}(ax)}\log\left(1+ie^{\text{arccosh}(ax)}\right)de^{\text{arccosh}(ax)}\right.}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\sqrt{ax-1}\left(2\text{arccosh}(ax)\arctan\left(e^{\text{arccosh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{arccosh}(ax)}\right)\right)}{\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(Sqrt[-1 + a*x]*(2*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]] - I*PolyLog[2, (-I)
*E^ArcCosh[a*x]] + I*PolyLog[2, I*E^ArcCosh[a*x]]))/Sqrt[1 - a*x]
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(126) = 252$.

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.62

method	result
default	$\frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\ \mathrm{arccosh}(ax)\ln\left(1+i\left(ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)\right)}{a^2x^2-1} - \frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\ \mathrm{arccosh}(ax)\ln(1-i(ax+}{a^2x^2-1}$

input
```
int(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*
ln(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/
2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*ln(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2)))+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*dil
og(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/
2)*(a*x+1)^(1/2)/(a^2*x^2-1)*dilog(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(acosh(a*x)/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.135 $\int \frac{\text{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\text{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx=-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{x}+\frac{a\sqrt{1-ax}\log(x)}{\sqrt{-1+ax}}$$

output
```
-(-a^2*x^2+1)^(1/2)*arccosh(a*x)/x+a*(-a*x+1)^(1/2)*ln(x)/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\text{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx=\frac{(-1+a^2x^2)\,\text{arccosh}(ax)-ax\sqrt{-1+ax}\sqrt{1+ax}\log(x)}{x\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
((-1 + a^2*x^2)*ArcCosh[a*x] - a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[x])/(x
*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6332, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6332}$$

$$-\frac{a\sqrt{ax-1}\int \frac{1}{x}dx}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{x}-\frac{a\sqrt{ax-1}\log(x)}{\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/x) - (a*Sqrt[-1 + a*x]*Log[x])/Sqrt[1 -
 a*x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(41) = 82.

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.57

method	result
default	$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)a}{a^2x^2-1}-\frac{\sqrt{-a^2x^2+1}\,(a^2x^2-\sqrt{ax-1}\,\sqrt{ax+1}\,ax-1)\operatorname{arccosh}(ax)}{x(a^2x^2-1)}+\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}}{}$

input
```
int(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)
*a-(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-1)*arccosh(
a*x)/x/(a^2*x^2-1)+(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2
-1)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(41) = 82.

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{ax\arctan\left(\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}(x^2-1)}{a^2x^4+(a^2-1)x^2-1}\right)-\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{x}$$

input
```
integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(a*x*arctan(sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*(x^2 - 1)/(a^2*x^4 + (a^2
 - 1)*x^2 - 1)) - sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1)))/x
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \frac{\operatorname{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx &= -\frac{1}{2}\left(a^2\sqrt{-\frac{1}{a^4}}\log\left(x^2-\frac{1}{a^2}\right)+i\,(-1)^{-2\,a^2x^2+2}\log\left(-2\,a^2+\frac{2}{x^2}\right)\right)a \\ &\quad -\frac{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)}{x}\end{aligned}$$

input
```
integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(a^2*sqrt(-1/a^4)*log(x^2 - 1/a^2) + I*(-1)^(-2*a^2*x^2 + 2)*log(-2*a
^2 + 2/x^2))*a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{1}{2}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\log\left(ax + \sqrt{a^2x^2-1}\right) + i\,a\log\left(|x|\right)$$

input
```
integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*
abs(a) + a)/(x*abs(a)))*log(a*x + sqrt(a^2*x^2 - 1)) + I*a*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(acosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.136 $\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{a\sqrt{1-ax}}{2x\sqrt{-1+ax}}-\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)}{2x^2}-\frac{a^2\sqrt{1-ax}\operatorname{arccosh}(ax)\arctan\left(e^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}+\frac{ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)}{2\sqrt{-1+ax}}-\frac{ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(ax)}\right)}{2\sqrt{-1+ax}}$$

output
```
-1/2*a*(-a*x+1)^(1/2)/x/(a*x-1)^(1/2)-1/2*(-a^2*x^2+1)^(1/2)*arccosh(a*x)/
x^2-a^2*(-a*x+1)^(1/2)*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)
)/(a*x-1)^(1/2)+1/2*I*a^2*(-a*x+1)^(1/2)*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2)))/(a*x-1)^(1/2)-1/2*I*a^2*(-a*x+1)^(1/2)*polylog(2,I*(a*x+(a*
x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{(1+ax)\left(ax\sqrt{\frac{-1+ax}{1+ax}} - \operatorname{arccosh}(ax) + ax\operatorname{arccosh}(ax) - ia^2x^2\sqrt{\frac{-1+ax}{1+ax}}\operatorname{arccosh}(ax)\log\left(1-ie^{-\operatorname{arccosh}(ax)}\right)\right.}{}$$

input
```
Integrate[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
((1 + a*x)*(a*x*Sqrt[(-1 + a*x)/(1 + a*x)] - ArcCosh[a*x] + a*x*ArcCosh[a*
x] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh
[a*x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^Arc
Cosh[a*x]] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, (-I)/E^ArcCos
h[a*x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I/E^ArcCosh[a*x]
]))/(2*x^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6347, 15, 6361, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{6347}$$
$$\frac{1}{2}a^2\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}dx - \frac{a\sqrt{ax-1}\int \frac{1}{x^2}dx}{2\sqrt{1-ax}} - \frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)}{2x^2}$$
$$\Big\downarrow \text{15}$$
$$\frac{1}{2}a^2\int \frac{\operatorname{arccosh}(ax)}{x\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)}{2x^2} + \frac{a\sqrt{ax-1}}{2x\sqrt{1-ax}}$$

$$\Big\downarrow \text{6361}$$

$$\frac{a^2\sqrt{ax-1}\int \frac{\text{arccosh}(ax)}{ax}d\text{arccosh}(ax)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2x^2}+\frac{a\sqrt{ax-1}}{2x\sqrt{1-ax}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{ax-1}\int \text{arccosh}(ax)\csc\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)d\text{arccosh}(ax)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2x^2}+\frac{a\sqrt{ax-1}}{2x\sqrt{1-ax}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{a^2\sqrt{ax-1}\left(-i\int \log\left(1-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+i\int \log\left(1+ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+2\text{arccosh}(ax)\,\text{arc}\right.}{2\sqrt{1-ax}}$$
$$\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2x^2}+\frac{a\sqrt{ax-1}}{2x\sqrt{1-ax}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{a^2\sqrt{ax-1}\left(-i\int e^{-\text{arccosh}(ax)}\log\left(1-ie^{\text{arccosh}(ax)}\right)de^{\text{arccosh}(ax)}+i\int e^{-\text{arccosh}(ax)}\log\left(1+ie^{\text{arccosh}(ax)}\right)de^{\text{arccosh}(a}\right.}{2\sqrt{1-ax}}$$
$$\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2x^2}+\frac{a\sqrt{ax-1}}{2x\sqrt{1-ax}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{a^2\sqrt{ax-1}\left(2\text{arccosh}(ax)\arctan\left(e^{\text{arccosh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{arccosh}(ax)}\right)\right)}{2\sqrt{1-ax}}-$$
$$\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{2x^2}+\frac{a\sqrt{ax-1}}{2x\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a*Sqrt[-1 + a*x])/(2*x*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/
(2*x^2) + (a^2*Sqrt[-1 + a*x]*(2*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]] - I*P
olyLog[2, (-I)*E^ArcCosh[a*x]] + I*PolyLog[2, I*E^ArcCosh[a*x]]))/(2*Sqrt[
1 - a*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.08

method	result
default	$-\frac{(a^2x^2\operatorname{arccosh}(ax)+\sqrt{ax-1}\,\sqrt{ax+1}\,ax-\operatorname{arccosh}(ax))\sqrt{-a^2x^2+1}}{2(a^2x^2-1)x^2}+\frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)\ln(1+i(ax+\sqrt{a}}{2a^2x^2-2}$

input
```
int(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a^2*x^2*arccosh(a*x)+(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-arccosh(a*x))*(
-a^2*x^2+1)^(1/2)/(a^2*x^2-1)/x^2+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+
1)^(1/2)*arccosh(a*x)*ln(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2
*x^2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*ln(1
-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)+I*(-a^2*x^2+1)^(1/
2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*dilog(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
)*a^2/(2*a^2*x^2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*dilog
(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)
```

Fricas [F]

$$\int\frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx=\int\frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(acosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.137 $\int \frac{\text{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{\text{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = -\frac{a\sqrt{1-ax}}{6x^2\sqrt{-1+ax}} - \frac{\sqrt{1-a^2x^2}\,\text{arccosh}(ax)}{3x^3} - \frac{2a^2\sqrt{1-a^2x^2}\,\text{arccosh}(ax)}{3x} + \frac{2a^3\sqrt{1-ax}\log(x)}{3\sqrt{-1+ax}}$$

output
```
-1/6*a*(-a*x+1)^(1/2)/x^2/(a*x-1)^(1/2)-1/3*(-a^2*x^2+1)^(1/2)*arccosh(a*x
)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)*arccosh(a*x)/x+2/3*a^3*(-a*x+1)^(1/2)*ln(
x)/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

$$\int \frac{\text{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\left(ax+6a^3x^3+2\sqrt{-1+ax}\sqrt{1+ax}\,(1+2a^2x^2)\,\text{arccosh}(ax)-4a^3x^3\log(-1+ax)-4a^3x^3}{6x^3\sqrt{-1+ax}\sqrt{1+ax}}$$

input
```
Integrate[ArcCosh[a*x]/(x^4*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/6*(Sqrt[1 - a^2*x^2]*(a*x + 6*a^3*x^3 + 2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
(1 + 2*a^2*x^2)*ArcCosh[a*x] - 4*a^3*x^3*Log[-1 + a*x] - 4*a^3*x^3*Log[1 +
 (-1 + a*x)^(-1)]))/(x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6347, 15, 6332, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6347}$$

$$\frac{2}{3}a^2\int \frac{\text{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{a\sqrt{ax-1}\int \frac{1}{x^3}dx}{3\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}a^2\int \frac{\text{arccosh}(ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3x^3}+\frac{a\sqrt{ax-1}}{6x^2\sqrt{1-ax}}$$

$$\Big\downarrow \text{6332}$$

$$\frac{2}{3}a^2\left(-\frac{a\sqrt{ax-1}\int \frac{1}{x}dx}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3x^3}+\frac{a\sqrt{ax-1}}{6x^2\sqrt{1-ax}}$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}a^2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{x}-\frac{a\sqrt{ax-1}\log(x)}{\sqrt{1-ax}}\right)-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)}{3x^3}+\frac{a\sqrt{ax-1}}{6x^2\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]/(x^4*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a*Sqrt[-1 + a*x])/(6*x^2*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x]
)/(3*x^3) + (2*a^2*(-((Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/x) - (a*Sqrt[-1 + a
*x]*Log[x])/Sqrt[1 - a*x]))/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\left(4\,\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}\,a^2x^2+4\,\operatorname{arccosh}(ax)a^3x^3-4\ln\left(1+\left(ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)^2\right)a^3x^3+2}{6x^3(a^2x^2-1)}$

input
```
int(arccosh(a*x)/x^4/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(4*arccosh(a*x)*(a*x-1
)^(1/2)*(a*x+1)^(1/2)*a^2*x^2+4*arccosh(a*x)*a^3*x^3-4*ln(1+(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))^2)*a^3*x^3+2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+
a*x)/x^3/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = -\frac{2\left(2\,a^4x^4-a^2x^2-1\right)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)-\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}(ax^3-ax)-4\left(a^5x\right.}{6\left(a^2x^5-x^3\right)}$$

input
```
integrate(arccosh(a*x)/x^4/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*(2*a^4*x^4 - a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^
2 - 1)) - sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*(a*x^3 - a*x) - 4*(a^5*x^5
- a^3*x^3)*arctan(sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*(x^2 - 1)/(a^2*x^4
+ (a^2 - 1)*x^2 - 1)))/(a^2*x^5 - x^3)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(ax)}{x^4\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)/x**4/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)/(x**4*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = -\frac{1}{6}\left(2\left(a^2\sqrt{-\frac{1}{a^4}}\log\left(x^2-\frac{1}{a^2}\right)+i(-1)^{-2a^2x^2+2}\log\left(-2a^2+\frac{2}{x^2}\right)\right)a^2-\frac{\sqrt{-a^4x^4+2a^2x^2-1}}{x^2}\right)a - \frac{1}{3}\left(\frac{2\sqrt{-a^2x^2+1}a^2}{x}+\frac{\sqrt{-a^2x^2+1}}{x^3}\right)\operatorname{arcosh}(ax)$$

input
```
integrate(arccosh(a*x)/x^4/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(2*(a^2*sqrt(-1/a^4)*log(x^2 - 1/a^2) + I*(-1)^(-2*a^2*x^2 + 2)*log(-
2*a^2 + 2/x^2))*a^2 - sqrt(-a^4*x^4 + 2*a^2*x^2 - 1)/x^2)*a - 1/3*(2*sqrt(
-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*arccosh(a*x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = \frac{1}{6}a^3\left(\frac{-2i\,a^2x^2-i}{a^2x^2}+2i\,\log\left(a^2x^2\right)\right) + \frac{1}{24}\left(\frac{\left(a^4+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} - \frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{x^3}}{a^2|a|}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)$$

input
```
integrate(arccosh(a*x)/x^4/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/6*a^3*((-2*I*a^2*x^2 - I)/(a^2*x^2) + 2*I*log(a^2*x^2)) + 1/24*((a^4 + 9
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a
) + a)^3*abs(a)) - (9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x + (sqrt(-a^2*x
^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*log(a*x + sqrt(a^2*x^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)}{x^4\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)/(x^4*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)/(x^4*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)}{\sqrt{-a^2x^2+1}\,x^4}dx$$

input
```
int(acosh(a*x)/x^4/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)/(sqrt( - a**2*x**2 + 1)*x**4),x)
```

3.138 $\int \frac{(fx)^{3/2}(a+b\operatorname{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{(fx)^{3/2}(a+b\operatorname{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx = \frac{2(fx)^{5/2}(a+b\operatorname{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)}{5f} - \frac{4bc(fx)^{7/2}\sqrt{1-cx}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{-1+cx}}$$

output
```
2/5*(f*x)^(5/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)/f-4
/35*b*c*(f*x)^(7/2)*(-c*x+1)^(1/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],c^2
*x^2)/f^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(fx)^{3/2}(a+b\operatorname{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx = \frac{2}{35}x(fx)^{3/2}\left(7(a+b\operatorname{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)+\right.$$

input
```
Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

output
```
(2*x*(f*x)^(3/2)*(7*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4,
c^2*x^2] + (2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4
, 7/4}, {9/4, 11/4}, c^2*x^2])/Sqrt[1 - c^2*x^2]))/35
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {6363}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx$$

$$\Big\downarrow \text{6363}$$

$$\frac{4bc\sqrt{cx-1}(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{1-cx}}+\frac{2(fx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)(a+b\text{arccosh}(cx))}{5f}$$

input
```
Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

output
```
(2*(f*x)^(5/2)*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x
^2])/(5*f) + (4*b*c*(f*x)^(7/2)*Sqrt[-1 + c*x]*HypergeometricPFQ[{1, 7/4,
7/4}, {9/4, 11/4}, c^2*x^2])/(35*f^2*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 6363
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}} (a + b \operatorname{arccosh}(cx))}{\sqrt{-c^2x^2 + 1}} dx$$

input
```
int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(a + b\text{arccosh}(cx))}{\sqrt{1 - c^2x^2}} dx = \int \frac{(fx)^{\frac{3}{2}} (b \operatorname{arcosh}(cx) + a)}{\sqrt{-c^2x^2 + 1}} dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="
fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2*x
^2 - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(fx)^{\frac{3}{2}}\,(b\,\text{arcosh}\,(cx)+a)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(fx)^{\frac{3}{2}}\,(b\,\text{arcosh}\,(cx)+a)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="
giac")
```

output
```
integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(f\,x)^{3/2}}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2),x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{1-c^2x^2}}\,dx = \frac{\sqrt{f}\,f\left(-2\sqrt{x}\,\sqrt{-c^2x^2+1}\,a-3\left(\int \frac{\sqrt{x}\,\sqrt{-c^2x^2+1}\,acosh(cx)x}{c^2x^2-1}dx\right)b\,c^2-\left(\int \frac{\sqrt{\cdots}}{\cdots}\right.\right.}{3c^2}$$

input
```
int((f*x)^(3/2)*(a+b*acosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

output
```
(sqrt(f)*f*( - 2*sqrt(x)*sqrt( - c**2*x**2 + 1)*a - 3*int((sqrt(x)*sqrt( -
 c**2*x**2 + 1)*acosh(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2 - int((sqrt(x)*sqr
t( - c**2*x**2 + 1))/(c**2*x**3 - x),x)*a))/(3*c**2)
```

3.139 $\int \frac{(fx)^{3/2}(a+b\mathbf{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{2(fx)^{5/2}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)}{5f\sqrt{d-c^2dx^2}} + \frac{4bc(fx)^{7/2}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}$$

output
```
2/5*(f*x)^(5/2)*(-c^2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 5/4]
,[9/4],c^2*x^2)/f/(-c^2*d*x^2+d)^(1/2)+4/35*b*c*(f*x)^(7/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)/f^2/(-c^2*d*x^2
+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{2x(fx)^{3/2}\left(7\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},\right.\right.}{35\sqrt{d-c^2dx^2}}$$

input
```
Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(2*x*(f*x)^(3/2)*(7*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2
F1[1/2, 5/4, 9/4, c^2*x^2] + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeo
metricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2]))/(35*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {6363}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \ 6363$$

$$\frac{4bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{1-c^2x^2}(fx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)(a+b\text{arccosh}(cx))}{5f\sqrt{d-c^2dx^2}}$$

input
```
Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(2*(f*x)^(5/2)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/
2, 5/4, 9/4, c^2*x^2])/(5*f*Sqrt[d - c^2*d*x^2]) + (4*b*c*(f*x)^(7/2)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2
*x^2])/(35*f^2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 6363
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}} (a+b\,\mathrm{arccosh}\,(cx))}{\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(a+b\mathrm{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}} (b\,\mathrm{arcosh}\,(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm
="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2
*d*x^2 - d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\,(b\,\text{arcosh}\,(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(fx)^{3/2}(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\,(b\,\text{arcosh}\,(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm
="giac")
```

output
```
integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}} \, dx = \int \frac{(a + b\,\text{acosh}(c\,x))\,(f\,x)^{3/2}}{\sqrt{d - c^2\,d\,x^2}} \, dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(fx)^{3/2}(a + b\text{arccosh}(cx))}{\sqrt{d - c^2dx^2}} \, dx = \frac{\sqrt{f}\,\sqrt{d}\,f\left(-2\sqrt{x}\,\sqrt{-c^2x^2+1}\,a - 3\left(\int \frac{\sqrt{x}\,\sqrt{-c^2x^2+1}\,acosh(cx)x}{c^2x^2-1}dx\right)b\,c^2 - \left(\right.\right.}{3c^2d}$$

input
```
int((f*x)^(3/2)*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(f)*sqrt(d)*f*( - 2*sqrt(x)*sqrt( - c**2*x**2 + 1)*a - 3*int((sqrt(x)
*sqrt( - c**2*x**2 + 1)*acosh(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2 - int((sqr
t(x)*sqrt( - c**2*x**2 + 1))/(c**2*x**3 - x),x)*a))/(3*c**2*d)
```

3.140 $\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 385

$$
\begin{aligned}
&\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx \\
&= \frac{bcd^3(2271+1329m+284m^2+27m^3+m^4)\,(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^2(3+m)^2(5+m)^2(7+m)^2} \\
&\quad - \frac{bc^3d^3(9+m)(13+2m)(fx)^{4+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^4(5+m)^2(7+m)^2} \\
&\quad + \frac{bc^5d^3(fx)^{6+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^6(7+m)^2} \\
&\quad + \frac{d^3(fx)^{1+m}(a+b\text{arccosh}(cx))}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}(a+b\text{arccosh}(cx))}{f^3(3+m)} \\
&\quad + \frac{3c^4d^3(fx)^{5+m}(a+b\text{arccosh}(cx))}{f^5(5+m)} - \frac{c^6d^3(fx)^{7+m}(a+b\text{arccosh}(cx))}{f^7(7+m)} \\
&\quad - \frac{3bcd^3(2161+1813m+455m^2+35m^3)\,(fx)^{2+m}\sqrt{1-cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{f^2(1+m)(2+m)(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}}
\end{aligned}
$$

output
```
b*c*d^3*(m^4+27*m^3+284*m^2+1329*m+2271)*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/f^2/(3+m)^2/(5+m)^2/(7+m)^2-b*c^3*d^3*(9+m)*(13+2*m)*(f*x)^(4+m)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/f^4/(5+m)^2/(7+m)^2+b*c^5*d^3*(f*x)^(6+m)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/f^6/(7+m)^2+d^3*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+
m)-3*c^2*d^3*(f*x)^(3+m)*(a+b*arccosh(c*x))/f^3/(3+m)+3*c^4*d^3*(f*x)^(5+m
)*(a+b*arccosh(c*x))/f^5/(5+m)-c^6*d^3*(f*x)^(7+m)*(a+b*arccosh(c*x))/f^7/
(7+m)-3*b*c*d^3*(35*m^3+455*m^2+1813*m+2161)*(f*x)^(2+m)*(-c*x+1)^(1/2)*hy
pergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/f^2/(1+m)/(2+m)/(3+m)^2/(5+m)^2/
(7+m)^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.01

$$\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$
$$= d^3x(fx)^m\left(\frac{a+b\text{arccosh}(cx)}{1+m}-\frac{3c^2x^2(a+b\text{arccosh}(cx))}{3+m}+\frac{3c^4x^4(a+b\text{arccosh}(cx))}{5+m}-\frac{c^6x^6(a+b\text{arccosh}(cx))}{7+m}+\frac{bc^7x^7\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},4+\frac{m}{2},5+\frac{m}{2},c^2x^2\right)}{(7+m)(8+m)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3bc^3x^3\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},c^2x^2\right)}{(12+7m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3bc^5x^5\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6+m}{2},\frac{8+m}{2},c^2x^2\right)}{(5+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
d^3*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (3*c^2*x^2*(a + b*ArcCosh[c*
x]))/(3 + m) + (3*c^4*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (c^6*x^6*(a + b*
ArcCosh[c*x]))/(7 + m) + (b*c^7*x^7*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/
2, 4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*x^3*
Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/(
(12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*x^5*Sqrt[1 - c^2
*x^2]*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 +
 m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6336, 27, 2113, 2340, 1590, 27, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^3(fx)^m(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6336}$$

$$-bc\int \frac{d^3(fx)^{m+1}\left(-\frac{c^6x^6}{m+7}+\frac{3c^4x^4}{m+5}-\frac{3c^2x^2}{m+3}+\frac{1}{m+1}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcd^3\int \frac{(fx)^{m+1}\left(-\frac{c^6x^6}{m+7}+\frac{3c^4x^4}{m+5}-\frac{3c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f}-\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{2113}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}\left(-\frac{c^6x^6}{m+7}+\frac{3c^4x^4}{m+5}-\frac{3c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{f\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\int \frac{(fx)^{m+1}\left(\frac{(m+9)(2m+13)x^4c^6}{(m+5)(m+7)}-\frac{3(m+7)x^2c^4}{m+3}+\frac{(m+7)c^2}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+7)}-\frac{c^4\sqrt{c^2x^2-1}(fx)^{m+6}}{f^5(m+7)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{1590}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\frac{\int \frac{c^4(fx)^{m+1}\left(\frac{(m+5)(m+7)}{m+1}-\frac{c^2\left(m^4+27m^3+284m^2+1329m+2271\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+5)}+\frac{c^4(m+9)(2m+13)\sqrt{c^2x^2-1}(fx)^{m+4}}{f^3(m+5)^2(m+7)}}{c^2(m+7)}-\frac{c^4\sqrt{c^2x^2}\cdots}{f^5(\cdots}\right.}{f\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\frac{c^2\int\frac{(fx)^{m+1}\left(\frac{(m+5)(m+7)}{m+1}-\frac{c^2\left(m^4+27m^3+284m^2+1329m+2271\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{c^2x^2-1}}dx}{m+5}+\frac{c^4(m+9)(2m+13)\sqrt{c^2x^2-1}(fx)^{m+4}}{f^3(m+5)^2(m+7)}}{c^2(m+7)}-\frac{c^4\sqrt{c^2x^2\cdots}}{f^5(\cdots}\right.}{f\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$\big\downarrow$ 363

$$-\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\frac{c^2\left(\frac{3\left(35m^3+455m^2+1813m+2161\right)\int\frac{(fx)^{m+1}}{\sqrt{c^2x^2-1}}dx}{(m+1)(m+3)^2(m+5)(m+7)}-\frac{\left(m^4+27m^3+284m^2+1329m+2271\right)\sqrt{c^2x^2-1}(fx)^{m+2}}{f(m+3)^2(m+5)(m+7)}\right)}{m+5}+\frac{c^4(m+9)(2m+13)\cdots}{f^3(m+5\cdots}}{c^2(m+7)}\right.}{f\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$\big\downarrow$ 279

$$-\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\frac{c^2\left(\frac{3\left(35m^3+455m^2+1813m+2161\right)\sqrt{1-c^2x^2}\int\frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx}{(m+1)(m+3)^2(m+5)(m+7)\sqrt{c^2x^2-1}}-\frac{\left(m^4+27m^3+284m^2+1329m+2271\right)\sqrt{c^2x^2-1}(fx)^{m+2}}{f(m+3)^2(m+5)(m+7)}\right)}{m+5}+\frac{c^4(m+9\cdots}{\cdots}}{c^2(m+7)}\right.}{f\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$\big\downarrow$ 278

$$-\frac{c^6d^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}-\frac{bcd^3\sqrt{c^2x^2-1}\left(\frac{\frac{c^2\left(\frac{3\left(35m^3+455m^2+1813m+2161\right)\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f(m+1)(m+2)(m+3)^2(m+5)(m+7)\sqrt{c^2x^2-1}}-\frac{\left(m^4+27m^3+284m^2+1329m+2271\cdots}{f(m+3)^2(m+5)(m\cdots}\right.}{m+5}}{c^2(m+7)}\cdots\right.}{f\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) - (3*c^2*d^3*(f*x)^(3
 + m)*(a + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (3*c^4*d^3*(f*x)^(5 + m)*(a +
b*ArcCosh[c*x]))/(f^5*(5 + m)) - (c^6*d^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x
]))/(f^7*(7 + m)) - (b*c*d^3*Sqrt[-1 + c^2*x^2]*(-((c^4*(f*x)^(6 + m)*Sqrt
[-1 + c^2*x^2])/(f^5*(7 + m)^2)) + ((c^4*(9 + m)*(13 + 2*m)*(f*x)^(4 + m)*
Sqrt[-1 + c^2*x^2])/(f^3*(5 + m)^2*(7 + m)) + (c^2*(-(((2271 + 1329*m + 28
4*m^2 + 27*m^3 + m^4)*(f*x)^(2 + m)*Sqrt[-1 + c^2*x^2])/(f*(3 + m)^2*(5 +
m)*(7 + m))) + (3*(2161 + 1813*m + 455*m^2 + 35*m^3)*(f*x)^(2 + m)*Sqrt[1
- c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f*(1 +
m)*(2 + m)*(3 + m)^2*(5 + m)*(7 + m)*Sqrt[-1 + c^2*x^2])))/(5 + m))/(c^2*(
7 + m))))/(f*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2113

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2340

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [F]

$$\int (fx)^m \left(-c^2 d\,x^2+d\right)^3 \left(a+b\,\operatorname{arccosh}\left(cx\right)\right)dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$
$$=\int -\left(c^2dx^2-d\right)^3(b\,\text{arcosh}\left(cx\right)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*
c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))*(f*
x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx=\text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$
$$=\int -\left(c^2dx^2-d\right)^3 (b\,\text{arcosh}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
-a*c^6*d^3*f^m*x^7*x^m/(m + 7) + 3*a*c^4*d^3*f^m*x^5*x^m/(m + 5) - 3*a*c^2
*d^3*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) - ((m^3 + 9*m^2
 + 23*m + 15)*b*c^6*d^3*f^m*x^7 - 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^4*d^3*f
^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^3*f^m*x^3 - (m^3 + 15*m^2 +
71*m + 105)*b*d^3*f^m*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^4 +
 16*m^3 + 86*m^2 + 176*m + 105) - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c
^7*d^3*f^m*x^7 - 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^5*d^3*f^m*x^5 + 3*(m^3 +
 13*m^2 + 47*m + 35)*b*c^3*d^3*f^m*x^3 - (m^3 + 15*m^2 + 71*m + 105)*b*c*d
^3*f^m*x)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 - (m^4 + 16*m
^3 + 86*m^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^
2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105)*sqrt(c*x + 1)*sqrt(c*x - 1)),
 x) + integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^8*d^3*f^m*x^8 - 3*(m^3 + 11
*m^2 + 31*m + 21)*b*c^6*d^3*f^m*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^4*d
^3*f^m*x^4 - (m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16
*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105
), x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \int (a+b\,\text{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \left(d-c^2dx^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \text{Too large to display}$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output

```
(f**m*d**3*( - x**m*a*c**6*m**3*x**7 - 9*x**m*a*c**6*m**2*x**7 - 23*x**m*a
*c**6*m*x**7 - 15*x**m*a*c**6*x**7 + 3*x**m*a*c**4*m**3*x**5 + 33*x**m*a*c
**4*m**2*x**5 + 93*x**m*a*c**4*m*x**5 + 63*x**m*a*c**4*x**5 - 3*x**m*a*c**
2*m**3*x**3 - 39*x**m*a*c**2*m**2*x**3 - 141*x**m*a*c**2*m*x**3 - 105*x**m
*a*c**2*x**3 + x**m*a*m**3*x + 15*x**m*a*m**2*x + 71*x**m*a*m*x + 105*x**m
*a*x - int(x**m*acosh(c*x)*x**6,x)*b*c**6*m**4 - 16*int(x**m*acosh(c*x)*x*
*6,x)*b*c**6*m**3 - 86*int(x**m*acosh(c*x)*x**6,x)*b*c**6*m**2 - 176*int(x
**m*acosh(c*x)*x**6,x)*b*c**6*m - 105*int(x**m*acosh(c*x)*x**6,x)*b*c**6 +
 3*int(x**m*acosh(c*x)*x**4,x)*b*c**4*m**4 + 48*int(x**m*acosh(c*x)*x**4,x
)*b*c**4*m**3 + 258*int(x**m*acosh(c*x)*x**4,x)*b*c**4*m**2 + 528*int(x**m
*acosh(c*x)*x**4,x)*b*c**4*m + 315*int(x**m*acosh(c*x)*x**4,x)*b*c**4 - 3*
int(x**m*acosh(c*x)*x**2,x)*b*c**2*m**4 - 48*int(x**m*acosh(c*x)*x**2,x)*b
*c**2*m**3 - 258*int(x**m*acosh(c*x)*x**2,x)*b*c**2*m**2 - 528*int(x**m*ac
osh(c*x)*x**2,x)*b*c**2*m - 315*int(x**m*acosh(c*x)*x**2,x)*b*c**2 + int(x
**m*acosh(c*x),x)*b*m**4 + 16*int(x**m*acosh(c*x),x)*b*m**3 + 86*int(x**m*
acosh(c*x),x)*b*m**2 + 176*int(x**m*acosh(c*x),x)*b*m + 105*int(x**m*acosh
(c*x),x)*b))/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

3.141 $\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx)) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 274

$$\begin{aligned}&\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx)) \, dx\\ &= \frac{bcd^2(38+13m+m^2)(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^2(3+m)^2(5+m)^2}\\ &\quad -\frac{bc^3d^2(fx)^{4+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^4(5+m)^2}+\frac{d^2(fx)^{1+m}(a+b\text{arccosh}(cx))}{f(1+m)}\\ &\quad -\frac{2c^2d^2(fx)^{3+m}(a+b\text{arccosh}(cx))}{f^3(3+m)}+\frac{c^4d^2(fx)^{5+m}(a+b\text{arccosh}(cx))}{f^5(5+m)}\\ &\quad -\frac{bcd^2(149+100m+15m^2)(fx)^{2+m}\sqrt{1-cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{f^2(1+m)(2+m)(3+m)^2(5+m)^2\sqrt{-1+cx}}\end{aligned}$$

output
```
b*c*d^2*(m^2+13*m+38)*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(3+m)^2/
(5+m)^2-b*c^3*d^2*(f*x)^(4+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^4/(5+m)^2+d^2*
(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)-2*c^2*d^2*(f*x)^(3+m)*(a+b*arccosh(
c*x))/f^3/(3+m)+c^4*d^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)-b*c*d^2*(
15*m^2+100*m+149)*(f*x)^(2+m)*(-c*x+1)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1
/2*m],c^2*x^2)/f^2/(1+m)/(2+m)/(3+m)^2/(5+m)^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int (fx)^m \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx \\
&= d^2x(fx)^m\left(\frac{a+b\text{arccosh}(cx)}{1+m}-\frac{2c^2x^2(a+b\text{arccosh}(cx))}{3+m}+\frac{c^4x^4(a+b\text{arccosh}(cx))}{5+m}\right. \\
&\quad -\frac{bcx\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad +\frac{2bc^3x^3\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},c^2x^2\right)}{(12+7m+m^2)\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad \left.-\frac{bc^5x^5\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6+m}{2},\frac{8+m}{2},c^2x^2\right)}{(5+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}}\right)
\end{aligned}$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
d^2*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (2*c^2*x^2*(a + b*ArcCosh[c*
x]))/(3 + m) + (c^4*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (b*c*x*Sqrt[1 - c^
2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + 3*m +
m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*x^3*Sqrt[1 - c^2*x^2]*Hyperg
eometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7*m + m^2)*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (b*c^5*x^5*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/
2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6336, 27, 1905, 1590, 27, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (fx)^m (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \ 6336$$

$$-bc\int \frac{d^2(fx)^{m+1}\left(\frac{c^4x^4}{m+5}-\frac{2c^2x^2}{m+3}+\frac{1}{m+1}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcd^2\int \frac{(fx)^{m+1}\left(\frac{c^4x^4}{m+5}-\frac{2c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f}+\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \ 1905$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}\left(\frac{c^4x^4}{m+5}-\frac{2c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \ 1590$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{\int \frac{c^2(fx)^{m+1}\left(\frac{m+5}{m+1}-\frac{c^2\left(m^2+13m+38\right)x^2}{(m+3)(m+5)}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+5)}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+4}}{f^3(m+5)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{\int \frac{(fx)^{m+1}\left(\frac{m+5}{m+1}-\frac{c^2(m^2+13m+38)x^2}{(m+3)(m+5)}\right)}{\sqrt{c^2x^2-1}}dx}{m+5}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+4}}{f^3(m+5)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \ 363$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{\frac{(15m^2+100m+149)\int \frac{(fx)^{m+1}}{\sqrt{c^2x^2-1}}dx}{(m+1)(m+3)^2(m+5)}-\frac{(m^2+13m+38)\sqrt{c^2x^2-1}(fx)^{m+2}}{f(m+3)^2(m+5)}}{m+5}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+4}}{f^3(m+5)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \ 279$$

$$-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{\frac{(15m^2+100m+149)\sqrt{1-c^2x^2}\int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx}{(m+1)(m+3)^2(m+5)\sqrt{c^2x^2-1}}-\frac{(m^2+13m+38)\sqrt{c^2x^2-1}(fx)^{m+2}}{f(m+3)^2(m+5)}}{m+5}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+4}}{f^3(m+5)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \ 278$$

$$\frac{c^4d^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}-\frac{bcd^2\sqrt{c^2x^2-1}\left(\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+4}}{f^3(m+5)^2}+\frac{\frac{(15m^2+100m+149)\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f(m+1)(m+2)(m+3)^2(m+5)\sqrt{c^2x^2-1}}-\frac{(m^2+13m+38)\sqrt{c^2x^2}}{f(m+3)^2(m+}}{m+5}\right.}{f\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) - (2*c^2*d^2*(f*x)^(3
 + m)*(a + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (c^4*d^2*(f*x)^(5 + m)*(a + b*
ArcCosh[c*x]))/(f^5*(5 + m)) - (b*c*d^2*Sqrt[-1 + c^2*x^2]*((c^2*(f*x)^(4
+ m)*Sqrt[-1 + c^2*x^2])/(f^3*(5 + m)^2) + (-(((38 + 13*m + m^2)*(f*x)^(2
+ m)*Sqrt[-1 + c^2*x^2])/(f*(3 + m)^2*(5 + m))) + ((149 + 100*m + 15*m^2)*
(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/
2, c^2*x^2])/(f*(1 + m)*(2 + m)*(3 + m)^2*(5 + m)*Sqrt[-1 + c^2*x^2]))/(5
+ m)))/(f*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [F]

$$\int (fx)^m \left(-c^2 d x^2 + d\right)^2 (a + b \operatorname{arccosh}(cx))\, dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx)) \, dx = \int (c^2dx^2-d)^2(b\,\text{arcosh}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx)) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx)) \, dx = \int (c^2dx^2-d)^2(b\,\text{arcosh}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
a*c^4*d^2*f^m*x^5*x^m/(m + 5) - 2*a*c^2*d^2*f^m*x^3*x^m/(m + 3) + (f*x)^(m
 + 1)*a*d^2/(f*(m + 1)) + ((m^2 + 4*m + 3)*b*c^4*d^2*f^m*x^5 - 2*(m^2 + 6*
m + 5)*b*c^2*d^2*f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(((m^2 + 4*
m + 3)*b*c^5*d^2*f^m*x^5 - 2*(m^2 + 6*m + 5)*b*c^3*d^2*f^m*x^3 + (m^2 + 8*
m + 15)*b*c*d^2*f^m*x)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^3*x^3 - (m^3 + 9*m
^2 + 23*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 2
3*m - 15)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate(((m^2 + 4*m + 3)*b*
c^6*d^2*f^m*x^6 - 2*(m^2 + 6*m + 5)*b*c^4*d^2*f^m*x^4 + (m^2 + 8*m + 15)*b
*c^2*d^2*f^m*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 2
3*m - 15), x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))\,dx = \int (a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \left(d - c^2dx^2\right)^2 (a + b\text{arccosh}(cx))\, dx$$
$$= \frac{f^m d^2 \left(x^m a\, c^4 m^2 x^5 + 4x^m a\, c^4 m\, x^5 + 3x^m a\, c^4 x^5 - 2x^m a\, c^2 m^2 x^3 - 12x^m a\, c^2 m\, x^3 - 10x^m a\, c^2 x^3 + x^m a\, m^2\right.}{}$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(f**m*d**2*(x**m*a*c**4*m**2*x**5 + 4*x**m*a*c**4*m*x**5 + 3*x**m*a*c**4*x
**5 - 2*x**m*a*c**2*m**2*x**3 - 12*x**m*a*c**2*m*x**3 - 10*x**m*a*c**2*x**
3 + x**m*a*m**2*x + 8*x**m*a*m*x + 15*x**m*a*x + int(x**m*acosh(c*x)*x**4,
x)*b*c**4*m**3 + 9*int(x**m*acosh(c*x)*x**4,x)*b*c**4*m**2 + 23*int(x**m*a
cosh(c*x)*x**4,x)*b*c**4*m + 15*int(x**m*acosh(c*x)*x**4,x)*b*c**4 - 2*int
(x**m*acosh(c*x)*x**2,x)*b*c**2*m**3 - 18*int(x**m*acosh(c*x)*x**2,x)*b*c*
*2*m**2 - 46*int(x**m*acosh(c*x)*x**2,x)*b*c**2*m - 30*int(x**m*acosh(c*x)
*x**2,x)*b*c**2 + int(x**m*acosh(c*x),x)*b*m**3 + 9*int(x**m*acosh(c*x),x)
*b*m**2 + 23*int(x**m*acosh(c*x),x)*b*m + 15*int(x**m*acosh(c*x),x)*b))/(m
**3 + 9*m**2 + 23*m + 15)
```

3.142 $\int (fx)^m (d-c^2dx^2) (a+b\text{arccosh}(cx)) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\begin{aligned}\int (fx)^m (d-c^2dx^2) (a+b\text{arccosh}(cx)) \, dx &= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^2(3+m)^2}+\frac{d(fx)^{1+m}(a+b\text{arccosh}(cx))}{f(1+m)} \\ &\quad -\frac{c^2d(fx)^{3+m}(a+b\text{arccosh}(cx))}{f^3(3+m)} \\ &\quad -\frac{bcd(7+3m)(fx)^{2+m}\sqrt{1-cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{f^2(1+m)(2+m)(3+m)^2\sqrt{-1+cx}}\end{aligned}$$

output

```
b*c*d*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(3+m)^2+d*(f*x)^(1+m)*(a
+b*arccosh(c*x))/f/(1+m)-c^2*d*(f*x)^(3+m)*(a+b*arccosh(c*x))/f^3/(3+m)-b*
c*d*(7+3*m)*(f*x)^(2+m)*(-c*x+1)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],
c^2*x^2)/f^2/(1+m)/(2+m)/(3+m)^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.12

$$\int (fx)^m \left(d-c^2dx^2\right)(a+b\text{arccosh}(cx))\,dx$$
$$=dx(fx)^m\left(\frac{a+b\text{arccosh}(cx)}{1+m}-\frac{c^2x^2(a+b\text{arccosh}(cx))}{3+m}-\frac{bcx\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3x^3\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},c^2x^2\right)}{(12+7m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
d*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (c^2*x^2*(a + b*ArcCosh[c*x]))
/(3 + m) - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 +
 m)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*x
^3*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2]
)/((12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6336, 27, 960, 136, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)(fx)^m(a+b\text{arccosh}(cx))\,dx$$
$$\Big\downarrow \text{6336}$$

$$-bc\int \frac{d(fx)^{m+1}\left(-c^2(m+1)x^2+m+3\right)}{f\left(m^2+4m+3\right)\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{c^2d(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcd\int \frac{(fx)^{m+1}\left(-c^2(m+1)x^2+m+3\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f\left(m^2+4m+3\right)}-\frac{c^2d(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{960}$$

$$-\frac{bcd\left(\frac{(3m+7)\int \frac{(fx)^{m+1}}{\sqrt{cx-1}\sqrt{cx+1}}dx}{m+3}-\frac{(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{f(m+3)}\right)}{f\left(m^2+4m+3\right)}-\frac{c^2d(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{136}$$

$$-\frac{bcd\left(\frac{(3m+7)\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}}{\sqrt{c^2x^2-1}}dx}{(m+3)\sqrt{cx-1}\sqrt{cx+1}}-\frac{(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{f(m+3)}\right)}{f\left(m^2+4m+3\right)}-\frac{c^2d(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{279}$$

$$-\frac{bcd\left(\frac{(3m+7)\sqrt{1-c^2x^2}\int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx}{(m+3)\sqrt{cx-1}\sqrt{cx+1}}-\frac{(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{f(m+3)}\right)}{f\left(m^2+4m+3\right)}-\frac{c^2d(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}$$

$$\Big\downarrow \text{278}$$

$$-\frac{c^2d(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}-\frac{bcd\left(\frac{(3m+7)\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f(m+2)(m+3)\sqrt{cx-1}\sqrt{cx+1}}-\frac{(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{f(m+3)}\right)}{f\left(m^2+4m+3\right)}$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) - (c^2*d*(f*x)^(3 + m)*
(a + b*ArcCosh[c*x]))/(f^3*(3 + m)) - (b*c*d*(-(((1 + m)*(f*x)^(2 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(f*(3 + m))) + ((7 + 3*m)*(f*x)^(2 + m)*Sqrt[1
- c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f*(2 +
m)*(3 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(f*(3 + 4*m + m^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 136
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^Fr
acPart[m])   Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b, c, d, f,
 m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [F]

$$\int (fx)^m \left(-c^2 d\,x^2 + d\right) (a + b\ \mathrm{arccosh}\,(cx))\, dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\mathrm{arccosh}(cx))\, dx = \int -\left(c^2dx^2 - d\right)(b\,\mathrm{arcosh}\,(cx) + a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*(f*x)^m,
x)
```

Sympy [F]

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\operatorname{arccosh}(cx))\,dx = -d\left(\int \left(-a(fx)^m\right)\,dx + \int \left(-b(fx)^m \operatorname{acosh}(cx)\right)\,dx + \int ac^2x^2(fx)^m\,dx + \int bc^2x^2(fx)^m \operatorname{acosh}(cx)\,dx\right)$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
-d*(Integral(-a*(f*x)**m, x) + Integral(-b*(f*x)**m*acosh(c*x), x) + Integ
ral(a*c**2*x**2*(f*x)**m, x) + Integral(b*c**2*x**2*(f*x)**m*acosh(c*x), x
))
```

Maxima [F]

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\operatorname{arccosh}(cx))\,dx = \int -\left(c^2dx^2-d\right)(b\operatorname{arcosh}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
-a*c^2*d*f^m*x^3*x^m/(m + 3) - (b*c^2*d*f^m*(m + 1)*x^3 - b*d*f^m*(m + 3)*
x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^2 + 4*m + 3) + (f*x)^(m +
 1)*a*d/(f*(m + 1)) - integrate((b*c^3*d*f^m*(m + 1)*x^3 - b*c*d*f^m*(m +
3)*x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 - (m^2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3
)*c^2*x^2 - m^2 - 4*m - 3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + integrate((b
*c^4*d*f^m*(m + 1)*x^4 - b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2
*x^2 - m^2 - 4*m - 3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right)(a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right)(a+b\operatorname{arccosh}(cx))\,dx = \int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \left(d-c^2dx^2\right)(a+b\operatorname{arccosh}(cx))\,dx$$
$$= \frac{f^m d\left(-x^m a\,c^2 m\,x^3 - x^m a\,c^2 x^3 + x^m a m x + 3x^m a x - \left(\int x^m acosh(cx)\,x^2 dx\right) b\,c^2 m^2 - 4\left(\int x^m acosh(cx)\,x\right.}{}$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(f**m*d*( - x**m*a*c**2*m*x**3 - x**m*a*c**2*x**3 + x**m*a*m*x + 3*x**m*a*
x - int(x**m*acosh(c*x)*x**2,x)*b*c**2*m**2 - 4*int(x**m*acosh(c*x)*x**2,x
)*b*c**2*m - 3*int(x**m*acosh(c*x)*x**2,x)*b*c**2 + int(x**m*acosh(c*x),x)
*b*m**2 + 4*int(x**m*acosh(c*x),x)*b*m + 3*int(x**m*acosh(c*x),x)*b))/(m**
2 + 4*m + 3)
```

3.143 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))}{d\,(1-c^2x^2)},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))/d/(-c^2*x^2+1),x)
```

Mathematica [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = \int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}dx$$

$$\Big\downarrow \ 6375$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\,\text{arccosh}\,(cx))}{-c^2d\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\int -\frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{c^2dx^2-d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Sympy [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=-\frac{\int \frac{a(fx)^m}{c^2x^2-1}\,dx+\int \frac{b(fx)^m\,\text{acosh}\,(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*(f*x)**m/(c**2*x**2 - 1), x) + Integral(b*(f*x)**m*acosh(c*x)
/(c**2*x**2 - 1), x))/d
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\int -\frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{c^2dx^2-d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\int -\frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{c^2dx^2-d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Mupad [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\,(f\,x)^m}{d-c^2\,d\,x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2),x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d-c^2dx^2}\,dx = -\frac{f^m\left(\left(\int \frac{x^m}{c^2x^2-1}dx\right)a+\left(\int \frac{x^m acosh(cx)}{c^2x^2-1}dx\right)b\right)}{d}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - f**m*(int(x**m/(c**2*x**2 - 1),x)*a + int((x**m*acosh(c*x))/(c**2*x**2
 - 1),x)*b))/d
```

3.144 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))}{d^2(1-c^2x^2)^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}dx$$

$$\Big\downarrow \ 6351$$

$$\frac{(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d(1-c^2x^2)}dx}{2d}+\frac{bc\int \frac{(fx)^{m+1}}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2f}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2d^2f(1-c^2x^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2d^2}+\frac{bc\int \frac{(fx)^{m+1}}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2d^2f}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2d^2f(1-c^2x^2)}$$

$$\Big\downarrow \ 136$$

$$\frac{(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2d^2}+\frac{bc\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}}{(c^2x^2-1)^{3/2}}dx}{2d^2f\sqrt{cx-1}\sqrt{cx+1}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2d^2f(1-c^2x^2)}$$

$$\Big\downarrow \ 279$$

$$\frac{(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2d^2}-\frac{bc\sqrt{1-c^2x^2}\int \frac{(fx)^{m+1}}{(1-c^2x^2)^{3/2}}dx}{2d^2f\sqrt{cx-1}\sqrt{cx+1}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2d^2f(1-c^2x^2)}$$

$$\Big\downarrow \ 278$$

$$\frac{(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2d^2}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2d^2f(1-c^2x^2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{2d^2f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 6375$$

$$\frac{(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{2d^2}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2d^2f(1-c^2x^2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{2d^2f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\,\text{arccosh}\,(cx))}{(-c^2d\,x^2+d)^2}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas
")
```

output
```
integral((b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
 x)
```

Sympy [N/A]

Not integrable

Time = 50.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\frac{\int \frac{a(fx)^m}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{b(fx)^m\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*(f*x)**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*(f*x)*
*m*acosh(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\,\operatorname{arcosh}(cx)+a)(fx)^m}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,\left(f\,x\right)^m}{\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^2,x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^2}\,dx=\frac{f^m\left(\left(\int \frac{x^m}{c^4x^4-2c^2x^2+1}dx\right)a+\left(\int \frac{x^m\,acosh(cx)}{c^4x^4-2c^2x^2+1}dx\right)b\right)}{d^2}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(f**m*(int(x**m/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a + int((x**m*acosh(c*x))
/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b))/d**2
```

3.145 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))}{d^3(1-c^2x^2)^3},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 11.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{6351}$$

$$\frac{(3-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d^2(1-c^2x^2)^2}dx}{4d}-\frac{bc\int \frac{(fx)^{m+1}}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4d^3f}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f(1-c^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4d^3}-\frac{bc\int \frac{(fx)^{m+1}}{(cx-1)^{5/2}(cx+1)^{5/2}}dx}{4d^3f}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f(1-c^2x^2)^2}$$

$$\Big\downarrow \text{136}$$

$$\frac{(3-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4d^3}-\frac{bc\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}}{(c^2x^2-1)^{5/2}}dx}{4d^3f\sqrt{cx-1}\sqrt{cx+1}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f(1-c^2x^2)^2}$$

$$\Big\downarrow \text{279}$$

$$\frac{(3-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4d^3}-\frac{bc\sqrt{1-c^2x^2}\int \frac{(fx)^{m+1}}{(1-c^2x^2)^{5/2}}dx}{4d^3f\sqrt{cx-1}\sqrt{cx+1}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f(1-c^2x^2)^2}$$

$$\Big\downarrow \text{278}$$

$$\frac{(3-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{4d^3}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f(1-c^2x^2)^2}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6351}$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx+\frac{bc\int \frac{(fx)^{m+1}}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2f}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2f(1-c^2x^2)}\right)}{4d^3}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f\left(1-c^2x^2\right)^2}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{136}$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx+\frac{bc\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}}{\left(c^2x^2-1\right)^{3/2}}dx}{2f\sqrt{cx-1}\sqrt{cx+1}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2f(1-c^2x^2)}\right)}{4d^3}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f\left(1-c^2x^2\right)^2}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{279}$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx-\frac{bc\sqrt{1-c^2x^2}\int \frac{(fx)^{m+1}}{\left(1-c^2x^2\right)^{3/2}}dx}{2f\sqrt{cx-1}\sqrt{cx+1}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2f(1-c^2x^2)}\right)}{4d^3}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f\left(1-c^2x^2\right)^2}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2f(1-c^2x^2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+}{2}\right.}{2f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}}{4d^3}$$

$$\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f\left(1-c^2x^2\right)^2}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6375}$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{1-c^2x^2}dx+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{2f(1-c^2x^2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+}{2}\right.}{2f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}}{4d^3}$$

$$\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{4d^3f\left(1-c^2x^2\right)^2}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b \operatorname{arccosh}(cx))}{(-c^2 d\, x^2 + d)^3} dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^3}\, dx = \int -\frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(c^2dx^2 - d)^3}\, dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas
")
```

output
```
integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^
2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima
")
```

output
```
-integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = \int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^m}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^3,x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{f^m\left(\left(\int \frac{x^m}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)a+\left(\int \frac{x^m\,acosh(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\right)}{d^3}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - f**m*(int(x**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a + int(
(x**m*acosh(c*x))/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b))/d**3
```

3.146 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 718

$$
\begin{aligned}
\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx &= -\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{15bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2(4+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{5bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(6+m)\left(8+6m+m^2\right)\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{5bc^3d^2(fx)^{4+m}\sqrt{d-c^2dx^2}}{f^4(4+m)^2(6+m)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3d^2(fx)^{4+m}\sqrt{d-c^2dx^2}}{f^4(4+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{bc^5d^2(fx)^{6+m}\sqrt{d-c^2dx^2}}{f^6(6+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{15d^2(fx)^{1+m}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{f(6+m)\left(8+6m+m^2\right)} \\
&+ \frac{5d(fx)^{1+m}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{f(4+m)(6+m)} \\
&+ \frac{(fx)^{1+m}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))}{f(6+m)} \\
&+ \frac{15d^3(fx)^{1+m}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(6+m)\left(8+14m+7m^2+m^3\right)\sqrt{d-c^2dx^2}} \\
&+ \frac{15bcd^3(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2(1+m)(2+m)^2(4+m)(6+m)\sqrt{d-c^2dx^2}}
\end{aligned}
$$

output
```
-b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)/(6+m)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-15*b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/
(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5*b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/
2)/f^2/(6+m)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*b*c^3*d^2*(f*x)^(4+
m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)^2/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*
c^3*d^2*(f*x)^(4+m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-b*c^5*d^2*(f*x)^(6+m)*(-c^2*d*x^2+d)^(1/2)/f^6/(6+m)^2/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+15*d^2*(f*x)^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c
*x))/f/(6+m)/(m^2+6*m+8)+5*d*(f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh
(c*x))/f/(4+m)/(6+m)+(f*x)^(1+m)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/f
/(6+m)+15*d^3*(f*x)^(1+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hypergeom(
[1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/f/(6+m)/(m^3+7*m^2+14*m+8)/(-c^2*d*x
^2+d)^(1/2)+15*b*c*d^3*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*hypergeom([
1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/f^2/(1+m)/(2+m)^2/(4+m)
/(6+m)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.49

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx = \frac{d^2x(fx)^m\sqrt{d-c^2dx^2}\left(\frac{5bcx\left(-\frac{1}{2+m}+\frac{c^2x^2}{4+m}\right)}{4+m}-bcx\left(\frac{1}{2+m}-\frac{2c^2x^2}{4+m}+\frac{c^4x^4}{6+m}\right)-\frac{5(-1+cx)^{3/2}(1+cx)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((5*b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4
 + m)))/(4 + m) - b*c*x*((2 + m)^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6
 + m)) - (5*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]))/(4 + m)
 + (-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) + (15*(-(b*c*x) +
 (2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]) - ((2 + m)*((Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*HypergeometricPF
Q[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + m)))/(1 + m)
))/((2 + m)^2*(4 + m))))/((6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {6345, 82, 244, 2009, 6345, 25, 82, 244, 2009, 6341, 17, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(fx)^m(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6345}$$

$$\frac{5d\int (fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}-\frac{bcd^2\sqrt{d-c^2dx^2}\int (fx)^{m+1}(1-cx)^2(cx+1)^2dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}$$

$$\Big\downarrow \text{82}$$

$$\frac{5d\int (fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}-\frac{bcd^2\sqrt{d-c^2dx^2}\int (fx)^{m+1}\left(1-c^2x^2\right)^2dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}$$

$$\Big\downarrow \text{244}$$

$$\frac{5d\int(fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}-\frac{bcd^2\sqrt{d-c^2dx^2}\int\left((fx)^{m+1}-\frac{2c^2(fx)^{m+3}}{f^2}+\frac{c^4(fx)^{m+5}}{f^4}\right)dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}$$

$\big\downarrow$ 2009

$$\frac{5d\int(fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 6345

$$\frac{5d\left(\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\int-(fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx}{f(m+4)}\right.}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 25

$$\frac{5d\left(\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bcd\sqrt{d-c^2dx^2}\int(fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}\right.}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 82

$$\frac{5d\left(\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bcd\sqrt{d-c^2dx^2}\int (fx)^{m+1}(1-c^2x^2)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(d-c^2dx^2)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}\right)}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{244}$$

$$\frac{5d\left(\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bcd\sqrt{d-c^2dx^2}\int \left((fx)^{m+1}-\frac{c^2(fx)^{m+3}}{f^2}\right)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(d-c^2dx^2)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}\right)}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5d\left(\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{(d-c^2dx^2)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6341}$$

$$\frac{5d\left(\frac{3d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int(fx)^{m+1}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}\right)}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)}{f}\right.}{m+6}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 17

$$\frac{5d\left(\frac{3d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{d-c^2dx^2}(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m}}{f(m}\right.}{m+6}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 6364

$$\frac{5d\left(\frac{3d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{bc(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)}+\frac{\sqrt{1-cx}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{arccosh}(cx))}{f(m+1)\sqrt{cx-1}}\right.}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right.}{m+4}\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-((b*c*d^2*Sqrt[d - c^2*d*x^2]*((f*x)^(2 + m)/(f*(2 + m)) - (2*c^2*(f*x)^(
4 + m))/(f^3*(4 + m)) + (c^4*(f*x)^(6 + m))/(f^5*(6 + m))))/(f*(6 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*(d - c^2*d*x^2)^(5/2)*(a + b*
ArcCosh[c*x]))/(f*(6 + m)) + (5*d*(-((b*c*d*Sqrt[d - c^2*d*x^2]*((f*x)^(2
+ m)/(f*(2 + m)) - (c^2*(f*x)^(4 + m))/(f^3*(4 + m))))/(f*(4 + m)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCo
sh[c*x]))/(f*(4 + m)) + (3*d*(-((b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f
^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(f*(2 + m)) - (Sqrt[d - c^2*d*x^2]*(((f*x)^(1
 + m)*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c*x]) + (b*c*(f*x)^(2 + m)*Hype
rgeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*
(1 + m)*(2 + m))))/((2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(4 + m)))/(6 +
 m)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (fx)^m \left(-c^2 d x^2+d\right)^{\frac{5}{2}} (a+b \operatorname{arccosh}(cx))\, dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d - c^2dx^2\right)^{5/2} (a + b\text{arccosh}(cx)) \, dx = \int \left(-c^2dx^2 + d\right)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fr
icas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d - c^2dx^2\right)^{5/2} (a + b\text{arccosh}(cx)) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \left(d - c^2dx^2\right)^{5/2} (a + b\text{arccosh}(cx)) \, dx = \int \left(-c^2dx^2 + d\right)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="ma
xima")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d - c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))\,dx = \int (a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))\,dx = f^m\sqrt{d}\,d^2\left(\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,x^4dx\right)b\,c^4 - 2\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,x^2dx\right)b\,c^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)b + \left(\int x^m\sqrt{-c^2x^2+1}\,x^4dx\right)a\,c^4 - 2\left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a\,c^2 + \left(\int x^m\sqrt{-c^2x^2+1}dx\right)a\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
f**m*sqrt(d)*d**2*(int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*b*c*
*4 - 2*int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*b*c**2 + int(x**
m*sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*b + int(x**m*sqrt( - c**2*x**2 + 1)
*x**4,x)*a*c**4 - 2*int(x**m*sqrt( - c**2*x**2 + 1)*x**2,x)*a*c**2 + int(x
**m*sqrt( - c**2*x**2 + 1),x)*a)
```

3.147 $\int (fx)^m (d - c^2dx^2)^{3/2} (a+b\text{arccosh}(cx)) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 454

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx)) \, dx = -\frac{3bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2\left(8+6m+m^2\right)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d(fx)^{4+m}\sqrt{d-c^2dx^2}}{f^4(4+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3d(fx)^{1+m}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{f\left(8+6m+m^2\right)} + \frac{(fx)^{1+m}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))}{f(4+m)} + \frac{3d^2(fx)^{1+m}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f\left(8+14m+7m^2+m^3\right)\sqrt{d-c^2dx^2}} + \frac{3bcd^2(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2(1+m)(2+m)^2(4+m)\sqrt{d-c^2dx^2}}$$

output
```
-3*b*c*d*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-b*c*d*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(m^2+6*m+8)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)+b*c^3*d*(f*x)^(4+m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)^
2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*d*(f*x)^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccosh(c*x))/f/(m^2+6*m+8)+(f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*
x))/f/(4+m)+3*d^2*(f*x)^(1+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hyperg
eom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/f/(m^3+7*m^2+14*m+8)/(-c^2*d*x^2
+d)^(1/2)+3*b*c*d^2*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*hypergeom([1,
1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/f^2/(1+m)/(2+m)^2/(4+m)/(-
c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.60

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))\,dx = -\frac{dx(fx)^m\sqrt{d-c^2dx^2}\left(\frac{3bcx}{(2+m)^2}+bcx\left(\frac{1}{2+m}-\frac{c^2x^2}{4+m}\right)-\frac{3\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))}{2+m}+(-1+cx)^{3/2}(1+cx)\cdots\right.}{(4+m)\cdots}$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-((d*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((3*b*c*x)/(2 + m)^2 + b*c*x*((2 + m)^(
-1) - (c^2*x^2)/(4 + m)) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[
c*x]))/(2 + m) + (-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + (
3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2])/((1 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*
b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x
^2])/((1 + m)*(2 + m)^2)))/((4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi [A] (warning: unable to verify)

Time = 1.17 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6345, 25, 82, 244, 2009, 6341, 17, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(fx)^m(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6345}$$

$$\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\int -(fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bcd\sqrt{d-c^2dx^2}\int (fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$$\Big\downarrow \text{82}$$

$$\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bcd\sqrt{d-c^2dx^2}\int (fx)^{m+1}\left(1-c^2x^2\right)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$$\Big\downarrow \text{244}$$

$$\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bcd\sqrt{d-c^2dx^2}\int \left((fx)^{m+1}-\frac{c^2(fx)^{m+3}}{f^2}\right)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6341}$$

$$\frac{3d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int (fx)^{m+1}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}\right)}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{3d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{d-c^2dx^2}(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6364}$$

$$\frac{3d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{bc(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)}+\frac{\sqrt{1-cx}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{arccosh}(cx))}{f(m+1)\sqrt{cx-1}}\right)}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+4}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-((b*c*d*Sqrt[d - c^2*d*x^2]*((f*x)^(2 + m)/(f*(2 + m)) - (c^2*(f*x)^(4 +
m))/(f^3*(4 + m))))/(f*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1
+ m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)) + (3*d*(-((b*
c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(2
+ m)) - (Sqrt[d - c^2*d*x^2]*(((f*x)^(1 + m)*Sqrt[1 - c*x]*(a + b*ArcCosh[
c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sq
rt[-1 + c*x]) + (b*c*(f*x)^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}
, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m))))/((2 + m)*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])))/(4 + m)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (fx)^m \left(-c^2 d\,x^2+d\right)^{\frac{3}{2}} (a+b\,\mathrm{arccosh}\,(cx))\,dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d - c^2dx^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))\, dx = \int \left(-c^2dx^2 + d\right)^{\frac{3}{2}} (b\operatorname{arcosh}(cx) + a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fr
icas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2
*d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d - c^2dx^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))\, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \left(d - c^2dx^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))\, dx = \int \left(-c^2dx^2 + d\right)^{\frac{3}{2}} (b\operatorname{arcosh}(cx) + a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="ma
xima")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx)) \, dx = \int (a+b\,\text{acosh}(c\,x)) \, \left(d-c^2\,d\,x^2\right)^{3/2} (f\,x)^m \, dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\operatorname{arccosh}(cx))\,dx = f^m\sqrt{d}\,d\left(-\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,x^2dx\right)b\,c^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)b - \left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a\,c^2 + \left(\int x^m\sqrt{-c^2x^2+1}dx\right)a\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
f**m*sqrt(d)*d*( - int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*b*c*
*2 + int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*b - int(x**m*sqrt( - c*
*2*x**2 + 1)*x**2,x)*a*c**2 + int(x**m*sqrt( - c**2*x**2 + 1),x)*a)
```

3.148 $\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 274

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx = -\frac{bc(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(fx)^{1+m}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{f(2+m)} + \frac{d(fx)^{1+m}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(2+3m+m^2)\sqrt{d-c^2dx^2}} + \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2(1+m)(2+m)^2\sqrt{d-c^2dx^2}}$$

output
```
-b*c*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1
/2)+(f*x)^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/f/(2+m)+d*(f*x)^(1
+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+
1/2*m],c^2*x^2)/f/(m^2+3*m+2)/(-c^2*d*x^2+d)^(1/2)+b*c*d*(f*x)^(2+m)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m
],c^2*x^2)/f^2/(1+m)/(2+m)^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.81

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx$$
$$= \frac{x(fx)^m\sqrt{d-c^2dx^2}\left((1+m)\left(-bcx\sqrt{-1+cx}\sqrt{1+cx}+a(2+m)\left(-1+c^2x^2\right)+b(2+m)\left(-1+c^2x^2\right)\right.\right.}{}$$

input
```
Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
(x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) -
 (2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeom
etricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*
(2 + m)^2*(-1 + c*x)*(1 + c*x))
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {6341, 17, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(fx)^m(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6341}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int (fx)^{m+1}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}$$

$$\Big\downarrow \text{17}$$

$$-\frac{\sqrt{d-c^2dx^2}\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{d-c^2dx^2}(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6364

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{bc(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)}+\frac{\sqrt{1-cx}(fx)^{m+1}\text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{arccos}}{f(m+1)\sqrt{cx-1}}\right.}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{d-c^2dx^2}(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

output
```
-((b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
f*(2 + m)) - (Sqrt[d - c^2*d*x^2]*(((f*x)^(1 + m)*Sqrt[1 - c*x]*(a + b*Arc
Cosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 +
m)*Sqrt[-1 + c*x]) + (b*c*(f*x)^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 +
 m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m))))/((2 + m)*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (fx)^m \sqrt{-c^2d\,x^2+d}\,(a+b\,\mathrm{arccosh}\,(cx))\,dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\mathrm{arccosh}(cx))\,dx=\int \sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fr
icas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int (fx)^m\sqrt{-d(cx-1)(cx+1)}(a+b\,\text{acosh}(cx))\,dx$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

output
```
Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\,dx=\int (a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\,dx = f^m\sqrt{d}\left(\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)b+\left(\int x^m\sqrt{-c^2x^2+1}dx\right)a\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x)),x)
```

output
```
f**m*sqrt(d)*(int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*b + int(x**m*s
qrt( - c**2*x**2 + 1),x)*a)
```

3.149 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 176

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{(fx)^{1+m}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(1+m)\sqrt{d-c^2dx^2}} + \frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2\left(2+3m+m^2\right)\sqrt{d-c^2dx^2}}$$

output
```
(f*x)^(1+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*
m],[3/2+1/2*m],c^2*x^2)/f/(1+m)/(-c^2*d*x^2+d)^(1/2)+b*c*(f*x)^(2+m)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m
],c^2*x^2)/f^2/(m^2+3*m+2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{x(fx)^m\left((2+m)\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)+bcx\sqrt{-1+cx}\right.}{(1+m)(2+m)\sqrt{d-c^2dx^2}}$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(x*(f*x)^m*((2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])
)/((1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {6363}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{ 6363}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{arccosh}(cx))}{f(m+1)\sqrt{d-c^2dx^2}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*
(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d - c^
2*d*x^2])
```

Defintions of rubi rules used

rule 6363
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^m (a + b \operatorname{arccosh}(cx))}{\sqrt{-c^2 d x^2 + d}} dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m (a + b\text{arccosh}(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{\sqrt{-c^2 dx^2 + d}} dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d
), x)
```

Sympy [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(fx)^m\,(a+b\operatorname{acosh}{(cx)})}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(f\,x)^m}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{f^m\left(\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}}dx\right)a+\left(\int \frac{x^m\,acosh(cx)}{\sqrt{-c^2x^2+1}}dx\right)b\right)}{\sqrt{d}}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(f**m*(int(x**m/sqrt( - c**2*x**2 + 1),x)*a + int((x**m*acosh(c*x))/sqrt(
- c**2*x**2 + 1),x)*b))/sqrt(d)
```

3.150 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 300

$$\begin{aligned}\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx &= \frac{(fx)^{1+m}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}\\ &- \frac{m(fx)^{1+m}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{df(1+m)\sqrt{d-c^2dx^2}}\\ &+ \frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{df^2(2+m)\sqrt{d-c^2dx^2}}\\ &- \frac{bcm(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{df^2(2+3m+m^2)\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
(f*x)^(1+m)*(a+b*arccosh(c*x))/d/f/(-c^2*d*x^2+d)^(1/2)-m*(f*x)^(1+m)*(-c^
2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c
^2*x^2)/d/f/(1+m)/(-c^2*d*x^2+d)^(1/2)+b*c*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)*hypergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)/d/f^2/(2+m)/(-c^2*d*x^2
+d)^(1/2)-b*c*m*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*hypergeom([1, 1+1/
2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/d/f^2/(m^2+3*m+2)/(-c^2*d*x^2+
d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.72

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(fx)^m\left(-m(2+m)\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\right.\right.}{}$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(f*x)^m*(-(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]) + (1 + m)*((2 + m)*(a + b*ArcC
osh[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, 1 + m/
2, 2 + m/2, c^2*x^2]) - b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometri
cPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(d*(1 + m)*(2
 + m)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {6351, 25, 82, 278, 6363}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6351}$$

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{df\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{df\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 82

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{1-c^2x^2}dx}{df\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 278

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{df^2(m+2)\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 6363

$$-\frac{m\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{arc}}{f(m+1)\sqrt{d-c^2dx^2}}\right.}{d}$$
$$\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{df^2(m+2)\sqrt{d-c^2dx^2}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*f*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x
)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2 + m)/2, (4
+ m)/2, c^2*x^2])/(d*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (m*(((f*x)^(1 + m)
*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2 + m)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6363
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^m (a + b \operatorname{arccosh}(cx))}{(-c^2 d\, x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 -
2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(fx)^m (a + b \operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)(fx)^m}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)(fx)^m}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(a + b\,\text{acosh}(c\,x))\,(f\,x)^m}{(d - c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{3/2}}\,dx = -\frac{f^m\left(\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)a + \left(\int \frac{x^m\,acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\right)}{\sqrt{d}\,d}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - f**m*(int(x**m/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 +
 1)),x)*a + int((x**m*acosh(c*x))/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt
( - c**2*x**2 + 1)),x)*b))/(sqrt(d)*d)
```

3.151 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 450

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{(fx)^{1+m}(a+b\text{arccosh}(cx))}{3df\,(d-c^2dx^2)^{3/2}}$$
$$+\frac{(2-m)(fx)^{1+m}(a+b\text{arccosh}(cx))}{3d^2f\sqrt{d-c^2dx^2}}$$
$$-\frac{(2-m)m(fx)^{1+m}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{3d^2f(1+m)\sqrt{d-c^2dx^2}}$$
$$+\frac{bc(2-m)(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{3d^2f^2(2+m)\sqrt{d-c^2dx^2}}$$
$$+\frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{3d^2f^2(2+m)\sqrt{d-c^2dx^2}}$$
$$-\frac{bc(2-m)m(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{3d^2f^2\left(2+3m+m^2\right)\sqrt{d-c^2dx^2}}$$

output
```
1/3*(f*x)^(1+m)*(a+b*arccosh(c*x))/d/f/(-c^2*d*x^2+d)^(3/2)+1/3*(2-m)*(f*x
)^(1+m)*(a+b*arccosh(c*x))/d^2/f/(-c^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*(f*x)^(1
+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+
1/2*m],c^2*x^2)/d^2/f/(1+m)/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*(f*x)^(2+m)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*hypergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)/d^2
/f^2/(2+m)/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*hypergeom([2, 1+1/2*m],[2+1/2*m],c^2*x^2)/d^2/f^2/(2+m)/(-c^2*d*x^2+
d)^(1/2)-1/3*b*c*(2-m)*m*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*hypergeom
([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/d^2/f^2/(m^2+3*m+2)/(
-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.71

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{x(fx)^m\sqrt{-1+cx}\sqrt{1+cx}\left(-\frac{a+b\text{arccosh}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bcx\,\text{Hypergeometric2F1}(2,1+}{2+m}\right.}{}$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x*(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/((-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))) + (b*c*x*Hypergeometric2F1[2, 1 + m/2, 2 + m/2,
c^2*x^2])/(2 + m) + ((-2 + m)*(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - (1 + m)*((2
+ m)*(a + b*ArcCosh[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeomet
ric2F1[1, 1 + m/2, 2 + m/2, c^2*x^2]) + b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]
))/((1 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(3*d^2*Sqrt[d - c^2*d*
x^2])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6351, 82, 278, 6351, 25, 82, 278, 6363}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6351}$$

$$\frac{(2-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{(1-cx)^2(cx+1)^2}dx}{3d^2f\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{82}$$

$$\frac{(2-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{(1-c^2x^2)^2}dx}{3d^2f\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(2-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df(d-c^2dx^2)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6351}$$

$$\frac{(2-m)\left(-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{df\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df(d-c^2dx^2)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 25

$$\frac{(2-m)\left(-\frac{m\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{df\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 82

$$\frac{(2-m)\left(-\frac{m\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{(fx)^{m+1}}{1-c^2x^2}dx}{df\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 278

$$\frac{(2-m)\left(-\frac{m\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{df\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+}{2}\right.}{df^2(m+2)\sqrt{d-c^2dx^2}}\right.}{3d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 6363

$$\frac{(2-m)\left(-\frac{m\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{ar}}{f(m+1)\sqrt{d-c^2dx^2}}\right.}{d}\right.}{3d}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3df\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d*f*(d - c^2*d*x^2)^(3/2)) + (b*c*
(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[2, (2 + m)/2,
 (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) + ((2 - m)*(
((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*f*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x
)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2 + m)/2, (4
+ m)/2, c^2*x^2])/(d*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (m*(((f*x)^(1 + m)
*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2 + m)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])))/d)
)/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6363
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^m (a + b \operatorname{arccosh}(cx))}{(-c^2 d\, x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d - c^2dx^2)^{5/2}}\, dx = \int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(-c^2dx^2 + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 -
 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(f\,x)^m}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(fx)^m(a+b\,\text{arccosh}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{f^m\left(\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)a+\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{}}\right.\right.}{\sqrt{d}\,d^2}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(f**m*(int(x**m/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 +
1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a + int((x**m*acosh(c*x))/(sqrt(
 - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( -
 c**2*x**2 + 1)),x)*b))/(sqrt(d)*d**2)
```

3.152 $\int (fx)^m(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}(a+b\mathbf{arccosh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 864

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx =$$
$$-\frac{15bc(fx)^{2+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}{f^2(2+m)^2(4+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)^2}$$
$$-\frac{5bc(fx)^{2+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}{f^2(6+m)\,(8+6m+m^2)\,\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)^2}$$
$$-\frac{bc(fx)^{2+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}{f^2\,(12+8m+m^2)\,\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)^2}$$
$$+\frac{5bc^3(fx)^{4+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}{f^4(4+m)^2(6+m)\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)^2}$$
$$+\frac{2bc^3(fx)^{4+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}{f^4(4+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)^2}$$
$$-\frac{bc^5(fx)^{6+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}{f^6(6+m)^2\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)^2}$$
$$+\frac{(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\operatorname{arccosh}(cx))}{f(6+m)}$$
$$+\frac{15(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\operatorname{arccosh}(cx))}{f(6+m)\,(8+6m+m^2)\,(1-c^2x^2)^2}$$
$$+\frac{5(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\operatorname{arccosh}(cx))}{f(4+m)(6+m)\,(1-c^2x^2)}$$
$$+\frac{15(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\operatorname{arccosh}(cx))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(6+m)\,(8+14m+7m^2+m^3)\,(1-c^2x^2)^{5/2}}$$
$$+\frac{15bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2(1+m)(2+m)^2(4+m)(6+m)\,(1-c^2x^2)^3}$$

output
```
-15*b*c*(f*x)^(2+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)/f^2/(2+m)^2/(4+m)
/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*x^2+1)^2-5*b*c*(f*x)^(2+m)*(c*d1*
x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)/f^2/(6+m)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)/(-c^2*x^2+1)^2-b*c*(f*x)^(2+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)
/f^2/(m^2+8*m+12)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*x^2+1)^2+5*b*c^3*(f*x)
^(4+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)/f^4/(4+m)^2/(6+m)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)/(-c^2*x^2+1)^2+2*b*c^3*(f*x)^(4+m)*(c*d1*x+d1)^(5/2)*(-c*d
2*x+d2)^(5/2)/f^4/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*x^2+1)^2-b
*c^5*(f*x)^(6+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)/f^6/(6+m)^2/(c*x-1)^
(1/2)/(c*x+1)^(1/2)/(-c^2*x^2+1)^2+(f*x)^(1+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+
d2)^(5/2)*(a+b*arccosh(c*x))/f/(6+m)+15*(f*x)^(1+m)*(c*d1*x+d1)^(5/2)*(-c*
d2*x+d2)^(5/2)*(a+b*arccosh(c*x))/f/(6+m)/(m^2+6*m+8)/(-c^2*x^2+1)^2+5*(f*
x)^(1+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x))/f/(4+m)/(
6+m)/(-c^2*x^2+1)+15*(f*x)^(1+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b
*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/f/(6+m)/(m^
3+7*m^2+14*m+8)/(-c^2*x^2+1)^(5/2)+15*b*c*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*hypergeom([1, 1+1/2*m, 1+1/2*
m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/f^2/(1+m)/(2+m)^2/(4+m)/(6+m)/(-c^2*x^2+1
)^3
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.45

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\operatorname{arccosh}(cx))\,dx = \frac{\mathrm{d1}^2\mathrm{d2}^2x(fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(-\frac{bcx\left(\frac{1}{2+m}-\frac{2c^2x^2}{4+m}+\frac{c^4x^4}{6+m}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}+(-1+\cdots\right.}{\cdots}$$

input
```
Integrate[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c
*x]),x]
```

output
```
(d1^2*d2^2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(-((b*c*x*((2 + m
)^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6 + m)))/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x])) + (-1 + c^2*x^2)^2*(a + b*ArcCosh[c*x]) + (5*((b*c*x*(-(2 + m)^(-
1) + (c^2*x^2)/(4 + m)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (-1 + c*x)*(1 +
c*x)*(a + b*ArcCosh[c*x]) + (3*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) -
 (2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeom
etricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*
(2 + m)^2*(-1 + c*x)*(1 + c*x))))/(4 + m)))/(6 + m)
```

Rubi [A] (warning: unable to verify)

Time = 3.66 (sec) , antiderivative size = 596, normalized size of antiderivative = 0.69, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {6346, 82, 244, 2009, 6346, 25, 82, 244, 2009, 6342, 17, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^m(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6346}$$

$$\frac{5\text{d1}\text{d2}\int (fx)^m(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int (fx)^{m+1}(1-cx)^2(cx+1)^2dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}$$

$$\Big\downarrow \text{82}$$

$$\frac{5\text{d1}\text{d2}\int (fx)^m(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int (fx)^{m+1}\left(1-c^2x^2\right)^2dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}$$

$$\Big\downarrow \text{244}$$

$$\frac{5\text{d1d2}\int(fx)^m(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int\left((fx)^{m+1}-\frac{2c^2(fx)^{m+3}}{f^2}+\frac{c^4(fx)^{m+5}}{f^4}\right)dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}$$

$\Big\downarrow$ 2009

$$\frac{5\text{d1d2}\int(fx)^m(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(a+b\text{arccosh}(cx))dx}{m+6}+\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6346

$$\frac{5\text{d1d2}\left(\frac{3\text{d1d2}\int(fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int-(fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}\right.}{m+6}$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 25

$$\frac{5\text{d1d2}\left(\frac{3\text{d1d2}\int(fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int(fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\right.}{m+6}$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 82

$$\frac{5\text{d1d2}\left(\frac{3\text{d1d2}\int(fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int(fx)^{m+1}(1-c^2x^2)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d}}{}\right.}{m+6}$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-$$

$$\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{244}$$

$$\frac{5\text{d1d2}\left(\frac{3\text{d1d2}\int(fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int\left((fx)^{m+1}-\frac{c^2(fx)^{m+3}}{f^2}\right)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}\right.}{m+6}$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-$$

$$\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\text{d1d2}\left(\frac{3\text{d1d2}\int(fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\right.}{m+6}$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-$$

$$\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6342}$$

$$5\text{d1d2}\left(\frac{3\text{d1d2}\left(-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int(fx)^{m+1}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(\ldots}{f(m\ldots}\right.}{m+4}\right.$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 17

$$5\text{d1d2}\left(\frac{3\text{d1d2}\left(-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c\text{d1}x+\text{d}\ldots}}{f^2(m+\ldots}\right.}{m+4}\right.$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6364

$$5\text{d1d2}\left(\frac{3\text{d1d2}\left(-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{bc(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)}+\frac{\sqrt{1-cx}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\ldots\right.}{f(m+1)\sqrt{cx-1}}\right.}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right.}{m+4}\right.$$

$$\frac{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+6)}-\frac{bc\text{d1}^2\text{d2}^2\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{c^4(fx)^{m+6}}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}}{f^3(m+4)}+\frac{(fx)^{m+2}}{f(m+2)}\right)}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]),x
]
```

output
```
-((b*c*d1^2*d2^2*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((f*x)^(2 + m)/(f*(2
+ m)) - (2*c^2*(f*x)^(4 + m))/(f^3*(4 + m)) + (c^4*(f*x)^(6 + m))/(f^5*(6
+ m))))/(f*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*(d1 + c
*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]))/(f*(6 + m)) + (5*d1
*d2*(-((b*c*d1*d2*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((f*x)^(2 + m)/(f*(2
 + m)) - (c^2*(f*x)^(4 + m))/(f^3*(4 + m))))/(f*(4 + m)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])) + ((f*x)^(1 + m)*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a +
 b*ArcCosh[c*x]))/(f*(4 + m)) + (3*d1*d2*(-((b*c*(f*x)^(2 + m)*Sqrt[d1 + c
*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) +
((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]))/(
f*(2 + m)) - (Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(((f*x)^(1 + m)*Sqrt[1 -
 c*x]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^
2*x^2])/(f*(1 + m)*Sqrt[-1 + c*x]) + (b*c*(f*x)^(2 + m)*HypergeometricPFQ[
{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m
))))/((2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(4 + m)))/(6 + m)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6342
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Simp[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-
1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
 c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]]   Int[(f*x)^(m + 1)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] &&
EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1
])
```

rule 6346
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Simp[2*d1*d2*(p/(m + 2*p + 1))   Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)
))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*
d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (fx)^m \left(c\,\mathrm{d1}\,x+\mathrm{d1}\right)^{\frac{5}{2}} \left(-c\,\mathrm{d2}\,x+\mathrm{d2}\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)dx$$

input
```
int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\mathrm{arccosh}(cx))\,dx = \int (cd_1x+d_1)^{\frac{5}{2}}(-cd_2x+d_2)^{\frac{5}{2}}(b\,\mathrm{arcosh}\left(cx\right)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),
x, algorithm="fricas")
```

output
```
integral((a*c^4*d1^2*d2^2*x^4 - 2*a*c^2*d1^2*d2^2*x^2 + a*d1^2*d2^2 + (b*c
^4*d1^2*d2^2*x^4 - 2*b*c^2*d1^2*d2^2*x^2 + b*d1^2*d2^2)*arccosh(c*x))*sqrt
(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\mathrm{arccosh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(c*d1*x+d1)**(5/2)*(-c*d2*x+d2)**(5/2)*(a+b*acosh(c*x))
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\mathrm{arccosh}(cx))\,dx = \int (cd_1x+d_1)^{\frac{5}{2}}(-cd_2x+d_2)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),
x, algorithm="maxima")
```

output
```
integrate((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^(5/2)*(b*arccosh(c*x) + a)*(f
*x)^m, x)
```

Giac [F]

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}(a+b\mathrm{arccosh}(cx))\,dx = \int (cd_1x+d_1)^{\frac{5}{2}}(-cd_2x+d_2)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),
x, algorithm="giac")
```

output
```
integrate((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^(5/2)*(b*arccosh(c*x) + a)*(f
*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (\mathrm{d1} + c\mathrm{d1}x)^{5/2} (\mathrm{d2} - c\mathrm{d2}x)^{5/2} (a + b\operatorname{arccosh}(cx))\,dx = \int (a + b\,\mathrm{acosh}(c\,x))\,(f\,x)^m\,(d_1 + c\,d_1\,x)^{5/2}\,(d_2 - c\,d_2\,x)^{5/2}\,dx$$

input
```
int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2),x)
```

output
```
int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2), x)
```

Reduce [F]

$$\int (fx)^m (\mathrm{d1} + c\mathrm{d1}x)^{5/2} (\mathrm{d2} - c\mathrm{d2}x)^{5/2} (a + b\operatorname{arccosh}(cx))\,dx = f^m \sqrt{d2}\,\sqrt{d1}\,d1^2 d2^2 \left(\left(\int x^m \sqrt{cx+1}\,\sqrt{-cx+1}\,acosh(cx)\,x^4 dx\right) b\,c^4 -$$

input
```
int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*acosh(c*x)),x)
```

output
```
f**m*sqrt(d2)*sqrt(d1)*d1**2*d2**2*(int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1
)*acosh(c*x)*x**4,x)*b*c**4 - 2*int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*ac
osh(c*x)*x**2,x)*b*c**2 + int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*acosh(c*
x),x)*b + int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*x**4,x)*a*c**4 - 2*int(x
**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*x**2,x)*a*c**2 + int(x**m*sqrt(c*x + 1)
*sqrt( - c*x + 1),x)*a)
```

3.153 $\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 546

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx = -\frac{3bc(fx)^{2+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)} - \frac{bc(fx)^{2+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}{f^2\,(8+6m+m^2)\,\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)} + \frac{bc^3(fx)^{4+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}{f^4(4+m)^2\sqrt{-1+cx}\sqrt{1+cx}\,(1-c^2x^2)} + \frac{(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))}{f(4+m)} + \frac{3(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))}{f\,(8+6m+m^2)\,(1-c^2x^2)} + \frac{3(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f\,(8+14m+7m^2+m^3)\,(1-c^2x^2)^{3/2}} + \frac{3bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2(1+m)(2+m)^2(4+m)\,(1-c^2x^2)^2}$$

output
```
-3*b*c*(f*x)^(2+m)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)/f^2/(2+m)^2/(4+m)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*x^2+1)-b*c*(f*x)^(2+m)*(c*d1*x+d1)^(3/2)
*(-c*d2*x+d2)^(3/2)/f^2/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*x^2+
1)+b*c^3*(f*x)^(4+m)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)/f^4/(4+m)^2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*x^2+1)+(f*x)^(1+m)*(c*d1*x+d1)^(3/2)*(-c*d2*
x+d2)^(3/2)*(a+b*arccosh(c*x))/f/(4+m)+3*(f*x)^(1+m)*(c*d1*x+d1)^(3/2)*(-c
*d2*x+d2)^(3/2)*(a+b*arccosh(c*x))/f/(m^2+6*m+8)/(-c^2*x^2+1)+3*(f*x)^(1+m
)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x))*hypergeom([1/2,
1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/f/(m^3+7*m^2+14*m+8)/(-c^2*x^2+1)^(3/2)+3*
b*c*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)
^(3/2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/f^2/(
1+m)/(2+m)^2/(4+m)/(-c^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.53

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx = \frac{\mathrm{d1}\mathrm{d2}x(fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(-\frac{3bcx}{(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcx\left(-\frac{1}{2+m}+\right.}{\sqrt{-1+cx}\sqrt{}}\right.}{}$$

input
```
Integrate[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c
*x]),x]
```

output
```
(d1*d2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((-3*b*c*x)/((2 + m)^
2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4 + m
)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*(a + b*ArcCosh[c*x]))/(2 + m) - (-
1 + c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]) - (3*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)*
(2 + m)*(-1 + c*x)*(1 + c*x)) - (3*b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/((1 + m)*(2 + m)^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])))/(4 + m)
```

Rubi [A] (warning: unable to verify)

Time = 1.59 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {6346, 25, 82, 244, 2009, 6342, 17, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^m(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6346}$$

$$\frac{3\text{d1}\text{d2}\int (fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{bc\text{d1}\text{d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int -(fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\text{d1}\text{d2}\int (fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bc\text{d1}\text{d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int (fx)^{m+1}(1-cx)(cx+1)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$$\Big\downarrow \text{82}$$

$$\frac{3\text{d1}\text{d2}\int (fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bc\text{d1}\text{d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int (fx)^{m+1}\left(1-c^2x^2\right)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$$\Big\downarrow \text{244}$$

$$\frac{3\text{d1d2}\int (fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int\left((fx)^{m+1}-\frac{c^2(fx)^{m+3}}{f^2}\right)dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}$$

$\Big\downarrow$ 2009

$$\frac{3\text{d1d2}\int (fx)^m\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))dx}{m+4}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6342

$$\frac{3\text{d1d2}\left(-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int (fx)^{m+1}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\ldots}{f\ldots}\right.}{m+4}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 17

$$\frac{3\text{d1d2}\left(-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int\frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c\text{d1}x+\ldots}}{f^2(m\ldots}\right.}{m+4}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6364

$$\frac{3\text{d1}\text{d2}\left(-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{bc(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)}+\frac{\sqrt{1-cx}(fx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(c}{f(m+1)\sqrt{cx-1}}\right.}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right.}{m+4}$$

$$\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+4)}-$$

$$\frac{bc\text{d1}\text{d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{(fx)^{m+2}}{f(m+2)}-\frac{c^2(fx)^{m+4}}{f^3(m+4)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]),x
]
```

output
```
-((b*c*d1*d2*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((f*x)^(2 + m)/(f*(2 + m)
) - (c^2*(f*x)^(4 + m))/(f^3*(4 + m))))/(f*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])) + ((f*x)^(1 + m)*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*Ar
cCosh[c*x]))/(f*(4 + m)) + (3*d1*d2*(-((b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x
]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x
)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]))/(f*(2
+ m)) - (Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(((f*x)^(1 + m)*Sqrt[1 - c*x]
*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2
])/(f*(1 + m)*Sqrt[-1 + c*x]) + (b*c*(f*x)^(2 + m)*HypergeometricPFQ[{1, 1
 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m))))/
((2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(4 + m)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6342
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Simp[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-
1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
 c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]]   Int[(f*x)^(m + 1)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] &&
EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1
])
```

rule 6346
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Simp[2*d1*d2*(p/(m + 2*p + 1))   Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)
))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*
d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (fx)^m (c\,\mathrm{d1}\,x+\mathrm{d1})^{\frac{3}{2}} (-c\,\mathrm{d2}\,x+\mathrm{d2})^{\frac{3}{2}} (a+b\,\mathrm{arccosh}\,(cx))\,dx$$

input
```
int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\mathrm{arccosh}(cx))\,dx = \int (cd_1x+d_1)^{\frac{3}{2}}(-cd_2x+d_2)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),
x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d1*d2*x^2 - a*d1*d2 + (b*c^2*d1*d2*x^2 - b*d1*d2)*arccosh
(c*x))*sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\mathrm{arccosh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(c*d1*x+d1)**(3/2)*(-c*d2*x+d2)**(3/2)*(a+b*acosh(c*x))
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m (\mathrm{d1} + c\mathrm{d1}x)^{3/2} (\mathrm{d2} - c\mathrm{d2}x)^{3/2} (a + b\mathrm{arccosh}(cx))\, dx = \int (cd_1x + d_1)^{\frac{3}{2}} (-cd_2x + d_2)^{\frac{3}{2}} (b \operatorname{arcosh}(cx) + a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),
x, algorithm="maxima")
```

output
```
integrate((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^(3/2)*(b*arccosh(c*x) + a)*(f
*x)^m, x)
```

Giac [F]

$$\int (fx)^m (\mathrm{d1} + c\mathrm{d1}x)^{3/2} (\mathrm{d2} - c\mathrm{d2}x)^{3/2} (a + b\mathrm{arccosh}(cx))\, dx = \int (cd_1x + d_1)^{\frac{3}{2}} (-cd_2x + d_2)^{\frac{3}{2}} (b \operatorname{arcosh}(cx) + a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),
x, algorithm="giac")
```

output
```
integrate((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^(3/2)*(b*arccosh(c*x) + a)*(f
*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx=\int (a+b\,\mathrm{acosh}(c\,x))\,\left(f\,x\right)^m\,{\left(d_1+c\,d_1\,x\right)}^{3/2}\,{\left(d_2-c\,d_2\,x\right)}^{3/2}\,dx$$

input
```
int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2),x)
```

output
```
int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2), x)
```

Reduce [F]

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}(a+b\operatorname{arccosh}(cx))\,dx=f^m\sqrt{d2}\,\sqrt{d1}\,d1\,d2\left(-\left(\int x^m\sqrt{cx+1}\,\sqrt{-cx+1}\,acosh(cx)\,x^2dx\right)b\,c^2+\right.$$

input
```
int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*acosh(c*x)),x)
```

output
```
f**m*sqrt(d2)*sqrt(d1)*d1*d2*( - int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*a
cosh(c*x)*x**2,x)*b*c**2 + int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*acosh(c
*x),x)*b - int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*x**2,x)*a*c**2 + int(x*
*m*sqrt(c*x + 1)*sqrt( - c*x + 1),x)*a)
```

3.154 $\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(a+b\mathrm{arccosh}(cx$

Optimal result

Integrand size = 35, antiderivative size = 308

$$\begin{aligned}
&\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(a+b\mathrm{arccosh}(cx))\,dx \\
&= -\frac{bc(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad+\frac{(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(a+b\mathrm{arccosh}(cx))}{f(2+m)} \\
&\quad+\frac{(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(a+b\mathrm{arccosh}(cx))\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f\,(2+3m+m^2)\,\sqrt{1-c^2x^2}} \\
&\quad+\frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2(1+m)(2+m)^2\,(1-c^2x^2)}
\end{aligned}$$

output
```
-b*c*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+(f*x)^(1+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*
arccosh(c*x))/f/(2+m)+(f*x)^(1+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+
b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/f/(m^2+3*m
+2)/(-c^2*x^2+1)^(1/2)+b*c*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c*d1*x
+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/
2+1/2*m],c^2*x^2)/f^2/(1+m)/(2+m)^2/(-c^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.74

$$\int (fx)^m\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{x(fx)^m\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}\left((1+m)\left(-bcx\sqrt{-1+cx}\sqrt{1+cx}+a(2+m)\left(-1+c^2x^2\right)+b(2+m)\right.\right.}{}$$

input
```
Integrate[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x])
,x]
```

output
```
(x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((1 + m)*(-(b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)
*ArcCosh[c*x]) - (2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*
x^2]))/((1 + m)*(2 + m)^2*(-1 + c*x)*(1 + c*x))
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {6342, 17, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^m(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6342}$$

$$-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int (fx)^{m+1}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}$$

$$\Big\downarrow \text{17}$$

$$-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6364

$$-\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\left(\frac{bc(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)}+\frac{\sqrt{1-cx}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)\ldots}{f(m+1)\sqrt{cx-1}}\right)}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]),x]
```

output
```
-((b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 -
 c*d2*x]*(a + b*ArcCosh[c*x]))/(f*(2 + m)) - (Sqrt[d1 + c*d1*x]*Sqrt[d2 -
c*d2*x]*(((f*x)^(1 + m)*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x])*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c*x]) + (b*c*
(f*x)^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2
}, c^2*x^2])/(f^2*(1 + m)*(2 + m))))/((2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6342
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Simp[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-
1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
 c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]]   Int[(f*x)^(m + 1)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] &&
EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1
])
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (fx)^m \sqrt{c\,\mathrm{d1}\,x+\mathrm{d1}}\,\sqrt{-c\,\mathrm{d2}\,x+\mathrm{d2}}\,(a+b\,\mathrm{arccosh}\,(cx))\,dx$$

input
```
int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))\,dx = \int \sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}(b\operatorname{arcosh}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(c*d1*x+d1)**(1/2)*(-c*d2*x+d2)**(1/2)*(a+b*acosh(c*x))
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}(a+b\text{arccosh}(cx))\,dx = \int \sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}(b\operatorname{arcosh}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^
m, x)
```

Giac [F]

$$\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(a+b\mathrm{arccosh}(cx))\,dx = \int \sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}(b\operatorname{arcosh}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),
x, algorithm="giac")
```

output
```
integrate(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^
m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(a+b\mathrm{arccosh}(cx))\,dx = \int (a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^m\,\sqrt{d_1+c\,d_1\,x}\,\sqrt{d_2-c\,d_2\,x}\,dx$$

input
```
int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2), x)
```

Reduce [F]

$$\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(a+b\mathrm{arccosh}(cx))\,dx$$
$$= f^m\sqrt{d2}\,\sqrt{d1}\left(\left(\left(\int x^m\sqrt{cx+1}\,\sqrt{-cx+1}\,acosh(cx)\,dx\right)b + \left(\int x^m\sqrt{cx+1}\,\sqrt{-cx+1}dx\right)a\right)$$

input
```
int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*acosh(c*x)),x)
```

output
```
f**m*sqrt(d2)*sqrt(d1)*(int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1)*acosh(c*x)
,x)*b + int(x**m*sqrt(c*x + 1)*sqrt( - c*x + 1),x)*a)
```

3.155 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 188

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}\,dx = \frac{(fx)^{1+m}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(1+m)\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}} + \frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2\left(2+3m+m^2\right)\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}$$

output
```
(f*x)^(1+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*
m],[3/2+1/2*m],c^2*x^2)/f/(1+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)+b*c*(
f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+
1/2*m, 3/2+1/2*m],c^2*x^2)/f^2/(m^2+3*m+2)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}\,dx = \frac{x(fx)^m\left((2+m)\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)+bcx\sqrt{-1+cx}\right.}{(1+m)(2+m)\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2
*x]),x]
```

output
```
(x*(f*x)^m*((2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])
)/((1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}\,dx$$

$$\Big\downarrow \ 6364$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{arccosh}(cx))}{f(m+1)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]),x
]
```

output
```
((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 -
c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricP
FQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2
+ m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Defintions of rubi rules used

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^m (a + b \operatorname{arccosh}(cx))}{\sqrt{c\,\mathrm{d1}\,x + \mathrm{d1}}\,\sqrt{-c\,\mathrm{d2}\,x + \mathrm{d2}}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{\sqrt{\mathrm{d1} + c\mathrm{d1}x}\sqrt{\mathrm{d2} - c\mathrm{d2}x}}\,dx = \int \frac{(b\,\operatorname{arcosh}(cx) + a)(fx)^m}{\sqrt{cd_1x + d_1}\sqrt{-cd_2x + d_2}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),
x, algorithm="fricas")
```

output
```
integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^
m/(c^2*d1*d2*x^2 - d1*d2), x)
```

Sympy [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{\sqrt{\text{d1} + c\text{d1}x}\sqrt{\text{d2} - c\text{d2}x}}\,dx = \int \frac{(fx)^m\,(a + b\,\text{acosh}\,(cx))}{\sqrt{d_1\,(cx+1)}\sqrt{-d_2\,(cx-1)}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(1/2)/(-c*d2*x+d2)**(1/2)
,x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))/(sqrt(d1*(c*x + 1))*sqrt(-d2*(c*x - 1
))), x)
```

Maxima [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{\sqrt{\text{d1} + c\text{d1}x}\sqrt{\text{d2} - c\text{d2}x}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)(fx)^m}{\sqrt{cd_1x + d_1}\sqrt{-cd_2x + d_2}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d
2)), x)
```

Giac [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{\text{d1}+\text{cd1}x}\sqrt{\text{d2}-\text{cd2}x}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),
x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d
2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{\text{d1}+\text{cd1}x}\sqrt{\text{d2}-\text{cd2}x}}\,dx=\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^m}{\sqrt{d_1+c\,d_1\,x}\,\sqrt{d_2-c\,d_2\,x}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2))
,x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2))
, x)
```

Reduce [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{\text{d1}+\text{cd1}x}\sqrt{\text{d2}-\text{cd2}x}}\,dx=\frac{f^m\left(\left(\int \frac{x^m}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)a+\left(\int \frac{x^m\,acosh(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b\right)}{\sqrt{d2}\,\sqrt{d1}}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

output
```
(f**m*(int(x**m/(sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a + int((x**m*acosh(c*
x))/(sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b))/(sqrt(d2)*sqrt(d1))
```

3.156 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 342

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}\,dx = \frac{(fx)^{1+m}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))}{f(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}$$
$$-\frac{m(fx)^{1+m}\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(1+m)(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}$$
$$+\frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{f^2(2+m)(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}$$
$$-\frac{bcm(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{f^2(1+m)(2+m)(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}$$

output
```
(f*x)^(1+m)*(-c^2*x^2+1)*(a+b*arccosh(c*x))/f/(c*d1*x+d1)^(3/2)/(-c*d2*x+d
2)^(3/2)-m*(f*x)^(1+m)*(-c^2*x^2+1)^(3/2)*(a+b*arccosh(c*x))*hypergeom([1/
2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/f/(1+m)/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^
(3/2)+b*c*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)*hypergeom([
1, 1+1/2*m],[2+1/2*m],c^2*x^2)/f^2/(2+m)/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3
/2)-b*c*m*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)*hypergeom([
1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/f^2/(1+m)/(2+m)/(c*d1*x
+d1)^(3/2)/(-c*d2*x+d2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.66

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}\,dx = \frac{x(fx)^m\left(-m(2+m)\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))\,\text{Hypergeometric2}\right.}{}$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x
)^(3/2)),x]
```

output
```
(x*(f*x)^m*(-(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]) + (1 + m)*((2 + m)*(a + b*ArcC
osh[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, 1 + m/
2, 2 + m/2, c^2*x^2]) - b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometri
cPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(d1*d2*(1 + m
)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6352, 25, 82, 278, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}\,dx$$

$$\Big\downarrow \text{6352}$$

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \text{82}$$

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{1-c^2x^2}dx}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{\text{d1d2}f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \text{6364}$$

$$-\frac{m\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{arc}}{f(m+1)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}\right.}{\text{d1d2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{\text{d1d2}f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2
)),x]
```

output
```
((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d1*d2*f*Sqrt[d1 + c*d1*x]*Sqrt[d2 -
c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2
F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d1*d2*f^2*(2 + m)*Sqrt[d1 + c*d1*x]
*Sqrt[d2 - c*d2*x]) - (m*(((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sq
rt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2},
c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])))/(d1*
d2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6352
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m +
1))*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d1*d
2*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d1*d2*(p + 1))   Int[(f*x)^m*(d1
 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
 + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m
}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && LtQ[p, -1] &&  !
GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^m\,(a+b\,\mathrm{arccosh}\,(cx))}{(c\,\mathrm{d1}\,x+\mathrm{d1})^{\frac{3}{2}}\,(-c\,\mathrm{d2}\,x+\mathrm{d2})^{\frac{3}{2}}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m(a+b\mathrm{arccosh}(cx))}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)(fx)^m}{(cd_1x+d_1)^{\frac{3}{2}}(-cd_2x+d_2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
/(c^4*d1^2*d2^2*x^4 - 2*c^2*d1^2*d2^2*x^2 + d1^2*d2^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(3/2)/(-c*d2*x+d2)**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{(cd_1x+d_1)^{\frac{3}{2}}(-cd_2x+d_2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)
^(3/2)), x)
```

Giac [F]

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{(cd_1x+d_1)^{\frac{3}{2}}(-cd_2x+d_2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),
x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)
^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,(f\,x)^m}{(d_1+c\,d_1\,x)^{3/2}\,(d_2-c\,d_2\,x)^{3/2}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2))
,x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2))
, x)
```

Reduce [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}\,dx=-\frac{f^m\left(\left(\int \frac{x^m}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)a+\left(\int \frac{x^m\,acosh(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b\right)}{\sqrt{d2}\,\sqrt{d1}\,d1\,d2}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

output
```
( - f**m*(int(x**m/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x +
1)*sqrt( - c*x + 1)),x)*a + int((x**m*acosh(c*x))/(sqrt(c*x + 1)*sqrt( - c
*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b))/(sqrt(d2)*sqrt(
d1)*d1*d2)
```

3.157 $\int \frac{(fx)^m(a+b\,\mathbf{arccosh}(cx))}{(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 526

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}\,dx=\frac{(fx)^{1+m}\left(1-c^2x^2\right)(a+b\operatorname{arccosh}(cx))}{3f(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}$$
$$+\frac{(2-m)(fx)^{1+m}\left(1-c^2x^2\right)^2(a+b\operatorname{arccosh}(cx))}{3f(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}$$
$$-\frac{(2-m)m(fx)^{1+m}\left(1-c^2x^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{3f(1+m)(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}$$
$$+\frac{bc(2-m)(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2\operatorname{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{3f^2(2+m)(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}$$
$$+\frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2\operatorname{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{3f^2(2+m)(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}$$
$$-\frac{bc(2-m)m(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{3f^2(1+m)(2+m)(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}$$

output
```
1/3*(f*x)^(1+m)*(-c^2*x^2+1)*(a+b*arccosh(c*x))/f/(c*d1*x+d1)^(5/2)/(-c*d2
*x+d2)^(5/2)+1/3*(2-m)*(f*x)^(1+m)*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))/f/(c*
d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2)-1/3*(2-m)*m*(f*x)^(1+m)*(-c^2*x^2+1)^(5/
2)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/f/(1
+m)/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2)+1/3*b*c*(2-m)*(f*x)^(2+m)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^2*hypergeom([1, 1+1/2*m],[2+1/2*m],c^2*x
^2)/f^2/(2+m)/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2)+1/3*b*c*(f*x)^(2+m)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^2*hypergeom([2, 1+1/2*m],[2+1/2*m],c
^2*x^2)/f^2/(2+m)/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2)-1/3*b*c*(2-m)*m*(f*
x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^2*hypergeom([1, 1+1/2*m,
 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/f^2/(1+m)/(2+m)/(c*d1*x+d1)^(5/2)/
(-c*d2*x+d2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.79

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}}\,dx = \frac{x(fx)^m\left((1+m)(2+m)\left(2+3m+m^2\right)(a+b\text{arccosh}(cx))-bc(1$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x
)^(5/2)),x]
```

output
```
(x*(f*x)^m*((1 + m)*(2 + m)*(2 + 3*m + m^2)*(a + b*ArcCosh[c*x]) - b*c*(1
+ m)*(2 + 3*m + m^2)*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*Hypergeometric2F1[
2, 1 + m/2, 2 + m/2, c^2*x^2] + (2 - m)*(2 + m)*(1 - c*x)*(1 + c*x)*((1 +
m)^2*(2 + m)*(a + b*ArcCosh[c*x]) + b*c*(1 + m)^2*x*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, c^2*x^2] - m*(a*(2 + 3*m + m
^2)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2
] + b*(2 + 3*m + m^2)*Sqrt[1 - c^2*x^2]*ArcCosh[c*x]*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*c*(1 + m)*x*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2
]))))/(3*d1^2*d2^2*(1 + m)*(2 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)*Sqr
t[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {6352, 82, 278, 6352, 25, 82, 278, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(c\text{d1}x+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}}dx$$

$$\Big\downarrow \text{ 6352}$$

$$\frac{(2-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}dx}{3\text{d1d2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{(1-cx)^2(cx+1)^2}dx}{3\text{d1}^2\text{d2}^2f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}$$

$$\Big\downarrow \text{ 82}$$

$$\frac{(2-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}dx}{3\text{d1d2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{(fx)^{m+1}}{(1-c^2x^2)^2}dx}{3\text{d1}^2\text{d2}^2f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(2-m)\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}dx}{3\text{d1d2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \text{ 6352}$$

$$\frac{(2-m)\left(-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}\right)}{3\text{d1d2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \; 25$$

$$\frac{(2-m)\left(-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{(fx)^{m+1}}{(1-cx)(cx+1)}dx}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}\right)}{3\text{d1d2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \; 82$$

$$\frac{(2-m)\left(-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{(fx)^{m+1}}{1-c^2x^2}dx}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}\right)}{3\text{d1d2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \; 278$$

$$\frac{(2-m)\left(-\frac{m\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{\sqrt{cx\text{d1}+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}dx}{\text{d1d2}}+\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{\text{d1d2}f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2}\right.}{\text{d1d2}f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}\right.}{3\text{d1d2}}$$

$$\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

$$\Big\downarrow \; 6364$$

$$\frac{(2-m)\left(-\frac{m\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\text{ar}\right.}{f(m+1)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}\right.}{\text{d1d2}}\right.}{3\text{d1d2}}$$

$$\frac{(fx)^{m+1}(a+b\text{arccosh}(cx))}{3\text{d1d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2
)),x]
```

output
```
((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1*d2*f*(d1 + c*d1*x)^(3/2)*(d2 -
c*d2*x)^(3/2)) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeom
etric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + ((2 - m)*(((f*x)^(1 + m)*(a + b*ArcCosh[
c*x]))/(d1*d2*f*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^
2*x^2])/(d1*d2*f^2*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (m*(((f*
x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (
1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2
*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{
1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])))/(d1*d2)))/(3*d1*d2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6352
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m +
1))*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d1*d
2*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d1*d2*(p + 1))   Int[(f*x)^m*(d1
 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
 + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m
}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && LtQ[p, -1] &&  !
GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^m (a + b\,\mathrm{arccosh}\,(cx))}{(c\,\mathrm{d1}\,x + \mathrm{d1})^{\frac{5}{2}} (-c\,\mathrm{d2}\,x + \mathrm{d2})^{\frac{5}{2}}} dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{(cd_1x+d_1)^{\frac{5}{2}}(-cd_2x+d_2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),
x, algorithm="fricas")
```

output
```
integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^
m/(c^6*d1^3*d2^3*x^6 - 3*c^4*d1^3*d2^3*x^4 + 3*c^2*d1^3*d2^3*x^2 - d1^3*d2
^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(5/2)/(-c*d2*x+d2)**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{(cd_1x+d_1)^{\frac{5}{2}}(-cd_2x+d_2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)
^(5/2)), x)
```

Giac [F]

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(\text{d1} + c\text{d1}x)^{5/2}(\text{d2} - c\text{d2}x)^{5/2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{(cd_1x + d_1)^{\frac{5}{2}}(-cd_2x + d_2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),
x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)
^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(\text{d1} + c\text{d1}x)^{5/2}(\text{d2} - c\text{d2}x)^{5/2}}\,dx = \int \frac{(a + b\,\mathrm{acosh}(c\,x))\,(f\,x)^m}{(d_1 + c\,d_1\,x)^{5/2}\,(d_2 - c\,d_2\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2))
,x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2))
, x)
```

Reduce [F]

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}\,dx = \frac{f^m\left(\left(\int \frac{x^m}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^4x^4-2\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2+\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)a+\left(\int\right.\right.}{\sqrt{d2}\,\sqrt{d1}\,d1^2\,d}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

output
```
(f**m*(int(x**m/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**4*x**4 - 2*sqrt(c*x + 1
)*sqrt( - c*x + 1)*c**2*x**2 + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a + int(
(x**m*acosh(c*x))/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**4*x**4 - 2*sqrt(c*x +
 1)*sqrt( - c*x + 1)*c**2*x**2 + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b))/(s
qrt(d2)*sqrt(d1)*d1**2*d2**2)
```

3.158 $\int \frac{(fx)^m \text{arccosh}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{(fx)^m \text{arccosh}(ax)}{\sqrt{1-a^2x^2}} \, dx = \frac{(fx)^{1+m} \text{arccosh}(ax) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right)}{f(1+m)} - \frac{a(fx)^{2+m}\sqrt{1-ax}\, {}_3F_2\left(1, 1+\frac{m}{2}, 1+\frac{m}{2}; \frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}; a^2x^2\right)}{f^2\left(2+3m+m^2\right)\sqrt{-1+ax}}$$

output
```
(f*x)^(1+m)*arccosh(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/f
/(1+m)-a*(f*x)^(2+m)*(-a*x+1)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2
*m, 3/2+1/2*m],a^2*x^2)/f^2/(m^2+3*m+2)/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \text{arccosh}(ax)}{\sqrt{1-a^2x^2}} \, dx = \frac{x(fx)^m \left(\text{arccosh}(ax) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right) + \frac{ax\sqrt{-1+ax}\sqrt{1+ax}\, {}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};a\right.}{(2+m)\sqrt{1-a^2x^2}}}{1+m}$$

input
```
Integrate[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(x*(f*x)^m*(ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*
x^2] + (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])))/(1 +
 m)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6363}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)(fx)^m}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6363}$$

$$\frac{a\sqrt{ax-1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;a^2x^2\right)}{f^2(m+1)(m+2)\sqrt{1-ax}}+\frac{\text{arccosh}(ax)(fx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{f(m+1)}$$

input
```
Int[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
((f*x)^(1 + m)*ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a
^2*x^2])/(f*(1 + m)) + (a*(f*x)^(2 + m)*Sqrt[-1 + a*x]*HypergeometricPFQ[{
1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/(f^2*(1 + m)*(2 + m)
*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 6363
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^m \operatorname{arccosh}(ax)}{\sqrt{-a^2x^2+1}}dx$$

input
```
int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m\operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(fx)^m \operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*(f*x)^m*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{(fx)^m \text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(fx)^m \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate((f*x)**m*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral((f*x)**m*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{(fx)^m \text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(fx)^m \operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(fx)^m \text{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(fx)^m \operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m \operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)\,\left(f\,x\right)^m}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((acosh(a*x)*(f*x)^m)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((acosh(a*x)*(f*x)^m)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(fx)^m \operatorname{arccosh}(ax)}{\sqrt{1-a^2x^2}}\,dx = f^m \left(\int \frac{x^m acosh(ax)}{\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int((f*x)^m*acosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
f**m*int((x**m*acosh(a*x))/sqrt( - a**2*x**2 + 1),x)
```

3.159 $\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 330

$$\begin{aligned}\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx &= -\frac{856b^2\sqrt{d-c^2dx^2}}{3375c^4}+\frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2}\\ &+\frac{2}{125}b^2x^4\sqrt{d-c^2dx^2}\\ &+\frac{4bx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{15c^3\sqrt{-1+cx}\sqrt{1+cx}}\\ &+\frac{2bx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{45c\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{2bcx^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{25\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{15c^4}\\ &-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{15c^2}\\ &+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\end{aligned}$$

output
```
-856/3375*b^2*(-c^2*d*x^2+d)^(1/2)/c^4+22/3375*b^2*x^2*(-c^2*d*x^2+d)^(1/2
)/c^2+2/125*b^2*x^4*(-c^2*d*x^2+d)^(1/2)+4/15*b*x*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccosh(c*x))/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/45*b*x^3*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/25*b*c*x^5*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/15*(-c^2*d
*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^4-1/15*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccosh(c*x))^2/c^2+1/5*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.72

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(225a^2(-1+c^2x^2)^2\,(2+3c^2x^2)-30abcx\sqrt{-1+cx}\sqrt{1+cx}(-30-5c^2x^2+9c^4x^4)+2b^2(4\right.}{}$$

input
```
Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2) - 30*a*b*c*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-30 - 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(428
- 439*c^2*x^2 - 16*c^4*x^4 + 27*c^6*x^6) + 30*b*(15*a*(-1 + c^2*x^2)^2*(2
+ 3*c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(30 + 5*c^2*x^2 - 9*c^4*
x^4))*ArcCosh[c*x] + 225*b^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2)*ArcCosh[c*x]
^2))/(3375*c^4*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {6341, 6298, 111, 27, 111, 27, 83, 6354, 6298, 111, 27, 83, 6330, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6341}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\int x^4(a+b\text{arccosh}(cx))dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 6298}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\int \frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 111}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{\int \frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 111}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{83}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6354}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}-\frac{2b\int x^2(a+b\text{arccosh}(cx))dx}{3c}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}\right)}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6298}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{3c}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx$$
$$\frac{\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)\Bigg)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 111

$$-\frac{\sqrt{d-c^2dx^2}\left(-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c}+\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\cdot}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)\Bigg)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 27

$$-\frac{\sqrt{d-c^2dx^2}\left(-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c}+\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-}}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)\Bigg)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 83

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}\right.\right.}{3c}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6330

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{c^2}-\frac{2b\int(a+b\text{arccosh}(cx))dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2009

$$\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}+\frac{2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c}\right)}{3c^2}-\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5\sqrt{cx-1}\sqrt{cx+1}}$$

input

```
Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/5 - (2*b*c*Sqrt[d - c^2*d
*x^2]*(-1/5*(b*c*((x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*c^2) + (4*((2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*
c^2)))/(5*c^2))) + (x^5*(a + b*ArcCosh[c*x]))/5))/(5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - (Sqrt[d - c^2*d*x^2]*((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*
ArcCosh[c*x])^2)/(3*c^2) - (2*b*(-1/3*(b*c*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2))) + (x^3*(a + b*Ar
cCosh[c*x]))/3))/(3*c) + (2*((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[
c*x])^2)/c^2 - (2*b*(a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCos
h[c*x]))/c))/(3*c^2)))/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.69

method	result
orering	$\frac{(1647c^8x^8-2131c^6x^6-8610c^4x^4+13060c^2x^2-5136)\sqrt{-c^2d\,x^2+d}\,(a+b\,\mathrm{arccosh}(cx))^2}{3375(c^2x^2-1)c^6x^2} - \frac{4(81c^6x^6-40c^4x^4-878c^2x^2+642)\left(3x^2\right.}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3375*(1647*c^8*x^8-2131*c^6*x^6-8610*c^4*x^4+13060*c^2*x^2-5136)/(c^2*x^
2-1)/c^6/x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2-4/3375*(81*c^6*x^6-
40*c^4*x^4-878*c^2*x^2+642)/c^6/x^4*(3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcco
sh(c*x))^2-x^4/(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*c^2*d+2*b*c*x^3*(
-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2))+1/3375
*(27*c^4*x^4+11*c^2*x^2-428)/c^6*(c*x-1)*(c*x+1)/x^3*(6*x*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccosh(c*x))^2-7*x^3/(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*
c^2*d+12*b*c*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-x^5/(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2*c^4*d^2-4*x^4/(-c
^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*c^3*d*b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2
*b^2*c^2*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)/(c*x+1)-b*c^2*x^3*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(3/2)/(c*x+1)^(1/2)-b*c^2*x^3*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.06

$$\int x^3\sqrt{d-c^2dx^2}(a+b\mathrm{arccosh}(cx))^2\,dx$$
$$= \frac{225\,(3\,b^2c^6x^6 - 4\,b^2c^4x^4 - b^2c^2x^2 + 2\,b^2)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2 - 30\,(9\,abc^5x^5 - 5\,abc^3x^3 -}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
1/3375*(225*(3*b^2*c^6*x^6 - 4*b^2*c^4*x^4 - b^2*c^2*x^2 + 2*b^2)*sqrt(-c^
2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 - 5*a*b*c^
3*x^3 - 30*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2*c^
5*x^5 - 5*b^2*c^3*x^3 - 30*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
 - 15*(3*a*b*c^6*x^6 - 4*a*b*c^4*x^4 - a*b*c^2*x^2 + 2*a*b)*sqrt(-c^2*d*x^
2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 - 4*(2
25*a^2 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b^2)*c^2*x^2 + 450*a^2 + 856*b^2)
*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^2\;dx$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.99

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$= -\frac{1}{15}b^2\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arcosh}(cx)^2$$
$$-\frac{2}{15}ab\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arcosh}(cx)$$
$$-\frac{1}{15}a^2\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)$$
$$+\frac{2}{3375}b^2\left(\frac{27\sqrt{c^2x^2-1}c^2\sqrt{-d}x^4+11\sqrt{c^2x^2-1}\sqrt{-d}x^2-\frac{428\sqrt{c^2x^2-1}\sqrt{-d}}{c^2}}{c^2}-\frac{15\left(9c^4\sqrt{-d}x^5-5c^2\sqrt{-c\right.}}{}\right.$$
$$-\frac{2\left(9c^4\sqrt{-d}x^5-5c^2\sqrt{-d}x^3-30\sqrt{-d}x\right)ab}{225c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
-1/15*b^2*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)
/(c^4*d))*arccosh(c*x)^2 - 2/15*a*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d)
+ 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d))*arccosh(c*x) - 1/15*a^2*(3*(-c^2*d*x^2
 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 2/3375*b^2*(
(27*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*x^4 + 11*sqrt(c^2*x^2 - 1)*sqrt(-d)*x^2
 - 428*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/c^2 - 15*(9*c^4*sqrt(-d)*x^5 - 5*c^
2*sqrt(-d)*x^3 - 30*sqrt(-d)*x)*arccosh(c*x)/c^3) - 2/225*(9*c^4*sqrt(-d)*
x^5 - 5*c^2*sqrt(-d)*x^3 - 30*sqrt(-d)*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\frac{\sqrt{d}\,\left(3\sqrt{-c^2x^2+1}\,a^2c^4x^4-\sqrt{-c^2x^2+1}\,a^2c^2x^2-2\sqrt{-c^2x^2+1}\,a^2+30\left(\int\sqrt{-c^2x^2+1}\,acosh(cx)\,x^3dx\right)\right.}{15c^4}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*(3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - sqrt( - c**2*x**2 + 1)
*a**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a**2 + 30*int(sqrt( - c**2*x**2
 + 1)*acosh(c*x)*x**3,x)*a*b*c**4 + 15*int(sqrt( - c**2*x**2 + 1)*acosh(c*
x)**2*x**3,x)*b**2*c**4))/(15*c**4)
```

3.160 $\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 319

$$\begin{aligned}\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx =& -\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d-c^2dx^2}\\ &-\frac{b^2\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{64c^3\sqrt{-1+cx}\sqrt{1+cx}}\\ &+\frac{bx^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{8c\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{bcx^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{8\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{8c^2}\\ &+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\\ &-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{24bc^3\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/64*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2+1/32*b^2*x^3*(-c^2*d*x^2+d)^(1/2)-1/6
4*b^2*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/
8*b*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-1/8*b*c*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-1/8*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^2+1/4*x^3*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2-1/24*(-c^2*d*x^2+d)^(1/2)*(a+b*arcco
sh(c*x))^3/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.94 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.76

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx =$$

$$-\frac{-96a^2cx(-1+2c^2x^2)\sqrt{d-c^2dx^2}+96a^2\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{12ab\sqrt{d-c^2dx^2}\left(8\text{arccosh}(cx)^2+\cosh(4a\right.}{\sqrt{}}}{}$$

input
```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
-1/768*(-96*a^2*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 96*a^2*Sqrt[d]*
ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (12*a*b*Sqrt[
d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*S
inh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d
 - c^2*d*x^2]*(32*ArcCosh[c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] -
3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x)))/c^3
```

Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {6341, 6298, 111, 27, 101, 43, 6354, 6298, 101, 43, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6341}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^3(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6298}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\int \frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{111}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{\int \frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{101}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$\Big\downarrow$ 43

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6354

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int x(a+b\text{arccosh}(cx))dx}{c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2c^2}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6298

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2c^2}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 101

$$-\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 43

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{}}{}\right.\right.}{c}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6308

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))^3}{6bc^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{}}{2c^2}\right.\right.}{c}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

input

```
Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (b*c*Sqrt[d - c^2*d*x
^2]*((x^4*(a + b*ArcCosh[c*x]))/4 - (b*c*((x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])/(4*c^2) + (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(
2*c^3)))/(4*c^2)))/4))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*
x^2]*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*c^2) + (a
 + b*ArcCosh[c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*ArcCosh[c*x]))/2 - (b*c*(
(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/2))/c))/
(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(271) = 542.

Time = 0.36 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.13

method	result
default	$-\frac{a^2x(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}+\frac{a^2x\sqrt{-c^2dx^2+d}}{8c^2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^3}{24\sqrt{cx-1}\,\sqrt{cx+1}\,c^3}+\frac{\sqrt{-d(c^2x^{}}}{}\right.$
parts	$-\frac{a^2x(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}+\frac{a^2x\sqrt{-c^2dx^2+d}}{8c^2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^3}{24\sqrt{cx-1}\,\sqrt{cx+1}\,c^3}+\frac{\sqrt{-d(c^2x^{}}}{}\right.$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a^2/c^2*x*(-c^2*d*x^2+d)^(1/2)+1
/8*a^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^
2*(-1/24*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*
x)^3+1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^
(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)/(c*x+1)/c^3/(c*x-1)
+1/512*(-d*(c^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^
5*x^5+8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+
1)^(1/2)+4*c*x)*(8*arccosh(c*x)^2+4*arccosh(c*x)+1)/(c*x+1)/c^3/(c*x-1))+2
*a*b*(-1/16*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh
(c*x)^2+1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-
1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c
^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*
x)*(1+4*arccosh(c*x))/(c*x+1)/c^3/(c*x-1))
```

Fricas [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^2\,dx$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
1/8*a^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) +
 sqrt(d)*arcsin(c*x)/c^3) + integrate(sqrt(-c^2*d*x^2 + d)*b^2*x^2*log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(-c^2*d*x^2 + d)*a*b*x^2*log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int x^2\,\left(a+b\,\mathrm{acosh}(c\,x)\right)^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\frac{\sqrt{d}\,\left(asin(cx)\,a^2+2\sqrt{-c^2x^2+1}\,a^2c^3x^3-\sqrt{-c^2x^2+1}\,a^2cx+16\left(\int\sqrt{-c^2x^2+1}\,acosh(cx)\,x^2dx\right)ab\,c^3+\right.}{8c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*(asin(c*x)*a**2 + 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 - sqrt(
 - c**2*x**2 + 1)*a**2*c*x + 16*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2
,x)*a*b*c**3 + 8*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**2,x)*b**2*c**
3))/(8*c**3)
```

3.161 $\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx = -\frac{14b^2\sqrt{d-c^2dx^2}}{27c^2}+\frac{2}{27}b^2x^2\sqrt{d-c^2dx^2} + \frac{2bx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{3c^2d}$$

output

```
-14/27*b^2*(-c^2*d*x^2+d)^(1/2)/c^2+2/27*b^2*x^2*(-c^2*d*x^2+d)^(1/2)+2/3*
b*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-
2/9*b*c*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(-6abcx\sqrt{-1+cx}\sqrt{1+cx}(-3+c^2x^2)+9a^2(-1+c^2x^2)^2+2b^2(7-8c^2x^2+c^4x^4)+6b\left(bc\right.\right.}{27c^2\left(-1+c\right.}$$

input
```
Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + c^2*x^
2) + 9*a^2*(-1 + c^2*x^2)^2 + 2*b^2*(7 - 8*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3*a*(-1 + c^2*x^2)^2)*ArcCos
h[c*x] + 9*b^2*(-1 + c^2*x^2)^2*ArcCosh[c*x]^2))/(27*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6329, 25, 6304, 6309, 27, 960, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{2b\sqrt{d-c^2dx^2}\int -((1-cx)(cx+1)(a+b\text{arccosh}(cx)))dx}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\sqrt{d-c^2dx^2}\int (1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 6304$$

$$\frac{2b\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)(a+\text{barccosh}(cx))dx}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 6309$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-bc\int\frac{x(3-c^2x^2)}{3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+\text{barccosh}(cx))+x(a+\text{barccosh}(cx))\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\int\frac{x(3-c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+\text{barccosh}(cx))+x(a+\text{barccosh}(cx))\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 960$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\left(\frac{7}{3}\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{3}c^2x^3(a+\text{barccosh}(cx))+x(a+\text{barccosh}(c\right.}{3c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 83$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+\text{barccosh}(cx))+x(a+\text{barccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+\text{barccosh}(cx))^2}{3c^2d}$$

input
```
Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(c^2*d) + (2*b*Sqrt[d
- c^2*d*x^2]*(-1/3*(b*c*((7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) - (x^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/3)) + x*(a + b*ArcCosh[c*x]) - (c^2*x^3*(a +
b*ArcCosh[c*x]))/3))/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(158) = 316$.

Time = 0.46 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.63

method	result
orering	$\frac{(19c^6x^6-71c^4x^4+48c^2x^2-14)\sqrt{-c^2dx^2+d}\,(a+b\,\operatorname{arccosh}(cx))^2}{27(c^2x^2-1)c^4x^2} - \frac{2(3c^4x^4-16c^2x^2+7)\left(\sqrt{-c^2dx^2+d}\,(a+b\,\operatorname{arccosh}(cx))^2-\frac{x^2(\cdots}{\cdots}\right.}{27c^4x\cdots}$
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx+1)\left(9\operatorname{arccosh}(cx)^2-6\operatorname{arc}\cdots\right.}{216(cx+1)c^2(cx-1)}\right.$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx+1)\left(9\operatorname{arccosh}(cx)^2-6\operatorname{arc}\cdots\right.}{216(cx+1)c^2(cx-1)}\right.$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/27*(19*c^6*x^6-71*c^4*x^4+48*c^2*x^2-14)/(c^2*x^2-1)/c^4/x^2*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccosh(c*x))^2-2/27*(3*c^4*x^4-16*c^2*x^2+7)/c^4/x^2*((-c^
2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2-x^2/(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))^2*c^2*d+2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*b*c/(c*x-1)^(1
/2)/(c*x+1)^(1/2))+1/27*(c^2*x^2-7)/c^4*(c*x-1)*(c*x+1)/x*(-3/(-c^2*d*x^2+
d)^(1/2)*(a+b*arccosh(c*x))^2*c^2*d*x+4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(
c*x))*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-x^3/(-c^2*d*x^2+d)^(3/2)*(a+b*arccos
h(c*x))^2*c^4*d^2-4*x^2/(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*c^3*d*b/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+2*x*(-c^2*d*x^2+d)^(1/2)*b^2*c^2/(c*x-1)/(c*x+1)
-x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(3/2)/(c*x+1)^(1/
2)-x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(1/2)/(c*x+1)^(
3/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.51

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\frac{9\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\left(abc^3x^3-3\,abcx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2}}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
1/27*(9*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 - 3*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 - 3*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(
c^2*x^2 - 1) - 3*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(-c^2*d*x^2 + d))
*log(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - 2*(9*a^2 + 8*b^
2)*c^2*x^2 + 9*a^2 + 14*b^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int x\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\frac{2}{27}b^2\left(\frac{\sqrt{c^2x^2-1}\sqrt{-d}dx^2-\frac{7\sqrt{c^2x^2-1}\sqrt{-d}d}{c^2}}{d}-\frac{3\left(c^2\sqrt{-d}dx^3-3\sqrt{-d}dx\right)\text{arcosh}\left(cx\right)}{cd}\right)$$
$$-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2\,\text{arcosh}\left(cx\right)^2}{3\,c^2d}-\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}ab\,\text{arcosh}\left(cx\right)}{3\,c^2d}$$
$$-\frac{2\left(c^2\sqrt{-d}dx^3-3\sqrt{-d}dx\right)ab}{9\,cd}-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}a^2}{3\,c^2d}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
2/27*b^2*((sqrt(c^2*x^2 - 1)*sqrt(-d)*d*x^2 - 7*sqrt(c^2*x^2 - 1)*sqrt(-d)
*d/c^2)/d - 3*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d*x)*arccosh(c*x)/(c*d)) -
1/3*(-c^2*d*x^2 + d)^(3/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/3*(-c^2*d*x^2 +
d)^(3/2)*a*b*arccosh(c*x)/(c^2*d) - 2/9*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d
*x)*a*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int x\,(a+b\,\text{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\frac{\sqrt{d}\,\left(\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a^2+6\left(\int\sqrt{-c^2x^2+1}\,acosh(cx)\,xdx\right)ab\,c^2+3\left(\int\sqrt{-c^2x^2+1}\,ac\right.}{3c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a
**2 + 6*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*a*b*c**2 + 3*int(sqrt(
- c**2*x**2 + 1)*acosh(c*x)**2*x,x)*b**2*c**2))/(3*c**2)
```

3.162 $\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 204

$$\begin{aligned}\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx &= \frac{1}{4}b^2x\sqrt{d-c^2dx^2}+\frac{b^2\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{4c\sqrt{-1+cx}\sqrt{1+cx}}\\ &\quad-\frac{bcx^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2\sqrt{-1+cx}\sqrt{1+cx}}\\ &\quad+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\\ &\quad-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)+1/4*b^2*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/c
/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*b*c*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh
(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh
(c*x))^2-1/6*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/c/(c*x-1)^(1/2)/(
c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.15

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\,dx$$
$$=\frac{1}{24}\left(12a^2x\sqrt{d-c^2dx^2}-\frac{12a^2\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{c}\right.$$
$$-\frac{6ab\sqrt{d-c^2dx^2}(2\operatorname{arccosh}(cx)^2+\cosh(2\operatorname{arccosh}(cx))-2\operatorname{arccosh}(cx)\sinh(2\operatorname{arccosh}(cx)))}{c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$
$$+\frac{b^2\sqrt{d-c^2dx^2}(-4\operatorname{arccosh}(cx)^3-6\operatorname{arccosh}(cx)\cosh(2\operatorname{arccosh}(cx))+(3+6\operatorname{arccosh}(cx)^2)\sinh(2\operatorname{arcco}}{c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(12*a^2*x*Sqrt[d - c^2*d*x^2] - (12*a^2*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d
*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]])/c - (6*a*b*Sqrt[d - c^2*d*x^2]*(2*ArcCos
h[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d - c^2*d*x^2]*(-4*ArcC
osh[c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 + 6*ArcCosh[c*x]^2)*
Sinh[2*ArcCosh[c*x]]))/(c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6310, 6298, 101, 43, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6310}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6298}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{101}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{43}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6308}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^3)/(6*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*Sqrt[d -
 c^2*d*x^2]*((x^2*(a + b*ArcCosh[c*x]))/2 - (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(172) = 344$.

Time = 0.00 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.58

method	result
default	$\frac{a^2 x\sqrt{-c^2 d\,x^2+d}}{2}+\frac{a^2 d \arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{2\sqrt{c^2 d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^3}{6\sqrt{cx-1}\,\sqrt{cx+1}\,c}+\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-2cx+2\sqrt{cx-}}{}\right.$
parts	$\frac{a^2 x\sqrt{-c^2 d\,x^2+d}}{2}+\frac{a^2 d \arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{2\sqrt{c^2 d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^3}{6\sqrt{cx-1}\,\sqrt{cx+1}\,c}+\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3x^3-2cx+2\sqrt{cx-}}{}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*a^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2
)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/6*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)/c*arccosh(c*x)^3+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x
+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arc
cosh(c*x)^2-2*arccosh(c*x)+1)/(c*x-1)/(c*x+1)/c+1/16*(-d*(c^2*x^2-1))^(1/2
)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^
(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)/(c*x-1)/(c*x+1)/c)+2*a*b*
(-1/4*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2+
1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))/(c*x-1)/(c*x+1)/
c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^
3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))/(c*x-1)/(c*x+1
)/c)
```

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2), x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^2\;dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a^2 + integrate(sqrt(
-c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(-c^2
*d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx = \int (a+b\,\text{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$= \frac{\sqrt{d}\left(asin(cx)\,a^2+\sqrt{-c^2x^2+1}\,a^2cx+4\left(\int \sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)abc+2\left(\int \sqrt{-c^2x^2+1}\,acosh(cx)^2\right.}{2c}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*(asin(c*x)*a**2 + sqrt( - c**2*x**2 + 1)*a**2*c*x + 4*int(sqrt( -
 c**2*x**2 + 1)*acosh(c*x),x)*a*b*c + 2*int(sqrt( - c**2*x**2 + 1)*acosh(c
*x)**2,x)*b**2*c))/(2*c)
```

3.163 $\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 402

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}\,dx \\
&= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad - \frac{2b^2cx\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2 \\
&\quad - \frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\arctan\left(e^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{2ib\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad - \frac{2ib\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad - \frac{2ib^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{2ib^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
2*b^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-2*b^2*c*x*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2-2*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b^2*(-c^2*d*x^2+d)^(1/2)*polylog(
3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^
2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}\,dx$$
$$= a^2\sqrt{d-c^2dx^2}+a^2\sqrt{d}\log(cx)-a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)$$
$$+\frac{2ab\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)+i\text{arccosh}(cx)\log\left(1-ie^{-\text{arccos}}\right.\right.}{\sqrt{\frac{-1+c}{1+c}}}$$
$$+b^2\sqrt{d-c^2dx^2}\left(2+\frac{2cx\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)}{1-cx}+\text{arccosh}(cx)^2\right.$$
$$+\frac{i\left(\text{arccosh}(cx)^2\log\left(1-ie^{-\text{arccosh}(cx)}\right)-\text{arccosh}(cx)^2\log\left(1+ie^{-\text{arccosh}(cx)}\right)+2\text{arccosh}(cx)\,\text{PolyLog}\left(\right.\right.}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

output
```
a^2*Sqrt[d - c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[
d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 +
c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]
 + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^A
rcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCos
h[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*Sqrt[d - c^2*d*x^2]
*(2 + (2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) + ArcCosh[
c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log
[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] -
2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[
c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)))
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.59, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6341, 2009, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{6341}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{6362}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 3042

$$-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 4668

$$-\frac{\sqrt{d-c^2dx^2}\left(-2ib\int (a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int (a+b\text{arccosh}(cx))\log\left(1+ie\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 3011

$$-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)\right.}{}$$

$$\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 2720

$$-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+\text{barco}\right.\right.}{}$$

$$\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 7143

$$-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arc}}\right.\right.\right.}{\sqrt{cx-1}}$$

$$\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

output
```
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a
*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]))/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*x])^2*ArcTan[
E^ArcCosh[c*x]] + (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCos
h[c*x]]) + b*PolyLog[3, (-I)*E^ArcCosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh[
c*x])*PolyLog[2, I*E^ArcCosh[c*x]]) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/(S
qrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\sqrt{-c^2 d\,x^2 + d}\,(a + b\,\mathrm{arccosh}\,(cx))^2}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x,x)
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\int \frac{\sqrt{-d(cx-1)(cx+1)}(a+b\operatorname{acosh}(cx))^2}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2/x,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima
")
```

output
```
-(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-
c^2*d*x^2 + d))*a^2 + integrate(sqrt(-c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))^2/x + 2*sqrt(-c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{x}\,dx = \int \frac{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{x}\,dx = \sqrt{d}\left(\sqrt{-c^2x^2+1}\,a^2 + 2\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x}dx\right)ab + \left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)^2}{x}dx\right)b^2 + \log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a^2 - a^2\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2/x,x)
```

output
```
sqrt(d)*(sqrt( - c**2*x**2 + 1)*a**2 + 2*int((sqrt( - c**2*x**2 + 1)*acosh
(c*x))/x,x)*a*b + int((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x,x)*b**2 + l
og(tan(asin(c*x)/2))*a**2 - a**2)
```

3.164 $\int \frac{\sqrt{d-c^2dx^2}(a+b\textbf{arccosh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 234

$$\begin{aligned}\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx &= -\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x} - \frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad + \frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{3b\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad + \frac{2bc\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\log\left(1+e^{2\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad + \frac{b^2c\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x-c*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/3*c*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))^3/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+b^2*c*(-c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.48 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx$$
$$= -\frac{a^2\sqrt{d-c^2dx^2}}{x} + a^2c\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)$$
$$+ abc\sqrt{d-c^2dx^2}\left(-\frac{2\text{arccosh}(cx)}{cx} + \frac{\text{arccosh}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)$$
$$+ \frac{1}{3}b^2c\sqrt{d-c^2dx^2}\left(\text{arccosh}(cx)\left(-\frac{3\text{arccosh}(cx)}{cx}\right.\right.$$
$$\left. + \frac{\text{arccosh}(cx)(3+\text{arccosh}(cx))+6\log\left(1+e^{-2\text{arccosh}(cx)}\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)$$
$$\left. + \frac{3\sqrt{\frac{-1+cx}{1+cx}}\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)}{1-cx}\right)$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

output
```
-((a^2*Sqrt[d - c^2*d*x^2])/x) + a^2*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*
x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + a*b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c
*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x))) + (b^2*c*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*((-3*ArcCosh[c*x])/(c*
x) + (ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])/(S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*Poly
Log[2, -E^(-2*ArcCosh[c*x])])/(1 - c*x)))/3
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.79, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {6339, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \text{6339}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{6297}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2c\sqrt{d-c^2dx^2}\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c\sqrt{d-c^2dx^2}\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 26$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ic\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 4201$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ic\sqrt{d-c^2dx^2}\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 2620$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 2715$$

$$\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+\text{barc}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 2838$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{6308}$$

$$\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{3b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x) + (c*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*c*Sqr
t[d - c^2*d*x^2]*((-1/2*I)*(a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*
ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCo
sh[c*x]])/4)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(232) = 464.

Time = 0.47 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.49

method	result
default	$-\frac{a^2(-c^2 d\,x^2+d)^{\frac{3}{2}}}{dx}-a^2c^2x\sqrt{-c^2d\,x^2+d}-\frac{a^2c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^3c}{3\sqrt{cx-1}\,\sqrt{cx+1}}-b^2\sqrt{}$
parts	$-\frac{a^2(-c^2 d\,x^2+d)^{\frac{3}{2}}}{dx}-a^2c^2x\sqrt{-c^2d\,x^2+d}-\frac{a^2c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^3c}{3\sqrt{cx-1}\,\sqrt{cx+1}}-b^2\sqrt{}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(3/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(1/2)-a^2*c^2*d/(c^
2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c-b^2*(-d*(c^2*x^2
-1))^(1/2)*arccosh(c*x)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*c-b^2*(-d*(c^2*x^2-1
))^(1/2)*arccosh(c*x)^2/(c*x-1)/(c*x+1)*x*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)*a
rccosh(c*x)^2/(c*x-1)/(c*x+1)/x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+b^
2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)*c+a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)*arccosh(c*x)^2*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)*arccosh(c*x)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-1)
/(c*x+1)*x*c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-1)/(c*x+1)/x
+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2)*c
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acosh}{(cx)})^2}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2/x**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxi
ma")
```

output
```
-(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a^2 + integrate(sqrt(-c^
2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*sqrt(-c^
2*d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{d-c^2\,d\,x^2}}{x^2} \,d x$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx$$
$$=\frac{\sqrt{d}\,\left(-asin(cx)\,a^2cx-\sqrt{-c^2x^2+1}\,a^2+2\left(\int\frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^2}dx\right)abx+\left(\int\frac{\sqrt{-c^2x^2+1}\,acosh(cx)^2}{x^2}dx\right)b^2x\right)}{x}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2/x^2,x)
```

output
```
(sqrt(d)*( - asin(c*x)*a**2*c*x - sqrt( - c**2*x**2 + 1)*a**2 + 2*int((sqr
t( - c**2*x**2 + 1)*acosh(c*x))/x**2,x)*a*b*x + int((sqrt( - c**2*x**2 + 1
)*acosh(c*x)**2)/x**2,x)*b**2*x))/x
```

3.165 $\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 427

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx \\
&= -\frac{bc\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2} \\
&\quad + \frac{c^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\arctan\left(e^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{b^2c^2\sqrt{d-c^2dx^2}\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad - \frac{ibc^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{ibc^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{ib^2c^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad - \frac{ib^2c^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
-b*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-1/2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^2+c^2*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+b^2*c^2*(-c^2*d*x^2+d)^(1/2)*arctan((c*x-1)^(1/2)*(c*x+1)
^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
osh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c
*x+1)^(1/2)+I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b^2*c^2*(-c
^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-I*b^2*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5035 vs. $2(427) = 854$.

Time = 62.67 (sec) , antiderivative size = 5035, normalized size of antiderivative = 11.79

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.59, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {6339, 6298, 103, 218, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{6339}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\left(bc\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{103}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\left(bc^2\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6362}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 4668$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(-2ib\int (a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int (a+b\text{arccosh}(cx))\log\left(1+ie\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{bc\sqrt{d-c^2dx^2}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 3011$$

$$c^2\sqrt{d-c^2dx^2}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)\right.$$
$$\frac{bc\sqrt{d-c^2dx^2}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 2720$$

$$c^2\sqrt{d-c^2dx^2}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+\text{barcc}\right.\right.$$
$$\frac{bc\sqrt{d-c^2dx^2}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 7143$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\operatorname{arccosh}(cx)}\right)(a+b\operatorname{arccosh}(cx))^2+2ib\left(b\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(cx)}\right)-\operatorname{PolyLog}\left(2,-ie^{\operatorname{arc}}\right.\right.\right.}{2\sqrt{cx-1}}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\operatorname{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{2x^2}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

output
```
-1/2*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2 + (b*c*Sqrt[d - c^2*
d*x^2]*(-((a + b*ArcCosh[c*x])/x) + b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*Sqrt[d - c^2*d*x^2]*(2*(a + b*A
rcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]] + (2*I)*b*(-((a + b*ArcCosh[c*x])*Po
lyLog[2, (-I)*E^ArcCosh[c*x]]) + b*PolyLog[3, (-I)*E^ArcCosh[c*x]]) - (2*I
)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]]) + b*PolyLog[3, I
*E^ArcCosh[c*x]])))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\sqrt{-c^2 d\,x^2+d}\,(a+b\,\mathrm{arccosh}\,(cx))^2}{x^3}dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\mathrm{arccosh}(cx))^2}{x^3}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^2}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2/x**3,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxi
ma")
```

output
```
1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a^2 + integrat
e(sqrt(-c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 +
2*sqrt(-c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx=\int \frac{\left(a+b\,\text{acosh}(c\,x)\right)^2\,\sqrt{d-c^2\,d\,x^2}}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx$$
$$=\frac{\sqrt{d}\left(-\sqrt{-c^2x^2+1}\,a^2+4\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^3}dx\right)ab\,x^2+2\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)^2}{x^3}dx\right)b^2x^2-\log\left(\tan\left(\frac{asi}{}\right.\right.\right.}{2x^2}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2/x^3,x)
```

output
```
(sqrt(d)*( - sqrt( - c**2*x**2 + 1)*a**2 + 4*int((sqrt( - c**2*x**2 + 1)*a
cosh(c*x))/x**3,x)*a*b*x**2 + 2*int((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)
/x**3,x)*b**2*x**2 - log(tan(asin(c*x)/2))*a**2*c**2*x**2))/(2*x**2)
```

3.166 $\int \frac{\sqrt{d-c^2dx^2}(a+b\,\text{arccosh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 326

$$\begin{aligned}\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx &= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x}-\frac{b^2c^3\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}}\\
&+\frac{bc\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3x^2}\\
&+\frac{c^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}\\
&-\frac{2bc^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\log\left(1+e^{2\text{arccosh}(cx)}\right)}{3\sqrt{-1+cx}\sqrt{1+cx}}\\
&-\frac{b^2c^3\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{3\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
1/3*b^2*c^2*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*(-c^2*d*x^2+d)^(1/2)*arccos
h(c*x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/3*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c
^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^2+1/3*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b
*arccosh(c*x))^2/d/x^3-2/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b
^2*c^3*(-c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.86 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx =$$

$$-\frac{d(1+cx)\left(a^2-a^2cx-a^2c^2x^2-b^2c^2x^2+a^2c^3x^3+b^2c^3x^3-abcx\sqrt{\frac{-1+cx}{1+cx}}-b^2\left(-1+cx+c^2x^2+c^3x\right.\right.}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

output
```
-1/3*(d*(1 + c*x)*(a^2 - a^2*c*x - a^2*c^2*x^2 - b^2*c^2*x^2 + a^2*c^3*x^3
 + b^2*c^3*x^3 - a*b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*(-1 + c*x + c^2*
x^2 + c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2 - b*ArcCos
h[c*x]*(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*a*(-1 + c*x)^2*(1 + c*x) + 2*
b*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x])]) - 2*a*b
*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[c*x] + b^2*c^3*x^3*Sqrt[(-1 + c*x)
/(1 + c*x)]*PolyLog[2, -E^(-2*ArcCosh[c*x])]))/(x^3*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.65, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {6332, 25, 6327, 6335, 108, 27, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{ 6332}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 6327}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 6335}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{1}{2}bc\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2}dx-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 108}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{1}{2}bc\left(\int \frac{c^2}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{1}{2}bc\left(c^2\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 43$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bc\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 6297$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(-\frac{c^2\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 25$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{c^2\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bc\left(ca\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}+$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{c^2\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bc\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 26$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}+$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(-\frac{ic^2\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bc\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 4201

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}+$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\int\frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}\right.}{2x^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2620

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}+$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{ar}\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2715

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}+$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2838

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3dx^3}+$$
$$\frac{2bc\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(d*x^3) + (2*b*c*Sqrt[
d - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/x^2 - (b*c*(-((S
qrt[-1 + c*x]*Sqrt[1 + c*x])/x) + c*ArcCosh[c*x]))/2 - (I*c^2*((-1/2*I)*(a
 + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*A
rcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b))/(3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

rule 6335
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*((-d)^p/(f*(m + 1)))   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(
m + 1)/2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. $2(304) = 608$.

Time = 0.54 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.37

method	result	size
default	Expression too large to display	1751
parts	Expression too large to display	1751

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c*x-1)^(1/2)/
(c*x+1)^(1/2)*arccosh(c*x)*c-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x
^2+1)*x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^5+b^2*(-d*(c^2*x^2-1)
)^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*
x)^2*c^5-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x-1)^(1
/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^7-3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x
^4-3*c^2*x^2+1)*x^3/(c*x-1)/(c*x+1)*arccosh(c*x)^2*c^6+10/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x-1)/(c*x+1)*arccosh(c*x)^2*c^4+
1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x-1)/(c*x+1)*a
rccosh(c*x)*c^4-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(
c*x-1)/(c*x+1)*arccosh(c*x)^2*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*
c^2*x^2+1)*x^5/(c*x-1)/(c*x+1)*arccosh(c*x)^2*c^8+1/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x-1)/(c*x+1)*arccosh(c*x)*c^8-2/3*b^
2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x-1)/(c*x+1)*arcco
sh(c*x)*c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3*c^6
-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(3/2)-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3+2
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c
^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)*c^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*...
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2/x**4,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxi
ma")
```

output
```
1/3*(c^4*d^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*
d)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d
)*d/x^2)*a*b*c/d + 1/3*b^2*((c^2*sqrt(d)*x^2 - sqrt(d))*sqrt(c*x + 1)*sqrt
(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 - 3*integrate(2/3*
((c*x + 1)*sqrt(c*x - 1)*c^2*sqrt(d)*x + (c^3*sqrt(d)*x^2 - c*sqrt(d))*sqr
t(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c*x^4 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*x^3), x)) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*b*ar
ccosh(c*x)/(d*x^3) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx$$
$$= \frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a^2+6\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)}{x^4}dx\right)ab\,x^3+3\left(\int \frac{\sqrt{-c^2x^2+1}\,acosh(cx)^2}{x^4}dx\right)\right.}{3x^3}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2/x^4,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a
**2 + 6*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**4,x)*a*b*x**3 + 3*int((
sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x**4,x)*b**2*x**3))/(3*x**3)
```

3.167 $\int x^3(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 453

$$\begin{aligned}\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx &= -\frac{37384b^2d\sqrt{d-c^2dx^2}}{385875c^4}\\
&+\frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2}+\frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875}-\frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2}\\
&+\frac{4bdx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{35c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bdx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{105c\sqrt{-1+cx}\sqrt{1+cx}}\\
&-\frac{16bcdx^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{175\sqrt{-1+cx}\sqrt{1+cx}}\\
&+\frac{2bc^3dx^7\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{49\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{35c^4}\\
&-\frac{dx^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{35c^2}+\frac{3}{35}dx^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\\
&+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\end{aligned}$$

output
```
-37384/385875*b^2*d*(-c^2*d*x^2+d)^(1/2)/c^4+3358/385875*b^2*d*x^2*(-c^2*d
*x^2+d)^(1/2)/c^2+484/42875*b^2*d*x^4*(-c^2*d*x^2+d)^(1/2)-2/343*b^2*c^2*d
*x^6*(-c^2*d*x^2+d)^(1/2)+4/35*b*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x
))/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/105*b*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-16/175*b*c*d*x^5*(-c^2*d*x^2+
d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/49*b*c^3*d*x^7*(
-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/35*d*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^4-1/35*d*x^2*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccosh(c*x))^2/c^2+3/35*d*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(
c*x))^2+1/7*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.58

$$\int x^3 (d - c^2dx^2)^{3/2} (a + b\text{arccosh}(cx))^2 \, dx =$$

$$-\frac{d\sqrt{d-c^2dx^2}\Big(11025a^2(-1+c^2x^2)^3\,(2+5c^2x^2) - 210abcx\sqrt{-1+cx}\sqrt{1+cx}(210+35c^2x^2-168c^4x^4+$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
-1/385875*(d*Sqrt[d - c^2*d*x^2]*(11025*a^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^
2) - 210*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*
x^4 + 75*c^6*x^6) + 2*b^2*(-18692 + 20371*c^2*x^2 + 499*c^4*x^4 - 3303*c^6
*x^6 + 1125*c^8*x^8) - 210*b*(-105*a*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) + b*
c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*
x^6))*ArcCosh[c*x] + 11025*b^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2)*ArcCosh[c*
x]^2))/(c^4*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 3.89 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.46, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.862$, Rules used = {6345, 25, 6327, 6336, 27, 960, 111, 27, 111, 27, 83, 6341, 6298, 111, 27, 111, 27, 83, 6354, 6298, 111, 27, 83, 6330, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6345}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int -x^4(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\int x^4(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6327}$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\int x^4\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6336}$$

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-bc\int \frac{x^5\left(7-5c^2x^2\right)}{35\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))\right)}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\int\frac{x^5(7-5c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))\right)}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 960

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\int\frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arc}\ldots}{7\sqrt{cx-1}\sqrt{cx+1}}\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 111

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{\int\frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2x^7(a+b\text{arccosh}(\ldots}{7\sqrt{cx-1}\sqrt{cx+1}}\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 27

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{4\int\frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c^2x^7(a+b\text{arccosh}\ldots}{7\sqrt{cx-1}\sqrt{cx+1}}\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 111

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{4\left(\frac{\int\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}c\ldots}{7\sqrt{cx-1}\sqrt{cx+1}}\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 27

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{4\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{7}\cdots}{7\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 83

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}\cdots}{3c^4}\cdots\right.}{\cdots}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6341

$$\frac{3}{7}d\left(-\frac{2bc\sqrt{d-c^2dx^2}\int x^4(a+b\text{arccosh}(cx))dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\cdots\right.$$
$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}\cdots}{3c^4}\cdots\right.}{\cdots}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6298

$$\frac{3}{7}d\left(-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\int\frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\cdots\right.$$
$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}\cdots}{3c^4}\cdots\right.}{\cdots}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 111

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{\int\frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+x\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 27

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\int\frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 111

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{\int\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\right.}{5c^2}\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\big\downarrow$ 27

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\right.}{5c^2}\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 83

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\right.}{}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}-\frac{2b\int x^2(a+b\text{arccosh}(cx))dx}{3c}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}\right)}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6298

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\int\frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{3c}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arcco}}{3c^2}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{111}$$

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\left(-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{\int\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c}+\frac{2\int\frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{}}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\left(-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c}+\frac{2\int\frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$$\big\downarrow \text{83}$$

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\sqrt{cx-1}}{3c}\right.\right.}{3c}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-}{}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$$\big\downarrow \text{6330}$$

$$\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{c^2}-\frac{2b\int(a+b\text{arccosh}(cx))dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-}{}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

$$\big\downarrow \text{2009}$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+$$

$$\frac{3}{7}d\left(\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{\sqrt{d-c^2dx^2}\left(\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{3c^2}+\frac{2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arcco}}{c^2}\right.}{}\right.}{}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx}}{3c^4}\right.}{}\right.\right.\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/7 - (2*b*c*d*Sqrt[d - c
^2*d*x^2]*(-1/35*(b*c*((-5*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/7 + (19*((x^4
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*c^2) + (4*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2)))/(5*c^2)))/7)) +
 (x^5*(a + b*ArcCosh[c*x]))/5 - (c^2*x^7*(a + b*ArcCosh[c*x]))/7))/(7*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*((x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/5 - (2*b*c*Sqrt[d - c^2*d*x^2]*(-1/5*(b*c*((x^4*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])/(5*c^2) + (4*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2)))/(5*c^2))) + (x^5*(a + b*ArcCosh[c
*x]))/5))/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^2]*((x^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*c^2) - (2*b*(-1/3*(b
*c*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(3*c^2))) + (x^3*(a + b*ArcCosh[c*x]))/3))/(3*c) + (2*((Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/c^2 - (2*b*(a*x - (b*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]))/c))/(3*c^2)))/(5*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.28

method	result
orering	$\frac{\left(47625c^{10}x^{10}-130566c^{8}x^{8}+68553c^{6}x^{6}+279840c^{4}x^{4}-260420c^{2}x^{2}+74768\right)\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arccosh}(cx)\right)^{2}}{128625c^{6}x^{2}\left(c^{2}x^{2}-1\right)^{2}}-\frac{2(10125c^{8}x^{8}-}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/128625*(47625*c^10*x^10-130566*c^8*x^8+68553*c^6*x^6+279840*c^4*x^4-2604
20*c^2*x^2+74768)/c^6/x^2/(c^2*x^2-1)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(
c*x))^2-2/385875*(10125*c^8*x^8-24174*c^6*x^6-863*c^4*x^4+118868*c^2*x^2-5
6076)/c^6/x^4/(c^2*x^2-1)*(3*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2
-3*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*c^2*d+2*x^3*(-c^2*d*x^2+d
)^(3/2)*(a+b*arccosh(c*x))*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))+1/385875*(1125
*c^6*x^6-2178*c^4*x^4-1679*c^2*x^2+18692)/c^6/x^3*(6*x*(-c^2*d*x^2+d)^(3/2
)*(a+b*arccosh(c*x))^2-21*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*c^
2*d+12*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))*b*c/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+3*x^5/(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*c^4*d^2-12*x^4*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*c^3*d*b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
2*x^3*(-c^2*d*x^2+d)^(3/2)*b^2*c^2/(c*x-1)/(c*x+1)-x^3*(-c^2*d*x^2+d)^(3/2
)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(3/2)/(c*x+1)^(1/2)-x^3*(-c^2*d*x^2+d)^
(3/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(1/2)/(c*x+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.95

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx =$$

$$-\frac{11025\,(5\,b^2c^8dx^8-13\,b^2c^6dx^6+9\,b^2c^4dx^4+b^2c^2dx^2-2\,b^2d)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-210\,(\ldots}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
-1/385875*(11025*(5*b^2*c^8*d*x^8 - 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 + b
^2*c^2*d*x^2 - 2*b^2*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^
2 - 210*(75*a*b*c^7*d*x^7 - 168*a*b*c^5*d*x^5 + 35*a*b*c^3*d*x^3 + 210*a*b
*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((75*b^2*c^7*d*x^7 -
168*b^2*c^5*d*x^5 + 35*b^2*c^3*d*x^3 + 210*b^2*c*d*x)*sqrt(-c^2*d*x^2 + d)
*sqrt(c^2*x^2 - 1) - 105*(5*a*b*c^8*d*x^8 - 13*a*b*c^6*d*x^6 + 9*a*b*c^4*d
*x^4 + a*b*c^2*d*x^2 - 2*a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x
^2 - 1)) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 - 9*(15925*a^2 + 734*b^2)*c^6*
d*x^6 + (99225*a^2 + 998*b^2)*c^4*d*x^4 + (11025*a^2 + 40742*b^2)*c^2*d*x^
2 - 2*(11025*a^2 + 18692*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d - c^2 d x^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.86

$$\int x^3 \left(d - c^2 d x^2\right)^{3/2} (a + b\operatorname{arccosh}(cx))^2\,dx =$$
$$-\frac{1}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)b^2\operatorname{arcosh}(cx)^2$$
$$-\frac{2}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)ab\operatorname{arcosh}(cx)$$
$$-\frac{1}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)a^2$$
$$-\frac{2}{385875}b^2\left(\frac{1125\sqrt{c^2x^2-1}c^4\sqrt{-d}dx^6-2178\sqrt{c^2x^2-1}c^2\sqrt{-d}dx^4-1679\sqrt{c^2x^2-1}\sqrt{-d}dx^2+\frac{18692\sqrt{c}}{}}{c^2}\right.$$
$$+\frac{2\left(75c^6\sqrt{-d}dx^7-168c^4\sqrt{-d}dx^5+35c^2\sqrt{-d}dx^3+210\sqrt{-d}dx\right)ab}{3675c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
-1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^
4*d))*b^2*arccosh(c*x)^2 - 2/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*
(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a*b*arccosh(c*x) - 1/35*(5*(-c^2*d*x^2 + d
)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 - 2/385875*b^2
*((1125*sqrt(c^2*x^2 - 1)*c^4*sqrt(-d)*d*x^6 - 2178*sqrt(c^2*x^2 - 1)*c^2*
sqrt(-d)*d*x^4 - 1679*sqrt(c^2*x^2 - 1)*sqrt(-d)*d*x^2 + 18692*sqrt(c^2*x^
2 - 1)*sqrt(-d)*d/c^2)/c^2 - 105*(75*c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(-d)
*d*x^5 + 35*c^2*sqrt(-d)*d*x^3 + 210*sqrt(-d)*d*x)*arccosh(c*x)/c^3) + 2/3
675*(75*c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(-d)*d*x^5 + 35*c^2*sqrt(-d)*d*x^
3 + 210*sqrt(-d)*d*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d - c^2 d x^2)^{3/2} (a + b\text{arccosh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 d x^2)^{3/2} (a + b\text{arccosh}(cx))^2 \, dx = \int x^3 \left(a + b\,\mathrm{acosh}(c\,x)\right)^2 \left(d - c^2\,d\,x^2\right)^{3/2} \,dx$$

input
```
int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^3 (d - c^2 d x^2)^{3/2} (a + b\,\text{arccosh}(cx))^2 \, dx = \frac{\sqrt{d}\, d\left(-5\sqrt{-c^2x^2+1}\, a^2c^6x^6 + 8\sqrt{-c^2x^2+1}\, a^2c^4x^4 - \sqrt{-c^2x^2+1}\, a^2c^2x^2 - 2\sqrt{-c^2x\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*d*( - 5*sqrt( - c**2*x**2 + 1)*a**2*c**6*x**6 + 8*sqrt( - c**2*x*
*2 + 1)*a**2*c**4*x**4 - sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( -
 c**2*x**2 + 1)*a**2 - 70*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**5,x)*a*
b*c**6 + 70*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**3,x)*a*b*c**4 - 35*in
t(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**5,x)*b**2*c**6 + 35*int(sqrt( -
c**2*x**2 + 1)*acosh(c*x)**2*x**3,x)*b**2*c**4))/(35*c**4)
```

3.168 $\int x^2(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 441

$$\begin{aligned}
\int x^2(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx &= \frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2} \\
&+ \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} + \frac{7b^2d\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{1152c^3\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{bdx^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{16c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{7bcdx^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{48\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{bc^3dx^6\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{18\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{dx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{16c^2} + \frac{1}{8}dx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2 \\
&+ \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2 - \frac{d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{48bc^3\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
7/1152*b^2*d*x*(-c^2*d*x^2+d)^(1/2)/c^2+43/1728*b^2*d*x^3*(-c^2*d*x^2+d)^(
1/2)-1/108*b^2*c^2*d*x^5*(-c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*(-c^2*d*x^2+d)^
(1/2)*arccosh(c*x)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*d*x^2*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/48*b*c*d*x^4
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/18*
b*c^3*d*x^6*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-1/16*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^2+1/8*d*x^3*(-c
^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2+1/6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*a
rccosh(c*x))^2-1/48*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/c^3/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.68 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.10

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx = \frac{-288a^2cdx\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}(3-14c^2x^2+8c^4x^4)-864a^2d^{3/2}\sqrt{\frac{-1+cx}{1+cx}}(1+\cdots}{\cdots}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(-288*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(
3 - 14*c^2*x^2 + 8*c^4*x^4) - 864*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 216*
a*b*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*Arc
Cosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 18*b^2*d*Sqrt[d - c^2*d*x^2]*(32*ArcCosh
[c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*
Sinh[4*ArcCosh[c*x]]) - 12*a*b*d*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 +
 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]]
 + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sin
h[6*ArcCosh[c*x]])) + b^2*d*Sqrt[d - c^2*d*x^2]*(288*ArcCosh[c*x]^3 + 12*A
rcCosh[c*x]*(-18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*ArcCosh[c*x]] + 2*Cosh[6*
ArcCosh[c*x]]) + 108*Sinh[2*ArcCosh[c*x]] - 27*Sinh[4*ArcCosh[c*x]] - 4*Si
nh[6*ArcCosh[c*x]] - 72*ArcCosh[c*x]^2*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4
*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(13824*c^3*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 4.77 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.25, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.724, Rules used = {6345, 25, 6327, 6336, 27, 960, 111, 27, 101, 43, 6341, 6298, 111, 27, 101, 43, 6354, 6298, 101, 43, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2 \, dx$$

$$\Big\downarrow \text{ 6345}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b\text{arccosh}(cx))^2 dx+\frac{b c d \sqrt{d-c^2 d x^2} \int -x^3 (1-c x) (c x+1) (a+b\text{arccosh}(cx)) dx}{3 \sqrt{c x-1} \sqrt{c x+1}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b\text{arccosh}(cx))^2 dx-\frac{b c d \sqrt{d-c^2 d x^2} \int x^3 (1-c x) (c x+1) (a+b\text{arccosh}(cx)) dx}{3 \sqrt{c x-1} \sqrt{c x+1}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 6327}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b\text{arccosh}(cx))^2 dx-\frac{b c d \sqrt{d-c^2 d x^2} \int x^3 \left(1-c^2 x^2\right) (a+b\text{arccosh}(cx)) dx}{3 \sqrt{c x-1} \sqrt{c x+1}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 6336}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b\text{arccosh}(cx))^2 dx-\frac{b c d \sqrt{d-c^2 d x^2} \left(-b c \int \frac{x^4 \left(3-2 c^2 x^2\right)}{12 \sqrt{c x-1} \sqrt{c x+1}} dx-\frac{1}{6} c^2 x^6 (a+b\text{arccosh}(cx))+\frac{1}{4} x^4 (a+b\text{arccosh}(cx))\right)}{3 \sqrt{c x-1} \sqrt{c x+1}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\int\frac{x^4(3-2c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 960

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\int\frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arcc}\ldots\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 111

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{\int\frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx\ldots\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 27

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2x^6(a+b\text{arccosh}(c\ldots\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 101

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{6}c^2x^{\ldots}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 43

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6341

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^3(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+ba\right.$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6298

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\int\frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}\right.$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 111

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{\int\frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{}}{}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 27

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 101

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x}{}\right.}{4c^2}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 43

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{ar}\right.}{}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\int x(a+b\text{arccosh}(cx))dx}{c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2c^2}\right)}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6298

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{c}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(\right.}{4\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 101

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 43

$$\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}\right.\right.}{c}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\downarrow \text{ 6308}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+$$

$$\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))^3}{6bc^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^2}{2c^2}-\frac{b\left(\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+}}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/6 - (b*c*d*Sqrt[d - c^2
*d*x^2]*((x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*x^6*(a + b*ArcCosh[c*x]))/6 -
 (b*c*(-1/3*(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*((x^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x])/(4*c^2) + (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + Arc
Cosh[c*x]/(2*c^3)))/(4*c^2)))/3))/12))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(d*((x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (b*c*Sqrt[d - c^2
*d*x^2]*((x^4*(a + b*ArcCosh[c*x]))/4 - (b*c*((x^3*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])/(4*c^2) + (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*
x]/(2*c^3)))/(4*c^2)))/4))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^
2*d*x^2]*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*c^2)
+ (a + b*ArcCosh[c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*ArcCosh[c*x]))/2 - (b
*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/2))/
c))/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(377) = 754$.

Time = 0.50 (sec) , antiderivative size = 1721, normalized size of antiderivative = 3.90

method	result	size
default	Expression too large to display	1721
parts	Expression too large to display	1721

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a^2/c^2*x*(-c^2*d*x^2+d)^(3/2)+
1/16*a^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a^2/c^2*d^2/(c^2*d)^(1/2)*arcta
n((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/48*(-d*(c^2*x^2-1))^(1/2)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^3*d-1/6912*(-d*(c^2*x^2-1))^(
1/2)*(32*c^7*x^7-64*c^5*x^5+32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+38*c^3*
x^3-48*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x+18*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(18*arccosh(c*x)^2-6*arccosh(c*
x)+1)*d/(c*x+1)/c^3/(c*x-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^
3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x
)+1)*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arcco
sh(c*x)^2-2*arccosh(c*x)+1)*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(
1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+
1)^(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)*d/(c*x+1)/c^3/(c*x-1)+
1/1024*(-d*(c^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^
5*x^5+8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+
1)^(1/2)+4*c*x)*(8*arccosh(c*x)^2+4*arccosh(c*x)+1)*d/(c*x+1)/c^3/(c*x-1)-
1/6912*(-d*(c^2*x^2-1))^(1/2)*(-32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+32*
c^7*x^7+48*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)-64*c^5*x^5-18*(c*x-1)^(1...
```

Fricas [F]

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(c x))^2 \, dx=\int \left(-c^2 d x^2+d\right)^{\frac{3}{2}} (b \operatorname{arcosh}(c x)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
integral(-(a^2*c^2*d*x^4 - a^2*d*x^2 + (b^2*c^2*d*x^4 - b^2*d*x^2)*arccosh
(c*x)^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(c x))^2 \, dx=\text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(c x))^2 \, dx=\int \left(-c^2 d x^2+d\right)^{\frac{3}{2}} (b \operatorname{arcosh}(c x)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
1/48*a^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2
*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + integr
ate((-c^2*d*x^2 + d)^(3/2)*b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^
2 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)
), x)
```

Giac [F]

$$\int x^2 (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2 \, dx = \int (-c^2dx^2+d)^{\frac{3}{2}} (b\,\text{arcosh}\,(cx)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2 \, dx = \int x^2 \left(a+b\,\text{acosh}(c\,x)\right)^2 \left(d-c^2\,d\,x^2\right)^{3/2} \, dx$$

input
```
int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b\text{arccosh}(cx))^2 \, dx = \frac{\sqrt{d}\, d\big(3asin(cx)\, a^2 - 8\sqrt{-c^2x^2+1}\, a^2c^5x^5 + 14\sqrt{-c^2x^2+1}\, a^2c^3x^3 - 3\sqrt{-c^2x^2+1}\, a^2}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a**2 - 8*sqrt( - c**2*x**2 + 1)*a**2*c**5*x**5 + 1
4*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 - 3*sqrt( - c**2*x**2 + 1)*a**2*c*
x - 96*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*a*b*c**5 + 96*int(sqr
t( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*a*b*c**3 - 48*int(sqrt( - c**2*x**2
 + 1)*acosh(c*x)**2*x**4,x)*b**2*c**5 + 48*int(sqrt( - c**2*x**2 + 1)*acos
h(c*x)**2*x**2,x)*b**2*c**3))/(48*c**3)
```

3.169 $\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 295

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx=-\frac{16b^2d\sqrt{d-c^2dx^2}}{75c^2}-\frac{8b^2d(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{225c^2}-\frac{2b^2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}}{125c^2}+\frac{2bdx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{15\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bc^3dx^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{25\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

output
```
-16/75*b^2*d*(-c^2*d*x^2+d)^(1/2)/c^2-8/225*b^2*d*(-c*x+1)*(c*x+1)*(-c^2*d
*x^2+d)^(1/2)/c^2-2/125*b^2*d*(-c*x+1)^2*(c*x+1)^2*(-c^2*d*x^2+d)^(1/2)/c^
2+2/5*b*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1
)^(1/2)-4/15*b*c*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+2/25*b*c^3*d*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/
c^2/d
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.71

$$\int x(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\left(225a^2(-1+c^2x^2)^3-30abcx\sqrt{-1+cx}\sqrt{1+cx}(15-10c^2x^2+3c^4x^4)+2b^2(-149+187c\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
-1/1125*(d*Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(-149 + 187
*c^2*x^2 - 47*c^4*x^4 + 9*c^6*x^6) - 30*b*(-15*a*(-1 + c^2*x^2)^3 + b*c*x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4))*ArcCosh[c*x] +
 225*b^2*(-1 + c^2*x^2)^3*ArcCosh[c*x]^2))/(c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6329, 6304, 6309, 27, 1905, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{6329}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\int(1-cx)^2(cx+1)^2(a+b\operatorname{arccosh}(cx))dx}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{6304}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\int(1-c^2x^2)^2(a+b\operatorname{arccosh}(cx))dx}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{6309}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{15\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arcc}\right.}{5c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-\frac{1}{15}bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{a}\right.}{5c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{1905}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{c^2x^2-1}}dx}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+\right.}{5c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{1576}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{c^2x^2-1}}dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\right.}{5c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{1140}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int \left(3(c^2x^2-1)^{3/2}-4\sqrt{c^2x^2-1}+\frac{8}{\sqrt{c^2x^2-1}}\right)dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arcc}\right.}{5c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2x^2-\cdots}{5c\cdots}\right.}{\cdots}\right.}{5c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

input
```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/(c^2*d) + (2*b*d*Sqrt[
d - c^2*d*x^2]*(-1/30*(b*c*Sqrt[-1 + c^2*x^2]*((16*Sqrt[-1 + c^2*x^2])/c^2
 - (8*(-1 + c^2*x^2)^(3/2))/(3*c^2) + (6*(-1 + c^2*x^2)^(5/2))/(5*c^2)))/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + x*(a + b*ArcCosh[c*x]) - (2*c^2*x^3*(a + b
*ArcCosh[c*x]))/3 + (c^4*x^5*(a + b*ArcCosh[c*x]))/5))/(5*c*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(255) = 510$.

Time = 0.77 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.75

method	result
orering	$\frac{(549c^8x^8-1982c^6x^6+4355c^4x^4-1420c^2x^2+298)(-c^2dx^2+d)^{\frac{3}{2}}(a+b\operatorname{arccosh}(cx))^2}{1125c^4x^2(c^2x^2-1)^2} - \frac{2(54c^6x^6-217c^4x^4+672c^2x^2-149)\left((-\right.}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/1125*(549*c^8*x^8-1982*c^6*x^6+4355*c^4*x^4-1420*c^2*x^2+298)/c^4/x^2/(c
^2*x^2-1)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-2/1125*(54*c^6*x^6-2
17*c^4*x^4+672*c^2*x^2-149)/c^4/x^2/(c^2*x^2-1)*((-c^2*d*x^2+d)^(3/2)*(a+b
*arccosh(c*x))^2-3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*c^2*d+2*x
*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))+
1/1125*(9*c^4*x^4-38*c^2*x^2+149)/c^4/x*(-9*c^2*d*x*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccosh(c*x))^2+4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))*b*c/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+3*x^3/(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*c^4*d^
2-12*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+2*x*(-c^2*d*x^2+d)^(3/2)*b^2*c^2/(c*x-1)/(c*x+1)-x*(-c^2*d*x^2+
d)^(3/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(3/2)/(c*x+1)^(1/2)-x*(-c^2*d*x^
2+d)^(3/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(1/2)/(c*x+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.24

$$\int x(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2\,dx =$$

$$-\frac{225\,(b^2c^6dx^6-3\,b^2c^4dx^4+3\,b^2c^2dx^2-b^2d)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\,(3\,abc^5dx^5-10\,abc}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
-1/1125*(225*(b^2*c^6*d*x^6 - 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 - b^2*d)*s
qrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(3*a*b*c^5*d*x^5 -
 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) -
 30*((3*b^2*c^5*d*x^5 - 10*b^2*c^3*d*x^3 + 15*b^2*c*d*x)*sqrt(-c^2*d*x^2 +
 d)*sqrt(c^2*x^2 - 1) - 15*(a*b*c^6*d*x^6 - 3*a*b*c^4*d*x^4 + 3*a*b*c^2*d*
x^2 - a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (9*(25*a
^2 + 2*b^2)*c^6*d*x^6 - (675*a^2 + 94*b^2)*c^4*d*x^4 + (675*a^2 + 374*b^2)
*c^2*d*x^2 - (225*a^2 + 298*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx=\int x(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))^2\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.94

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx=-\frac{(-c^2dx^2+d)^{\frac{5}{2}}b^2\operatorname{arcosh}(cx)^2}{5c^2d} -\frac{2}{1125}b^2\left(\frac{9\sqrt{c^2x^2-1}c^2\sqrt{-d}d^2x^4-38\sqrt{c^2x^2-1}\sqrt{-d}d^2x^2+\frac{149\sqrt{c^2x^2-1}\sqrt{-d}d^2}{c^2}}{d}-\frac{15\left(3c^4\sqrt{-d}d^2x^5-10\right.}{}\right. -\frac{2(-c^2dx^2+d)^{\frac{5}{2}}ab\operatorname{arcosh}(cx)}{5c^2d}-\frac{(-c^2dx^2+d)^{\frac{5}{2}}a^2}{5c^2d} +\frac{2\left(3c^4\sqrt{-d}d^2x^5-10c^2\sqrt{-d}d^2x^3+15\sqrt{-d}d^2x\right)ab}{75cd}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-1/5*(-c^2*d*x^2 + d)^(5/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/1125*b^2*((9*sq
rt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^2*x^4 - 38*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2*x
^2 + 149*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2/c^2)/d - 15*(3*c^4*sqrt(-d)*d^2*x^
5 - 10*c^2*sqrt(-d)*d^2*x^3 + 15*sqrt(-d)*d^2*x)*arccosh(c*x)/(c*d)) - 2/5
*(-c^2*d*x^2 + d)^(5/2)*a*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(5
/2)*a^2/(c^2*d) + 2/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 +
 15*sqrt(-d)*d^2*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2\,dx = \int x\,(a+b\,\text{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx=\frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a^2c^4x^4+2\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a^2-10\left(\int\sqrt{-c^2x^2\right.\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 + 2*sqrt( - c**2*x**2
 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2 - 10*int(sqrt( - c**2*x
**2 + 1)*acosh(c*x)*x**3,x)*a*b*c**4 + 10*int(sqrt( - c**2*x**2 + 1)*acosh
(c*x)*x,x)*a*b*c**2 - 5*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**3,x)*b
**2*c**4 + 5*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x,x)*b**2*c**2))/(5*
c**2)
```

3.170 $\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 324

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2 \, dx = \frac{15}{64}b^2dx\sqrt{d-c^2dx^2}$$
$$+\frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}+\frac{9b^2d\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{64c\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{3bcdx^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{8\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{bd(-1+cx)^{3/2}(1+cx)^{3/2}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{8c}$$
$$+\frac{3}{8}dx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx}{8bc\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
15/64*b^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/32*b^2*d*x*(-c*x+1)*(c*x+1)*(-c^2*d*x
^2+d)^(1/2)+9/64*b^2*d*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/c/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-3/8*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)
^(1/2)/(c*x+1)^(1/2)+1/8*b*d*(c*x-1)^(3/2)*(c*x+1)^(3/2)*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccosh(c*x))/c+3/8*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2
+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-1/8*d*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccosh(c*x))^3/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.83 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.15

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx = \frac{-96a^2cdx\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(-5+2c^2x^2\right)\sqrt{d-c^2dx^2}-288a^2d^{3/2}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\,\text{arc}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(-96*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-5 + 2*c^2*x^2)*Sqrt[
d - c^2*d*x^2] - 288*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcT
an[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 192*a*b*d*Sqrt[d
- c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2
*ArcCosh[c*x]])) - 32*b^2*d*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcC
osh[c*x]*Cosh[2*ArcCosh[c*x]] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*
x]]) + 12*a*b*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x
]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b^2*d*Sqrt[d - c^2*d*x^2]*(32*
ArcCosh[c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c
*x]^2)*Sinh[4*ArcCosh[c*x]]))/(768*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6312, 25, 6310, 6298, 101, 43, 6308, 6327, 6329, 40, 40, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \; 6312$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int -x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+ b\text{arccosh}(cx))^2dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+ b\text{arccosh}(cx))^2dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6310}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arcco}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6298}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{101}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arcc}}{\sqrt{cx-}}}{2\sqrt{cx-1}\sqrt{cx}}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{43}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arcco}\right.}{\sqrt{}}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\downarrow 6308$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+$$
$$b\text{arccosh}(cx))^2+$$
$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}\right.}{\sqrt{c}}\right.$$

$$\downarrow 6327$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+$$
$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}\right.}{\sqrt{c}}\right.$$

$$\downarrow 6329$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\int (cx-1)^{3/2}(cx+1)^{3/2}dx}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+$$
$$b\text{arccosh}(cx))^2+$$
$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}\right.}{\sqrt{c}}\right.$$

$$\downarrow 40$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+$$
$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}\right.}{\sqrt{c}}\right.$$

$$\downarrow 40$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+$$

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+$$

$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}\right.}{\sqrt{c}}\right.$$

$$\Big\downarrow \text{ 43}$$

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right)\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+$$

$$\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}\right.}{\sqrt{c}}\right.$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*
x])^3)/(6*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*Sqrt[d - c^2*d*x^2]*((x
^2*(a + b*ArcCosh[c*x]))/2 - (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2
) + ArcCosh[c*x]/(2*c^3)))/2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/4 - (b*c*d
*Sqrt[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/c^2 + (b
*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6312
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p
)]   Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && GtQ[p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(276) = 552$.

Time = 0.00 (sec) , antiderivative size = 1061, normalized size of antiderivative = 3.27

method	result	size
default	Expression too large to display	1061
parts	Expression too large to display	1061

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)+3/8*a^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a^2*d^
2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/8*(-d
*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^3*d-1/512*(
-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1
)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)*d/(c*x-1)/(c*x+1)/c+1/16*(-d*(c
^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c
*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*arccosh(c*x)+1)*d/(c*x-1)/(
c*x+1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x
^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccos
h(c*x)+1)*d/(c*x-1)/(c*x+1)/c-1/512*(-d*(c^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*
x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-1
2*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(8*arccosh(c*x)^2+4*arccosh(c
*x)+1)*d/(c*x-1)/(c*x+1)/c)+2*a*b*(-3/16*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x
^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d/(
c*x-1)/(c*x+1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))
*d/(c*x-1)/(c*x+1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x...
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2
+ 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2\,dx=\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{3}{2}} \left(a+b\operatorname{acosh}\left(cx\right)\right)^2\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*a
rcsin(c*x)/c)*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx=\int \left(a+b\,\text{acosh}(c\,x)\right)^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2\,dx=\frac{\sqrt{d}\,d\left(3asin(cx)\,a^2-2\sqrt{-c^2x^2+1}\,a^2c^3x^3+5\sqrt{-c^2x^2+1}\,a^2cx-16\left(\int\sqrt{-c^2x^2+1}\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 + 5
*sqrt( - c**2*x**2 + 1)*a**2*c*x - 16*int(sqrt( - c**2*x**2 + 1)*acosh(c*x
)*x**2,x)*a*b*c**3 + 16*int(sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*a*b*c - 8
*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt(
- c**2*x**2 + 1)*acosh(c*x)**2,x)*b**2*c))/(8*c)
```

3.171 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\mathbf{arccosh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 573

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}\,dx &= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} \\
&- \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{2b^2cdx\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcdx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{2bc^3dx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{9\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2 + \frac{1}{3}(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2 \\
&- \frac{2d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\arctan\left(e^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{2ibd\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{2ibd\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{2ib^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{2ib^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,ie^{\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
68/27*b^2*d*(-c^2*d*x^2+d)^(1/2)-2/27*b^2*c^2*d*x^2*(-c^2*d*x^2+d)^(1/2)-2
*a*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b^2*c*d*x*(-
c^2*d*x^2+d)^(1/2)*arccosh(c*x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/3*b*c*d*x*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/9*b*c^
3*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2+1/3*(-c^2*d*x^2+d)^(3/2)*(a+
b*arccosh(c*x))^2-2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*arctan(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b*d*(-c^2*d*
x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-2*I*b^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*d*(-c^2*d*x^2+d)^(1/2)*p
olylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.04 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.13

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}\,dx =$$

$$-\frac{1}{3}a^2d(-4+c^2x^2)\sqrt{d-c^2dx^2}-\frac{1}{54}b^2d\sqrt{d-c^2dx^2}\left(2(-13+\cosh(2\text{arccosh}(cx)))\right.$$
$$+9\text{arccosh}(cx)^2(-1+\cosh(2\text{arccosh}(cx)))$$
$$\left.+\frac{3\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)(9cx-\cosh(3\text{arccosh}(cx)))}{-1+cx}\right)$$
$$-\frac{abd\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\text{arccosh}(cx)-\cosh(3\text{arccosh}(cx))\right)}{18\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$
$$+a^2d^{3/2}\log(cx)-a^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)+\frac{2abd\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\text{arccosh}(cx)+cx\sqrt{\frac{-1}{1+}}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

output
```
-1/3*(a^2*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (b^2*d*Sqrt[d - c^2*d*x^
2]*(2*(-13 + Cosh[2*ArcCosh[c*x]]) + 9*ArcCosh[c*x]^2*(-1 + Cosh[2*ArcCosh
[c*x]]) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*(9*c*x - Cosh[3*ArcCo
sh[c*x]]))/(-1 + c*x)))/54 - (a*b*d*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 +
 c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(
18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3
/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d*Sqrt[d - c^2*d*x^2]*(-
(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh
[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*Po
lyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*
d*Sqrt[d - c^2*d*x^2]*(2 + (2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])
/(1 - c*x) + ArcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]]
 - ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-
I)/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyL
og[3, (-I)/E^ArcCosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)))
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.69, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {6345, 25, 6304, 6309, 27, 960, 83, 6341, 2009, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx$$

$$\Big\downarrow \text{6345}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int -((1-cx)(cx+1)(a+b\text{arccosh}(cx)))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+ d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\int(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 6304

$$-\frac{2bcd\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 6309

$$d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-bc\int\frac{x(3-c^2x^2)}{3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 27

$$d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\int\frac{x(3-c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 960

$$d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\left(\frac{7}{3}\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 83

$$d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6341

$$d\left(-\frac{2bc\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}(cx\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 6362}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bxa\right.}{\sqrt{cx-}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 4668}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2ib\int(a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int(a+b\text{arccosh}(cx))\log\left(1-\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 3011

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx\right.\right.\right.}{}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 2720

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{a}\right.\right.}{}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 7143

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(2,-ie\right.\right.\right.}{\sqrt{cx-}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{7\sqrt{cx-1}\sqrt{cx+1}}{3c^2}-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

output
```
((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/3 - (2*b*c*d*Sqrt[d - c^2*d
*x^2]*(-1/3*(b*c*((7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) - (x^2*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/3)) + x*(a + b*ArcCosh[c*x]) - (c^2*x^3*(a + b*ArcCo
sh[c*x]))/3))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*(Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x - (b*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (S
qrt[d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]] + (2*I
)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) + b*PolyLog[3
, (-I)*E^ArcCosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, I*E^A
rcCosh[c*x]]) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(-c^2d\,x^2+d)^{\frac{3}{2}} (a+b\,\mathrm{arccosh}\,(cx))^2}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2} (a+b\mathrm{arccosh}(cx))^2}{x}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas
")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2
+ 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2} (a+b\mathrm{arccosh}(cx))^2}{x}\,dx = \int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^2}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima
")
```

output
```
-1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
(-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a^2 + integrate((-c^2*d
*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x + 2*(-c^2*d
*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\mathrm{arccosh}(cx)\right)^2}{x}\,dx=\frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a^2c^2x^2+4\sqrt{-c^2x^2+1}\,a^2+6\left(\int\frac{\sqrt{-c^2x^2+1}}{x}\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2/x,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 + 4*sqrt( - c**2*x**2
 + 1)*a**2 + 6*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x,x)*a*b + 3*int((s
qrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x,x)*b**2 - 6*int(sqrt( - c**2*x**2 +
 1)*acosh(c*x)*x,x)*a*b*c**2 - 3*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*
x,x)*b**2*c**2 + 3*log(tan(asin(c*x)/2))*a**2 - 4*a**2))/3
```

3.172 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 444

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx &= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2}\\
&-\frac{5b^2cd\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{4\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3bc^3dx^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2\sqrt{-1+cx}\sqrt{1+cx}}\\
&-bcd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\\
&-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{cd\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}+\frac{cd\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{2b\sqrt{-1+cx}\sqrt{1+cx}}\\
&+\frac{2bcd\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\log\left(1+e^{2\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&+\frac{b^2cd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/4*b^2*c^2*d*x*(-c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*(-c^2*d*x^2+d)^(1/2)*arc
cosh(c*x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/2*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*d*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-3/2*c^2*d*x*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccosh(c*x))^2-c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x+1/
2*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/(c*x-1)^(1/2)/(c*x+1)^(1
/2)+2*b*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b^2*c*d*(-c^2*d*x^2+d)^(1/
2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.09 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.98

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx = \frac{-12a^2d\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(2+c^2x^2)\sqrt{d-c^2dx^2}+36a^2cd^{3/2}x\sqrt{}}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

output
```
(-12*a^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2)*Sqrt[d - c^2
*d*x^2] + 36*a^2*c*d^(3/2)*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(
c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 24*a*b*d*Sqrt[d - c^2
*d*x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCos
h[c*x]^2 + 2*Log[c*x])) - 8*b^2*d*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*(3*Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]*(3 + Ar
cCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*PolyLog[2, -E^(-2*A
rcCosh[c*x])]) + 6*a*b*c*d*x*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2
*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + b^2*c*d*x*Sqrt[d -
c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] - 3*(1
+ 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/(24*x*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.89, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {6343, 25, 6310, 6298, 101, 43, 6308, 6327, 6334, 40, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{6343}$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{25}$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{6310}$$

$$-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+\text{bar}\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{6298}$$

$$-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{101}$$

$$-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b}{\sqrt{c}}}{2\sqrt{cx-1}\sqrt{}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{43}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arcc}\right.}{}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{6308}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \text{6327}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \text{6334}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))\right)}{\sqrt{cx-1}\sqrt{cx+1}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \; 40$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arcc}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \; 43$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arccosh}(cx)\right)+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(cx)}{2c}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \; 6297$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arccosh}(cx\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\downarrow 3042$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(c\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\downarrow 26$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{i\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\downarrow 4201$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\int\frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+\text{barc}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \text{2620}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arcc}\right.}{b}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \text{2715}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\right.}{b}\right.}{\sqrt{cx-1}\sqrt{cx+}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccos}\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \text{2838}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\operatorname{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\operatorname{arccosh}(cx)}+1\right)(a+b\operatorname{arccosh}(cx))\right)-\frac{1}{2}i(a+b\operatorname{arccosh}(cx))^2\right)}{b}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))^2}{x}-$$

$$3c^2d\left(-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\operatorname{arccos}\right.}{\sqrt{}}\right.$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x) - 3*c^2*d*((x*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^3)/(6*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*Sqrt[d - c^2*d*x^2]
*((x^2*(a + b*ArcCosh[c*x]))/2 - (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2
*c^2) + ArcCosh[c*x]/(2*c^3)))/2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (2*b*
c*d*Sqrt[d - c^2*d*x^2]*(((1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 + (b*c*((x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/2 + (I*((-1/2*I)*(
a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*
ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6334
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Simp[b*c*((-d
)^p/(2*p))   Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.05

method	result
default	$-\frac{a^2(-c^2 d x^2+d)^{\frac{5}{2}}}{dx} - a^2c^2x(-c^2dx^2+d)^{\frac{3}{2}} - \frac{3a^2c^2dx\sqrt{-c^2dx^2+d}}{2} - \frac{3a^2c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + \frac{b^2\sqrt{-d(c^2}}{}$
parts	$-\frac{a^2(-c^2 d x^2+d)^{\frac{5}{2}}}{dx} - a^2c^2x(-c^2dx^2+d)^{\frac{3}{2}} - \frac{3a^2c^2dx\sqrt{-c^2dx^2+d}}{2} - \frac{3a^2c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + \frac{b^2\sqrt{-d(c^2}}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(5/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a^2*c^2*d
*x*(-c^2*d*x^2+d)^(1/2)-3/2*a^2*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)/x*(-2*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x
^3*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*arccosh(c*x)^3*c*x-4
*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-4*arccosh(c*x)^2*c*x+8*arccosh
(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c-c*x*arccosh(c*x)+4*pol
ylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c)*d+1/4*a*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x*(-4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*c^2*x^2+2*c^3*x^3+6*arccosh(c*x)^2*c*x-8*arccosh(c*x)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)-8*c*x*arccosh(c*x)+8*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)*x*c-c*x)*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="fric
as")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2
+ 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\text{acosh}\,(cx))^2}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxi
ma")
```

output
```
-1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)/x)*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx = \frac{\sqrt{d}\,d\Big(-3asin(cx)\,a^2cx-\sqrt{-c^2x^2+1}\,a^2c^2x^2-2\sqrt{-c^2x^2+1}\,a$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2/x^2,x)
```

output
```
(sqrt(d)*d*( - 3*asin(c*x)*a**2*c*x - sqrt( - c**2*x**2 + 1)*a**2*c**2*x**
2 - 2*sqrt( - c**2*x**2 + 1)*a**2 + 4*int((sqrt( - c**2*x**2 + 1)*acosh(c*
x))/x**2,x)*a*b*x + 2*int((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x**2,x)*b
**2*x - 4*int(sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*a*b*c**2*x - 2*int(sqrt
( - c**2*x**2 + 1)*acosh(c*x)**2,x)*b**2*c**2*x))/(2*x)
```

3.173 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 630

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2}{x^3}\,dx = -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\operatorname{arccosh}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3dx\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2 - \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2}{2x^2} + \frac{3c^2d\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b^2c^2d\sqrt{d-c^2dx^2}\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3ibc^2d\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3ibc^2d\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3ib^2c^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3ib^2c^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-2*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)+3*a*b*c^3*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)+3*b^2*c^3*d*x*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-b*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^3*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x
))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh
(c*x))^2-1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2+3*c^2*d*(-c^2*d
*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)*arctan((c*x-1)
^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*I*b*c^2*d*(-c^2*d*x^2+
d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+3*I*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*I*b^2*c^2*d*(-c^2*d*x^2+d)
^(1/2)*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5444 vs. $2(630) = 1260$.

Time = 62.10 (sec) , antiderivative size = 5444, normalized size of antiderivative = 8.64

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.61, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.552, Rules used = {6343, 25, 6327, 6336, 25, 960, 103, 218, 6341, 2009, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2}{x^3} \, dx$$

$$\Big\downarrow \text{6343}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6327}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x^2}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6336}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\left(-bc\int -\frac{c^2x^2+1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(bc\int \frac{c^2x^2+1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\big\downarrow$ 960

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(bc\left(\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\sqrt{cx-1}\sqrt{cx+1}\right)+c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\big\downarrow$ 103

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(bc\left(c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)+c^2(-x)(a+b\text{arccosh}(cx))-\frac{a}{}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\big\downarrow$ 218

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\big\downarrow$ 6341

$$-\frac{3}{2}c^2d\left(-\frac{2bc\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arcco}\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 2009

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arcco}\right.}{\sqrt{cx-1}\sqrt{c}}\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 6362

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax\right.}{\sqrt{}}\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 3042

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(c\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 4668

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2ib\int(a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int(a+b\text{arccosh}(cx))\text{lo}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 3011

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccos}\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 2720

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 7143

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(\right.\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+bc\left(\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

output
```
-1/2*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2 + (b*c*d*Sqrt[d -
c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/x) - c^2*x*(a + b*ArcCosh[c*x]) + b*c*(
Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])))/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])/c + b*x*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*
d*x^2]*(2*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]] + (2*I)*b*(-((a +
b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) + b*PolyLog[3, (-I)*E^Arc
Cosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]]
) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(-c^2 d\,x^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccosh}(cx))^2}{x^3} dx$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b\text{arccosh}(cx))^2}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \operatorname{arcosh}(cx) + a)^2}{x^3} dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="fric
as")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2
+ 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b\text{arccosh}(cx))^2}{x^3} dx = \int \frac{(-d(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \operatorname{acosh}(cx))^2}{x^3} dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**3,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxi
ma")
```

output
```
1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2
 + d)^(5/2)/(d*x^2))*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}{x^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx=\frac{\sqrt{d}\,d\Big(-8\sqrt{-c^2x^2+1}\,a^2c^2x^2-4\sqrt{-c^2x^2+1}\,a^2+16\Big(\int \frac{\sqrt{-c^2x^2}}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2/x^3,x)
```

output
```
(sqrt(d)*d*( - 8*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 4*sqrt( - c**2*x*
*2 + 1)*a**2 + 16*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**3,x)*a*b*x**2
 - 16*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x,x)*a*b*c**2*x**2 + 8*int((
sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x**3,x)*b**2*x**2 - 8*int((sqrt( - c
**2*x**2 + 1)*acosh(c*x)**2)/x,x)*b**2*c**2*x**2 - 12*log(tan(asin(c*x)/2)
)*a**2*c**2*x**2 + 9*a**2*c**2*x**2))/(8*x**2)
```

3.174 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 416

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\,\mathrm{arccosh}(cx))^2}{x^4}\,dx &= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} \\
&- \frac{b^2c^3d\sqrt{d-c^2dx^2}\,\mathrm{arccosh}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{3x^2} \\
&+ \frac{c^2d\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2}{x} + \frac{4c^3d\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2}{3\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{(d-c^2dx^2)^{3/2}\,(a+b\,\mathrm{arccosh}(cx))^2}{3x^3} - \frac{c^3d\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^3}{3b\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{8bc^3d\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))\log\left(1+e^{2\mathrm{arccosh}(cx)}\right)}{3\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{4b^2c^3d\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{arccosh}(cx)}\right)}{3\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
1/3*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*d*(-c^2*d*x^2+d)^(1/2)*ar
ccosh(c*x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/3*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^2+c^2*d*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccosh(c*x))^2/x+4/3*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2
/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2
/x^3-1/3*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-8/3*b*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*ln(1+(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/3*b^2*c^3*d
*(-c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.77 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.40

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2}{x^4}\,dx = \frac{-abcd^2x+abc^2d^2x^2-a^2d^2\sqrt{\frac{-1+cx}{1+cx}}+5a^2c^2d^2x^2\sqrt{\frac{-1+cx}{1+cx}}+b^2c}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

output
```
(-(a*b*c*d^2*x) + a*b*c^2*d^2*x^2 - a^2*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 5
*a^2*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + b^2*c^2*d^2*x^2*Sqrt[(-1 + c
*x)/(1 + c*x)] - 4*a^2*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*c^4*d^
2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b*d^2*(-1 + c*x)*(-3*a*c^3*x^3 + b*(-Sq
rt[(-1 + c*x)/(1 + c*x)] - c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*c^2*x^2*Sqrt
[(-1 + c*x)/(1 + c*x)] + 4*c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)])))*Arc
Cosh[c*x]^2 + b^2*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 3*a^2*c^3*d^(3/2
)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d -
c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d^2*(-1 + c*x)*ArcCosh[c*x]*(b*c
*x + 2*a*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x - 4*c^2*x^2 - 4*c^3*x^3) + 8*
b*c^3*x^3*Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*b*c^3*d^2*x^3*Log[c*x] + 8*a
*b*c^4*d^2*x^4*Log[c*x] - 4*b^2*c^3*d^2*x^3*(-1 + c*x)*PolyLog[2, -E^(-2*A
rcCosh[c*x])])/(3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.68 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.97, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.862, Rules used = {6343, 25, 6327, 6335, 108, 27, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838, 6339, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{6343}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{2bcd\sqrt{d-c^2dx^2}\int -\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6327}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6335}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{1}{2}bc\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2}dx-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 108$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{1}{2}bc\left(\int \frac{c^2}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}} \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 27$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{1}{2}bc\left(c^2\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}} \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 43$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\text{arccosh}(cx)}{x}dx\right)-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bc\left(c\text{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}}{x}\right)\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 6297$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\right.}{3\sqrt{cx-1}\sqrt{cx+1}} \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 25$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{c^2\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}bc\left(c\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{c^2\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}b\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{26}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}-\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{2x^2}-\frac{1}{2}b\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{4201}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}-\frac{(1-c^2x^2)(a+b\text{arccos}}{2x^2}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2620}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+ba\right.}{b}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2715$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx)\right.}{b}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2838$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 6339$$

$$c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc\sqrt{d-c^2dx^2}\int \frac{a+b\text{arccosh}(cx)}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(\right.}{x}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 6297$$

$$\frac{c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2c\sqrt{d-c^2dx^2}\int-\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 25

$$\frac{c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d}{\sqrt{cx-1}\sqrt{cx+1}}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 3042

$$\frac{c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c\sqrt{d-c^2dx^2}\int-i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx)}{b}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 26

$$\frac{c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ic\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{\sqrt{cx-1}\sqrt{cx+1}}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 4201

$$c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ic\sqrt{d-c^2dx^2}\left(2i\int\frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2620

$$c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(c\right.\right.}{\sqrt{}}\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2715

$$c^2(-d)\left(\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2838

$$c^2(-d)\left(\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(\right.\right.\right.}{\sqrt{cx-1}}\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 6308

$$c^2(-d)\left(\frac{2ic\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{ic^2\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))\right.}{b}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3 + (2*b*c*d*Sqrt[d
 - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/x^2 - (b*c*(-((Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/x) + c*ArcCosh[c*x]))/2 - (I*c^2*((-1/2*I)*(a +
 b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*Arc
Cosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/b))/(3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - c^2*d*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
2)/x) + (c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + ((2*I)*c*Sqrt[d - c^2*d*x^2]*((-1/2*I)*(a + b*ArcCosh[c*
x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])
+ (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6335
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*((-d)^p/(f*(m + 1)))   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(
m + 1)/2, 0]
```

rule 6339
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.20

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx^3}+\frac{2a^2c^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx}+\frac{2a^2c^4x(-c^2dx^2+d)^{\frac{3}{2}}}{3}+a^2c^4dx\sqrt{-c^2dx^2+d}+\frac{a^2c^4d^2\arctan\left(\frac{\sqrt{}}{\sqrt{-c}}\right.}{\sqrt{c^2d}}$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx^3}+\frac{2a^2c^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx}+\frac{2a^2c^4x(-c^2dx^2+d)^{\frac{3}{2}}}{3}+a^2c^4dx\sqrt{-c^2dx^2+d}+\frac{a^2c^4d^2\arctan\left(\frac{\sqrt{}}{\sqrt{-c}}\right.}{\sqrt{c^2d}}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+2
/3*a^2*c^4*x*(-c^2*d*x^2+d)^(3/2)+a^2*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a^2*c^4
*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/3*b^2*(-
d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x^3*(arccosh(c*x)^3*x^3*c
^3-4*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-4*arccosh(c*x)^2*c
^3*x^3+8*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3+4*
polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3-(c*x-1)^(1/2)*(c*x
+1)^(1/2)*c^2*x^2+c^3*x^3+arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x*a
rccosh(c*x))*d-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/
x^3*(3*arccosh(c*x)^2*c^3*x^3-8*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c
^2*x^2-8*c^3*x^3*arccosh(c*x)+8*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*
x^3*c^3+2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="fric
as")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2
+ 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\text{acosh}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**4,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxi
ma")
```

output
```
1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*
d*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a^2 + integrate((
-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^4 + 2
*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \frac{\sqrt{d}\,d\Big(3asin(cx)\,a^2c^3x^3+4\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2/x^4,x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a**2*c**3*x**3 + 4*sqrt( - c**2*x**2 + 1)*a**2*c**
2*x**2 - sqrt( - c**2*x**2 + 1)*a**2 + 6*int((sqrt( - c**2*x**2 + 1)*acosh
(c*x))/x**4,x)*a*b*x**3 - 6*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**2,x
)*a*b*c**2*x**3 + 3*int((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x**4,x)*b**
2*x**3 - 3*int((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x**2,x)*b**2*c**2*x*
*3))/(3*x**3)
```

3.175 $\int x^3(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 731

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=-\frac{33464b^2d^2\sqrt{d-c^2dx^2}}{694575c^4}+\frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2}+\frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175}-\frac{10b^2c^2d^2x^6\sqrt{d-c^2dx^2}}{3087}+\frac{8b^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{8505c^4}+\frac{2b^2d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}}{4725c^4}-\frac{20b^2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}}{3969c^4}+\frac{2b^2d^2(1-cx)^4(1+cx)^4\sqrt{d-c^2dx^2}}{729c^4}+\frac{4bd^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{63c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^2x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{189c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{21\sqrt{-1+cx}\sqrt{1+cx}}+\frac{38bc^3d^2x^7\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{441\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bc^5d^2x^9\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{81\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{63c^4}-\frac{d^2x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{63c^2}+\frac{1}{21}d^2x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2+\frac{5}{63}dx^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

output
```
-33464/694575*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^4+3358/694575*b^2*d^2*x^2*(-c
^2*d*x^2+d)^(1/2)/c^2+484/77175*b^2*d^2*x^4*(-c^2*d*x^2+d)^(1/2)-10/3087*b
^2*c^2*d^2*x^6*(-c^2*d*x^2+d)^(1/2)+8/8505*b^2*d^2*(-c*x+1)*(c*x+1)*(-c^2*
d*x^2+d)^(1/2)/c^4+2/4725*b^2*d^2*(-c*x+1)^2*(c*x+1)^2*(-c^2*d*x^2+d)^(1/2
)/c^4-20/3969*b^2*d^2*(-c*x+1)^3*(c*x+1)^3*(-c^2*d*x^2+d)^(1/2)/c^4+2/729*
b^2*d^2*(-c*x+1)^4*(c*x+1)^4*(-c^2*d*x^2+d)^(1/2)/c^4+4/63*b*d^2*x*(-c^2*d
*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/189*b*d
^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^(1/
2)-2/21*b*c*d^2*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+38/441*b*c^3*d^2*x^7*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))
/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/81*b*c^5*d^2*x^9*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/63*d^2*(-c^2*d*x^2+d)^(1/2)*(a
+b*arccosh(c*x))^2/c^4-1/63*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)
)^2/c^2+1/21*d^2*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2+5/63*d*x^4*
(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/9*x^4*(-c^2*d*x^2+d)^(5/2)*(a+
b*arccosh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.39

$$\int x^3(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2\,dx=\frac{d^2\sqrt{d-c^2dx^2}\Big(3969a^2(-1+c^2x^2)^4(2+7c^2x^2)-126abcx\sqrt{-1+cx}\sqrt{1+cx}(-126+\cdots}{\cdots}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(3969*a^2*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2) - 126*
a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-126 - 21*c^2*x^2 + 189*c^4*x^4 - 17
1*c^6*x^6 + 49*c^8*x^8) + 2*b^2*(6140 - 7039*c^2*x^2 - 106*c^4*x^4 + 2152*
c^6*x^6 - 1490*c^8*x^8 + 343*c^10*x^10) + 126*b*(63*a*(-1 + c^2*x^2)^4*(2
+ 7*c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(126 + 21*c^2*x^2 - 189*
c^4*x^4 + 171*c^6*x^6 - 49*c^8*x^8))*ArcCosh[c*x] + 3969*b^2*(-1 + c^2*x^2
)^4*(2 + 7*c^2*x^2)*ArcCosh[c*x]^2))/(250047*c^4*(-1 + c^2*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6345}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int x^4(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+ b\text{arccosh}(cx))^2dx+\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 6327}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int x^4\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))dx}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+ b\text{arccosh}(cx))^2dx+\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 6336}$$

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-bc\int \frac{x^5\left(35c^4x^4-90c^2x^2+63\right)}{315\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{315}bc\int \frac{x^5\left(35c^4x^4-90c^2x^2+63\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 1905}$$

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int\frac{x^5\left(35c^4x^4-90c^2x^2+63\right)}{\sqrt{c^2x^2-1}}dx}{315\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}\ldots\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 1578

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int\frac{x^4\left(35c^4x^4-90c^2x^2+63\right)}{\sqrt{c^2x^2-1}}dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\ldots\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 1195

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int\left(\frac{35\left(c^2x^2-1\right)^{7/2}}{c^4}+\frac{50\left(c^2x^2-1\right)^{5/2}}{c^4}+\frac{3\left(c^2x^2-1\right)^{3/2}}{c^4}-\frac{4\sqrt{c^2x^2-1}}{c^4}+\frac{8}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{9}c^4x^9(a+b\ldots\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 2009

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\ldots\right.}{\ldots}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6345

$$\frac{5}{9}d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int -x^4(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(7\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 25

$$\frac{5}{9}d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int x^4(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(7\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6327

$$\frac{5}{9}d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int x^4\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{7}x\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(7\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6336

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-bc\int\frac{x^5(7-5c^2x^2)}{35\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^2x^7(a+b\text{arccos}\right.}{7\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{\ldots}\right.}{\ldots}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 27$$

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\int\frac{x^5(7-5c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^2x^7(a+b\text{arccos}\right.}{7\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{\ldots}\right.}{\ldots}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 960$$

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\int\frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{5}{7}x^6\sqrt{cx-1}\sqrt{\ldots}\right.\right.}{7\sqrt{cx-\ldots}}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{\ldots}\right.}{\ldots}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 111$$

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{\int\frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right.\right.\right.}{}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{}\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 27

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{4\int\frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+}}{5c^2}\right.\right.\right.}{}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{}\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 111

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{4\left(\frac{\int\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right.}{5c^2}\right.\right.\right.}{}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(70\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 27

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\left(\frac{19}{7}\left(\frac{4\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right.}{5c^2}\right.\right.\right.}{}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(70\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 83

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\right.}{}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{}\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6341

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{2bc\sqrt{d-c^2dx^2}\int x^4(a+b\text{arccosh}(cx))dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{}\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6298

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\int\frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}d\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{\ldots}\right.}{\ldots}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 111

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{\int\frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{\ldots}\right.}{\ldots}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 27

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}d\cdots}{5c^2}\cdots\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\cdots\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{9\sqrt{cx-1}\sqrt{cx+1}\cdots}$$

$$\Big\downarrow \; 111$$

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+\cdots}}}{3c^2}\cdots\right.}{\cdots}\cdots\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\cdots\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{9\sqrt{cx-1}\sqrt{cx+1}\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx}}}{3c^2}\right.}{}\right.\right.}{5\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(7\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 83

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a\right.}{}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\text{arccosh}(cx))-\frac{2}{7}c^2x^7(a+b\text{arccosh}(cx))+\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(7\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 6354

$$\frac{1}{9}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2x^4-$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4(a+b\text{arccosh}(cx))x^9-\frac{2}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(cx))x^5-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{-}\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{5}{9}d\left(\frac{1}{7}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2x^4-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(cx\right.}{}\right.$$

$\downarrow$ 6298

$$\frac{1}{9}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2x^4-$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4(a+b\text{arccosh}(cx))x^9-\frac{2}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(cx))x^5-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{-}\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{5}{9}d\left(\frac{1}{7}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2x^4-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(cx\right.}{}\right.$$

$\downarrow$ 111

$$\frac{1}{9}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2x^4-$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4(a+b\text{arccosh}(cx))x^9-\frac{2}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(cx))x^5-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70}{-}\right.}{}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{5}{9}d\left(\frac{1}{7}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2x^4-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(cx\right.}{}\right.$$

$$\downarrow 27$$

$$\frac{1}{9}(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2x^4 - \frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4(a+b\text{arccosh}(cx))x^9-\frac{2}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(cx))x^5-\frac{bc\sqrt{c^2x^2-1}\left(\frac{70\cdots}{\cdots}\right.}{\cdots}\right.}{9\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{5}{9}d\left(\frac{1}{7}(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2x^4-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2(a+b\text{arccosh}(cx))x^7+\frac{1}{5}(a+b\text{arccosh}(c\cdots\right.}{\cdots}\right.$$

input
```
Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.89

method	result
orering	$\frac{(74431x^{12}c^{12}-287825c^{10}x^{10}+383719c^8x^8-112267c^6x^6-485758c^4x^4+324980c^2x^2-73680)(-c^2dx^2+d)^{\frac{5}{2}}(a+b\,\text{arccosh}(cx))^2}{250047c^6(cx-1)(cx+1)x^2(c^2x^2-1)^2} - \cdots$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/250047*(74431*c^12*x^12-287825*c^10*x^10+383719*c^8*x^8-112267*c^6*x^6-4
85758*c^4*x^4+324980*c^2*x^2-73680)/c^6/(c*x-1)/(c*x+1)/x^2/(c^2*x^2-1)^2*
(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2-4/83349*(686*c^10*x^10-2503*c^8*
x^8+2822*c^6*x^6+511*c^4*x^4-8786*c^2*x^2+3070)/c^6/(c*x-1)/(c*x+1)/x^4/(c
^2*x^2-1)*(3*x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2-5*x^4*(-c^2*d*x
^2+d)^(3/2)*(a+b*arccosh(c*x))^2*c^2*d+2*x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arc
cosh(c*x))*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))+1/250047*(343*c^8*x^8-1147*c^6
*x^6+1005*c^4*x^4+899*c^2*x^2-6140)/c^6/(c*x-1)/(c*x+1)/x^3*(6*x*(-c^2*d*x
^2+d)^(5/2)*(a+b*arccosh(c*x))^2-35*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(
c*x))^2*c^2*d+12*x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))*b*c/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+15*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*c^4*d^
2-20*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))*c^3*d*b/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+2*x^3*(-c^2*d*x^2+d)^(5/2)*b^2*c^2/(c*x-1)/(c*x+1)-x^3*(-c^2*d*
x^2+d)^(5/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(3/2)/(c*x+1)^(1/2)-x^3*(-c^
2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1)^(1/2)/(c*x+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.76

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} (a+b\operatorname{arccosh}(cx))^2\,dx = \frac{3969\,(7\,b^2c^{10}d^2x^{10} - 26\,b^2c^8d^2x^8 + 34\,b^2c^6d^2x^6 - 16\,b^2c^4d^2x^4 - b^2c^2d^2x^2 + 2\,b^2d^2)\sqrt{-}}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
1/250047*(3969*(7*b^2*c^10*d^2*x^10 - 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*
x^6 - 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 + 2*b^2*d^2)*sqrt(-c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 126*(49*a*b*c^9*d^2*x^9 - 171*a*b*c^7*
d^2*x^7 + 189*a*b*c^5*d^2*x^5 - 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt
(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 126*((49*b^2*c^9*d^2*x^9 - 171*b^2*c^
7*d^2*x^7 + 189*b^2*c^5*d^2*x^5 - 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sq
rt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 63*(7*a*b*c^10*d^2*x^10 - 26*a*b*c^
8*d^2*x^8 + 34*a*b*c^6*d^2*x^6 - 16*a*b*c^4*d^2*x^4 - a*b*c^2*d^2*x^2 + 2*
a*b*d^2)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (343*(81*a^2
 + 2*b^2)*c^10*d^2*x^10 - 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*
a^2 + 2152*b^2)*c^6*d^2*x^6 - 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a
^2 + 14078*b^2)*c^2*d^2*x^2 + 2*(3969*a^2 + 6140*b^2)*d^2)*sqrt(-c^2*d*x^2
 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b\text{arccosh}(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.64

$$\int x^3 \left(d - c^2 d x^2\right)^{5/2} (a + b\text{arccosh}(cx))^2\, dx =$$
$$-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b^2\operatorname{arcosh}\left(cx\right)^2$$
$$-\frac{2}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)ab\operatorname{arcosh}\left(cx\right)$$
$$-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)a^2$$
$$+\frac{2}{250047}b^2\left(\frac{343\sqrt{c^2x^2-1}c^6\sqrt{-d}d^2x^8-1147\sqrt{c^2x^2-1}c^4\sqrt{-d}d^2x^6+1005\sqrt{c^2x^2-1}c^2\sqrt{-d}d^2x^4+899}{c^2}\right.$$
$$-\frac{2\left(49\,c^8\sqrt{-d}d^2x^9-171\,c^6\sqrt{-d}d^2x^7+189\,c^4\sqrt{-d}d^2x^5-21\,c^2\sqrt{-d}d^2x^3-126\sqrt{-d}d^2x\right)ab}{3969\,c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
-1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^
4*d))*b^2*arccosh(c*x)^2 - 2/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*
(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a*b*arccosh(c*x) - 1/63*(7*(-c^2*d*x^2 + d
)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 + 2/250047*b^2
*((343*sqrt(c^2*x^2 - 1)*c^6*sqrt(-d)*d^2*x^8 - 1147*sqrt(c^2*x^2 - 1)*c^4
*sqrt(-d)*d^2*x^6 + 1005*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^2*x^4 + 899*sqrt
(c^2*x^2 - 1)*sqrt(-d)*d^2*x^2 - 6140*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2/c^2)/
c^2 - 63*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqrt(-d)*d^2*x^7 + 189*c^4*sqr
t(-d)*d^2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sqrt(-d)*d^2*x)*arccosh(c*x)
/c^3) - 2/3969*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqrt(-d)*d^2*x^7 + 189*c
^4*sqrt(-d)*d^2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sqrt(-d)*d^2*x)*a*b/c^
3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2\,dx = \int x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} (a +b\text{arccosh}(cx))^2\,dx = \frac{\sqrt{d}\,d^2\left(7\sqrt{-c^2x^2+1}\,a^2c^8x^8-19\sqrt{-c^2x^2+1}\,a^2c^6x^6+15\sqrt{-c^2x^2+1}\,a^2c^4x^4-\sqrt{-c^2\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2,x)
```

output

```
(sqrt(d)*d**2*(7*sqrt( - c**2*x**2 + 1)*a**2*c**8*x**8 - 19*sqrt( - c**2*x
**2 + 1)*a**2*c**6*x**6 + 15*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - sqrt(
 - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a**2 + 126*int
(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**7,x)*a*b*c**8 - 252*int(sqrt( - c**2
*x**2 + 1)*acosh(c*x)*x**5,x)*a*b*c**6 + 126*int(sqrt( - c**2*x**2 + 1)*ac
osh(c*x)*x**3,x)*a*b*c**4 + 63*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x*
*7,x)*b**2*c**8 - 126*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**5,x)*b**
2*c**6 + 63*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**3,x)*b**2*c**4))/(
63*c**4)
```

3.176 $\int x^2(d-c^2dx^2)^{5/2}(a+b\textbf{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 653

$$\begin{aligned}\int x^2(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2\,dx &= \frac{359b^2d^2x\sqrt{d-c^2dx^2}}{36864c^2}\\
&+\frac{1079b^2d^2x^3\sqrt{d-c^2dx^2}}{55296}-\frac{209b^2c^2d^2x^5\sqrt{d-c^2dx^2}}{13824}+\frac{1}{256}b^2c^4d^2x^7\sqrt{d-c^2dx^2}\\
&+\frac{35b^2d^2\sqrt{d-c^2dx^2}\operatorname{arccosh}(cx)}{9216c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5bd^2x^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{128c\sqrt{-1+cx}\sqrt{1+cx}}\\
&-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{384\sqrt{-1+cx}\sqrt{1+cx}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{144\sqrt{-1+cx}\sqrt{1+cx}}\\
&-\frac{bc^5d^2x^8\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{32\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5d^2x\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{128c^2}\\
&+\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2+\frac{5}{48}dx^3(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2\\
&+\frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2-\frac{5d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^3}{384bc^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{73b^2d^2\sqrt{-1+c^2x^2}\sqrt{d-c^2 \cdots}}{12288c^3(1-cx \cdots}\end{aligned}$$

output
```
359/36864*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^2+1079/55296*b^2*d^2*x^3*(-c^2*
d*x^2+d)^(1/2)-209/13824*b^2*c^2*d^2*x^5*(-c^2*d*x^2+d)^(1/2)+1/256*b^2*c^
4*d^2*x^7*(-c^2*d*x^2+d)^(1/2)+35/9216*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*arccos
h(c*x)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/128*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/384*b*c*d^2*x^4*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/144*b*c
^3*d^2*x^6*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-1/32*b*c^5*d^2*x^8*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-5/128*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^
2+5/64*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2+5/48*d*x^3*(-c^2*
d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/8*x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcc
osh(c*x))^2-5/384*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/c^3/(c*x
-1)^(1/2)/(c*x+1)^(1/2)-73/12288*b^2*d^2*(c^2*x^2-1)^(1/2)*(-c^2*d*x^2+d)^
(1/2)*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^3/(-c*x+1)/(c*x+1)
```

Mathematica [A] (warning: unable to verify)

Time = 5.76 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.39

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx =$$

$$-\frac{d^2\left(34560a^2cx\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}+34560a^2c^2x^2\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}-271872a^2c^3x^3\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx}\right.}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
-1/884736*(d^2*(34560*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^
2] + 34560*a^2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 27
1872*a^2*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 271872*a
^2*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^5
*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^6*x^6*S
qrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^7*x^7*Sqrt[(-
1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^8*x^8*Sqrt[(-1 + c*
x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 11520*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[
c*x]^3 + 34560*a^2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d -
 c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 34560*a^2*c*Sqrt[d]*x*Sqrt[(-1 +
c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]
 + 13824*a*b*Sqrt[d - c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 3456*a*b*Sqrt[d -
c^2*d*x^2]*Cosh[4*ArcCosh[c*x]] - 1536*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcC
osh[c*x]] + 216*a*b*Sqrt[d - c^2*d*x^2]*Cosh[8*ArcCosh[c*x]] - 6912*b^2*Sq
rt[d - c^2*d*x^2]*Sinh[2*ArcCosh[c*x]] - 864*b^2*Sqrt[d - c^2*d*x^2]*Sinh[
4*ArcCosh[c*x]] + 256*b^2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] - 27*b^
2*Sqrt[d - c^2*d*x^2]*Sinh[8*ArcCosh[c*x]] + 24*b*Sqrt[d - c^2*d*x^2]*ArcC
osh[c*x]*(576*b*Cosh[2*ArcCosh[c*x]] + 144*b*Cosh[4*ArcCosh[c*x]] - 64*b*C
osh[6*ArcCosh[c*x]] + 9*b*Cosh[8*ArcCosh[c*x]] - 1152*a*Sinh[2*ArcCosh[c*x
]] - 576*a*Sinh[4*ArcCosh[c*x]] + 384*a*Sinh[6*ArcCosh[c*x]] - 72*a*Sin...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6345}$$

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2} \int x^3 (1-cx)^2 (cx+1)^2 (a+b\text{arccosh}(cx)) dx}{4 \sqrt{cx-1} \sqrt{cx+1}}+\frac{5}{8} d \int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2 dx+\frac{1}{8} x^3 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6327}$$

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2} \int x^3 \left(1-c^2 x^2\right)^2 (a+b\text{arccosh}(cx)) dx}{4 \sqrt{cx-1} \sqrt{cx+1}}+\frac{5}{8} d \int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^2 dx+\frac{1}{8} x^3 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))^2$$

$\downarrow$ 6336

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-bc\int\frac{x^4(3c^4x^4-8c^2x^2+6)}{24\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+ba\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 27

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{24}bc\int\frac{x^4(3c^4x^4-8c^2x^2+6)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 1905

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int\frac{x^4\left(3c^4x^4-8c^2x^2+6\right)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 1590

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int\frac{c^2x^4\left(48-43c^2x^2\right)}{\sqrt{c^2x^2-1}}dx}{8c^2}+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccos}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 27

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\int\frac{x^4\left(48-43c^2x^2\right)}{\sqrt{c^2x^2-1}}dx+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+\text{barccos}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 363

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\int\frac{x^4}{\sqrt{c^2x^2-1}}dx-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\int\frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4x^8(a+\text{barcco}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}c^4x\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 224

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)-\frac{43}{6}x^5\sqrt{c^2x^2-1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2-1}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 219

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx-$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 6345

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\int -x^3(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\downarrow 25$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^3(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\downarrow 6327$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\int x^3\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3(d\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 6336

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-bc\int\frac{x^4\left(3-2c^2x^2\right)}{12\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{6}c^2x^6(a+b\text{arccosh}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 27

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\int\frac{x^4(3-2c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{6}c^2x^6(a+b\text{arccosh}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{960}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\int\frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^5\sqrt{cx-1}\sqrt{ca\right.\right.}{3\sqrt{cx-1}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{111}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{\int\frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right.\right.}{\cdots}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\cdots\right.}{\cdots}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \; 27$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right.\right.}{\cdots}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\cdots\right.}{\cdots}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \; 101$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\right.\right.\right.}{}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{43}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(-\right.}{}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\left(\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6341}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\int x^3(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 6298

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\int\frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{2\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$\downarrow$ 111

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{\int \frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{8}\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{101}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}}{2c^2}\right.}{4}\right.\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 43}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\text{arccosh}(cx))-\frac{1}{3}c^2x^6(a+b\text{arccosh}(cx))+\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\left(\frac{1}{8}\right.\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 6354}$$

$$\frac{1}{8}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2x^3-$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4(a+b\text{arccosh}(cx))x^8-\frac{1}{3}c^2(a+b\text{arccosh}(cx))x^6+\frac{1}{4}(a+b\text{arccosh}(cx))x^4-\frac{bc\sqrt{c^2x^2-1}\left(\frac{3}{8}c^2\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{5}{8}d\left(\frac{1}{6}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2x^3-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2(a+b\text{arccosh}(cx))x^6+\frac{1}{4}(a+b\text{arccosh}(cx)\right.}{}\right.$$

$\downarrow$ 6298

$$\frac{1}{8}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2x^3-$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4(a+b\text{arccosh}(cx))x^8-\frac{1}{3}c^2(a+b\text{arccosh}(cx))x^6+\frac{1}{4}(a+b\text{arccosh}(cx))x^4-\frac{bc\sqrt{c^2x^2-1}\left(\frac{3}{8}c^2\right.}{}\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{5}{8}d\left(\frac{1}{6}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2x^3-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2(a+b\text{arccosh}(cx))x^6+\frac{1}{4}(a+b\text{arccosh}(cx)\right.}{}\right.$$

input
```
Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(565) = 1130$.

Time = 0.59 (sec) , antiderivative size = 2528, normalized size of antiderivative = 3.87

method	result	size
default	Expression too large to display	2528
parts	Expression too large to display	2528

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a^2/c^2*x*(-c^2*d*x^2+d)^(5/2)+
5/192*a^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a^2/c^2*d^2*x*(-c^2*d*x^2+d)^
(1/2)+5/128*a^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d
)^(1/2))+b^2*(-5/384*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^
3*arccosh(c*x)^3*d^2+1/65536*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x
^7+128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+272*c^5*x^5-256*c^6*x^6*(c*x-1)
^(1/2)*(c*x+1)^(1/2)-88*c^3*x^3+160*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*
c*x-32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(3
2*arccosh(c*x)^2-8*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)-1/6912*(-d*(c^2
*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/
2)+38*c^3*x^3-48*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x+18*(c*x-1)^(1/2
)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(18*arccosh(c*x)^2-6*
arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(8*c
^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4
*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c
^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(
1/2))*(2*arccosh(c*x)^2-2*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-
d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*
x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)*d^2...
```

Fricas [F]

$$\int x^2 (d - c^2 d x^2)^{5/2} (a + b\text{arccosh}(cx))^2 \, dx = \int (-c^2 d x^2 + d)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
integral((a^2*c^4*d^2*x^6 - 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2
*x^6 - 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^
6 - 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x
)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d - c^2 d x^2)^{5/2} (a + b\text{arccosh}(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (d - c^2 d x^2)^{5/2} (a + b\text{arccosh}(cx))^2 \, dx = \int (-c^2 d x^2 + d)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d
) + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2
+ 15*d^(5/2)*arcsin(c*x)/c^3)*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*x
^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2 + d)^(5/2)*a*b
*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F]

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2 \, dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} (b \operatorname{arcosh}(cx)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2 \, dx = \int x^2 \left(a+b \operatorname{acosh}(c x)\right)^2 \left(d-c^2 d x^2\right)^{5/2} \, dx$$

input
```
int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} (a + b\text{arccosh}(cx))^2 \, dx = \frac{\sqrt{d}\, d^2 \left(15 asin(cx)\, a^2 + 48\sqrt{-c^2x^2+1}\, a^2c^7x^7 - 136\sqrt{-c^2x^2+1}\, a^2c^5x^5 + 118\sqrt{-c^2x}\right.}{}$$

input

```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2,x)
```

output

```
(sqrt(d)*d**2*(15*asin(c*x)*a**2 + 48*sqrt( - c**2*x**2 + 1)*a**2*c**7*x**
7 - 136*sqrt( - c**2*x**2 + 1)*a**2*c**5*x**5 + 118*sqrt( - c**2*x**2 + 1)
*a**2*c**3*x**3 - 15*sqrt( - c**2*x**2 + 1)*a**2*c*x + 768*int(sqrt( - c**
2*x**2 + 1)*acosh(c*x)*x**6,x)*a*b*c**7 - 1536*int(sqrt( - c**2*x**2 + 1)*
acosh(c*x)*x**4,x)*a*b*c**5 + 768*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x*
*2,x)*a*b*c**3 + 384*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**6,x)*b**2
*c**7 - 768*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**4,x)*b**2*c**5 + 3
84*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**2,x)*b**2*c**3))/(384*c**3)
```

3.177 $\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 405

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=-\frac{32b^2d^2\sqrt{d-c^2dx^2}}{245c^2}-\frac{16b^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{735c^2}-\frac{12b^2d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}}{1225c^2}-\frac{2b^2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}}{343c^2}+\frac{2bd^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{6bc^3d^2x^5\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{35\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bc^5d^2x^7\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{49\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

output
```
-32/245*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^2-16/735*b^2*d^2*(-c*x+1)*(c*x+1)*(
-c^2*d*x^2+d)^(1/2)/c^2-12/1225*b^2*d^2*(-c*x+1)^2*(c*x+1)^2*(-c^2*d*x^2+d
)^(1/2)/c^2-2/343*b^2*d^2*(-c*x+1)^3*(c*x+1)^3*(-c^2*d*x^2+d)^(1/2)/c^2+2/
7*b*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-2/7*b*c*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+6/35*b*c^3*d^2*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))
/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/49*b*c^5*d^2*x^7*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*ar
ccosh(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.58

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))^2\,dx=\frac{d^2\sqrt{d-c^2dx^2}\left(3675a^2(-1+c^2x^2)^4-210abcx\sqrt{-1+cx}\sqrt{1+cx}(-35+35c^2x^2-2\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(3675*a^2*(-1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(
2161 - 2918*c^2*x^2 + 1108*c^4*x^4 - 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35
*a*(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(35 - 35*c^2*x^2
+ 21*c^4*x^4 - 5*c^6*x^6))*ArcCosh[c*x] + 3675*b^2*(-1 + c^2*x^2)^4*ArcCos
h[c*x]^2))/(25725*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6329, 25, 6304, 6309, 27, 2113, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{2bd^2\sqrt{d-c^2dx^2}\int -(1-cx)^3(cx+1)^3(a+b\text{arccosh}(cx))dx}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\int (1-cx)^3(cx+1)^3(a+b\text{arccosh}(cx))dx}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{6304}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))dx}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{6309}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-bc\int \frac{x\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)}{35\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^6x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arccosh}(cx))-c^2\cdots\right.}{7c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\int \frac{x\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{7}c^6x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arccosh}(cx))-\cdots\right.}{7c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{2113}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int \frac{x\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)}{\sqrt{c^2x^2-1}}dx}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{7}c^6x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arccosh}(cx))\cdots\right.}{7c\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{2331}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int\frac{-5c^6x^6+21c^4x^4-35c^2x^2+35}{\sqrt{c^2x^2-1}}dx^2}{70\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{7}c^6x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arccosh}(cx))-\right.}{7c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{2389}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int\left(-5\left(c^2x^2-1\right)^{5/2}+6\left(c^2x^2-1\right)^{3/2}-8\sqrt{c^2x^2-1}+\frac{16}{\sqrt{c^2x^2-1}}\right)dx^2}{70\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{7}c^6x^7(a+b\text{arccosh}(cx))+\frac{3}{5}\right.}{7c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^6x^7(a+b\text{arccosh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arccosh}(cx))-c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arcc}\right.}{7c\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\text{arccosh}(cx))^2}{7c^2d}$$

input

```
Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output

```
-1/7*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x])^2)/(c^2*d) + (2*b*d^2*Sqr
t[d - c^2*d*x^2]*(-1/70*(b*c*Sqrt[-1 + c^2*x^2]*((32*Sqrt[-1 + c^2*x^2])/c
^2 - (16*(-1 + c^2*x^2)^(3/2))/(3*c^2) + (12*(-1 + c^2*x^2)^(5/2))/(5*c^2)
 - (10*(-1 + c^2*x^2)^(7/2))/(7*c^2)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + x*
(a + b*ArcCosh[c*x]) - c^2*x^3*(a + b*ArcCosh[c*x]) + (3*c^4*x^5*(a + b*Ar
cCosh[c*x]))/5 - (c^6*x^7*(a + b*ArcCosh[c*x]))/7))/(7*c*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.43

method	result
orering	$\frac{(9525c^{10}x^{10}-41691c^8x^8+76515c^6x^6-124979c^4x^4+26152c^2x^2-4322)(-c^2dx^2+d)^{\frac{5}{2}}(a+b\,\mathrm{arccosh}(cx))^2}{25725c^4(cx-1)(cx+1)x^2(c^2x^2-1)^2} - \frac{2(675c^8x^8-3108c^6}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/25725*(9525*c^10*x^10-41691*c^8*x^8+76515*c^6*x^6-124979*c^4*x^4+26152*c
^2*x^2-4322)/c^4/(c*x-1)/(c*x+1)/x^2/(c^2*x^2-1)^2*(-c^2*d*x^2+d)^(5/2)*(a
+b*arccosh(c*x))^2-2/25725*(675*c^8*x^8-3108*c^6*x^6+6352*c^4*x^4-14480*c^
2*x^2+2161)/c^4/(c*x-1)/(c*x+1)/x^2/(c^2*x^2-1)*((-c^2*d*x^2+d)^(5/2)*(a+b
*arccosh(c*x))^2-5*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2*c^2*d+2*x
*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2))+
1/25725*(75*c^6*x^6-351*c^4*x^4+757*c^2*x^2-2161)/c^4/(c*x-1)/(c*x+1)/x*(-
15*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+4*(-c^2*d*x^2+d)^(5/2
)*(a+b*arccosh(c*x))*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15*x^3*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccosh(c*x))^2*c^4*d^2-20*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcco
sh(c*x))*c^3*d*b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*x*(-c^2*d*x^2+d)^(5/2)*b^2*
c^2/(c*x-1)/(c*x+1)-x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))*b*c^2/(c*x-1
)^(3/2)/(c*x+1)^(1/2)-x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))*b*c^2/(c*x
-1)^(1/2)/(c*x+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.18

$$\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2\,dx = \frac{3675\,(b^2c^8d^2x^8-4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4-4\,b^2c^2d^2x^2+b^2d^2)\sqrt{-c^2dx^2+d}\log\left(cx+\right.}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
1/25725*(3675*(b^2*c^8*d^2*x^8 - 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 - 4
*b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
1))^2 - 210*(5*a*b*c^7*d^2*x^7 - 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 -
 35*a*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((5*b^2*c^7*
d^2*x^7 - 21*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3 - 35*b^2*c*d^2*x)*sqrt(-
c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 35*(a*b*c^8*d^2*x^8 - 4*a*b*c^6*d^2*x^6
 + 6*a*b*c^4*d^2*x^4 - 4*a*b*c^2*d^2*x^2 + a*b*d^2)*sqrt(-c^2*d*x^2 + d))*
log(c*x + sqrt(c^2*x^2 - 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 - 12*(1225
*a^2 + 71*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 - 4*(367
5*a^2 + 1459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2)*sqrt(-c^2*d*x^2
 + d))/(c^4*x^2 - c^2)
```

Sympy [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.83

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx =$$

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b^2\,\text{arcosh}\left(cx\right)^2}{7\,c^2d}-\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}ab\,\text{arcosh}\left(cx\right)}{7\,c^2d}$$
$$+\frac{2}{25725}\,b^2\left(\frac{75\,\sqrt{c^2x^2-1}c^4\sqrt{-d}d^3x^6-351\,\sqrt{c^2x^2-1}c^2\sqrt{-d}d^3x^4+757\,\sqrt{c^2x^2-1}\sqrt{-d}d^3x^2-\frac{2161\,\sqrt{c^2x^2-}}{c^2}}{d}\right.$$
$$-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}a^2}{7\,c^2d}$$
$$-\frac{2\left(5\,c^6\sqrt{-d}d^3x^7-21\,c^4\sqrt{-d}d^3x^5+35\,c^2\sqrt{-d}d^3x^3-35\,\sqrt{-d}d^3x\right)ab}{245\,cd}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-1/7*(-c^2*d*x^2 + d)^(7/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/7*(-c^2*d*x^2 +
 d)^(7/2)*a*b*arccosh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 - 1)*c^
4*sqrt(-d)*d^3*x^6 - 351*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^3*x^4 + 757*sqrt
(c^2*x^2 - 1)*sqrt(-d)*d^3*x^2 - 2161*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^3/c^2)/
d - 105*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 + 35*c^2*sqrt(-d
)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*arccosh(c*x)/(c*d)) - 1/7*(-c^2*d*x^2 + d)^
(7/2)*a^2/(c^2*d) - 2/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^
5 + 35*c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=\int x\,(a+b\,\text{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=\frac{\sqrt{d}\,d^2\left(\sqrt{-c^2x^2+1}\,a^2c^6x^6-3\sqrt{-c^2x^2+1}\,a^2c^4x^4+3\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2}\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*d**2*(sqrt( - c**2*x**2 + 1)*a**2*c**6*x**6 - 3*sqrt( - c**2*x**2
 + 1)*a**2*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c
**2*x**2 + 1)*a**2 + 14*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**5,x)*a*b*
c**6 - 28*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**3,x)*a*b*c**4 + 14*int(
sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*a*b*c**2 + 7*int(sqrt( - c**2*x**2
+ 1)*acosh(c*x)**2*x**5,x)*b**2*c**6 - 14*int(sqrt( - c**2*x**2 + 1)*acosh
(c*x)**2*x**3,x)*b**2*c**4 + 7*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x,
x)*b**2*c**2))/(7*c**2)
```

3.178 $\int (d-c^2dx^2)^{5/2} (a+b\,\mathrm{arccosh}(cx))^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 462

$$\int (d-c^2dx^2)^{5/2} (a+b\,\mathrm{arccosh}(cx))^2 \, dx = \frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152} + \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2} + \frac{115b^2d^2\sqrt{d-c^2dx^2}\mathrm{arccosh}(cx)}{1152c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5bcd^2x^2\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{16\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bd^2(-1+cx)^{3/2}(1+cx)^{3/2}\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{48c} - \frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))}{18c} + \frac{5}{16}d^2x\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2 + \frac{5}{24}dx(d-c^2dx^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))^2 + \frac{1}{6}x(d-c^2dx^2)^{5/2}(a+\mathrm{barc}$$

output
```
245/1152*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+65/1728*b^2*d^2*x*(-c*x+1)*(c*x+1)
*(-c^2*d*x^2+d)^(1/2)+1/108*b^2*d^2*x*(-c*x+1)^2*(c*x+1)^2*(-c^2*d*x^2+d)^
(1/2)+115/1152*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/c/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-5/16*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x
-1)^(1/2)/(c*x+1)^(1/2)+5/48*b*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccosh(c*x))/c-1/18*b*d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c+5/16*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))^2+5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/6*x*
(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2-5/48*d^2*(-c^2*d*x^2+d)^(1/2)*(a
+b*arccosh(c*x))^3/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.56 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.60

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))^2\,dx = \frac{d^2\left(9504a^2cx\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}+9504a^2c^2x^2\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}-7488a^2c^3x^3\sqrt{\cdots}\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(d^2*(9504*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 9504*a
^2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^3*x
^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^4*x^4*Sqrt[
(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[(-1 + c*
x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^6*x^6*Sqrt[(-1 + c*x)/(1 +
c*x)]*Sqrt[d - c^2*d*x^2] - 1440*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^3 -
4320*a^2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2
])/(Sqrt[d]*(-1 + c^2*x^2))] - 4320*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c
*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 3240*a*b
*Sqrt[d - c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 324*a*b*Sqrt[d - c^2*d*x^2]*Co
sh[4*ArcCosh[c*x]] - 24*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcCosh[c*x]] + 162
0*b^2*Sqrt[d - c^2*d*x^2]*Sinh[2*ArcCosh[c*x]] - 81*b^2*Sqrt[d - c^2*d*x^2
]*Sinh[4*ArcCosh[c*x]] + 4*b^2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] -
12*b*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*(270*b*Cosh[2*ArcCosh[c*x]] - 27*b*C
osh[4*ArcCosh[c*x]] + 2*b*Cosh[6*ArcCosh[c*x]] - 540*a*Sinh[2*ArcCosh[c*x]
] + 108*a*Sinh[4*ArcCosh[c*x]] - 12*a*Sinh[6*ArcCosh[c*x]]) + 72*b*Sqrt[d
- c^2*d*x^2]*ArcCosh[c*x]^2*(-60*a + 45*b*Sinh[2*ArcCosh[c*x]] - 9*b*Sinh[
4*ArcCosh[c*x]] + b*Sinh[6*ArcCosh[c*x]])))/(13824*c*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6312, 6312, 25, 6310, 6298, 101, 43, 6308, 6327, 6329, 40, 40, 40, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2dx$$

$$\Big\downarrow \text{ 6312}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{ 6312}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\int -x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\int\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{4}x\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \ 25$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\int\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{1}{4}x\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \ 6310$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \ 6298$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))\right.}{\sqrt{cx-1}\sqrt{cx+}}\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \ 101$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))\right.}{\sqrt{cx-}}\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \ 43$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6308}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+\frac{3}{4}d\left(\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6327}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+\frac{3}{4}d\left(-\frac{\sqrt{d}}{}\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6329}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\int(cx-1)^{5/2}(cx+1)^{5/2}dx}{6c}-\frac{\left(1-c^2x^2\right)^3(a+b\text{arccosh}(cx))}{6c^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\int(cx-1)^{3/2}(cx+1)^{3/2}dx}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{40}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\int(cx-1)^{3/2}(cx+1)^{3/2}dx\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\text{arccosh}(cx))}{6c^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int\sqrt{cx-1}\sqrt{cx+1}dx\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/}\right.$$

$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow 40$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int\sqrt{cx-1}\sqrt{cx+1}dx\right)\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\text{arccosh}(cx))}{6c^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\right.$$

$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow 40$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)\right)}{6c}-\frac{(1-c^2x^2}{}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\right.$$

$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow 43$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^3(a+b\text{arccosh}(cx))}{6c^2}-\frac{b\left(\frac{1}{6}x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{5}{6}\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\right.\right.\right.}{6c}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2+$$

$$\frac{5}{6}d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}(c}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

input
```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
(x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/6 - (b*c*d^2*Sqrt[d - c^2
*d*x^2]*(-1/6*((1 - c^2*x^2)^3*(a + b*ArcCosh[c*x]))/c^2 - (b*((x*(-1 + c*
x)^(5/2)*(1 + c*x)^(5/2))/6 - (5*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4 -
 (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/6))/(6*
c)))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d*((x*(d - c^2*d*x^2)^(3/2)*(a
+ b*ArcCosh[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b*c*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcCosh[c*x]))/2 -
 (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/2
))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/4 - (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/4*(
(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/c^2 + (b*((x*(-1 + c*x)^(3/2)*(1 + c
*x)^(3/2))/4 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)
))/4))/(4*c)))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6312
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p
)]   Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && GtQ[p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1736 vs. $2(398) = 796$.

Time = 0.50 (sec) , antiderivative size = 1737, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	1737
parts	Expression too large to display	1737

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(-c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a^2*
d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-5/48*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)/c*arccosh(c*x)^3*d^2+1/6912*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^
7-64*c^5*x^5+32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+38*c^3*x^3-48*c^4*x^4*
(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x+18*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(
c*x-1)^(1/2)*(c*x+1)^(1/2))*(18*arccosh(c*x)^2-6*arccosh(c*x)+1)*d^2/(c*x-
1)/(c*x+1)/c-3/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4
*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)*d^2/(c*x-1
)/(c*x+1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*ar
ccosh(c*x)+1)*d^2/(c*x-1)/(c*x+1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*
x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)*d^2/(c*x-1)/(c*x+1)/c-3/1024*(-d*(c
^2*x^2-1))^(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*
x)*(8*arccosh(c*x)^2+4*arccosh(c*x)+1)*d^2/(c*x-1)/(c*x+1)/c+1/6912*(-d*(c
^2*x^2-1))^(1/2)*(-32*c^6*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)+32*c^7*x^7+48*c^
4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)-64*c^5*x^5-18*(c*x-1)^(1/2)*(c*x+1)^(...
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt
(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a^2 + integrate((-c^2*d
*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x
^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))^2\,dx=\int (a+b\operatorname{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))^2\,dx=\frac{\sqrt{d}\,d^2\left(15asin(cx)\,a^2+8\sqrt{-c^2x^2+1}\,a^2c^5x^5-26\sqrt{-c^2x^2+1}\,a^2c^3x^3+33\sqrt{-c^2x^2+}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2,x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a**2 + 8*sqrt( - c**2*x**2 + 1)*a**2*c**5*x**5
 - 26*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 + 33*sqrt( - c**2*x**2 + 1)*a*
*2*c*x + 96*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*a*b*c**5 - 192*i
nt(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*a*b*c**3 + 96*int(sqrt( - c**
2*x**2 + 1)*acosh(c*x),x)*a*b*c + 48*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)
**2*x**4,x)*b**2*c**5 - 96*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**2,x
)*b**2*c**3 + 48*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2,x)*b**2*c))/(48*
c)
```

3.179 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 758

$$
\begin{aligned}
\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{x}\,dx &= \frac{1844}{675}b^2d^2\sqrt{d-c^2dx^2} \\
&- \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{8}{225}b^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{2}{125}b^2d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2} \\
&- \frac{2b^2cd^2x\sqrt{d-c^2dx^2}\operatorname{arccosh}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{16bcd^2x\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{15\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{22bc^3d^2x^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{45\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bc^5d^2x^5\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{25\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2 + \frac{1}{3}d(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2 \\
&+ \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2 - \frac{2d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2ibd^2\sqrt{d-}
\end{aligned}
$$

output
```
1844/675*b^2*d^2*(-c^2*d*x^2+d)^(1/2)-2/27*b^2*c^2*d^2*x^2*(-c^2*d*x^2+d)^
(1/2)-2*a*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/225
*b^2*d^2*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)+2/125*b^2*d^2*(-c*x+1)^2*(c
*x+1)^2*(-c^2*d*x^2+d)^(1/2)-2*b^2*c*d^2*x*(-c^2*d*x^2+d)^(1/2)*arccosh(c*
x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-16/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+22/45*b*c^3*d^2*x^3*(-c^2*d*x^2+d
)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/25*b*c^5*d^2*x^5*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d^2*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2+1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*ar
ccosh(c*x))^2+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2-2*d^2*(-c^2*d*
x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/
(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*
x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-2*I*b*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*d^2*(-c^2
*d*x^2+d)^(1/2)*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-2*I*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.64 (sec) , antiderivative size = 963, normalized size of antiderivative = 1.27

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

output
```
(a^2*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 - (b^2*d^2*
Sqrt[d - c^2*d*x^2]*(2*(-13 + Cosh[2*ArcCosh[c*x]]) + 9*ArcCosh[c*x]^2*(-1
 + Cosh[2*ArcCosh[c*x]]) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*(9*c
*x - Cosh[3*ArcCosh[c*x]]))/(-1 + c*x)))/27 - (a*b*d^2*Sqrt[d - c^2*d*x^2]
*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[
3*ArcCosh[c*x]]))/(9*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a^2*d^(5/2)*L
og[c*x] - a^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d^2*Sq
rt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*
Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCo
sh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^
ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)) + b^2*d^2*Sqrt[d - c^2*d*x^2]*(2 + (2*c*x*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x])/(1 - c*x) + ArcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log
[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcC
osh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^A
rcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] - 2*PolyLog[3, I/E^ArcCos
h[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) - (a*b*d^2*Sqrt[d - c^2*
d*x^2]*(25*Cosh[3*ArcCosh[c*x]] + 9*(-50*c*x + Cosh[5*ArcCosh[c*x]]) + 15*
ArcCosh[c*x]*(30*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - 5*Sinh[3*ArcCosh[c
*x]] - 3*Sinh[5*ArcCosh[c*x]])))/(1800*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + ...
```

Rubi [A] (verified)

Time = 4.76 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.81, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.793$, Rules used = {6345, 6304, 6309, 27, 1905, 1576, 1140, 2009, 6345, 25, 6304, 6309, 27, 960, 83, 6341, 2009, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}dx$$

$$\Big\downarrow \text{ 6345}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int (1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{5\sqrt{cx-1}\sqrt{cx+1}}+ d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6304}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))dx}{5\sqrt{cx-1}\sqrt{cx+1}}+$$
$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{6309}$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx-$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-bc\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{15\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+ba\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx-$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{15}bc\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{1905}$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx-$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{\sqrt{c^2x^2-1}}dx}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\Big\downarrow \text{1576}$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx-$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{c^2x^2-1}}dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\downarrow 1140$$

$$d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx - \frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int\left(3(c^2x^2-1)^{3/2}-4\sqrt{c^2x^2-1}+\frac{8}{\sqrt{c^2x^2-1}}\right)dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+\text{barc}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2$$

$$\downarrow 2009$$

$$d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2 - \frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\downarrow 6345$$

$$d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int-((1-cx)(cx+1)(a+b\text{arccosh}(cx)))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}(d\right.$$

$$\frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2 - \frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\downarrow 25$$

$$d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}(d-\right.$$

$$\frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2 - \frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6304$$

$$d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}(d-c^2dx\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\cdots}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 6309$$

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-bc\int \frac{x(3-c^2x^2)}{3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))\right.}{3\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\cdots}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 27$$

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\int \frac{x(3-c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+b\text{arccosh}(cx))\right.}{3\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\cdots}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 960$$

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\left(\frac{7}{3}\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx+1}\right)\right.}{3\sqrt{cx-1}\sqrt{cx+}}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 83

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a\right.}{}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6341

$$d\left(d\left(-\frac{2bc\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 2009

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\text{arccosh}\right.}{\sqrt{cx-1}\sqrt{cx}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6\left(c^2\right.}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 6362

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+\right.}{\sqrt{c}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6\left(c^2\right.}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 3042

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx)\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6\left(c^2\right.}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 4668

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2ib\int (a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int (a+b\text{arccosh}(cx))\log\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 3011$$

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(\right.\right.}{}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2720$$

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a-\right.\right.}{}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6(c^2\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 7143$$

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(2,\right.\right.\right.}{\sqrt{}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\text{arccosh}(cx))-\frac{2}{3}c^2x^3(a+b\text{arccosh}(cx))+x(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6\left(c^2\right.}{}\right.}{}\right.}{5\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

output
```
((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/5 - (2*b*c*d^2*Sqrt[d - c^2
*d*x^2]*(-1/30*(b*c*Sqrt[-1 + c^2*x^2]*((16*Sqrt[-1 + c^2*x^2])/c^2 - (8*(
-1 + c^2*x^2)^(3/2))/(3*c^2) + (6*(-1 + c^2*x^2)^(5/2))/(5*c^2)))/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + x*(a + b*ArcCosh[c*x]) - (2*c^2*x^3*(a + b*ArcCos
h[c*x]))/3 + (c^4*x^5*(a + b*ArcCosh[c*x]))/5))/(5*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) + d*(((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/3 - (2*b*c*d*Sq
rt[d - c^2*d*x^2]*(-1/3*(b*c*((7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) - (
x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/3)) + x*(a + b*ArcCosh[c*x]) - (c^2*x^3*
(a + b*ArcCosh[c*x]))/3))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*(Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x - (b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (Sqrt[d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[
c*x]] + (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) +
 b*PolyLog[3, (-I)*E^ArcCosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh[c*x])*Poly
Log[2, I*E^ArcCosh[c*x]]) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^2}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2} (a+b\mathrm{arccosh}(cx))^2}{x}\,dx = \int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas
")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2} (a+b\mathrm{arccosh}(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima
")
```

output
```
-1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
- 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x
^2 + d)*d^2)*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))^2/x + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x}\,dx = \frac{\sqrt{d}\,d^2\Big(3\sqrt{-c^2x^2+1}\,a^2c^4x^4 - 11\sqrt{-c^2x^2+1}\,a^2c^2x^2 + 23\sqrt{-c^2}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2/x,x)
```

output
```
(sqrt(d)*d**2*(3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 11*sqrt( - c**2*x
**2 + 1)*a**2*c**2*x**2 + 23*sqrt( - c**2*x**2 + 1)*a**2 + 30*int((sqrt( -
 c**2*x**2 + 1)*acosh(c*x))/x,x)*a*b + 15*int((sqrt( - c**2*x**2 + 1)*acos
h(c*x)**2)/x,x)*b**2 + 30*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**3,x)*a*
b*c**4 - 60*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*a*b*c**2 + 15*int(s
qrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**3,x)*b**2*c**4 - 30*int(sqrt( - c**
2*x**2 + 1)*acosh(c*x)**2*x,x)*b**2*c**2 + 15*log(tan(asin(c*x)/2))*a**2 -
 23*a**2))/15
```

3.180 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 586

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx =$$

$$-\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2}-\frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}$$

$$-\frac{89b^2cd^2\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{64\sqrt{-1+cx}\sqrt{1+cx}}+\frac{15bc^3d^2x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{8\sqrt{-1+cx}\sqrt{1+cx}}$$

$$-bcd^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{1}{8}bcd^2(-1$$

$$+cx)^{3/2}(1+cx)^{3/2}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{cd^2\sqrt{d-c^2dx^2}(}{\sqrt{-1+}}$$

output
```
-31/64*b^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-1/32*b^2*c^2*d^2*x*(-c*x+1)*(c*x
+1)*(-c^2*d*x^2+d)^(1/2)-89/64*b^2*c*d^2*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/8*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-1/8*b*c*d^2*(c*x-1)^(3/2)*(c*x
+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-15/8*c^2*d^2*x*(-c^2*d*x
^2+d)^(1/2)*(a+b*arccosh(c*x))^2-c*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c
*x))^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*a
rccosh(c*x))^2-(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x+5/8*c*d^2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c*d
^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b^2*c*d^2*(-c^2*d*x^2+d)^(1/2)*poly
log(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.99 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.95

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx = \frac{d^2\left(96a^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}(-8-9c^2x^2+2c^4x^4)+14\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

output
```
(d^2*(96*a^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-8
- 9*c^2*x^2 + 2*c^4*x^4) + 1440*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 76
8*a*b*Sqrt[d - c^2*d*x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[
c*x] - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) - 256*b^2*Sqrt[d - c^2*d*x^2]*(A
rcCosh[c*x]*(3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(Ar
cCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*P
olyLog[2, -E^(-2*ArcCosh[c*x])]) + 384*a*b*c*x*Sqrt[d - c^2*d*x^2]*(Cosh[2
*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + 6
4*b^2*c*x*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*Ar
cCosh[c*x]] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]) - 12*a*b*c*x*
Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c
*x]*Sinh[4*ArcCosh[c*x]]) - b^2*c*x*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[c*x]^3
 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4*
ArcCosh[c*x]])))/(768*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.65 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.13, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.966$, Rules used = {6343, 6312, 25, 6310, 6298, 101, 43, 6308, 6327, 6329, 40, 40, 43, 6334, 40, 40, 43, 6334, 40, 43, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2}{x^2} \, dx$$

$$\Big\downarrow \text{6343}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-5c^2d\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2dx-\frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \text{6312}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} - 5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\int -x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}} + \frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx + \frac{1}{4}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$\downarrow$ 25

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} - 5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}} + \frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx + \frac{1}{4}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$\downarrow$ 6310

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} - 5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}} + \frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\text{arccosh}(cx))d}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$\downarrow$ 6298

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} - 5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}} + \frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx)\right.}{\sqrt{cx-1}\sqrt{cx+}}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$\downarrow$ 101

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} -$$
$$5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx)\right.}{\sqrt{cx\ }}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 43$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} -$$
$$5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{4}d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 6308$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} -$$
$$5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+\frac{3}{4}d\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 6327$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}} -$$
$$5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2+\frac{3}{4}d\left(-\frac{\sqrt{\ }}{\ }\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 6329$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\int (cx-1)^{3/2}(cx+1)^{3/2}dx}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(c\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 40$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 40$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-5c^2d\left(-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{4c^2}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}$$

$$\Big\downarrow \; 43$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\Big\downarrow \; 6334$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bc\int (cx-1)^{3/2}(cx+1)^{3/2}dx+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(c\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 40

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\int \sqrt{cx-1}\sqrt{cx+1}dx\right)+\frac{1}{4}\left(\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 40

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\right.\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 43

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 6334

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))+\frac{1}{2}\left(1-c^2\right.\right.}{\sqrt{cx-1}\sqrt{cx+}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 40

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+bar\right.}{\sqrt{cx}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 43

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(cx)\right)+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arccosh}(cx)\right)+\frac{1}{2}b\right.}{\sqrt{cx-}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\text{arccosh}(cx)\right)^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 6297

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{\int -\left(\left(a+b\text{arccosh}(cx)\right)\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\text{arccosh}(cx)\right)^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 25

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(cx)\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\text{arccosh}(cx)\right)^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 3042

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\Big\downarrow \ 26$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{i\int(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\text{arccosh}(cx\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\Big\downarrow \ 4201$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\int\frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\Big\downarrow \ 2620$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arc}}{b}\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 2715

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-}{b}\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$\downarrow$ 2838

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x}-$$

$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{1}{4}x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{4}\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arccosh}}{2c}\right.\right.}{4c}\right.}{2\sqrt{cx-1}\sqrt{cx+1}}\right.$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x) - 5*c^2*d*((x*(d - c^2
*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b
*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*A
rcCosh[c*x]))/2 - (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh
[c*x]/(2*c^3)))/2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/4 - (b*c*d*Sqrt[d - c
^2*d*x^2]*(-1/4*((1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/c^2 + (b*((x*(-1 +
c*x)^(3/2)*(1 + c*x)^(3/2))/4 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - A
rcCosh[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (2*b*c
*d^2*Sqrt[d - c^2*d*x^2]*(((1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 + ((1 - c
^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 + (b*c*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
/2 - ArcCosh[c*x]/(2*c)))/2 - (b*c*((x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/4
 - (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/2 - ArcCosh[c*x]/(2*c)))/4))/4 + (
I*((-1/2*I)*(a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*L
og[1 + E^(-2*ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))/
b))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6310
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Simp[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(a + b*ArcC
osh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Simp[b*c*(n/2)*Simp[Sq
rt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[x*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0]
```

rule 6312
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p
)]   Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && GtQ[p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6334
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Simp[b*c*((-d
)^p/(2*p))   Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.01

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-a^2c^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5a^2c^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15a^2c^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15a^2c^2d^3\arctan}{8\sqrt{}}$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-a^2c^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5a^2c^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15a^2c^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15a^2c^2d^3\arctan}{8\sqrt{}}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(7/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a^2*c^2*d
*x*(-c^2*d*x^2+d)^(3/2)-15/8*a^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a^2*c
^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/64*b^2
*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x*(16*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*arccosh(c*x)^2*x^4*c^4-8*arccosh(c*x)*c^5*x^5+2*c^4*x^4*(c*x-1
)^(1/2)*(c*x+1)^(1/2)-72*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^
2+72*c^3*x^3*arccosh(c*x)-33*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+40*arccos
h(c*x)^3*c*x-64*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-64*arccosh(c*x)
^2*c*x+128*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c-33*c
*x*arccosh(c*x)+64*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c)*d^
2+1/64*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x*(32*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4-8*c^5*x^5-144*arccosh(c*x)*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+72*c^3*x^3+120*arccosh(c*x)^2*c*x-128*arcc
osh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-128*c*x*arccosh(c*x)+128*ln(1+(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c-33*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="fric
as")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxi
ma")
```

output
```
-1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*
x + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a^2 + integrate
((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^2 +
 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\operatorname{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{x^2}\,dx = \frac{\sqrt{d}\,d^2\left(-15asin(cx)\,a^2cx+2\sqrt{-c^2x^2+1}\,a^2c^4x^4-9\sqrt{-c^2x^2+\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2/x^2,x)
```

output
```
(sqrt(d)*d**2*( - 15*asin(c*x)*a**2*c*x + 2*sqrt( - c**2*x**2 + 1)*a**2*c*
*4*x**4 - 9*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 8*sqrt( - c**2*x**2 +
1)*a**2 + 16*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**2,x)*a*b*x + 8*int
((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x**2,x)*b**2*x + 16*int(sqrt( - c*
*2*x**2 + 1)*acosh(c*x)*x**2,x)*a*b*c**4*x - 32*int(sqrt( - c**2*x**2 + 1)
*acosh(c*x),x)*a*b*c**2*x + 8*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x**
2,x)*b**2*c**4*x - 16*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2,x)*b**2*c**
2*x))/(8*x)
```

3.181 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 777

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{x^3}\,dx = -\frac{122}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{2}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5b^2c^3d^2x\sqrt{d-c^2dx^2}\operatorname{arccosh}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bc^5d^2x^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2 - \frac{5}{6}c^2d(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^2 - \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{2x^2} + \frac{5c^2d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b^2c^2d^2\sqrt{d-c^2dx^2}\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5ibc^2d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5ibc^2d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5ib^2c^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5ib^2c^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-122/27*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)+2/27*b^2*c^4*d^2*x^2*(-c^2*d*x^2+
d)^(1/2)+5*a*b*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
5*b^2*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-b*c*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x/(c*x-1)^(1/2)/(c*x+1
)^(1/2)-1/3*b*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-2/9*b*c^5*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))^2-5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-1/2*(-c^2*d*
x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2+5*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*arctan((c*x-1)^(1/2)*(c*x+1)^(1
/2))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*I*b*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-5*I*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5*I*b*c^2*d^2*(-c^2*d*
x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*I*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*pol
ylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5694 vs. 2(777) = 1554.

Time = 62.96 (sec) , antiderivative size = 5694, normalized size of antiderivative = 7.33

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 5.46 (sec) , antiderivative size = 609, normalized size of antiderivative = 0.78, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.828$, Rules used = {6343, 6327, 6336, 27, 1905, 1578, 1192, 1467, 2009, 6345, 25, 6304, 6309, 27, 960, 83, 6341, 2009, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{6343}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x^2}dx}{\sqrt{cx-1}\sqrt{cx+1}} - \frac{5}{2}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx - \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6327}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{x^2}dx}{\sqrt{cx-1}\sqrt{cx+1}} - \frac{5}{2}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx - \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6336}$$

$$-\frac{5}{2}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx + \frac{bcd^2\sqrt{d-c^2dx^2}\left(-bc\int -\frac{-c^4x^4+6c^2x^2+3}{3x\sqrt{cx-1}\sqrt{cx+1}}dx + \frac{1}{3}c^4x^3(a+b\text{arccosh}(cx)) - 2c^2x(a+b\text{arccosh}(cx)) - \frac{a+b\text{arccosh}(cx}{x}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}bc\int \frac{-c^4x^4+6c^2x^2+3}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{1905}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{bc\sqrt{c^2x^2-1}\int \frac{-c^4x^4+6c^2x^2+3}{x\sqrt{c^2x^2-1}}dx}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{1578}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{bc\sqrt{c^2x^2-1}\int \frac{-c^4x^4+6c^2x^2+3}{x^2\sqrt{c^2x^2-1}}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{b\sqrt{c^2x^2-1}\int \frac{-c^4x^8+4c^4x^4+8c^4}{x^4+1}d\sqrt{c^2x^2-1}}{3c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{b\sqrt{c^2x^2-1}\int\left(-x^4c^4+\frac{3c^4}{x^4+1}+5c^4\right)d\sqrt{c^2x^2-1}}{3c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a\cdots}{}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 2009

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\cdots}\right.\right.}{3c^3\sqrt{cx-\cdots}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 6345

$$-\frac{5}{2}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int-((1-cx)(cx+1)(a+b\text{arccosh}(cx)))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx\cdots\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\cdots}\right.\right.}{3c^3\sqrt{cx-\cdots}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 25

$$-\frac{5}{2}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\cdots\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\cdots}\right.\right.}{3c^3\sqrt{cx-\cdots}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 6304

$$-\frac{5}{2}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{3\sqrt{cx-1}\sqrt{cx+1}}+d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}(d-\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 6309

$$-\frac{5}{2}c^2d\left(d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-bc\int\frac{x(3-c^2x^2)}{3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+b\text{arccosh}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 27

$$-\frac{5}{2}c^2d\left(d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\int\frac{x(3-c^2x^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}c^2x^3(a+b\text{arccosh}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\downarrow$ 960

$$-\frac{5}{2}c^2d\left(d\int\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\left(\frac{7}{3}\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{1}{3}x^2\sqrt{cx-1}\sqrt{cx\right.\right.}{3\sqrt{cx-1}\sqrt{\right.}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{5}{2}c^2d\left(d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 6341}$$

$$-\frac{5}{2}c^2d\left(d\left(d\left(-\frac{2bc\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+ba\right.\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{5}{2}c^2d\left(d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bxar\right.}{\sqrt{cx-1}}\right.\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 6362}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2-\frac{2bc\sqrt{d-c^2dx^2}\left(\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}(a+b\text{arccosl}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2ib\int(a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int(a+b\text{arccosh}(cx\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arc}\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\big\downarrow$ 2720

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right.\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

$\big\downarrow$ 7143

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLo}\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\text{arccosh}(cx))-2c^2x(a+b\text{arccosh}(cx))-\frac{a+b\text{arccosh}(cx)}{x}+\frac{b\sqrt{c^2x^2-1}\left(3c^4\arctan\left(\sqrt{c^2x\right.\right.}{3c^3\sqrt{cx-}}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{2x^2}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

output
```
-1/2*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2 + (b*c*d^2*Sqrt[d
- c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/x) - 2*c^2*x*(a + b*ArcCosh[c*x]) + (
c^4*x^3*(a + b*ArcCosh[c*x]))/3 + (b*Sqrt[-1 + c^2*x^2]*(-1/3*(c^4*x^6) +
5*c^4*Sqrt[-1 + c^2*x^2] + 3*c^4*ArcTan[Sqrt[-1 + c^2*x^2]]))/(3*c^3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d*(((d
- c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/3 - (2*b*c*d*Sqrt[d - c^2*d*x^2
]*(-1/3*(b*c*((7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) - (x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/3)) + x*(a + b*ArcCosh[c*x]) - (c^2*x^3*(a + b*ArcCosh[c
*x]))/3))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*(Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x - (b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/c + b*x*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[
d - c^2*d*x^2]*(2*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]] + (2*I)*b*
(-((a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) + b*PolyLog[3, (-
I)*E^ArcCosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCo
sh[c*x]]) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
]))))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6309
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6336
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6341
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1
+ c*x]*Sqrt[-1 + c*x])), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])]   Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
 && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6343
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)
*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && G
tQ[p, 0] && LtQ[m, -1]
```

rule 6345
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m
 + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{2}}{x^{3}}d x$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2}{x^3} \, dx = \int \frac{{\left(-c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{2}}{x^{3}} \,d x$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="fric
as")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2}{x^3} \, dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2}{x^3} \, dx = \int \frac{{\left(-c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{2}}{x^{3}} \,d x$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxi
ma")
```

output
```
1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)
) - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqr
t(-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a^2 + integr
ate((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^
3 + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^
3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^2}{x^3}\,dx = \frac{\sqrt{d}\,d^2\left(8\sqrt{-c^2x^2+1}\,a^2c^4x^4-56\sqrt{-c^2x^2+1}\,a^2c^2x^2-12\sqrt{-c^2\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2/x^3,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 56*sqrt( - c**2*x
**2 + 1)*a**2*c**2*x**2 - 12*sqrt( - c**2*x**2 + 1)*a**2 + 48*int((sqrt( -
 c**2*x**2 + 1)*acosh(c*x))/x**3,x)*a*b*x**2 - 96*int((sqrt( - c**2*x**2 +
 1)*acosh(c*x))/x,x)*a*b*c**2*x**2 + 24*int((sqrt( - c**2*x**2 + 1)*acosh(
c*x)**2)/x**3,x)*b**2*x**2 - 48*int((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)
/x,x)*b**2*c**2*x**2 + 48*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)*x,x)*a*b*c
**4*x**2 + 24*int(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*x,x)*b**2*c**4*x**2
 - 60*log(tan(asin(c*x)/2))*a**2*c**2*x**2 + 65*a**2*c**2*x**2))/(24*x**2)
```

3.182 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\,\text{arccosh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 616

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2}$$
$$+\frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x}+\frac{23b^2c^3d^2\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{12\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{5bc^5d^2x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{7}{3}bc^3d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))$$
$$-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3x^2}$$
$$+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2+\frac{7c^3d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5c^2d(d-c^2dx^2)^{3/2}(a+b}{3x}$$

output
```
7/12*b^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+1/3*b^2*c^2*d^2*(-c*x+1)*(c*x+1)*(
-c^2*d*x^2+d)^(1/2)/x+23/12*b^2*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/2*b*c^5*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2)+7/3*b*c^3*d^2*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))-1/3*b*c*d^2*(c*x-1)^(3/2)*
(c*x+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/x^2+5/2*c^4*d^2*x*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2+7/3*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccosh(c*x))^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/3*c^2*d*(-c^2*d*x^2+d)^(
3/2)*(a+b*arccosh(c*x))^2/x-1/3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/
x^3-5/6*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-14/3*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*ln(1+
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/3*b^2*c
^3*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.61 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.30

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \frac{-8abcd^3x+8abc^2d^3x^2-8a^2d^3\sqrt{\frac{-1+cx}{1+cx}}+64a^2c^2d^3x^2\sqrt{\frac{-1+cx}{1+cx}}\ -$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

output
```
(-8*a*b*c*d^3*x + 8*a*b*c^2*d^3*x^2 - 8*a^2*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]
 + 64*a^2*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*b^2*c^2*d^3*x^2*Sqrt[
(-1 + c*x)/(1 + c*x)] - 44*a^2*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 8*
b^2*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 12*a^2*c^6*d^3*x^6*Sqrt[(-1 +
 c*x)/(1 + c*x)] + 20*b^2*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 60*a^2*c
^3*d^(5/2)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 6*a*b*c^3*d^3*x^3*Cosh[2*
ArcCosh[c*x]] + 6*a*b*c^4*d^3*x^4*Cosh[2*ArcCosh[c*x]] - 112*a*b*c^3*d^3*x
^3*Log[c*x] + 112*a*b*c^4*d^3*x^4*Log[c*x] - 56*b^2*c^3*d^3*x^3*(-1 + c*x)
*PolyLog[2, -E^(-2*ArcCosh[c*x])] + 3*b^2*c^3*d^3*x^3*Sinh[2*ArcCosh[c*x]]
 - 3*b^2*c^4*d^3*x^4*Sinh[2*ArcCosh[c*x]] + 2*b*d^3*(-1 + c*x)*ArcCosh[c*x
]*(4*b*c*x + 8*a*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*a*c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)] - 56*a*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 56*a*c^3*x^3*Sqrt[(-1 +
 c*x)/(1 + c*x)] + 3*b*c^3*x^3*Cosh[2*ArcCosh[c*x]] + 56*b*c^3*x^3*Log[1 +
 E^(-2*ArcCosh[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcCosh[c*x]]) - 2*b*d^3*(-1 +
c*x)*ArcCosh[c*x]^2*(-30*a*c^3*x^3 + 4*b*(-Sqrt[(-1 + c*x)/(1 + c*x)] - c*
x*Sqrt[(-1 + c*x)/(1 + c*x)] + 7*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 7*c^
3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)])) + 3*b*c^3*x^3*Sinh[2*ArcCosh[c*x]
]))/(24*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \text{6343}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6327}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{x^3}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-$$
$$\frac{5}{3}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6335}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{2}bc\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2}dx-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{2x^2}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-$$
$$\frac{5}{3}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{108}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{2}bc\left(\int 3c^2\sqrt{cx-1}\sqrt{cx+1}dx-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)-\frac{(1-\ldots}{}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{5}{3}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{2}bc\left(3c^2\int \sqrt{cx-1}\sqrt{cx+1}dx-\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x}\right)-\frac{(1-\ldots}{}\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{5}{3}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{40}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int \frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx+\frac{1}{2}bc\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)-\frac{(c\ldots}{}\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{5}{3}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{43}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx-\frac{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}{2x^2}+\frac{1}{2}bc\left(3c^2\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}\right.\right.\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 6334

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{2}(1-c^2x^2)(a+b\text{arccosh}(cx))\right)-\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 40

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{2}(1-c^2x^2)(a\right.\right.}{3\sqrt{cx-}}$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 43

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}(1-c^2x^2)(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{arc}}{}\right.\right.\right.}{3\sqrt{cx-1}\sqrt{}}$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 6297

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{\int-\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}(1-c^2x^2)(a+\text{barc}\right.\right.$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 25

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-\frac{\int(a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arccos}\right.\right.\right.}{\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-\frac{\int-i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{ar}\right.\right.\right.}{\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}}$$

$$\Big\downarrow \text{26}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\int(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\text{arcco}\right.\right.\right.}{\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\int\frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{b}+\frac{1}{2}\left(1-c^2x^2\right)\right.\right.}{\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i\right.}{b}\right.\right.}{\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}}$$

$\Big\downarrow$ 2715

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccos}\right.\right.}{b}\right.\right.}{}$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2838

$$-\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccos}\right.}{b}\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 6343

$$-\frac{5}{3}c^2d\left(-3c^2d\int\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\int-\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d}{}\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccos}\right.}{b}\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 25

$$-\frac{5}{3}c^2d\left(-3c^2d\int\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-}{}\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccos}\right.}{b}\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 6310

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\text{arccosh}(cx))dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx\right.\right.$$

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}\right.}{b}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 6298

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(c}{\sqrt{cx-1}\sqrt{cx+}}}{2\sqrt{cx-1}\sqrt{cx+1}}\right.\right.$$

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}\right.}{b}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 101

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2d}}{2\sqrt{}}\right.\right.$$

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}\right.}{b}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 43

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-3c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}}{b}\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 6308

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-cx)(cx+1)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-3c^2d\left(-\frac{\sqrt{d-}}{}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}}{b}\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 6327

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\text{arccosh}(cx))}{x}dx}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}{x}-3c^2d\left(-\frac{\sqrt{d-c^2}}{6bc}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}}{b}\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\downarrow$ 6334

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{cx-1}\sqrt{cx+1}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}}{b}\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 40

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{1}{2}\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)+\frac{1}{2}(1-c^2x^2)\,(\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}\right.}{b}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 43

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\text{arccosh}(cx)}{x}dx+\frac{1}{2}(1-c^2x^2)\,(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(\frac{1}{2}x\sqrt{cx-1}\sqrt{cx+1}-\frac{\text{ar}}{}\right.\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}\right.}{b}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 6297

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{\int-\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{b}+\frac{1}{2}(1-c^2x^2)\,(a+b\text{arc}\right.}{\sqrt{cx-1}\sqrt{cx+1}}\right.$$
$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}i(a+b\text{arccosh}\right.}{b}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{3x^3}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.06

method	result
default	$-\frac{a^{2}\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{3 d\,x^{3}}+\frac{4 a^{2} c^{2}\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{3 d x}+\frac{4 a^{2} c^{4} x\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3}+\frac{5 a^{2} c^{4} d x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+\frac{5 a^{2} c^{4} d^{2} x\sqrt{-c^{2} d\,x^{2}+d}}{2}$
parts	$-\frac{a^{2}\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{3 d\,x^{3}}+\frac{4 a^{2} c^{2}\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{3 d x}+\frac{4 a^{2} c^{4} x\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3}+\frac{5 a^{2} c^{4} d x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+\frac{5 a^{2} c^{4} d^{2} x\sqrt{-c^{2} d\,x^{2}+d}}{2}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(7/2)+4/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(7/2)+4
/3*a^2*c^4*x*(-c^2*d*x^2+d)^(5/2)+5/3*a^2*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2
*a^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*a^2*c^4*d^3/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/12*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)/x^3*(-6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)^2*
x^4*c^4+6*arccosh(c*x)*c^5*x^5-3*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+10*ar
ccosh(c*x)^3*x^3*c^3-28*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2
-28*arccosh(c*x)^2*c^3*x^3+56*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2)*x^3*c^3-3*c^3*x^3*arccosh(c*x)+28*polylog(2,-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*x^3*c^3-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+4*c^3*x^3
+4*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x*arccosh(c*x))*d^2-1/12
*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/x^3*(-12*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4+6*c^5*x^5+30*arccosh(c*x)^2*c^3*x^
3-56*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2-56*c^3*x^3*arccosh(c
*x)+56*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3-3*c^3*x^3+8*arcco
sh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*d^2
```

Fricas [F]

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx=\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="fric
as")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^2}{x^4}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxi
ma")
```

output
```
1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
 + 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d
*x^2 + d)^(7/2)/(d*x^3))*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^4 + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}{x^4}\,dx = \frac{\sqrt{d}\,d^2\Big(15asin(cx)\,a^2c^3x^3+3\sqrt{-c^2x^2+1}\,a^2c^4x^4+14\sqrt{-c^2x^2}}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2/x^4,x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a**2*c**3*x**3 + 3*sqrt( - c**2*x**2 + 1)*a**2
*c**4*x**4 + 14*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( - c**2*x**
2 + 1)*a**2 + 12*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**4,x)*a*b*x**3
- 24*int((sqrt( - c**2*x**2 + 1)*acosh(c*x))/x**2,x)*a*b*c**2*x**3 + 6*int
((sqrt( - c**2*x**2 + 1)*acosh(c*x)**2)/x**4,x)*b**2*x**3 - 12*int((sqrt(
- c**2*x**2 + 1)*acosh(c*x)**2)/x**2,x)*b**2*c**2*x**3 + 12*int(sqrt( - c*
*2*x**2 + 1)*acosh(c*x),x)*a*b*c**4*x**3 + 6*int(sqrt( - c**2*x**2 + 1)*ac
osh(c*x)**2,x)*b**2*c**4*x**3))/(6*x**3)
```

3.183 $\int \frac{x^5(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 365

$$\begin{aligned}\int \frac{x^5(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = &-\frac{4144b^2\sqrt{d-c^2dx^2}}{3375c^6d} \\ &-\frac{272b^2x^2\sqrt{d-c^2dx^2}}{3375c^4d}-\frac{2b^2x^4\sqrt{d-c^2dx^2}}{125c^2d} \\ &+\frac{16bx\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{15c^5d\sqrt{-1+cx}\sqrt{1+cx}} \\ &+\frac{8bx^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{45c^3d\sqrt{-1+cx}\sqrt{1+cx}} \\ &+\frac{2bx^5\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{25cd\sqrt{-1+cx}\sqrt{1+cx}} \\ &-\frac{8\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{15c^6d} \\ &-\frac{4x^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{15c^4d} \\ &-\frac{x^4\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{5c^2d}\end{aligned}$$

output
```
-4144/3375*b^2*(-c^2*d*x^2+d)^(1/2)/c^6/d-272/3375*b^2*x^2*(-c^2*d*x^2+d)^
(1/2)/c^4/d-2/125*b^2*x^4*(-c^2*d*x^2+d)^(1/2)/c^2/d+16/15*b*x*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccosh(c*x))/c^5/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/45*b*x^3*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^3/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
2/25*b*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/d/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-8/15*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^6/d-4/15*x^2*(-c
^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^4/d-1/5*x^4*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccosh(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.70

$$\int \frac{x^5(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{\sqrt{d-c^2dx^2}\left(30abcx\sqrt{-1+cx}\sqrt{1+cx}(120+20c^2x^2+9c^4x^4)-225a^2(-8+4c^2x^2+c^4x^4+3c^6x^6)-2\right.}{}$$

input
```
Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(30*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(120 + 20*c^
2*x^2 + 9*c^4*x^4) - 225*a^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 2*b^
2*(-2072 + 1936*c^2*x^2 + 109*c^4*x^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(120 + 20*c^2*x^2 + 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2
 + c^4*x^4 + 3*c^6*x^6))*ArcCosh[c*x] - 225*b^2*(-8 + 4*c^2*x^2 + c^4*x^4
+ 3*c^6*x^6)*ArcCosh[c*x]^2))/(3375*c^6*d*(-1 + c*x)*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {6353, 6298, 111, 27, 111, 27, 83, 6353, 6298, 111, 27, 83, 6329, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6353}$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int x^4(a+b\text{arccosh}(cx))dx}{5c\sqrt{d-c^2dx^2}}+\frac{4\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{6298}$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\int \frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{5c\sqrt{d-c^2dx^2}}+\frac{4\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{111}$$

$$\frac{4\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{\int \frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{111}$$

$$\frac{4\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{4\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{83}$$

$$\frac{4\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6353}$$

$$\frac{4\left(-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int x^2(a+b\text{arccosh}(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6298}$$

$$\frac{4\left(\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}\right)}{5c^2}$$

$$\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}-$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 111

$$\frac{4\left(\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(\ldots}{\ldots}\right.}{5c^2}$$

$$\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}-$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 27

$$\frac{4\left(\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx\ldots}}{\ldots}\right.}{5c^2}$$

$$\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}-$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 83

$$\frac{4\left(\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\cdots}{\cdots}\right.\right.}{3c\sqrt{d-c^2dx^2}}\cdots\right.}{5c^2}$$

$$\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}-$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)\Bigg)}{5c\sqrt{d-c^2dx^2}}$$

$$\downarrow \text{6329}$$

$$\frac{4\left(\frac{2\left(-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}\cdots\right.}{\cdots}\right.}{5c^2}$$

$$\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}-$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)\Bigg)}{5c\sqrt{d-c^2dx^2}}$$

$$\downarrow \text{2009}$$

$$-\frac{x^4\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{5c^2d}+$$

$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2\cdots}{\cdots}\right.}{5c^2}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{5}x^5(a+b\text{arccosh}(cx))-\frac{1}{5}bc\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\right)\Bigg)}{5c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/5*(x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d) - (2*b*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(-1/5*(b*c*((x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*
c^2) + (4*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(3*c^2)))/(5*c^2))) + (x^5*(a + b*ArcCosh[c*x]))/5))/(5*c*S
qrt[d - c^2*d*x^2]) + (4*(-1/3*(x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x
])^2)/(c^2*d) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1/3*(b*c*((2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2))
) + (x^3*(a + b*ArcCosh[c*x]))/3))/(3*c*Sqrt[d - c^2*d*x^2]) + (2*(-((Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d)) - (2*b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]))/
(c*Sqrt[d - c^2*d*x^2])))/(3*c^2)))/(5*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.54

method	result
orering	$\frac{(1647c^8x^8+1684c^6x^6+34306c^4x^4-102032c^2x^2+62160)(a+b\operatorname{arccosh}(cx))^2}{3375c^8x^2\sqrt{-c^2dx^2+d}} - \frac{2(cx-1)(cx+1)(162c^6x^6+491c^4x^4+7472c^2x^2-1}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3375*(1647*c^8*x^8+1684*c^6*x^6+34306*c^4*x^4-102032*c^2*x^2+62160)/c^8/
x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2)-2/3375*(c*x-1)*(c*x+1)*(162*
c^6*x^6+491*c^4*x^4+7472*c^2*x^2-10360)/x^6/c^8*(5*x^4*(a+b*arccosh(c*x))^
2/(-c^2*d*x^2+d)^(1/2)+2*x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c/(
c*x-1)^(1/2)/(c*x+1)^(1/2)+x^6*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2)*c
^2*d)+1/3375*(27*c^4*x^4+136*c^2*x^2+2072)/c^8*(c*x+1)^2*(c*x-1)^2/x^5*(20
*x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2)+20*x^4*(a+b*arccosh(c*x))/(
-c^2*d*x^2+d)^(1/2)*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11*x^5*(a+b*arccosh(c*
x))^2/(-c^2*d*x^2+d)^(3/2)*c^2*d+2*x^5*b^2*c^2/(c*x-1)/(c*x+1)/(-c^2*d*x^2
+d)^(1/2)+4*x^6*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2)*b*c^3/(c*x-1)^(1/2
)/(c*x+1)^(1/2)*d-x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c^2/(c*x-1
)^(3/2)/(c*x+1)^(1/2)-x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c^2/(c
*x-1)^(1/2)/(c*x+1)^(3/2)+3*x^7*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2)*
c^4*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.95

$$\int \frac{x^5(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx =$$
$$-\frac{225\,(3\,b^2c^6x^6+b^2c^4x^4+4\,b^2c^2x^2-8\,b^2)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\,(9\,abc^5x^5+20\,abc^3x^3$$

input
```
integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
-1/3375*(225*(3*b^2*c^6*x^6 + b^2*c^4*x^4 + 4*b^2*c^2*x^2 - 8*b^2)*sqrt(-c
^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 + 20*a*b*
c^3*x^3 + 120*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2
*c^5*x^5 + 20*b^2*c^3*x^3 + 120*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2
 - 1) - 15*(3*a*b*c^6*x^6 + a*b*c^4*x^4 + 4*a*b*c^2*x^2 - 8*a*b)*sqrt(-c^2
*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 +
 (225*a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 + 968*b^2)*c^2*x^2 - 1800*a^2 -
4144*b^2)*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

Sympy [F]

$$\int \frac{x^5(a + b\operatorname{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x^5(a + b\operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \frac{x^5(a + b\operatorname{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx \\
&= -\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d} + \frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d} + \frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)b^2\operatorname{arcosh}(cx)^2 \\
&\quad - \frac{2}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d} + \frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d} + \frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)ab\operatorname{arcosh}(cx) \\
&\quad - \frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d} + \frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d} + \frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)a^2 \\
&\quad - \frac{2}{3375}b^2\left(\frac{27\sqrt{c^2x^2-1}c^2\sqrt{-d}x^4 + 136\sqrt{c^2x^2-1}\sqrt{-d}x^2 + \frac{2072\sqrt{c^2x^2-1}\sqrt{-d}}{c^2}}{c^4d} - \frac{15\left(9c^4\sqrt{-d}x^5 + 20c^2\sqrt{\right.}}{}\right. \\
&\quad + \frac{2\left(9c^4\sqrt{-d}x^5 + 20c^2\sqrt{-d}x^3 + 120\sqrt{-d}x\right)ab}{225c^5d}
\end{aligned}$$

input
```
integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^
4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b^2*arccosh(c*x)^2 - 2/15*(3*sqrt(-
c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-
c^2*d*x^2 + d)/(c^6*d))*a*b*arccosh(c*x) - 1/15*(3*sqrt(-c^2*d*x^2 + d)*x^
4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c
^6*d))*a^2 - 2/3375*b^2*((27*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*x^4 + 136*sqrt
(c^2*x^2 - 1)*sqrt(-d)*x^2 + 2072*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/(c^4*d)
- 15*(9*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*sqrt(-d)*x)*arccosh(c
*x)/(c^5*d)) + 2/225*(9*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*sqrt(
-d)*x)*a*b/(c^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^5\,(a+b\,\text{acosh}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{-3\sqrt{-c^2x^2+1}\,a^2c^4x^4 - 4\sqrt{-c^2x^2+1}\,a^2c^2x^2 - 8\sqrt{-c^2x^2+1}\,a^2 + 30\left(\int \frac{acosh(cx)x^5}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^6 + 15\left(\int \frac{acos}{\sqrt{-}}\right.}{15\sqrt{d}\,c^6}$$

input
```
int(x^5*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a**
2*c**2*x**2 - 8*sqrt( - c**2*x**2 + 1)*a**2 + 30*int((acosh(c*x)*x**5)/sqr
t( - c**2*x**2 + 1),x)*a*b*c**6 + 15*int((acosh(c*x)**2*x**5)/sqrt( - c**2
*x**2 + 1),x)*b**2*c**6)/(15*sqrt(d)*c**6)
```

3.184 $\int \frac{x^4(a+b\,\mathbf{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 351

$$\begin{aligned}\int \frac{x^4(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx =& -\frac{15b^2x\sqrt{d-c^2dx^2}}{64c^4d}-\frac{b^2x^3\sqrt{d-c^2dx^2}}{32c^2d}\\&-\frac{15b^2\sqrt{d-c^2dx^2}\operatorname{arccosh}(cx)}{64c^5d\sqrt{-1+cx}\sqrt{1+cx}}\\&+\frac{3bx^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{8c^3d\sqrt{-1+cx}\sqrt{1+cx}}\\&+\frac{bx^4\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{8cd\sqrt{-1+cx}\sqrt{1+cx}}\\&-\frac{3x\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{8c^4d}\\&-\frac{x^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{4c^2d}\\&-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^3}{8bc^5d\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-15/64*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/32*b^2*x^3*(-c^2*d*x^2+d)^(1/2)/
c^2/d-15/64*b^2*(-c^2*d*x^2+d)^(1/2)*arccosh(c*x)/c^5/d/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+3/8*b*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^3/d/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+1/8*b*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/d/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/8*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^
2/c^4/d-1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^2/d-1/8*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/c^5/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.51 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.84

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{32a^2c\sqrt{d}x(-1+c^2x^2)(3+2c^2x^2)-96a^2\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+b^2\sqrt{d}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\,(32a}{}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(32*a^2*c*Sqrt[d]*x*(-1 + c^2*x^2)*(3 + 2*c^2*x^2) - 96*a^2*Sqrt[d - c^2*d
*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b^2*Sqr
t[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(32*ArcCosh[c*x]^3 - 4*ArcCosh[c
*x]*(16*Cosh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]]) + 32*Sinh[2*ArcCosh[c
*x]] + Sinh[4*ArcCosh[c*x]] + 8*ArcCosh[c*x]^2*(8*Sinh[2*ArcCosh[c*x]] + S
inh[4*ArcCosh[c*x]])) - 4*a*b*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*(16*Cosh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*(6*ArcCo
sh[c*x] + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x]])))/(256*c^5*Sqrt[d
]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {6353, 6298, 111, 27, 101, 43, 6353, 6298, 101, 43, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int x^3(a+b\text{arccosh}(cx))dx}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\int \frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{111}$$

$$\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{\int \frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{101}$$

$$\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{43}$$

$$\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6353}$$

$$\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int x(a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6298}$$

$$\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 101

$$\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 43

$$\frac{3\left(\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6307

$$-\frac{x^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{4c^2d}+\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}\right)}{4c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/4*(x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d) - (b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*((x^4*(a + b*ArcCosh[c*x]))/4 - (b*c*((x^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(4*c^2) + (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2
) + ArcCosh[c*x]/(2*c^3)))/(4*c^2)))/4))/(2*c*Sqrt[d - c^2*d*x^2]) + (3*(-
1/2*(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d) + (Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) - (
b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((x^2*(a + b*ArcCosh[c*x]))/2 - (b*c*((x*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/2))/(c*Sqrt[d
 - c^2*d*x^2])))/(4*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. $2(303) = 606$.

Time = 0.47 (sec) , antiderivative size = 1092, normalized size of antiderivative = 3.11

method	result	size
default	Expression too large to display	1092
parts	Expression too large to display	1092

input
```
int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(-c^2*d*x^2+d)^(1/
2)+3/8*a^2/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+
b^2*(-1/8*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^
2-1)*arccosh(c*x)^3-1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c
^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2
*x^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)/d/c^
5/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*ar
ccosh(c*x)+1)/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*
arccosh(c*x)^2+2*arccosh(c*x)+1)/d/c^5/(c^2*x^2-1)-1/512*(-d*(c^2*x^2-1))^
(1/2)*(-8*c^4*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c^5*x^5+8*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*c^2*x^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(8*arcco
sh(c*x)^2+4*arccosh(c*x)+1)/d/c^5/(c^2*x^2-1))+2*a*b*(-3/16*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1)*arccosh(c*x)^2-1/25
6*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*c^4*x^4*(c*x-1)^(1/2)*(c*
x+1)^(1/2)+4*c*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*(-1+4*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2
)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*
x+1)^(1/2))*(-1+2*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))...
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt
(-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^4(a+b\,\text{acosh}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/8*a^2*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c
^4*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + integrate(b^2*x^4*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))^2/sqrt(-c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x^4/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^4\,(a+b\,\text{acosh}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{3asin(cx)\,a^2-2\sqrt{-c^2x^2+1}\,a^2c^3x^3-3\sqrt{-c^2x^2+1}\,a^2cx+16\left(\int \frac{acosh(cx)x^4}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^5+8\left(\int \frac{acosh(cx)^2x^4}{\sqrt{-c^2x^2+1}}dx\right.}{8\sqrt{d}\,c^5}$$

input
```
int(x^4*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(3*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 - 3*sqrt( - c*
*2*x**2 + 1)*a**2*c*x + 16*int((acosh(c*x)*x**4)/sqrt( - c**2*x**2 + 1),x)
*a*b*c**5 + 8*int((acosh(c*x)**2*x**4)/sqrt( - c**2*x**2 + 1),x)*b**2*c**5
)/(8*sqrt(d)*c**5)
```

3.185 $\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 241

$$\begin{aligned}\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx =& -\frac{40b^2\sqrt{d-c^2dx^2}}{27c^4d}-\frac{2b^2x^2\sqrt{d-c^2dx^2}}{27c^2d}\\ &+\frac{4bx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{3c^3d\sqrt{-1+cx}\sqrt{1+cx}}\\ &+\frac{2bx^3\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{9cd\sqrt{-1+cx}\sqrt{1+cx}}\\ &-\frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^4d}\\ &-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}\end{aligned}$$

output

```
-40/27*b^2*(-c^2*d*x^2+d)^(1/2)/c^4/d-2/27*b^2*x^2*(-c^2*d*x^2+d)^(1/2)/c^
2/d+4/3*b*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c^3/d/(c*x-1)^(1/2)/(c
*x+1)^(1/2)+2/9*b*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/c/d/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-2/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^4/d-1/
3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.83

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{\sqrt{d-c^2dx^2}\left(6abcx\sqrt{-1+cx}\sqrt{1+cx}(6+c^2x^2)-9a^2(-2+c^2x^2+c^4x^4)-2b^2(-20+19c^2x^2+c^4x^4)+\right.}{27c^4d(}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2)
 - 9*a^2*(-2 + c^2*x^2 + c^4*x^4) - 2*b^2*(-20 + 19*c^2*x^2 + c^4*x^4) + 6
*b*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3*a*(-2 + c^2*x^2 +
 c^4*x^4))*ArcCosh[c*x] - 9*b^2*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]^2))/
(27*c^4*d*(-1 + c*x)*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {6353, 6298, 111, 27, 83, 6329, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \text{6353}$$
$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int x^2(a+b\text{arccosh}(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}$$
$$\Big\downarrow \text{6298}$$

$$\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{111}$$

$$\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{83}$$

$$\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6329}$$

$$\frac{2\left(-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{3}x^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\right)}{3c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d) - (2*b*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(-1/3*(b*c*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^
4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2))) + (x^3*(a + b*ArcCosh[c*
x]))/3))/(3*c*Sqrt[d - c^2*d*x^2]) + (2*(-((Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])^2)/(c^2*d)) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a*x - (b*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/
(3*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(209) = 418.

Time = 0.59 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.21

method	result
orering	$\frac{(19c^6x^6+100c^4x^4-380c^2x^2+240)(a+b\ \mathrm{arccosh}(cx))^2}{27c^6x^2\sqrt{-c^2d\,x^2+d}}-\frac{2(cx-1)(cx+1)(c^4x^4+12c^2x^2-20)\left(\frac{3x^2(a+b\ \mathrm{arccosh}(cx))^2}{\sqrt{-c^2d\,x^2+d}}+\frac{2x^3(a+b}{\sqrt{-c^2d\,x^2-}}\right.}{9c^6x^4}$
default	$a^2\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx}{216c^4d(c^2x^2-1)}\right.$
parts	$a^2\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(4c^4x^4-5c^2x^2+4c^3x^3\sqrt{cx-1}\,\sqrt{cx+1}-3\sqrt{cx-1}\,\sqrt{cx+1}\,cx}{216c^4d(c^2x^2-1)}\right.$

input
```
int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*(19*c^6*x^6+100*c^4*x^4-380*c^2*x^2+240)/c^6/x^2*(a+b*arccosh(c*x))^2
/(-c^2*d*x^2+d)^(1/2)-2/9*(c*x-1)*(c*x+1)*(c^4*x^4+12*c^2*x^2-20)/c^6/x^4*
(3*x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2)+2*x^3*(a+b*arccosh(c*x))/
(-c^2*d*x^2+d)^(1/2)*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+x^4*(a+b*arccosh(c*x)
)^2/(-c^2*d*x^2+d)^(3/2)*c^2*d)+1/27*(c^2*x^2+20)/c^6*(c*x+1)^2*(c*x-1)^2/
x^3*(6*x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2)+12*x^2*(a+b*arccosh(c*x
))/(-c^2*d*x^2+d)^(1/2)*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+7*x^3*(a+b*arccosh
(c*x))^2/(-c^2*d*x^2+d)^(3/2)*c^2*d+2*x^3*b^2*c^2/(c*x-1)/(c*x+1)/(-c^2*d*
x^2+d)^(1/2)+4*x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2)*b*c^3/(c*x-1)^(
1/2)/(c*x+1)^(1/2)*d-x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c^2/(c*
x-1)^(3/2)/(c*x+1)^(1/2)-x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c^2
/(c*x-1)^(1/2)/(c*x+1)^(3/2)+3*x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/
2)*c^4*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.17

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx =$$
$$-\frac{9\,(b^2c^4x^4+b^2c^2x^2-2\,b^2)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\,(abc^3x^3+6\,abcx)\sqrt{-c^2dx^2+d}\sqrt{c^2}}{}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
-1/27*(9*(b^2*c^4*x^4 + b^2*c^2*x^2 - 2*b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x
+ sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 + 6*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 + 6*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 3*(a*b*c^4*x^4 + a*b*c^2*x^2 - 2*a*b)*sqrt(-c^2*d*x^2 + d)
)*log(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + (9*a^2 + 38*b^
2)*c^2*x^2 - 18*a^2 - 40*b^2)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^3(a+b\operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int \frac{x^3(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx \\
&= -\frac{1}{3}b^2\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\sqrt{-c^2dx^2+d}}{c^4d}\right)\operatorname{arcosh}(cx)^2 \\
&\quad -\frac{2}{3}ab\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\sqrt{-c^2dx^2+d}}{c^4d}\right)\operatorname{arcosh}(cx) \\
&\quad -\frac{1}{3}a^2\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\sqrt{-c^2dx^2+d}}{c^4d}\right) \\
&\quad -\frac{2}{27}b^2\left(\frac{\sqrt{c^2x^2-1}\sqrt{-d}x^2+\frac{20\sqrt{c^2x^2-1}\sqrt{-d}}{c^2}}{c^2d}-\frac{3\left(c^2\sqrt{-d}x^3+6\sqrt{-d}x\right)\operatorname{arcosh}(cx)}{c^3d}\right) \\
&\quad +\frac{2\left(c^2\sqrt{-d}x^3+6\sqrt{-d}x\right)ab}{9\,c^3d}
\end{aligned}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/3*b^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d
))*arccosh(c*x)^2 - 2/3*a*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^
2*d*x^2 + d)/(c^4*d))*arccosh(c*x) - 1/3*a^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^
2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)) - 2/27*b^2*((sqrt(c^2*x^2 - 1)*sqrt
(-d)*x^2 + 20*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/(c^2*d) - 3*(c^2*sqrt(-d)*x^
3 + 6*sqrt(-d)*x)*arccosh(c*x)/(c^3*d)) + 2/9*(c^2*sqrt(-d)*x^3 + 6*sqrt(-
d)*x)*a*b/(c^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\, dx = \int \frac{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\, dx$$

input
```
int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a + b\operatorname{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx$$
$$= \frac{-\sqrt{-c^2x^2+1}\,a^2c^2x^2 - 2\sqrt{-c^2x^2+1}\,a^2 + 6\left(\int \frac{acosh(cx)x^3}{\sqrt{-c^2x^2+1}}dx\right) ab\,c^4 + 3\left(\int \frac{acosh(cx)^2x^3}{\sqrt{-c^2x^2+1}}dx\right) b^2c^4}{3\sqrt{d}\,c^4}$$

input
```
int(x^3*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a**2
+ 6*int((acosh(c*x)*x**3)/sqrt( - c**2*x**2 + 1),x)*a*b*c**4 + 3*int((acos
h(c*x)**2*x**3)/sqrt( - c**2*x**2 + 1),x)*b**2*c**4)/(3*sqrt(d)*c**4)
```

3.186 $\int \frac{x^2(a+b\,\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 227

$$\begin{aligned}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = & -\frac{b^2x\sqrt{d-c^2dx^2}}{4c^2d}-\frac{b^2\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{4c^3d\sqrt{-1+cx}\sqrt{1+cx}} \\ & +\frac{bx^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{2cd\sqrt{-1+cx}\sqrt{1+cx}} \\ & -\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d} \\ & -\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{6bc^3d\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output

```
-1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2/d-1/4*b^2*(-c^2*d*x^2+d)^(1/2)*arccosh
(c*x)/c^3/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*x^2*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccosh(c*x))/c/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*x*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccosh(c*x))^2/c^2/d-1/6*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/
b/c^3/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$

$$= \frac{-\frac{12a^2cx\sqrt{d-c^2dx^2}}{d}-\frac{12a^2\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}}+\frac{b^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(4\text{arccosh}(cx)^3-6\text{arccosh}(cx)\cosh(2\text{arccosh}(cx))+\left(3+\right.\right.}{\sqrt{d-c^2dx^2}}}{}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
((-12*a^2*c*x*Sqrt[d - c^2*d*x^2])/d - (12*a^2*ArcTan[(c*x*Sqrt[d - c^2*d*
x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b^2*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*(4*ArcCosh[c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 +
6*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (6*a*b*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(
ArcCosh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d - c^2*d*x^2])/(24*c^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {6353, 6298, 101, 43, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{6353}$$

$$-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int x(a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{6298}$$

$$-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \textbf{101}$$

$$-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \textbf{43}$$

$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \textbf{6307}$$

$$-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/2*(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d) + (Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) -
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((x^2*(a + b*ArcCosh[c*x]))/2 - (b*c*((x*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/2))/(c*Sqrt[
d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(195) = 390$.

Time = 0.36 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.48

method	result
default	$-\frac{a^2x\sqrt{-c^2dx^2+d}}{2c^2d}+\frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^3}{6d\,c^3(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3\right.$
parts	$-\frac{a^2x\sqrt{-c^2dx^2+d}}{2c^2d}+\frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^3}{6d\,c^3(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,(2c^3\right.$

input
```
int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a^2/c^2/(c^2*d)^(1/2)*arctan((c^
2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^3-1/16*(-d*(c^2*x^2-1
))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1
/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*arccosh(c*x)+1)/d/c^3/(c^2*x^2-1)-1
/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x
^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)/
d/c^3/(c^2*x^2-1))+2*a*b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^2-1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^
3*x^3-2*c*x+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1
/2))*(-1+2*arccosh(c*x))/d/c^3/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-2
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)
-2*c*x)*(1+2*arccosh(c*x))/d/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt
(-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^2(a+b\operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/2*a^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + in
tegrate(b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/sqrt(-c^2*d*x^2 +
 d) + 2*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d
), x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^2\,(a+b\,\text{acosh}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{asin(cx)\,a^2-\sqrt{-c^2x^2+1}\,a^2cx+4\left(\int \frac{acosh(cx)x^2}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^3+2\left(\int \frac{acosh(cx)^2x^2}{\sqrt{-c^2x^2+1}}dx\right)b^2c^3}{2\sqrt{d}\,c^3}$$

input
```
int(x^2*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output

```
(asin(c*x)*a**2 - sqrt( - c**2*x**2 + 1)*a**2*c*x + 4*int((acosh(c*x)*x**2
)/sqrt( - c**2*x**2 + 1),x)*a*b*c**3 + 2*int((acosh(c*x)**2*x**2)/sqrt( -
c**2*x**2 + 1),x)*b**2*c**3)/(2*sqrt(d)*c**3)
```

3.187 $\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = -\frac{2b^2\sqrt{d-c^2dx^2}}{c^2d}+\frac{2bx\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))}{cd\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}$$

output
```
-2*b^2*(-c^2*d*x^2+d)^(1/2)/c^2/d+2*b*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(
c*x))/c/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*
x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.34

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(2abcx\sqrt{-1+cx}\sqrt{1+cx}+a^2(1-c^2x^2)-2b^2(-1+c^2x^2)+2b\left(a-ac^2x^2+bcx\sqrt{-1+cx}\right.\right.}{c^2d(-1+cx)(1+cx)}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(2*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + a^2*(1 - c^
2*x^2) - 2*b^2*(-1 + c^2*x^2) + 2*b*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])*ArcCosh[c*x] + b^2*(1 - c^2*x^2)*ArcCosh[c*x]^2))/(c^2*d*(-
1 + c*x)*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6329, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{ 6329}$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d)) - (2*b*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[
c*x]))/(c*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(101) = 202$.

Time = 0.46 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.83

method	result
default	$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{c^2d}+b^2\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(\operatorname{arccosh}(cx)^2-2\operatorname{arccosh}(cx)+2\right)}{2c^2d(c^2x^2-1)}-\sqrt{-d(c^2x^2-1)}$
parts	$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{c^2d}+b^2\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(\operatorname{arccosh}(cx)^2-2\operatorname{arccosh}(cx)+2\right)}{2c^2d(c^2x^2-1)}-\sqrt{-d(c^2x^2-1)}$
orering	$\frac{(c^4x^4-4c^2x^2+2)(a+b\operatorname{arccosh}(cx))^2}{c^4x^2\sqrt{-c^2d\,x^2+d}}+\frac{2(cx-1)(cx+1)\Bigg(\frac{(a+b\operatorname{arccosh}(cx))^2}{\sqrt{-c^2d\,x^2+d}}+\frac{2x(a+b\operatorname{arccosh}(cx))bc}{\sqrt{-c^2d\,x^2+d}\,\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{x^2(a+b\operatorname{arccosh}(cx))^2c^2d}{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}}{c^4x^2}$

input
```
int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2/c^2/d*(-c^2*d*x^2+d)^(1/2)+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x-1)^
(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^2/d
/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+
c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/c^2/d/(c^2*x^2-1))+2*a*b*(-1/
2*(-d*(c^2*x^2-1))^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-1+a
rccosh(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x-1)^(1/2)*
(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(1+arccosh(c*x))/c^2/d/(c^2*x^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(101) = 202$.

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

$$\int \frac{x(a + b\text{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx$$
$$= \frac{2\sqrt{-c^2dx^2 + d}\sqrt{c^2x^2 - 1}abcx - (b^2c^2x^2 - b^2)\sqrt{-c^2dx^2 + d}\log\left(cx + \sqrt{c^2x^2 - 1}\right)^2 + 2\left(\sqrt{-c^2dx^2 + d}\sqrt{\right.}{c^4d}$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a*b*c*x - (b^2*c^2*x^2 - b^2)*sq
rt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*(sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1)*b^2*c*x - (a*b*c^2*x^2 - a*b)*sqrt(-c^2*d*x^2 + d))*lo
g(c*x + sqrt(c^2*x^2 - 1)) - ((a^2 + 2*b^2)*c^2*x^2 - a^2 - 2*b^2)*sqrt(-c
^2*d*x^2 + d))/(c^4*d*x^2 - c^2*d)
```

Sympy [F]

$$\int \frac{x(a + b\text{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x(a + b\,\text{acosh}\,(cx))^2}{\sqrt{-d\,(cx - 1)\,(cx + 1)}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = 2\,b^2\left(\frac{\sqrt{-d}x\,\text{arcosh}\,(cx)}{cd}-\frac{\sqrt{c^2x^2-1}\sqrt{-d}}{c^2d}\right)+\frac{2\,ab\sqrt{-d}x}{cd}-\frac{\sqrt{-c^2dx^2+d}b^2\,\text{arcosh}\,(cx)^2}{c^2d}-\frac{2\,\sqrt{-c^2dx^2+d}ab\,\text{arcosh}\,(cx)}{c^2d}-\frac{\sqrt{-c^2dx^2+d}a^2}{c^2d}$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
2*b^2*(sqrt(-d)*x*arccosh(c*x)/(c*d) - sqrt(c^2*x^2 - 1)*sqrt(-d)/(c^2*d))
 + 2*a*b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b^2*arccosh(c*x)^2/(c^2*d
) - 2*sqrt(-c^2*d*x^2 + d)*a*b*arccosh(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)
*a^2/(c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x\,(a+b\,\text{acosh}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a^2+2\left(\int \frac{acosh(cx)x}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^2+\left(\int \frac{acosh(cx)^2x}{\sqrt{-c^2x^2+1}}dx\right)b^2c^2}{\sqrt{d}\,c^2}$$

input
```
int(x*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a**2 + 2*int((acosh(c*x)*x)/sqrt( - c**2*x**2 +
 1),x)*a*b*c**2 + int((acosh(c*x)**2*x)/sqrt( - c**2*x**2 + 1),x)*b**2*c**
2)/(sqrt(d)*c**2)
```

3.188 $\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{3bcd\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/c/d/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^3}{3bc\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*
d*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx$$
$$\Big\downarrow \text{ 6307}$$
$$\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*
d*x^2])
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(48) = 96.

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.66

method	result
default	$\frac{a^2 \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d\, x^2+d}}\right)}{\sqrt{c^2 d}} - \frac{b^2 \sqrt{-d(cx-1)(cx+1)}\, \sqrt{cx-1}\, \sqrt{cx+1}\, \operatorname{arccosh}(cx)^3}{3d(c^2x^2-1)c} - \frac{ab\sqrt{-d(cx-1)(cx+1)}\, \sqrt{cx-1}\, \sqrt{cx+1}\, \operatorname{arccos}}{d(c^2x^2-1)c}$
parts	$\frac{a^2 \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d\, x^2+d}}\right)}{\sqrt{c^2 d}} - \frac{b^2 \sqrt{-d(cx-1)(cx+1)}\, \sqrt{cx-1}\, \sqrt{cx+1}\, \operatorname{arccosh}(cx)^3}{3d(c^2x^2-1)c} - \frac{ab\sqrt{-d(cx-1)(cx+1)}\, \sqrt{cx-1}\, \sqrt{cx+1}\, \operatorname{arccos}}{d(c^2x^2-1)c}$

input
```
int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/3*b^2*(-d
*(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)/c*arccos
h(c*x)^3-a*b*(-d*(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2
*x^2-1)/c*arccosh(c*x)^2
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\operatorname{acosh}(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
a^2*arcsin(c*x)/(c*sqrt(d)) + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1))^2/sqrt(-c^2*d*x^2 + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(cx))^2}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(a + b\,\text{acosh}(c\,x))^2}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{asin(cx)\,a^2+2\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}}dx\right)abc+\left(\int \frac{acosh(cx)^2}{\sqrt{-c^2x^2+1}}dx\right)b^2c}{\sqrt{d}\,c}$$

input
```
int((a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(asin(c*x)*a**2 + 2*int(acosh(c*x)/sqrt( - c**2*x**2 + 1),x)*a*b*c + int(a
cosh(c*x)**2/sqrt( - c**2*x**2 + 1),x)*b**2*c)/(sqrt(d)*c)
```

3.189 $\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 288

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx &= -\frac{2\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2\arctan\left(e^{\mathrm{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad+\frac{2ib\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad-\frac{2ib\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad-\frac{2ib^2\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(3,-ie^{\mathrm{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} \\ &\quad+\frac{2ib^2\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(3,ie^{\mathrm{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b^2*
(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \frac{a^2\log(cx)}{\sqrt{d}} - \frac{a^2\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{2iab\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\text{arccosh}(cx)\left(\log\left(1-ie^{-\text{arccosh}(cx)}\right)-\log\left(1+ie^{-\text{arccosh}(cx)}\right)\right)+\text{PolyLog}\left(2,-ie\right.\right.}{\sqrt{d-c^2dx^2}} + \frac{ib^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(-\text{arccosh}(cx)^2\left(\log\left(1-ie^{-\text{arccosh}(cx)}\right)-\log\left(1+ie^{-\text{arccosh}(cx)}\right)\right)-2\text{arccosh}(cx)\left(\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d
] - ((2*I)*a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 -
 I/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCos
h[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (I*b^2*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-(ArcCosh[c*x]^2*(Log[1 - I/E^ArcCosh[c*
x]] - Log[1 + I/E^ArcCosh[c*x]])) - 2*ArcCosh[c*x]*(PolyLog[2, (-I)/E^ArcC
osh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c*
x]] + 2*PolyLog[3, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.48, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6361, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{ 6361}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-2ib\int (a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int (a+b\text{arccosh}(cx))\log\left(1-\right.\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx\right.\right.}{}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+ba\right.\right.}{}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(2,-ie\right.\right.\right.}{\sqrt{d-}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c
*x]] + (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) +
b*PolyLog[3, (-I)*E^ArcCosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyL
og[2, I*E^ArcCosh[c*x]]) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/Sqrt[d - c^2*
d*x^2]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2}{x\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2}{\sqrt{-c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acosh}{(cx)})^2}{x\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}{(cx)}+a)^2}{\sqrt{-c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
-a^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) + int
egrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*
x) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x)
, x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}{(cx)}+a)^2}{\sqrt{-c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \frac{2\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x}dx\right)ab+\left(\int \frac{acosh(cx)^2}{\sqrt{-c^2x^2+1}\,x}dx\right)b^2+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a^2}{\sqrt{d}}$$

input
```
int((a+b*acosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(2*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*x),x)*a*b + int(acosh(c*x)**2/(s
qrt( - c**2*x**2 + 1)*x),x)*b**2 + log(tan(asin(c*x)/2))*a**2)/sqrt(d)
```

3.190 $\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 195

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx} - \frac{c\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{d\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\log\left(1+e^{2\text{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b^2c\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/d/x-c*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))^2/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+b^2*c*(-c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2)/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = -\frac{a^2\sqrt{-d\,(-1+c^2x^2)}}{dx} - 2abc\left(\frac{\sqrt{d-c^2dx^2}\text{arccosh}(cx)}{cdx} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(\log\left(-1+\sqrt{1+cx}\right)+\log\left(1+\sqrt{1+cx}\right)\right)}{\sqrt{d-c^2dx^2}}\right) + \frac{b^2c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\text{arccosh}(cx)\left(-\text{arccosh}(cx)+\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{arccosh}(cx)}{cx}-2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right.\right.}{\sqrt{-d(-1+cx)(1+cx)}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((a^2*Sqrt[-(d*(-1 + c^2*x^2))])/(d*x)) - 2*a*b*c*((Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x])/(c*d*x) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(Log[-1 + Sqrt[1 + c*
x]] + Log[1 + Sqrt[1 + c*x]]))/Sqrt[d - c^2*d*x^2]) + (b^2*c*Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(-ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*ArcCosh[c*x])/(c*x) - 2*Log[1 + E^(-2*ArcCosh[c*x])]) +
 PolyLog[2, -E^(-2*ArcCosh[c*x])]))/Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {6332, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \; 6332$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}$$

$$\Big\downarrow \text{6297}$$

$$-\frac{2c\sqrt{cx-1}\sqrt{cx+1}\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}$$

$$\Big\downarrow \text{25}$$

$$\frac{2c\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}+\frac{2c\sqrt{cx-1}\sqrt{cx+1}\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)d(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\left(2i\int \frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))-\frac{1}{2}i(a+b\text{arccosh}(cx))^2\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}(2i(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccos}}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-2\text{arccosh}(cx)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))\right)-\frac{1}{2}\right.}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(d*x)) - ((2*I)*c*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*((-1/2*I)*(a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a +
 b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*Ar
cCosh[c*x]])/4)))/Sqrt[d - c^2*d*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(201) = 402.

Time = 0.58 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.44

method	result
default	$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\,\mathrm{arccosh}(cx)^2}{x(c^2x^2-1)d}-\frac{2\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arc}}{d(c^2x^2-1)}\right.$
parts	$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\,\mathrm{arccosh}(cx)^2}{x(c^2x^2-1)d}-\frac{2\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arc}}{d(c^2x^2-1)}\right.$

input
```
int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(1/2)+b^2*(-(-d*(c^2*x^2-1))^(1/2)*(-(c*x-1)^(1/2)
*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*arccosh(c*x)^2/x/(c^2*x^2-1)/d-2*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*c+2*(
-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*
x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+(-d*(c^2*x^2-1))^(1/2)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2)*c)+2*a*b*(-2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/(c^2*x^2-1)*arccosh(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(-(c*x-1)^(1/2)*(c*x
+1)^(1/2)*c*x+c^2*x^2-1)*arccosh(c*x)/x/(c^2*x^2-1)/d+(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)*c)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\operatorname{acosh}(cx))^2}{x^2\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
-(c^2*d*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d)*sq
rt(d)*log(-2*c^2*d + 2*d/x^2))*a*b*c/d + b^2*integrate(log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x^2), x) - 2*sqrt(-c^2*d*x^2 +
 d)*a*b*arccosh(c*x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a^2/(d*x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a^2+2\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x^2}dx\right)abx+\left(\int \frac{acosh(cx)^2}{\sqrt{-c^2x^2+1}\,x^2}dx\right)b^2x}{\sqrt{d}\,x}$$

input
```
int((a+b*acosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a**2 + 2*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)
*x**2),x)*a*b*x + int(acosh(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**2),x)*b**2*
x)/(sqrt(d)*x)
```

3.191 $\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 452

$$\begin{aligned}
\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx
&= -\frac{bc\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{dx\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{2dx^2}\\
&\quad-\frac{c^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}\\
&\quad+\frac{b^2c^2\sqrt{d-c^2dx^2}\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}\\
&\quad+\frac{ibc^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}\\
&\quad-\frac{ibc^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}\\
&\quad-\frac{ib^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}\\
&\quad+\frac{ib^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
-b*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))/d/x/(c*x-1)^(1/2)/(c*x+1)^(1/
2)-1/2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/d/x^2-c^2*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d/(c*x-
1)^(1/2)/(c*x+1)^(1/2)+b^2*c^2*(-c^2*d*x^2+d)^(1/2)*arctan((c*x-1)^(1/2)*(
c*x+1)^(1/2))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*poly
log(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I
*b^2*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b^2*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(3
,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5036 vs. $2(452) = 904$.

Time = 61.28 (sec) , antiderivative size = 5036, normalized size of antiderivative = 11.14

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^3\sqrt{d - c^2dx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.56, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {6347, 6298, 103, 218, 6361, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{ 6347}$$

$$\frac{1}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x^2}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{1}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{ 103}$$

$$\frac{1}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc^2\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{ 6361}$$

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$\downarrow$ 4668

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\left(-2ib\int (a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int (a+b\text{arccosh}(cx))\log\left(\right.\right.}{2\sqrt{d-c^2dx^2}}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$\downarrow$ 3011

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(c\right.\right.}{}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$\downarrow$ 2720

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+\right.\right.}{}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

$\downarrow$ 7143

$$\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(2,-\right.\right.\right.}{2\sqrt{d}}$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2dx^2}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-1/2*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(d*x^2) - (b*c*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/x) + b*c*ArcTan[Sqrt[-1 + c*x]
*Sqrt[1 + c*x]]))/Sqrt[d - c^2*d*x^2] + (c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(2*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]] + (2*I)*b*(-((a + b*ArcCo
sh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) + b*PolyLog[3, (-I)*E^ArcCosh[c*
x]]) - (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]]) + b*P
olyLog[3, I*E^ArcCosh[c*x]])))/(2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2}{x^3\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acosh}(cx))^2}{x^3\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)
+ sqrt(-c^2*d*x^2 + d)/(d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x^3) + 2*a*b*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2}{x^3\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a^2+4\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x^3}dx\right)ab\,x^2+2\left(\int \frac{acosh(cx)^2}{\sqrt{-c^2x^2+1}\,x^3}dx\right)b^2x^2+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a^2c^2x^2}{2\sqrt{d}\,x^2}$$

input
```
int((a+b*acosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a**2 + 4*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)
*x**3),x)*a*b*x**2 + 2*int(acosh(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**3),x)*
b**2*x**2 + log(tan(asin(c*x)/2))*a**2*c**2*x**2)/(2*sqrt(d)*x**2)
```

3.192 $\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 332

$$\begin{aligned}
\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx &= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3dx} - \frac{bc\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))}{3dx^2\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad - \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{3dx^3} \\
&\quad - \frac{2c^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{3dx} \\
&\quad - \frac{2c^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{3d\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{4bc^3\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\log\left(1+e^{2\operatorname{arccosh}(cx)}\right)}{3d\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{2b^2c^3\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)}{3d\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
1/3*b^2*c^2*(-c^2*d*x^2+d)^(1/2)/d/x-1/3*b*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cosh(c*x))/d/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccosh(c*x))^2/d/x^3-2/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/d
/x-2/3*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/d/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+4/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)/d/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/3*b^2*c^3*(-c^2
*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d/(c*x-1)^
(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.11

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^4\sqrt{d - c^2dx^2}}\,dx$$
$$= \frac{-a^2 - a^2c^2x^2 + b^2c^2x^2 + 2a^2c^4x^4 - b^2c^4x^4 + abcx\sqrt{-1 + cx}\sqrt{1 + cx} - b^2(1 + cx)\left(1 - cx + 2c^2x^2 + 2c\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(-a^2 - a^2*c^2*x^2 + b^2*c^2*x^2 + 2*a^2*c^4*x^4 - b^2*c^4*x^4 + a*b*c*x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x] - b^2*(1 + c*x)*(1 - c*x + 2*c^2*x^2 + 2*c^3*
x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2 + b*(1 + c*x)*ArcCos
h[c*x]*(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*a*(-1 + c*x - 2*c^2*x^2 + 2*c
^3*x^3) - 4*b*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x
])]) - 4*a*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[-1 + Sqrt[1 + c*x]]
- 4*a*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 + Sqrt[1 + c*x]] + 2*b^
2*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c
*x])])/(3*x^3*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.79, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {6347, 6298, 106, 6332, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^4\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{6347}$$

$$\frac{2}{3}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}dx-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x^3}dx}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{6298}$$

$$\frac{2}{3}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}dx-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}bc\int \frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{106}$$

$$\frac{2}{3}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\sqrt{d-c^2dx^2}}dx-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{6332}$$

$$\frac{2}{3}c^2\left(-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}\right)-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 6297

$$\frac{2}{3}c^2\left(-\frac{2c\sqrt{cx-1}\sqrt{cx+1}\int -\left((a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\right)d(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2d\ldots}}{}\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 25

$$\frac{2}{3}c^2\left(\frac{2c\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+\ldots}{d\ldots}\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 3042

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}+\frac{2c\sqrt{cx-1}\sqrt{cx+1}\int -i(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\ldots\right.}{\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 26

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)\ldots}{\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 4201

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\left(2i\int\frac{e^{-2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1+e^{-2\text{arccosh}(cx)}}d(a+b\text{arccosh}(\ldots\right.}{\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}(cx)}\right)d(a+b\text{arccosh}(cx\right.\right.}{\sqrt{c}}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\left(2i\left(-\frac{1}{4}b^2\int e^{2\text{arccosh}(cx)}\log\left(1+e^{-2\text{arccosh}(cx)}\right)de^{-}\right.\right.}{}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{dx}-\frac{2ic\sqrt{cx-1}\sqrt{cx+1}\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arccosh}(cx))-\frac{1}{2}b\log\left(\right.\right.\right.}{\sqrt{d-c^2c}}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\text{arccosh}(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{3dx^3}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-1/3*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(d*x^3) - (2*b*c*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*((b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (a + b*A
rcCosh[c*x])/(2*x^2)))/(3*Sqrt[d - c^2*d*x^2]) + (2*c^2*(-((Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(d*x)) - ((2*I)*c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*((-1/2*I)*(a + b*ArcCosh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c*x])*
Log[1 + E^(-2*ArcCosh[c*x])]) + (b^2*PolyLog[2, -a - b*ArcCosh[c*x]])/4)))
/Sqrt[d - c^2*d*x^2]))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1519 vs. $2(314) = 628$.

Time = 0.67 (sec) , antiderivative size = 1520, normalized size of antiderivative = 4.58

method	result	size
default	Expression too large to display	1520
parts	Expression too large to display	1520

input
```
int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8+1/3*b^2*(
-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*c^6-2/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2
)/d/(3*c^4*x^4-2*c^2*x^2-1)/x*c^2-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*(4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+4*c
^3*x^3*arccosh(c*x)-4*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3+2*
arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)/d/x^3/(c^2*x^2-1)+a^2*(-1/3/
d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*c^2/d/x*(-c^2*d*x^2+d)^(1/2))+4/3*b^2*(-d*(
c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*arccosh(c*x)*c^8-2*b^2*(-d
*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*arccosh(c*x)^2*c^6-2/3*b
^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+1
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)^2*c
^4-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)
*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*c^3+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-
1)/x*arccosh(c*x)^2*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*
x^2-1)/x^3*arccosh(c*x)^2-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^
2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2
*c^2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+2/3*b^2*(-d*(c^2*x^2-1))^(
1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/...
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{\sqrt{-c^2dx^2+dx^4}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^4\sqrt{d - c^2dx^2}}\,dx = \int \frac{(a + b\operatorname{acosh}(cx))^2}{x^4\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^4\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx) + a)^2}{\sqrt{-c^2dx^2 + dx^4}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
1/3*(4*c^2*sqrt(-d)*log(x)/d - sqrt(-d)/(d*x^2))*a*b*c - 2/3*a*b*(2*sqrt(-
c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccosh(c*x) - 1/
3*a^2*(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3)) +
b^2*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 +
d)*x^4), x)
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^4\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx) + a)^2}{\sqrt{-c^2dx^2 + dx^4}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^4\sqrt{d - c^2dx^2}}\,dx = \int \frac{(a + b\,\text{acosh}(c\,x))^2}{x^4\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^4\sqrt{d - c^2dx^2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\,a^2c^2x^2 - \sqrt{-c^2x^2+1}\,a^2 + 6\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,x^4}dx\right)ab\,x^3 + 3\left(\int \frac{acosh(cx)^2}{\sqrt{-c^2x^2+1}\,x^4}dx\right)b^2x^3}{3\sqrt{d}\,x^3}$$

input
```
int((a+b*acosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2
+ 6*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*x**4),x)*a*b*x**3 + 3*int(acosh
(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**4),x)*b**2*x**3)/(3*sqrt(d)*x**3)
```

3.193 $\int \frac{x^4(a+b\,\mathbf{arccosh}(cx))^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 440

$$\begin{aligned}\int \frac{x^4(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx &= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} \\ &- \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{arccosh}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))}{2c^3d\sqrt{d-c^2dx^2}} \\ &+ \frac{x^3(a+b\operatorname{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))^2}{c^5d\sqrt{d-c^2dx^2}} \\ &+ \frac{3x\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2}{2c^4d^2} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))^3}{2bc^5d\sqrt{d-c^2dx^2}} \\ &- \frac{2b\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))\log\left(1-e^{2\operatorname{arccosh}(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}} \\ &- \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,e^{2\operatorname{arccosh}(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
1/4*b^2*x*(-c*x+1)*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)-1/4*b^2*(c*x-1)^(1/2
)*(c*x+1)^(1/2)*arccosh(c*x)/c^5/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*x^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b
*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a
+b*arccosh(c*x))^2/c^5/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccosh(c*x))^2/c^4/d^2-1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c
*x))^3/b/c^5/d/(-c^2*d*x^2+d)^(1/2)-2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*a
rccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^5/d/(-c^2*d*x^2+d
)^(1/2)-b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)/c^5/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.58 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.78

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-4a^2c\sqrt{d}x(-3+c^2x^2)+12a^2\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+2ab\sqrt{d}\Big(8}{}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(-4*a^2*c*Sqrt[d]*x*(-3 + c^2*x^2) + 12*a^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*b*Sqrt[d]*(8*c*x*Ar
cCosh[c*x] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh
[2*ArcCosh[c*x]] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh
[c*x]*Sinh[2*ArcCosh[c*x]])) + b^2*Sqrt[d]*(8*c*x*ArcCosh[c*x]^2 + 8*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])] - Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(4*ArcCosh[c*x]^3 - 2*ArcCosh[c*x]*(Cosh[2*Ar
cCosh[c*x]] - 8*Log[1 - E^(-2*ArcCosh[c*x])]) + Sinh[2*ArcCosh[c*x]] + 2*A
rcCosh[c*x]^2*(4 + Sinh[2*ArcCosh[c*x]]))))/(8*c^5*d^(3/2)*Sqrt[d - c^2*d*
x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {6349, 25, 6327, 6353, 101, 43, 6298, 101, 43, 6307, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6349}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{x^3(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 6327}$$

$$-\frac{3\int \frac{x^2(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 6353}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int x(a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{101}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2c}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int x(a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{43}$$

$$-\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\int x(a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}\right)}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6298}$$

$$-\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}\right)}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 101$$

$$-\frac{3\left(-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a}{}\right.}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 43$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-$$

$$\frac{3\left(\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}}{2c^{}}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 6307$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\downarrow$ 6328

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^4}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\downarrow$ 3042

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\right.}{2c}\right.}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\downarrow$ 26

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\int(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{c}}{}\right.}{2c}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\downarrow$ 4199

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(2i\int-\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(a\right.}{}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\downarrow$ 25

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(-2i\int\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}-\frac{x^2(a+b\text{arccosh}(cx))}{2c^2}+\frac{b\left(a\right.}{}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\downarrow$ 2620

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(-2i\left(\frac{1}{2}b\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccosh}(c}{2b}}{c^4}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-}{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\big\downarrow$ 2715

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(}{}}{c^4}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-}{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$\big\downarrow$ 2838

$$\frac{\frac{x^3(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-}{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}x^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{\text{arccosh}(}{2c^3}\right.\right.}{c\sqrt{d-c^2dx^2}}\right.}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}-\frac{x}{}\right.}{cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (3*(-1/2*(x*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d) + (Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*((x^2*(a + b*ArcCosh[c*x]))/2 - (b*c*((x*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/2))/(c*Sqrt[d - c^2*d*x
^2])))/(c^2*d) + (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1/2*(x^2*(a + b*ArcCo
sh[c*x]))/c^2 + (b*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x
]/(2*c^3)))/(2*c) + (I*(((-1/2*I)*(a + b*ArcCosh[c*x])^2)/b - (2*I)*(-1/2*
((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])]) - (b*PolyLog[2, E^(2*Ar
cCosh[c*x])])/4)))/c^4))/(c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.72

method	result
default	$-\frac{a^{2} x^{3}}{2 c^{2} d \sqrt{-c^{2} d \,x^{2}+d}}+\frac{3 a^{2} x}{2 c^{4} d \sqrt{-c^{2} d \,x^{2}+d}}-\frac{3 a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d \,x^{2}+d}}\right)}{2 c^{4} d \sqrt{c^{2} d}}+\frac{b^{2} \sqrt{c x -1}\, \sqrt{c x +1}\, \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(2 \operatorname{arccosh}\left(c x \right)^{2} \sqrt{c}\right.}{}$
parts	$-\frac{a^{2} x^{3}}{2 c^{2} d \sqrt{-c^{2} d \,x^{2}+d}}+\frac{3 a^{2} x}{2 c^{4} d \sqrt{-c^{2} d \,x^{2}+d}}-\frac{3 a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d \,x^{2}+d}}\right)}{2 c^{4} d \sqrt{c^{2} d}}+\frac{b^{2} \sqrt{c x -1}\, \sqrt{c x +1}\, \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(2 \operatorname{arccosh}\left(c x \right)^{2} \sqrt{c}\right.}{}$

input
```
int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(-c^2*d*x^2+d)^(1/
2)-3/2*a^2/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)+1/4*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(2*arccosh(c*
x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-2*arccosh(c*x)*c^4*x^4+c^3*x^3*(c
*x-1)^(1/2)*(c*x+1)^(1/2)+2*arccosh(c*x)^3*x^2*c^2-6*arccosh(c*x)^2*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c-4*arccosh(c*x)^2*x^2*c^2+8*arccosh(c*x)*ln(1+c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2+8*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))*x^2*c^2+3*c^2*x^2*arccosh(c*x)+8*polylog(2,-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))*x^2*c^2+8*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x
^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-2*arccosh(c*x)^3+4*arccosh(c*x)^2-8
*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-8*arccosh(c*x)*ln(1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-arccosh(c*x)-8*polylog(2,-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))-8*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(c^2*x^2-1)^
2/d^2/c^5+1/4*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(4*ar
ccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-2*c^4*x^4+6*arccosh(c*x)^2*
x^2*c^2-12*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x-8*c^2*x^2*arccosh(
c*x)+8*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^2*c^2+3*c^2*x^2-6*arcco
sh(c*x)^2+8*arccosh(c*x)-8*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)-1)/(c
^2*x^2-1)^2/d^2/c^5
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fric
as")
```

output
```
integral((b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(
-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\,\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^4(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\,\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxi
ma")
```

output
```
-1/2*a^2*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4
*d) + 3*arcsin(c*x)/(c^5*d^(3/2))) + integrate(b^2*x^4*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) + 2*a*b*x^4*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\,\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^4\,(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-3\sqrt{-c^2x^2+1}\,asin(cx)\,a^2-4\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^4}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)a}{2\sqrt{d}\,\sqrt{-c^2x}}$$

input
```
int(x^4*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*asin(c*x)*a**2 - 4*sqrt( - c**2*x**2 + 1)*int
((acosh(c*x)*x**4)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 +
 1)),x)*a*b*c**5 - 2*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)**2*x**4)/(sqrt
( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2*c**5 - a**2
*c**3*x**3 + 3*a**2*c*x)/(2*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**5*d)
```

3.194 $\int \frac{x^3(a+b\,\mathrm{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 346

$$\begin{aligned}
\int \frac{x^3(a+b\,\mathrm{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx &= -\frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} \\
&+ \frac{4b^2\sqrt{d-c^2dx^2}}{c^4d^2} + \frac{2bx\sqrt{-1+cx}\sqrt{1+cx}(a+b\,\mathrm{arccosh}(cx))}{c^3d\sqrt{d-c^2dx^2}} \\
&+ \frac{x^2(a+b\,\mathrm{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2}{c^4d^2} \\
&+ \frac{4b\sqrt{-1+cx}\sqrt{1+cx}(a+b\,\mathrm{arccosh}(cx))\,\mathrm{arctanh}\left(e^{\mathrm{arccosh}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}} \\
&+ \frac{2b^2\sqrt{-1+cx}\sqrt{1+cx}\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arccosh}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}} \\
&- \frac{2b^2\sqrt{-1+cx}\sqrt{1+cx}\,\mathrm{PolyLog}\left(2,e^{\mathrm{arccosh}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}
\end{aligned}$$

output
```
-2*b^2*(-c*x+1)*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+4*b^2*(-c^2*d*x^2+d)^(1
/2)/c^4/d^2+2*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))/c^3/d/(-c
^2*d*x^2+d)^(1/2)+x^2*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2/c^4/d^2+4*b*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^4/d/(-
c^2*d*x^2+d)^(1/2)+2*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/c^4/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^4/d/(-c^2*d*x^2+d)^
(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-2a^2(-2+c^2x^2)+2ab\Big(3\text{arccosh}(cx)-\text{arccosh}(cx)\cosh(2\text{arccosh}(cx))+$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(-2*a^2*(-2 + c^2*x^2) + 2*a*b*(3*ArcCosh[c*x] - ArcCosh[c*x]*Cosh[2*ArcCo
sh[c*x]] + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(Log[Cosh[ArcCosh[c*x]/2
]] - Log[Sinh[ArcCosh[c*x]/2]]) + Sinh[2*ArcCosh[c*x]]) + b^2*(2 + 3*ArcCo
sh[c*x]^2 - 2*Cosh[2*ArcCosh[c*x]] - ArcCosh[c*x]^2*Cosh[2*ArcCosh[c*x]] -
 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c
*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-Ar
cCosh[c*x])] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-ArcC
osh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-ArcCosh
[c*x])] + 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(2*c^4*d*Sqrt[d - c^2*d*x^
2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.83, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {6349, 25, 6327, 6329, 2009, 6353, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6349}$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{x^2(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 6327}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 6329}$$

$$-\frac{2\left(-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}\right)}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\Big\downarrow \text{6353}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2}+\frac{b\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{x(a+b\text{arccosh}(cx))}{c^2}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\Big\downarrow \text{83}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\Big\downarrow \text{6318}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 26

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^3}-\frac{x(a+b\text{arccosh}(cx))}{c^2}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 4670

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)\right.}{c^3}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 2715

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2\right.}{c^3}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 2838

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{c^3}-\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2}{c^2d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(ax+bx\text{arccosh}(cx)-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^2*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*(-((Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(c^2*d)) - (2*b*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]))/(c*S
qrt[d - c^2*d*x^2])))/(c^2*d) + (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/c^3 - (x*(a + b*ArcCosh[c*x]))/c^2 - (I*((2*I)*(
a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*
x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/c^3))/(c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.87

method	result
default	$a^2\left(-\frac{x^2}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2d\,x^2+d}}\right)+\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^2x^2-1)}\,\left(\operatorname{arccosh}(cx)^2\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-2c^3x^3\operatorname{ar}\right.}{}$
parts	$a^2\left(-\frac{x^2}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2d\,x^2+d}}\right)+\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-d(c^2x^2-1)}\,\left(\operatorname{arccosh}(cx)^2\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-2c^3x^3\operatorname{ar}\right.}{}$

input
```
int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b^2*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arccosh(c*x)^2*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^2*c^2-2*c^3*x^3*arccosh(c*x)+2*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*c^2*x^2-2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2+
2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2-2*polylog(2,-
c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2+2*polylog(2,c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*x^2*c^2-2*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x*arc
cosh(c*x)-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))-2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*
polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*polylog(2,c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)))/(c^2*x^2-1)^2/d^2/c^4+2*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-
d*(c^2*x^2-1))^(1/2)*(arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2-c^3
*x^3+ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)*x^2*c^2-ln(1+c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*x^2*c^2-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-ln((
c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)+ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/(
c^2*x^2-1)^2/d^2/c^4
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2x^3}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fric
as")
```

output
```
integral((b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt(
-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^3(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^3}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxi
ma")
```

output
```
-a*b*c*(2*sqrt(-d)*x/(c^4*d^2) + sqrt(-d)*log(c*x + 1)/(c^5*d^2) - sqrt(-d
)*log(c*x - 1)/(c^5*d^2)) - 2*a*b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(s
qrt(-c^2*d*x^2 + d)*c^4*d))*arccosh(c*x) - a^2*(x^2/(sqrt(-c^2*d*x^2 + d)*
c^2*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d)) - b^2*((c^2*x^2 - 2)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))^2/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^4*d^(3/2)) -
integrate(2*(c^4*x^4 - 3*c^2*x^2 + (c^3*x^3 - 2*c*x)*sqrt(c*x + 1)*sqrt(c*
x - 1) + 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c^5*d^(3/2)*x^2 - c^
3*d^(3/2))*(c*x + 1)*sqrt(c*x - 1) + (c^6*d^(3/2)*x^3 - c^4*d^(3/2)*x)*sqr
t(c*x + 1))*sqrt(-c*x + 1)), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{3/2}}\, dx = \int \frac{x^3\,(a + b\,\text{acosh}(c\,x))^2}{(d - c^2\,d\,x^2)^{3/2}}\, dx$$

input
```
int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{3/2}}\, dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^3}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab\,c^4 - \sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2x}}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^4d}$$

input
```
int(x^3*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output

```
( - 2*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)
*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c**4 - sqrt( - c**2*x**2 + 1)*
int((acosh(c*x)**2*x**3)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*
x**2 + 1)),x)*b**2*c**4 - a**2*c**2*x**2 + 2*a**2)/(sqrt(d)*sqrt( - c**2*x
**2 + 1)*c**4*d)
```

3.195 $\int \frac{x^2(a+b\,\mathbf{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 257

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\operatorname{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))^2}{c^3d\sqrt{d-c^2dx^2}} + \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^3}{3bc^3d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))\log\left(1-e^{2\operatorname{arccosh}(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,e^{2\operatorname{arccosh}(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+(c*x-1)^(1/2)*(c*x+1)^(1
/2)*(a+b*arccosh(c*x))^2/c^3/d/(-c^2*d*x^2+d)^(1/2)+1/3*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccosh(c*x))^3/b/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(
2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.97 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{3a^2cdx+3a^2\sqrt{d}\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+3abd\Big(2cx\text{arccosh}(cx)-$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(3*a^2*c*d*x + 3*a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*
d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 3*a*b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)])) - b^2*d*(ArcCosh[c*x]*(-3*c*x*ArcCosh[c*x] + Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 - E^
(-2*ArcCosh[c*x])])) - 3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E
^(-2*ArcCosh[c*x])]))/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.77, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {6349, 25, 6307, 6327, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6349$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6307}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6327}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6328}$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int-i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4199}$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\int-\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2620

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)\right.}{c^3d\sqrt{d-c^2dx^2}}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2715

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+\text{bar}\right.\right.}{c^3d\sqrt{d-c^2dx^2}}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2838

$$\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arc}}{}\right.}{c^3d\sqrt{d-c^2dx^2}}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]) + ((2
*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(((-1/2*I)*(a + b*ArcCosh[c*x])^2)/b -
(2*I)*(-1/2*((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])]) - (b*PolyLo
g[2, E^(2*ArcCosh[c*x])])/4)))/(c^3*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(255) = 510$.

Time = 0.64 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.87

method	result
default	$\frac{a^2 x}{c^2 d\sqrt{-c^2 d x^2+d}} - \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d x^2+d}}\right)}{c^2 d\sqrt{c^2 d}} + \frac{b^2 \sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^3}{3d^2c^3(c^2x^2-1)} - \frac{b^2\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{c}}{d^2c^3(c^2x^2-}$
parts	$\frac{a^2 x}{c^2 d\sqrt{-c^2 d x^2+d}} - \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d x^2+d}}\right)}{c^2 d\sqrt{c^2 d}} + \frac{b^2 \sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^3}{3d^2c^3(c^2x^2-1)} - \frac{b^2\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{c}}{d^2c^3(c^2x^2-}$

input
```
int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1
/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^3-b^2*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c
^2*x^2-1))^(1/2)*arccosh(c*x)^2/d^2/c^2/(c^2*x^2-1)*x+2*b^2*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^
(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c
^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-2*a*b*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-2*a*b*(-d
*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+2*a*b*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fric
as")
```

output
```
integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^2(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxi
ma")
```

output
```
a^2*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + integra
te(b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2)
 + 2*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2)
, x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^2\,(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\frac{-\sqrt{-c^2x^2+1}\,asin(cx)\,a^2-2\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^2}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c}$$

input
```
int(x^2*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*int((
acosh(c*x)*x**2)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1
)),x)*a*b*c**3 - sqrt( - c**2*x**2 + 1)*int((acosh(c*x)**2*x**2)/(sqrt( -
c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2*c**3 + a**2*c*x
)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*c**3*d)
```

3.196 $\int \frac{x(a+b\mathbf{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 196

$$\begin{aligned}\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx &= \frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\ &+ \frac{4b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}} \\ &+ \frac{2b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}} \\ &- \frac{2b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}\end{aligned}$$

output

```
(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+4*b*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^2/d/(-
c^2*d*x^2+d)^(1/2)+2*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/c^2/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^2/d/(-c^2*d*x^2+d)^
(1/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.12

$$\int \frac{x(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{a^2+2ab\left(\operatorname{arccosh}(cx)+\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\log\left(\cosh\left(\frac{1}{2}\operatorname{arccosh}(cx)\right)\right)-\log\left(\right.\right.\right.}{}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a^2 + 2*a*b*(ArcCosh[c*x] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(Log[Cos
h[ArcCosh[c*x]/2]] - Log[Sinh[ArcCosh[c*x]/2]])) - b^2*(-(ArcCosh[c*x]*(Ar
cCosh[c*x] - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(Log[1 - E^(-ArcCosh[c
*x])] - Log[1 + E^(-ArcCosh[c*x])]))) + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*PolyLog[2, -E^(-ArcCosh[c*x])] - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*PolyLog[2, E^(-ArcCosh[c*x])]))/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6329, 25, 6304, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6329}$$

$$\frac{(a+b\operatorname{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{a+b\operatorname{arccosh}(cx)}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\operatorname{arccosh}(cx)}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{(a+b\operatorname{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6304

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6318

$$\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3042

$$\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 26

$$\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 4670

$$\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$
$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\right.}{c^2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$
$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{a}\right.}{c^2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$
$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{c^2d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCosh[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b
*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/(c^2*d*Sqr
t[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.74

method	result
default	$\frac{a^2}{c^2 d\sqrt{-c^2 d\,x^2+d}} - \frac{b^2\sqrt{-d(c^2x^2-1)}\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-2\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\ln(\right.}{}$
parts	$\frac{a^2}{c^2 d\sqrt{-c^2 d\,x^2+d}} - \frac{b^2\sqrt{-d(c^2x^2-1)}\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)-2\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\ln(\right.}{}$

input
```
int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)*(2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+ar
ccosh(c*x)^2)/d^2/c^2/(c^2*x^2-1)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*((c*x-1)^(1
/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-(c*x-1)^(1/2)*(c*x
+1)^(1/2)*ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)+arccosh(c*x))/d^2/c^2/(c^2
*x^2-1)
```

Fricas [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
 + a^2*x)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
a^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab\,c^2-\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2x^2}}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^2d}$$

input
```
int(x*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c*
*2*x**2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c**2 - sqrt( - c**2*x**2 + 1)*int
((acosh(c*x)**2*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 +
 1)),x)*b**2*c**2 + a**2)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*c**2*d)
```

3.197 $\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 198

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\log\left(1-e^{2\text{arccosh}(cx)}\right)}{cd\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{cd\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+(c*x-1)^(1/2)*(c*x+1)^(1/2)*
(a+b*arccosh(c*x))^2/c/d/(-c^2*d*x^2+d)^(1/2)-2*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c/d/(-c^2
*d*x^2+d)^(1/2)-b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\text{arccosh}(cx))^2 + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left((a+b\text{arccosh}(cx))\left(a+b\text{arccosh}(cx)-2b\log\left(1-e^{\text{a}\cdots}\right.\right.\right.}{\cdots}}{d\sqrt{d-\cdots}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCosh[c*x])^2 + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[
c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] - 2*b*Log[1 + E^Ar
cCosh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*PolyLog[2, E^ArcC
osh[c*x]]))/c)/(d*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {6314, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6314}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6328}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 4199}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+bar\right.\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+$$
$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arc}}{}\right.}{cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(((-1/2*I)*(a + b*ArcCosh[c*x])^2)/b - (2*I)*(-1/2*((a +
b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])]) - (b*PolyLog[2, E^(2*ArcCosh[
c*x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6314
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]   Int[x*((a
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(204) = 408$.

Time = 0.53 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.92

method	result
default	$\frac{a^2 x}{d\sqrt{-c^2 d x^2+d}} - \frac{b^2 \sqrt{cx+1}\,\sqrt{cx-1}\,\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2}{d^2 c(c^2x^2-1)} - \frac{b^2 \sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2 x}{d^2(c^2x^2-1)} + \frac{2b^2\sqrt{cx+1}\,\sqrt{cx-1}\,\sqrt{-}}{}$
parts	$\frac{a^2 x}{d\sqrt{-c^2 d x^2+d}} - \frac{b^2 \sqrt{cx+1}\,\sqrt{cx-1}\,\sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2}{d^2 c(c^2x^2-1)} - \frac{b^2 \sqrt{-d(c^2x^2-1)}\,\operatorname{arccosh}(cx)^2 x}{d^2(c^2x^2-1)} + \frac{2b^2\sqrt{cx+1}\,\sqrt{cx-1}\,\sqrt{-}}{}$

input
```
int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*x/d/(-c^2*d*x^2+d)^(1/2)-b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-
1))^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c^2*x^2-1))^(1/2)*arcc
osh(c*x)^2/d^2/(c^2*x^2-1)*x+2*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^
2-1))^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^
(1/2))+2*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2
*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^
2-1))^(1/2)/d^2/c/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-
2*a*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2*x^2-1)
*arccosh(c*x)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x+
2*a*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2*x^2-1)
*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-a*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b^2*integrate(log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2), x) + 2*a*b*x*arccosh(c*x
)/(sqrt(-c^2*d*x^2 + d)*d) + a^2*x/(sqrt(-c^2*d*x^2 + d)*d)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab-\sqrt{-c^2x^2+1}\left(\int \frac{acos}{\sqrt{-c^2x^2+1}\,c^2}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x
**2 - sqrt( - c**2*x**2 + 1)),x)*a*b - sqrt( - c**2*x**2 + 1)*int(acosh(c*
x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2
+ a**2*x)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.198 $\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 471

$$\begin{aligned}\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx &= \frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}} \\ &- \frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\arctan\left(e^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} \\ &+ \frac{4b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\ &+ \frac{2b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\ &+ \frac{2ib\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} \\ &- \frac{2ib\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} \\ &- \frac{2b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\ &- \frac{2ib^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} \\ &+ \frac{2ib^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,ie^{\text{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2/(c*x-1)^(1/2)/(c
*x+1)^(1/2)+4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))*arctanh(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d/(-c^2*d*x^2+d)^
(1/2)+2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polyl
og(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*(-c^2
*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.04 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.30

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=$$

$$-\frac{\frac{a^2\sqrt{d-c^2dx^2}}{-1+c^2x^2}-a^2\sqrt{d}\log(cx)+a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)+\frac{2iabd\left(i\text{arccosh}(cx)+\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{arccosh}(cx)\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-(((a^2*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a^2*Sqrt[d]*Log[c*x] + a^2*S
qrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + ((2*I)*a*b*d*(I*ArcCosh[c*x]
 + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c
*x]] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCo
sh[c*x]] + I*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Cosh[ArcCosh[c*x]/2]
] - I*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Sinh[ArcCosh[c*x]/2]] + Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2
*d*x^2] + (b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x]^2)/(1 - c*x) + 2*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*
x])] + I*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]^2*Log[1
 + I/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 2*PolyL
og[2, -E^(-ArcCosh[c*x])] + (2*I)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c
*x]] - (2*I)*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] - 2*PolyLog[2, E^(-
ArcCosh[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcCosh[c*x]] - (2*I)*PolyLog[3,
I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.55, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {6351, 25, 6304, 6318, 3042, 26, 4670, 2715, 2838, 6361, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x (d-c^2dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{6351}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6304}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6318}$$

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctan}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)d\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d} - \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6361

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3042

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 4668

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-2ib\int (a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int (a+b\text{arccosh}(cx))\log\left(1-\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3011

$$\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx\right.\right.$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2720

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+ba\right.\right.}{}$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 7143$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(2,-ie\right.\right.\right.}{d\sqrt{d-}}$$

$$\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{a}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(a + b*ArcCosh[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*Pol
yLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/(d*Sqrt[d - c^
2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*(a + b*ArcCosh[c*x])^2*ArcTan
[E^ArcCosh[c*x]] + (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCo
sh[c*x]]) + b*PolyLog[3, (-I)*E^ArcCosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh
[c*x])*PolyLog[2, I*E^ArcCosh[c*x]]) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/(
d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2}{x\,(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arccosh}(cx))^2}{x\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\mathrm{arccosh}(cx))^2}{x\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x\,(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^2}{(-c^2dx^2 + d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
-a^2*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/
(sqrt(-c^2*d*x^2 + d)*d)) + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1))^2/((-c^2*d*x^2 + d)^(3/2)*x) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x\,(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^2}{(-c^2dx^2 + d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x\,(d - c^2dx^2)^{3/2}}\,dx = \int \frac{(a + b\,\text{acosh}(c\,x))^2}{x\,(d - c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^3-\sqrt{-c^2x^2+1}\,x}dx\right)ab-\sqrt{-c^2x^2+1}\left(\int \frac{ac}{\sqrt{-c^2x^2+1}\,\ldots}\right.}{\sqrt{d}\,\sqrt{-\ldots}}$$

input
```
int((a+b*acosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x
**3 - sqrt( - c**2*x**2 + 1)*x),x)*a*b - sqrt( - c**2*x**2 + 1)*int(acosh(
c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**3 - sqrt( - c**2*x**2 + 1)*x),x)*b
**2 + sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a**2 - sqrt( - c**2*x**
2 + 1)*a**2 + a**2)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.199 $\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 341

$$\begin{aligned}\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx &= -\frac{(a+b\operatorname{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}} \\ &+ \frac{2c^2x(a+b\operatorname{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}} \\ &- \frac{4bc\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\ &- \frac{4bc\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))\log\left(1-e^{2\operatorname{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\ &- \frac{b^2c\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\ &- \frac{b^2c\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,e^{2\operatorname{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-(a+b*arccosh(c*x))^2/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2*x*(a+b*arccosh(c*x))^
2/d/(-c^2*d*x^2+d)^(1/2)+2*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x)
)^2/d/(-c^2*d*x^2+d)^(1/2)-4*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(
c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)-
4*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)-b^2*c*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)
-b^2*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2)/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.37

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^2 (d - c^2dx^2)^{3/2}}\, dx = \frac{2ab(-1 + 2c^2x^2)\,\text{arccosh}(cx) + b^2\text{arccosh}(cx)^2 + a\big(-a + 2ac^2x^2 - 2bcx\sqrt{-1}}{dx\sqrt{d - c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(2*a*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x] + b^2*ArcCosh[c*x]^2 + a*(-a + 2*a*c^
2*x^2 - 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[c*x] - b*c*x*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2]))/(d*x*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.44 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.82, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {6347, 25, 6314, 6327, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838, 6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2 \left(d-c^2dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6347}$$

$$2c^2\int \frac{(a+b\text{arccosh}(cx))^2}{\left(d-c^2dx^2\right)^{3/2}}dx+\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{a+b\text{arccosh}(cx)}{x(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$2c^2\int \frac{(a+b\text{arccosh}(cx))^2}{\left(d-c^2dx^2\right)^{3/2}}dx-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6314}$$

$$2c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6327}$$

$$2c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6328}$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}\right)-$$
$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 26$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}\right)-$$
$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 4199$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}}{2b}\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 25$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}}{2b}\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 2620$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{2}b\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 6331$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 5984$$

$$\frac{4bc\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{4bc\sqrt{cx-1}\sqrt{cx+1}\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \text{ 26}$$
$$\frac{4ibc\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \text{ 4670}$$
$$\frac{4ibc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+i\text{arct}\right.}{d\sqrt{d-c^2dx^2}}$$
$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \text{ 2715}$$
$$\frac{4ibc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e^{2\text{arccosh}}\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \text{ 2838}$$

$$\frac{4ibc\sqrt{cx-1}\sqrt{cx+1}\left(i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$2c^2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcCosh[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + ((4*I)*b*c*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(I*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])] +
(I/4)*b*PolyLog[2, -E^(2*ArcCosh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcCosh[
c*x])]))/(d*Sqrt[d - c^2*d*x^2]) + 2*c^2*((x*(a + b*ArcCosh[c*x])^2)/(d*Sq
rt[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(((-1/2*I)*(a +
 b*ArcCosh[c*x])^2)/b - (2*I)*(-1/2*((a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])]) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/4)))/(c*d*Sqrt[d - c^2*d
*x^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6314
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]   Int[x*((a
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1270 vs. $2(359) = 718$.

Time = 0.67 (sec) , antiderivative size = 1271, normalized size of antiderivative = 3.73

method	result	size
default	Expression too large to display	1271
parts	Expression too large to display	1271

input
```
int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))-b^2*(-4*a
rccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2-4*arccosh(c*x)*l
n(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^2*c^2-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3+4*arccosh(
c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^4*c^4+4*arccosh(c*x)*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^4*c^4+4*arccosh(c*x)*ln(1-c*x-(c*x-1)^
(1/2)*(c*x+1)^(1/2))*x^4*c^4-4*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2)*x^2*c^2-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3+arccosh(c*x)^2-4*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^3*c^3-4*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^3*c^3+(c*x-1)^(1/2
)*(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x*c+2*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*x*c+2*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*x*c
+2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^4*c^4+4*polylog(2,-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*x^4*c^4+4*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*x^4*c^4-2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^2*c^
2-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*x^3*c^3-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*ln(1-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))*x^3*c^3+2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c...
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\,(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\operatorname{acosh}(cx))^2}{x^2(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxi
ma")
```

output
```
a*b*c*(sqrt(-d)*log(c*x + 1)/d^2 + sqrt(-d)*log(c*x - 1)/d^2 + 2*sqrt(-d)*
log(x)/d^2) + 2*(2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d
)*d*x))*a*b*arccosh(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^
2*d*x^2 + d)*d*x))*a^2 + b^2*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))^2/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\,\text{acosh}(c\,x)\right)^2}{x^2\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^4-\sqrt{-c^2x^2+1}\,x^2}dx\right)abx-\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2x^2}}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,dx}$$

input
```
int((a+b*acosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x
**4 - sqrt( - c**2*x**2 + 1)*x**2),x)*a*b*x - sqrt( - c**2*x**2 + 1)*int(a
cosh(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**4 - sqrt( - c**2*x**2 + 1)*x*
*2),x)*b**2*x + 2*a**2*c**2*x**2 - a**2)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d
*x)
```

3.200 $\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 650

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^3(d-c^2dx^2)^{3/2}}\,dx = \frac{bc\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2(a+b\operatorname{arccosh}(cx))^2}{2d\sqrt{d-c^2dx^2}} - \frac{(a+b\operatorname{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}} - \frac{3c^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^2\arctan\left(e^{\operatorname{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d\sqrt{d-c^2dx^2}} + \frac{4bc^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{2b^2c^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,-e^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{3ibc^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3ibc^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2c^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,e^{\operatorname{arccosh}(cx)}\right)}{d\sqrt{d-c^2dx^2}} - \frac{3ib^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3ib^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(cx)}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))/d/x/(-c^2*d*x^2+d)^(1/2
)+3/2*c^2*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x
))^2/d/x^2/(-c^2*d*x^2+d)^(1/2)-3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*
x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-b^2*c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2)
)/d/(-c^2*d*x^2+d)^(1/2)+4*b*c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(
c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)+2*b^
2*c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/d/(-c^2*d*x^2+d)^(1/2)+3*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x
))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-3*I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b^2*c^2*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d/(
-c^2*d*x^2+d)^(1/2)-3*I*b^2*c^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*I*b^2*c^2*(-c
^2*d*x^2+d)^(1/2)*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^2/(c*x-
1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5362 vs. $2(650) = 1300$.

Time = 64.73 (sec) , antiderivative size = 5362, normalized size of antiderivative = 8.25

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 6.15 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.67, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.931$, Rules used = {6347, 25, 6327, 6347, 103, 218, 6318, 3042, 26, 4670, 2715, 2838, 6351, 25, 6304, 6318, 3042, 26, 4670, 2715, 2838, 6361, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3 (d-c^2dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \ 6347$$

$$\frac{3}{2}c^2 \int \frac{(a+b\text{arccosh}(cx))^2}{x (d-c^2dx^2)^{3/2}}dx+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{a+b\text{arccosh}(cx)}{x^2(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{3}{2}c^2 \int \frac{(a+b\text{arccosh}(cx))^2}{x (d-c^2dx^2)^{3/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x^2(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 6327$$

$$\frac{3}{2}c^2 \int \frac{(a+b\text{arccosh}(cx))^2}{x (d-c^2dx^2)^{3/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x^2(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 6347$$

$$\frac{3}{2}c^2 \int \frac{(a+b\text{arccosh}(cx))^2}{x (d-c^2dx^2)^{3/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{a+b\text{arccosh}(cx)}{x}\right)}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 103$$

$$\frac{3}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+bc^2\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\text{arccosh}(cx)}{x}\right)}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 218

$$-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(c^2\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 6318

$$-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-c\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 3042

$$-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-c\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx\right.}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{3}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 26

$$-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)-\frac{a+b\text{arccosh}(cx)}{x}+bc\arctan\left(\sqrt{cx\right.}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{3}{2}c^2\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 4670

$$-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arc}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{3}{2}c^2\int\frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(c}\right.\right.\right.}{d\sqrt{d-}}$$

$$\frac{3}{2}c^2\int\frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$\frac{3}{2}c^2\int\frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6351

$$\frac{3}{2}c^2\left(-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int-\frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 25

$$\frac{3}{2}c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6304

$$\frac{3}{2}c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-$$
$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 6318

$$\frac{3}{2}c^2\left(-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-$$
$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3042

$$\frac{3}{2}c^2\left(\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b}{d\surd}\right.$$
$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 26

$$\frac{3}{2}c^2\left(\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+ba}{d\surd}\right.$$
$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 4670

$$\frac{3}{2}c^2\left(-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ia\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3}{2}c^2\left(-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{3}{2}c^2\left(\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{6361}$$

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\int\frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+\text{barcc}\right.}{}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arcta}\right.}{}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 4668$$

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-2ib\int(a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int(a+b\text{arccosh}(cx))\,\text{lo}\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3011$$

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccos}\right.\right.}{}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2720$$

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)\left(\right.\right.\right.}{}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 7143$$

$$\frac{3}{2}c^2\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(\right.\right.\right.}{}\right.$$

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(-ic\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/2*(a + b*ArcCosh[c*x])^2/(d*x^2*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/x) + b*c*ArcTan[Sqrt[-1 + c*x]*S
qrt[1 + c*x]] - I*c*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] +
I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]])))/(d*Sqr
t[d - c^2*d*x^2]) + (3*c^2*((a + b*ArcCosh[c*x])^2/(d*Sqrt[d - c^2*d*x^2])
 - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((2*I)*(a + b*ArcCosh[c*x])*ArcTa
nh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^Ar
cCosh[c*x]]))/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*(
a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]] + (2*I)*b*(-((a + b*ArcCosh[c
*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) + b*PolyLog[3, (-I)*E^ArcCosh[c*x]])
 - (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]]) + b*PolyL
og[3, I*E^ArcCosh[c*x]])))/(d*Sqrt[d - c^2*d*x^2])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2}{x^3\,(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3 (d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3 (d-c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{acosh}\,(cx))^2}{x^3\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3 (d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxi
ma")
```

output
```
-1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2
) - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a^2 +
 integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^
(3/2)*x^3) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d
)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2}{x^3\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = \frac{-16\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^5-\sqrt{-c^2x^2+1}\,x^3}dx\right)ab\,x^2-8\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-}}\right.}{}$$

input
```
int((a+b*acosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 16*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*
x**5 - sqrt( - c**2*x**2 + 1)*x**3),x)*a*b*x**2 - 8*sqrt( - c**2*x**2 + 1)
*int(acosh(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**5 - sqrt( - c**2*x**2 +
 1)*x**3),x)*b**2*x**2 + 12*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a
**2*c**2*x**2 - 9*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 + 12*a**2*c**2*x**
2 - 4*a**2)/(8*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**2)
```

3.201 $\int \frac{x^4(a+b\,\mathrm{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 437

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{b^2x}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{arccosh}(cx)}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{bx^2(a+b\text{arccosh}(cx))}{3c^3d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}} - \frac{x(a+b\text{arccosh}(cx))^2}{c^4d^2\sqrt{d-c^2dx^2}} - \frac{4\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^2}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^3}{3bc^5d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{8b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\log\left(1-e^{2\text{arccosh}(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{4b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/3*b^2*x/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*arccosh(c*x)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x^2*(a+b*arccosh(c*x))/c
^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arcco
sh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-x*(a+b*arccosh(c*x))^2/c^4/d^2/(-c^2
*d*x^2+d)^(1/2)-4/3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^2/c^5/d
^2/(-c^2*d*x^2+d)^(1/2)-1/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^3/b/c^
5/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*a
rccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^5/d^2/(-c^2*d*x^2
+d)^(1/2)+4/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.85 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.87

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\frac{a^2cx(-3+4c^2x^2)\sqrt{d-c^2dx^2}}{(-1+c^2x^2)^2} - 3a^2\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right) + \frac{abd\left(-8cx\text{arccosh}(cx) - \right.}{}}{}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
((a^2*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 3*a^2*S
qrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (a*b*d
*(-8*c*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcC
osh[c*x])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh
[c*x]^2 + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^
2] + (b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((c*x*(-1 + c^2*x^2 + (
-3 + 4*c^2*x^2)*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3
)) + ArcCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x]*(4 + ArcCosh[c*x]) +
8*Log[1 - E^(-2*ArcCosh[c*x])]) - 4*PolyLog[2, E^(-2*ArcCosh[c*x])]))/Sqrt
[d - c^2*d*x^2])/(3*c^5*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.05, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {6349, 6327, 6349, 25, 100, 27, 87, 43, 6307, 6327, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6349}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3(a+b\text{arccosh}(cx))}{(1-cx)^2(cx+1)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{6327}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{6349}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\int \frac{x^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\int\frac{x^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 100

$$-\frac{\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{\int\frac{c^2x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c^3}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int\frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{\int\frac{x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 87

$$-\frac{\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{43}$$

$$-\frac{\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6307}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6327}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 6328

$$-\frac{-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arcco}}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{}{c^2}}{c}\right.}{}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 26

$$-\frac{\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\int(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-}{c}\right.}{}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 4199

$$-\frac{\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\int-\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx-}}{3b}}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\int-\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b}{}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 25

$$-\frac{\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\int\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx-}}{3b}}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\int\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b}{}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 2620

$$-\frac{\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{2}b\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\left(\frac{1}{2}b\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccosh}}{2b}}{c^4}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 2715

$$-\frac{\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arc}}{}\right.}{c^3d\sqrt{d-c^2dx^2}}}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\right.}{c^4}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 2838

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}-$$

$$\frac{\frac{x(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arccc}}{2b}\right.}{c^3d\sqrt{d-c^2dx^2}}}{c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}+\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCosh[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*((x^2*(a + b*ArcCosh[c*x]))/(2*c^2*(1 - c^2*x^2)) +
 (b*(-(1/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (-(Sqrt[-1 + c*x]/(c^2*Sqrt
[1 + c*x])) + ArcCosh[c*x]/c^2)/c))/(2*c) - (I*(((-1/2*I)*(a + b*ArcCosh[c
*x])^2)/b - (2*I)*(-1/2*((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])
 - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/4)))/c^4))/(3*c*d^2*Sqrt[d - c^2*d*x
^2]) - ((x*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2
]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(((-1/2*I)*(a + b*ArcCosh[c*x])
^2)/b - (2*I)*(-1/2*((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])]) - (
b*PolyLog[2, E^(2*ArcCosh[c*x])])/4)))/(c^3*d*Sqrt[d - c^2*d*x^2]))/(c^2*d
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2900 vs. $2(405) = 810$.

Time = 0.79 (sec) , antiderivative size = 2901, normalized size of antiderivative = 6.64

method	result	size
default	Expression too large to display	2901
parts	Expression too large to display	2901

input
```
int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/c^4/d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-a^2
/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+1/3*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)+20/
3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2
+16)/d^3*c^2*x^7+43/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+11
8*c^4*x^4-71*c^2*x^2+16)/d^3/c^2*x^3-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*
x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3/c^4*x-76*b^2*(-d*(c^2*x^2-1)
)^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)
^2*x^5-44/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-
71*c^2*x^2+16)/d^3*arccosh(c*x)*x^5-17*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*
x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*x^5-16/3*b^2*(-d*(c^2*x^2-1)
)^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*x-1)*(c*x
+1)*arccosh(c*x)*x^5+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)^2-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d^3/c^5/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/3*b^2*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arcco
sh(c*x)^3-8*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-
71*c^2*x^2+16)/d^3*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^6-8/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3/c^2*(c*...
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt
(-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^4(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxi
ma")
```

output
```
1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c
^4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a
^2 + integrate(b^2*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^
2 + d)^(5/2) + 2*a*b*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^
2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^4\,(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\frac{3\sqrt{-c^2x^2+1}\,asin(cx)\,a^2c^2x^2-3\sqrt{-c^2x^2+1}\,asin(cx)\,a^2+6\sqrt{-c^2x^2+1}\,\left(\right.}{}$$

input
```
int(x^4*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*asin(c*x)*a**2*c**2*x**2 - 3*sqrt( - c**2*x**2 +
 1)*asin(c*x)*a**2 + 6*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**4)/(sqrt(
 - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( -
 c**2*x**2 + 1)),x)*a*b*c**7*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acosh(c*
x)*x**4)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2
*x**2 + sqrt( - c**2*x**2 + 1)),x)*a*b*c**5 + 3*sqrt( - c**2*x**2 + 1)*int
((acosh(c*x)**2*x**4)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x
**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**7*x**2 - 3*sqrt( -
 c**2*x**2 + 1)*int((acosh(c*x)**2*x**4)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c*
*5 - 4*a**2*c**3*x**3 + 3*a**2*c*x)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**5
*d**2*(c**2*x**2 - 1))
```

3.202 $\int \frac{x^3(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 324

$$\begin{aligned}\int \frac{x^3(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx &= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}}\\
&\quad-\frac{bx(a+b\operatorname{arccosh}(cx))}{3c^3d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}}\\
&\quad+\frac{x^2(a+b\operatorname{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}-\frac{2(a+b\operatorname{arccosh}(cx))^2}{3c^4d^2\sqrt{d-c^2dx^2}}\\
&\quad-\frac{10b\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arccosh}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}\\
&\quad-\frac{5b^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,-e^{\operatorname{arccosh}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}\\
&\quad+\frac{5b^2\sqrt{-1+cx}\sqrt{1+cx}\operatorname{PolyLog}\left(2,e^{\operatorname{arccosh}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/3*b^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x*(a+b*arccosh(c*x))/c^3/d^2/(
c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^2*(a+b*arccosh(c*x))
^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3*(a+b*arccosh(c*x))^2/c^4/d^2/(-c^2*d*x^2
+d)^(1/2)-10/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))*arctanh(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*b^2*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^4/d^2/
(-c^2*d*x^2+d)^(1/2)+5/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.64 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.10

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{4a^2(-2+3c^2x^2)-ab\Big(\text{arccosh}(cx)(4-12\cosh(2\text{arccosh}(cx)))-2\sinh(2a$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(4*a^2*(-2 + 3*c^2*x^2) - a*b*(ArcCosh[c*x]*(4 - 12*Cosh[2*ArcCosh[c*x]])
- 2*Sinh[2*ArcCosh[c*x]] + 5*(Log[Cosh[ArcCosh[c*x]/2]] - Log[Sinh[ArcCosh
[c*x]/2]])*(3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - Sinh[3*ArcCosh[c*x]])
) - b^2*(2 + 2*ArcCosh[c*x]^2 - 2*(1 + 3*ArcCosh[c*x]^2)*Cosh[2*ArcCosh[c*
x]] - 15*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*(Log[1 - E^(-Ar
cCosh[c*x])] - Log[1 + E^(-ArcCosh[c*x])]) + 20*((-1 + c*x)/(1 + c*x))^(3/
2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])] - 20*((-1 + c*x)/(1 + c*x))^
(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])] - 2*ArcCosh[c*x]*Sinh[2*Ar
cCosh[c*x]] + 5*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])]*Sinh[3*ArcCosh[c*x
]] - 5*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*Sinh[3*ArcCosh[c*x]]))/(12*
c^4*d*(d - c^2*d*x^2)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.81 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.02, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {6349, 6327, 6329, 25, 6304, 6318, 3042, 26, 4670, 2715, 2838, 6349, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \; 6349$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-cx)^2(cx+1)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 6327$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 6329$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -\frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{cd\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 6304$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6318}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arcta}\right.}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx}\right.\right.}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{x^2(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 6349$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2c^2}+\frac{b\int\frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 83$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int\frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{2c^2}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}-$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 6318$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c^3}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}-$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c^3}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}-$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\int(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c^3}+\frac{x(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}-$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(ib\int\log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int\log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a\right.}{2c^3}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2ia\right.}{2c^3}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2\left(\frac{(a+b\text{arccosh}(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c^3}+\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^2(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^2*(a + b*ArcCosh[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(-1/2*b/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a
+ b*ArcCosh[c*x]))/(2*c^2*(1 - c^2*x^2)) + ((I/2)*((2*I)*(a + b*ArcCosh[c*
x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLo
g[2, E^ArcCosh[c*x]]))/c^3))/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (2*((a + b*Ar
cCosh[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[
2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh[c*x]]))/(c^2*d*Sqrt[d - c^2
*d*x^2])))/(3*c^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.96

method	result
default	$a^2\left(\frac{x^2}{c^2 d(-c^2 d\,x^2+d)^{\frac{3}{2}}}-\frac{2}{3d\,c^4(-c^2 d\,x^2+d)^{\frac{3}{2}}}\right)+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(3\,\mathrm{arccosh}(cx)^2x^2c^2+\mathrm{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,cx+\right.}{3(c^2x^2-1)^2d^3c^4}\right.$
parts	$a^2\left(\frac{x^2}{c^2 d(-c^2 d\,x^2+d)^{\frac{3}{2}}}-\frac{2}{3d\,c^4(-c^2 d\,x^2+d)^{\frac{3}{2}}}\right)+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(3\,\mathrm{arccosh}(cx)^2x^2c^2+\mathrm{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,cx+\right.}{3(c^2x^2-1)^2d^3c^4}\right.$

input
```
int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b^2*(1
/3*(-d*(c^2*x^2-1))^(1/2)*(3*arccosh(c*x)^2*x^2*c^2+arccosh(c*x)*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*c*x+c^2*x^2-2*arccosh(c*x)^2-1)/(c^2*x^2-1)^2/d^3/c^4+5
/3*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*
arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/3*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)
^(1/2)*(c*x+1)^(1/2))-5/3*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/d^3/c^4/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-
5/3*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)
*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+2*a*b*(1/6*(-d*(c^2*x^2-1))^(
1/2)*(6*c^2*x^2*arccosh(c*x)+(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x-4*arccosh(c*x
))/(c^2*x^2-1)^2/d^3/c^4+5/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/c^4/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-5/6*(-d*(c
^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln((c*x-1
)^(1/2)*(c*x+1)^(1/2)+c*x-1))
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^3}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt
(-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{5/2}}\,dx = \int \frac{x^3(a + b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^2 x^3}{(-c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxi
ma")
```

output
```
1/6*a*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/
(c^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 2/3*a*b*(3*x^2/((-c^2*d*x
^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/
3*a^2*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^
4*d)) + b^2*integrate(x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d
*x^2 + d)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^3}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^6x^2-6\sqrt{-c^2}}{}$$

input
```
int(x^3*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)*c*
*4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*
a*b*c**6*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**3)/(sqrt( - c*
*2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2
*x**2 + 1)),x)*a*b*c**4 + 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)**2*x**3
)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 +
 sqrt( - c**2*x**2 + 1)),x)*b**2*c**6*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(
(acosh(c*x)**2*x**3)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x*
*2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**4 - 3*a**2*c**2*x**
2 + 2*a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**4*d**2*(c**2*x**2 - 1))
```

3.203 $\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 344

$$\begin{aligned}\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx &= -\frac{b^2x}{3c^2d^2\sqrt{d-c^2dx^2}}\\ &+\frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\text{arccosh}(cx)}{3c^3d^2\sqrt{d-c^2dx^2}}-\frac{bx^2(a+b\text{arccosh}(cx))}{3cd^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}}\\ &+\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\,(d-c^2dx^2)^{3/2}}-\frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^2}{3c^3d^2\sqrt{d-c^2dx^2}}\\ &+\frac{2b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\log\left(1-e^{2\text{arccosh}(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}}\\ &+\frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/3*b^2*x/c^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*arccosh(c*x)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x^2*(a+b*arccosh(c*x))/c
/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arccosh
(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arcco
sh(c*x))^2/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+2/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^3/d^2/(-c^2
*d*x^2+d)^(1/2)+1/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.78 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.77

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\frac{a^2c^3x^3}{1-c^2x^2}+ab\left(\frac{2c^3x^3\operatorname{arccosh}(cx)}{1-c^2x^2}+\frac{\sqrt{\frac{-1+cx}{1+cx}}\left(-1+2(-1+c^2x^2)\log\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right)}{-1+cx}\right)+b}{}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
((a^2*c^3*x^3)/(1 - c^2*x^2) + a*b*((2*c^3*x^3*ArcCosh[c*x])/(1 - c^2*x^2)
 + (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + 2*(-1 + c^2*x^2)*Log[Sqrt[(-1 + c*x)/
(1 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*(-((c*x*(-1 + c^2*x^2 + c^2*x^2*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*
x))^(3/2)*(1 + c*x)^3)) + ArcCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x]
+ 2*Log[1 - E^(-2*ArcCosh[c*x])]) - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c
^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.72, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {6332, 6327, 6349, 100, 27, 87, 43, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 6332}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\operatorname{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6327}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x^3(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 6349

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\int \frac{x^2}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 100

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{\int \frac{c^2x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c^3}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{\int \frac{x}{\sqrt{cx-1}(cx+1)^{3/2}}dx}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 87

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{c}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 43$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{c^2}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 6328$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-\frac{\sqrt{cx-1}}{c^2\sqrt{cx+1}}}{c}-\frac{1}{c^3\sqrt{cx-1}\sqrt{cx+1}}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-}{c}\right.}{}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 26$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\int(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{\frac{\text{arccosh}(cx)}{c^2}-}{c}\right.}{}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 4199$$

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 25

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}+\frac{x^2(a+b\text{arccosh}(cx))}{2c^2(1-c^2x^2)}+\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2620

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-\frac{i(a+b\text{arccos}\cdots}{2b}\right.}{c^4}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)\cdots\right.}{c^4}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$\frac{x^3(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{c^4}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCosh[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*((x^2*(a + b*ArcCosh[c*x]))/(2*c^2*(1 - c^2*x^2)) + (
b*(-(1/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (-(Sqrt[-1 + c*x]/(c^2*Sqrt[1
 + c*x])) + ArcCosh[c*x]/c^2)/c))/(2*c) - (I*(((-1/2*I)*(a + b*ArcCosh[c*x
])^2)/b - (2*I)*(-1/2*((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])]) -
 (b*PolyLog[2, E^(2*ArcCosh[c*x])])/4)))/c^4))/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

rule 6349
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0]
&& GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2656 vs. $2(322) = 644$.

Time = 0.70 (sec) , antiderivative size = 2657, normalized size of antiderivative = 7.72

method	result	size
default	Expression too large to display	2657
parts	Expression too large to display	2657

input

```
int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)/d^3*c^2*(c*x-1)*(c*x+1)*arccosh(c*x)*x^5+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)
/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)^2*x^3+1/3*b
^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3
*arccosh(c*x)*x^3+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)/d^3*c^4*x^7-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^
6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*c^2*x^5-2/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))-2/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1
))^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2
+1)/d^3/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^2+b^2*(-d*(c^2*x^2-1)
)^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*c*(c*x-1)^(1/2)*(
c*x+1)^(1/2)*arccosh(c*x)*x^4+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c^3*x^3+(c*x
-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-2*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^5*c^5+2*ln(
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^6*c^6+6*arccosh(c*x)*c^4*x^4+2*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^3*c^
3-6*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^4*c^4+c^3*x^3*(c*x-1)^(1/2
)*(c*x+1)^(1/2)-c^4*x^4-6*c^2*x^2*arccosh(c*x)+6*ln((c*x+(c*x-1)^(1/2)*...
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt
(-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^2(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxi
ma")
```

output
```
1/3*a*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d
^3) - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 2/3*a*b*(x/(sqrt(-c^2*d*x^2 + d)*
c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a^2*(x/(sq
rt(-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d)) + b^2*inte
grate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2),
 x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^2\,(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x^2}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^2x^2-6\sqrt{-c^2}$$

input
```
int(x^2*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**2)/(sqrt( - c**2*x**2 + 1)*c*
*4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*
a*b*c**2*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x**2)/(sqrt( - c*
*2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2
*x**2 + 1)),x)*a*b + 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)**2*x**2)/(sq
rt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt
( - c**2*x**2 + 1)),x)*b**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos
h(c*x)**2*x**2)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 +
1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2 - a**2*x**3)/(3*sqrt(d)*sqr
t( - c**2*x**2 + 1)*d**2*(c**2*x**2 - 1))
```

3.204 $\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 286

$$\begin{aligned}\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}} dx &= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} \\ &- \frac{bx(a+b\text{arccosh}(cx))}{3cd^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{(a+b\text{arccosh}(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\ &+ \frac{2b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}} \\ &+ \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}} \\ &- \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/3*b^2/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x*(a+b*arccosh(c*x))/c/d^2/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccosh(c*x))^2/c^2
/d/(-c^2*d*x^2+d)^(3/2)+2/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x
))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)+1
/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*poly
log(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.36

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{4a^2+b^2\Big(-2+4\text{arccosh}(cx)^2+2\cosh(2\text{arccosh}(cx))-3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{ar}}{}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(4*a^2 + b^2*(-2 + 4*ArcCosh[c*x]^2 + 2*Cosh[2*ArcCosh[c*x]] - 3*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 3*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]
 + 4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x]
)] - 4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x
])] + 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] + ArcCosh[c*x]*Log[1 - E^(-ArcCo
sh[c*x])]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*S
inh[3*ArcCosh[c*x]]) + a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(4*c*x + 3
*(Log[Cosh[ArcCosh[c*x]/2]] - Log[Sinh[ArcCosh[c*x]/2]]) + (8*ArcCosh[c*x]
 + (-Log[Cosh[ArcCosh[c*x]/2]] + Log[Sinh[ArcCosh[c*x]/2]])*Sinh[3*ArcCosh
[c*x]])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(12*c^2*d*(d - c^2*d*x^2)
^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.66, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6329, 6304, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6329}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{(1-cx)^2(cx+1)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6304}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6316}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6318}$$

$$\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 4670$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)\right.}{2c}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2\right.}{2c}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}+\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCosh[c*x])^2/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(-1/2*b/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcC
osh[c*x]))/(2*(1 - c^2*x^2)) - ((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[
E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCo
sh[c*x]]))/c))/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(285) = 570$.

Time = 0.62 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.03

method	result
default	$\frac{a^2}{3c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,cx+c^2x^2+\operatorname{arccosh}(cx)^2-1\right)}{3(c^2x^2-1)^2d^3c^2}-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{}}{}\right.$
parts	$\frac{a^2}{3c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,cx+c^2x^2+\operatorname{arccosh}(cx)^2-1\right)}{3(c^2x^2-1)^2d^3c^2}-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{}}{}\right.$

input
```
int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2/c^2/d/(-c^2*d*x^2+d)^(3/2)+b^2*(1/3*(-d*(c^2*x^2-1))^(1/2)*(arccos
h(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x+c^2*x^2+arccosh(c*x)^2-1)/(c^2*x^2-
1)^2/d^3/c^2-1/3*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^
2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/3*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*polylog(
2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))+1/3*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c
^2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+2*a*b*(1/6*(-d*
(c^2*x^2-1))^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+2*arccosh(c*x))/(c^2*x
^2-1)^2/d^3/c^2+1/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3
/c^2/(c^2*x^2-1)*ln((c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x-1)-1/6*(-d*(c^2*x^2-1)
)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{5/2}} \, dx = \int \frac{(b \operatorname{arcosh}(cx) + a)^2 x}{(-c^2dx^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x
) + a^2*x)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x(a + b\text{arccosh}(cx))^2}{(d - c^2dx^2)^{5/2}} \, dx = \int \frac{x(a + b \operatorname{acosh}(cx))^2}{(-d(cx - 1)(cx + 1))^{\frac{5}{2}}} \, dx$$

input
```
integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
1/3*a^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x\,(a+b\,\mathrm{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)x}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^4x^2-6\sqrt{-c^2x}$$

input
```
int(x*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c**4*
x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a*b
*c**4*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)*x)/(sqrt( - c**2*x**
2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2
+ 1)),x)*a*b*c**2 + 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*x)**2*x)/(sqrt(
- c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( -
c**2*x**2 + 1)),x)*b**2*c**4*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acosh(c*
x)**2*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2
*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**2 - a**2)/(3*sqrt(d)*sqrt( - c*
*2*x**2 + 1)*c**2*d**2*(c**2*x**2 - 1))
```

3.205 $\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 319

$$\begin{aligned}\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = & -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} \\ & -\frac{b(a+b\text{arccosh}(cx))}{3cd^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\,(d-c^2dx^2)^{3/2}} \\ & +\frac{2x(a+b\text{arccosh}(cx))^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^2}{3cd^2\sqrt{d-c^2dx^2}} \\ & -\frac{4b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\log\left(1-e^{2\text{arccosh}(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}} \\ & -\frac{2b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/3*b^2*x/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*(a+b*arccosh(c*x))/c/d^2/(c*x-1)
^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arccosh(c*x))^2/d/(-c
^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+2/3*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^2/c/d^2/(-c^2*d*x^2+d)^(1/2
)-4/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)/c/d^2/(-c^2*d*x^2+d)^(1/2)-2/3*b^2*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c/d^2/(-c^2*d*x
^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\frac{a^2cx(-3+2c^2x^2)}{-1+c^2x^2}+ab\left(2cx\left(2+\frac{1}{1-c^2x^2}\right)\text{arccosh}(cx)+\frac{\sqrt{\frac{-1+cx}{1+cx}}\left(-1+(4-4c^2x^2)\log\left(\sqrt{\frac{-1+cx}{1+cx}}\right.\right.}{-1+cx}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

output
```
((a^2*c*x*(-3 + 2*c^2*x^2))/(-1 + c^2*x^2) + a*b*(2*c*x*(2 + (1 - c^2*x^2)
^(-1))*ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + (4 - 4*c^2*x^2)*Lo
g[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*(-((ArcCosh[c*
x]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + c*x*ArcCosh[c*x]))/(-1 + c^2*x^
2)) + c*x*(-1 + 2*ArcCosh[c*x]^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 - E^(-2*ArcCosh[c*x])]) + 2*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c*d^2*Sq
rt[d - c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6316, 6314, 6327, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838, 6329, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6316}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{6314}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6327}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6328}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int -i(a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\tan\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 4199

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arccosh}(cx))}{1-e^{2\text{arccosh}(cx)}}d\text{arccosh}(cx)-\frac{i(a+b\text{arccosh}(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 2620

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\cdots\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 2715

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+\right.}{cd\sqrt{d-c^2dx^2}}}{3d}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{a}}{}\right.}{cd\sqrt{d-c^2dx^2}}}{3d}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 6329

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{b\int \frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2c}+\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{a}}{}\right.}{cd\sqrt{d-c^2dx^2}}}{3d}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 41

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2\text{arccosh}(cx)}\right)\right)-\frac{i(a+b\text{a}}{}\right.}{cd\sqrt{d-c^2dx^2}}}{3d}$$

$$\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcCosh[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(-1/2*(b*x)/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*A
rcCosh[c*x])/(2*c^2*(1 - c^2*x^2))))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*((x*
(a + b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(((-1/2*I)*(a + b*ArcCosh[c*x])^2)/b - (2*I)*(-1/2*((a + b*A
rcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])]) - (b*PolyLog[2, E^(2*ArcCosh[c*x
])])/4)))/(c*d*Sqrt[d - c^2*d*x^2])))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6314
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]   Int[x*((a
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2434 vs. $2(301) = 602$.

Time = 0.59 (sec) , antiderivative size = 2435, normalized size of antiderivative = 7.63

method	result	size
default	Expression too large to display	2435
parts	Expression too large to display	2435

input
```
int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^
3*arccosh(c*x)*x^5+17/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+1
1*c^2*x^2-4)*c^2/d^3*arccosh(c*x)^2*x^3-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3
*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*x^3-4/3*b^2*(-d*(c^
2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*(c*x-1)^(1/2)*(c
*x+1)^(1/2)+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^
2-4)/d^3*(c*x-1)*(c*x+1)*x-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^
4*x^4+11*c^2*x^2-4)*c^6/d^3*arccosh(c*x)*x^7-2*b^2*(-d*(c^2*x^2-1))^(1/2)/
(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)^2*x^5+4/3*b^2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(
c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c
^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(
c*x)*x^2-10/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-
4)*c^2/d^3*(c*x-1)*(c*x+1)*arccosh(c*x)*x^3-14/3*b^2*(-d*(c^2*x^2-1))^(1/2
)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ar
ccosh(c*x)^2*x^2+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c^4/d^3*(c*x-1)*(c*x+1)*arccosh(c*x)*x^5+4/3*b^2*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*
c^4*x^4+11*c^2*x^2-4)*c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)^...
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{acosh}{(cx)})^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2
*d^3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 2/3*a*b*(2*x/(sqrt(-c^2*d*x^2
 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a^2*(2*x/(sq
rt(-c^2*d*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^2x^2-6\sqrt{-c^2x^2}}{}$$

input
```
int((a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a*b*c**
2*x**2 - 6*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c
**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)
*a*b + 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)**2/(sqrt( - c**2*x**2 + 1)*
c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x
)*b**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)**2/(sqrt( - c**
2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*
x**2 + 1)),x)*b**2 + 2*a**2*c**2*x**3 - 3*a**2*x)/(3*sqrt(d)*sqrt( - c**2*
x**2 + 1)*d**2*(c**2*x**2 - 1))
```

3.206 $\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 585

$$\begin{aligned}
\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{5/2}}\,dx &= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} \\
&- \frac{bcx(a+b\text{arccosh}(cx))}{3d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} + \frac{(a+b\text{arccosh}(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
&+ \frac{(a+b\text{arccosh}(cx))^2}{d^2\sqrt{d-c^2dx^2}} - \frac{2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\arctan\left(e^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{14b\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} \\
&+ \frac{7b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} \\
&+ \frac{2ib\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{2ib\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{7b^2\sqrt{-1+cx}\sqrt{1+cx}\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} \\
&- \frac{2ib^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2ib^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(3,ie^{\text{arccosh}(cx)}\right)}{d^3\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
-1/3*b^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*x*(a+b*arccosh(c*x))/d^2/(c*x-1)
^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccosh(c*x))^2/d/(-c^2
*d*x^2+d)^(3/2)+(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^
3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+14/3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arcc
osh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2
)+7/3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)+2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c
*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^3/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))*polylog(2,I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/3*b^2*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2/(-c
^2*d*x^2+d)^(1/2)-2*I*b^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*(-c^2*d*x^2+d
)^(1/2)*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))/d^3/(c*x-1)^(1/2)/(
c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.39 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\,(d-c^2dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
Sqrt[-(d*(-1 + c^2*x^2))]*(a^2/(3*d^3*(-1 + c^2*x^2)^2) - a^2/(d^3*(-1 + c
^2*x^2))) + (a^2*Log[c*x])/d^(5/2) - (a^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c
^2*x^2))]])/d^(5/2) + (a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCos
h[c*x]*Coth[ArcCosh[c*x]/2] - Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh
[c*x]*Log[1 - I/E^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[
c*x]] + 28*Log[Cosh[ArcCosh[c*x]/2]] - 28*Log[Sinh[ArcCosh[c*x]/2]] - (24*
I)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (24*I)*PolyLog[2, I/E^ArcCosh[c*x]] -
 Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^4)/(((-1 +
c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) - 14*ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2])
)/(12*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]) + (b^2*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)*(-4*Coth[ArcCosh[c*x]/2] + 14*ArcCosh[c*x]^2*Coth[ArcCosh[c*x
]/2] - 2*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*ArcCosh[c*x]^2*Csch[ArcCosh[c*x]/2]^4)/2 - 56*ArcCosh[c*x]*Log[
1 - E^(-ArcCosh[c*x])] - (24*I)*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] +
 (24*I)*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 56*ArcCosh[c*x]*Log[1 +
 E^(-ArcCosh[c*x])] - 56*PolyLog[2, -E^(-ArcCosh[c*x])] - (48*I)*ArcCosh[c
*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (48*I)*ArcCosh[c*x]*PolyLog[2, I/E^A
rcCosh[c*x]] + 56*PolyLog[2, E^(-ArcCosh[c*x])] - (48*I)*PolyLog[3, (-I)/E
^ArcCosh[c*x]] + (48*I)*PolyLog[3, I/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*S...
```

Rubi [A] (verified)

Time = 5.32 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.77, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.862$, Rules used = {6351, 6304, 6316, 83, 6318, 3042, 26, 4670, 2715, 2838, 6351, 25, 6304, 6318, 3042, 26, 4670, 2715, 2838, 6361, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6351}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6304}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6316}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6318}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{2c}+\frac{x(a+b\text{arccosh}(cx))}{2(1-c^2x^2)}-\frac{b}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right.\right.}{2c}}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+\right.}{2c}}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6351}$$

$$\frac{-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int -\frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{(1-cx)(cx+1)}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 6304

$$\frac{\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 6318

$$\frac{-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int i(a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 26

$$\frac{\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d} - \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\csc(i\text{arccosh}(cx))d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}}{d} + \frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 4670

$$-\frac{\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int \log\left(1-e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-ib\int \log\left(1+e^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arcc}\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 2715

$$-\frac{\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(ib\int e^{-\text{arccosh}(cx)}\log\left(1-e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-ib\int e^{-\text{arccosh}(cx)}\log\left(1+e^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}+2i\text{arctanh}\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 2838

$$\frac{\frac{\int \frac{(a+b\text{arccosh}(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d} - \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 6361

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\int \frac{(a+b\text{arccosh}(cx))^2}{cx}d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{ar}}\right.\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^2\csc\left(i\text{arccosh}(cx)+\frac{\pi}{2}\right)d\text{arccosh}(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(\right.}{d\sqrt{}}}{d}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 4668

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-2ib\int (a+b\text{arccosh}(cx))\log\left(1-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)+2ib\int (a+b\text{arccosh}(cx))\log\left(1+ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 3011

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-2ib\left(b\int \text{PolyLog}\left(2,ie^{\text{ar}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 2720$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ib\left(b\int e^{-\text{arccosh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)de^{\text{arccosh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))\right)-2ib\left(b\int e^{-}\right.\right.}{d\sqrt{}}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 7143$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\arctan\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(c\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\frac{i\left(2i\text{arctanh}\left(e^{\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arccosh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arccosh}(cx)}\right)\right)}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(a + b*ArcCosh[c*x])^2/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(-1/2*b/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCos
h[c*x]))/(2*(1 - c^2*x^2)) - ((I/2)*((2*I)*(a + b*ArcCosh[c*x])*ArcTanh[E^
ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b*PolyLog[2, E^ArcCosh
[c*x]]))/c))/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((a + b*ArcCosh[c*x])^2/(d*Sqrt
[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((2*I)*(a + b*Arc
Cosh[c*x])*ArcTanh[E^ArcCosh[c*x]] + I*b*PolyLog[2, -E^ArcCosh[c*x]] - I*b
*PolyLog[2, E^ArcCosh[c*x]]))/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(2*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]] + (2*I)*b*(-(
(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]]) + b*PolyLog[3, (-I)*
E^ArcCosh[c*x]]) - (2*I)*b*(-((a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[
c*x]]) + b*PolyLog[3, I*E^ArcCosh[c*x]])))/(d*Sqrt[d - c^2*d*x^2]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6318
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2}{x\,(-c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arccosh}(cx))^2}{x\,(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]

$$\int \frac{(a+b\mathrm{arccosh}(cx))^2}{x\,(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x\,(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acosh(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

output
```
-1/3*a^2*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2
) - 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + integra
te(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x)
 + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x)
, x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^2}{x\,(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^5-2\sqrt{-c^2x^2+1}\,c^2x^3+\sqrt{-c^2x^2+1}\,x}dx\right)ab\,c^2x^2-6\sqrt{-c^2x}$$

input
```
int((a+b*acosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**5
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x),x)*a*b*c
**2*x**2 - 6*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)
*c**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x
),x)*a*b + 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)**2/(sqrt( - c**2*x**2 +
 1)*c**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1
)*x),x)*b**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)**2/(sqrt(
 - c**2*x**2 + 1)*c**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( -
 c**2*x**2 + 1)*x),x)*b**2 + 3*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2)
)*a**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a**2 - 4
*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 + 4*sqrt( - c**2*x**2 + 1)*a**2 + 3
*a**2*c**2*x**2 - 4*a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*(c**2*x**
2 - 1))
```

3.207 $\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 464

$$\begin{aligned}
\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx &= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} \\
&- \frac{bc(a+b\,\mathrm{arccosh}(cx))}{3d^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d-c^2dx^2}} - \frac{(a+b\,\mathrm{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}} \\
&+ \frac{4c^2x(a+b\,\mathrm{arccosh}(cx))^2}{3d\,(d-c^2dx^2)^{3/2}} + \frac{8c^2x(a+b\,\mathrm{arccosh}(cx))^2}{3d^2\sqrt{d-c^2dx^2}} \\
&+ \frac{8c\sqrt{-1+cx}\sqrt{1+cx}(a+b\,\mathrm{arccosh}(cx))^2}{3d^2\sqrt{d-c^2dx^2}} \\
&- \frac{4bc\sqrt{-1+cx}\sqrt{1+cx}(a+b\,\mathrm{arccosh}(cx))\,\mathrm{arctanh}\left(e^{2\mathrm{arccosh}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} \\
&- \frac{16bc\sqrt{-1+cx}\sqrt{1+cx}(a+b\,\mathrm{arccosh}(cx))\log\left(1-e^{2\mathrm{arccosh}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} \\
&- \frac{b^2c\sqrt{-1+cx}\sqrt{1+cx}\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{arccosh}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} \\
&- \frac{5b^2c\sqrt{-1+cx}\sqrt{1+cx}\,\mathrm{PolyLog}\left(2,e^{2\mathrm{arccosh}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}
\end{aligned}$$

output
```
-1/3*b^2*c^2*x/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arccosh(c*x))/d^2/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*x))^2/d/x/(-c
^2*d*x^2+d)^(3/2)+4/3*c^2*x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)+8/
3*c^2*x*(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+8/3*c*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-4*b*c*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a
+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2/(-c^2*d*x^2
+d)^(1/2)-b^2*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*b^2*c*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(
1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.76 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx = \frac{c\left(\frac{a^2(3-12c^2x^2+8c^4x^4)}{cx(-1+c^2x^2)}+ab\left(10cx\text{arccosh}(cx)-\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)+2cx\text{arccosh}(cx)}{-1+c^2x^2}-2\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(c*((a^2*(3 - 12*c^2*x^2 + 8*c^4*x^4))/(c*x*(-1 + c^2*x^2)) + a*b*(10*c*x*
ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[c*x])
/(-1 + c^2*x^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((-3*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/(c*x) + 3*Log[c*x] + 5*Log[Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)])) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*((c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + ArcCosh[c*x]/(1 - c^2*x^2)
- 8*ArcCosh[c*x]^2 - (c*x*ArcCosh[c*x]^2)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1
 + c*x)^3) + (5*c*x*ArcCosh[c*x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
 + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c*x) - 10*ArcC
osh[c*x]*Log[1 - E^(-2*ArcCosh[c*x])] - 6*ArcCosh[c*x]*Log[1 + E^(-2*ArcCo
sh[c*x])] + 3*PolyLog[2, -E^(-2*ArcCosh[c*x])] + 5*PolyLog[2, E^(-2*ArcCos
h[c*x])])))/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.88 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.01, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.793, Rules used = {6347, 6316, 6314, 6327, 6328, 3042, 26, 4199, 25, 2620, 2715, 2838, 6329, 41, 6351, 41, 6331, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6347}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-cx)^2(cx+1)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\int \frac{(a+b\text{arccosh}(cx))^2}{\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6316}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\int \frac{(a+b\text{arccosh}(cx))^2}{\left(d-c^2dx^2\right)^{3/2}}dx}{3d}+\frac{x(a+b\text{arccosh}(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}\right)-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-cx)^2(cx+1)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6314}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-cx)^2(cx+1)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\right.$$
$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-cx)^2(cx+1)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 6327}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 6328

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int\frac{cx(a+b\text{arccosh}(cx))}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arcc}}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\int-i(a+b\text{arccosh}(cx))\tan\left(i\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 26

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))\tan\left(i\text{ar}\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 4199

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(2i\int -\frac{e^{2\text{arccosh}(cx)}(a+b\text{arc}}{1-e^{2\text{arccosh}(c}}\right.}{cd\sqrt{d}}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 25

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\int \frac{e^{2\text{arccosh}(cx)}(a+b\text{arc}}{1-e^{2\text{arccosh}(c}}\right.}{cd\sqrt{d}}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 2620

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{2}b\int \log\left(1-e^{2\text{arccosh}(c}\right.\right.\right.}{}\right)}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{4}b\int e^{-2\text{arccosh}(cx)}\log\right.\right.\right.}{}\right)}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{x(a+b\text{arccosh}(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{arccosh}(cx}\right.\right.\right.}{}\right)}{}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{6329}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{b\int\frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx}{2c}+\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\right.}{}\right.}{}\right.$$

$$\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{41}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\int\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(\right.\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6351}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\int\frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int\frac{1}{(cx-1)^{3/2}(cx+1)^{3/2}}dx+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(\right.\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{41}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\int \frac{a+b\text{arccosh}(cx)}{x(1-c^2x^2)}dx + \frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2\sqrt{d-c^2dx^2}} +$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}} + \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(\right.\right.\right.}{}}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 6331

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-\int \frac{a+b\text{arccosh}(cx)}{cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)}d\text{arccosh}(cx) + \frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2\sqrt{d-c^2dx^2}} +$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}} + \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(\right.\right.\right.}{}}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5984

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-2\int (a+b\text{arccosh}(cx))\text{csch}(2\text{arccosh}(cx))d\text{arccosh}(cx) + \frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{cx-1}\sqrt{cx+1}}\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}} + \frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(\right.\right.\right.}{}}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-2\int i(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+}}\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\downarrow \text{26}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-2i\int(a+b\text{arccosh}(cx))\csc(2i\text{arccosh}(cx))d\text{arccosh}(cx)+\frac{a+b\text{arccosh}(cx)}{2(1-c^2x^2)}-\frac{bcx}{2\sqrt{cx-1}\sqrt{cx+}}\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\downarrow \text{4670}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arccosh}(cx)}\right)d\text{arccosh}(cx)\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\downarrow \text{2715}$$

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1-e^{2\text{arccosh}(cx)}\right)de^{2\text{arccosh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arccosh}(cx)}\log\left(1+e\right.\right.\right.}{d^2\sqrt{d-c^2}}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(\right.\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(i\text{arctanh}\left(e^{2\text{arccosh}(cx)}\right)(a+b\text{arccosh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)-\frac{1}{4}ib\,\text{Pol}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\left(\frac{a+b\text{arccosh}(cx)}{2c^2(1-c^2x^2)}-\frac{bx}{2c\sqrt{cx-1}\sqrt{cx+1}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\text{arccosh}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\left(-2i\left(-\frac{1}{2}\log\left(\right.\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arccosh}(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcCosh[c*x])^2/(d*x*(d - c^2*d*x^2)^(3/2))) - (2*b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(-1/2*(b*c*x)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*Ar
cCosh[c*x])/(2*(1 - c^2*x^2)) - (2*I)*(I*(a + b*ArcCosh[c*x])*ArcTanh[E^(2
*ArcCosh[c*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcCosh[c*x])] - (I/4)*b*PolyLo
g[2, E^(2*ArcCosh[c*x])])))/(d^2*Sqrt[d - c^2*d*x^2]) + 4*c^2*((x*(a + b*A
rcCosh[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(-1/2*(b*x)/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x]
)/(2*c^2*(1 - c^2*x^2))))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*((x*(a + b*ArcC
osh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(((-1/2*I)*(a + b*ArcCosh[c*x])^2)/b - (2*I)*(-1/2*((a + b*ArcCosh[c*x]
)*Log[1 - E^(2*ArcCosh[c*x])]) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/4)))/(
c*d*Sqrt[d - c^2*d*x^2])))/(3*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6314
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]   Int[x*((a
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 6316
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcCosh[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
 c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6328
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6331
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IG
tQ[n, 0]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6351
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[
(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] ||
EqQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. $2(458) = 916$.

Time = 0.82 (sec) , antiderivative size = 2857, normalized size of antiderivative = 6.16

method	result	size
default	Expression too large to display	2857
parts	Expression too large to display	2857

input
```
int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(16*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4*c^4+16*arccosh(c*x)*c^5*x^5-6*ln(1+(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^5*c^5-10*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2-1)*x^5*c^5-24*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2-32
*c^3*x^3*arccosh(c*x)+12*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*x^3*c^3
+20*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x^3*c^3-c^3*x^3+6*arccosh(c*
x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+16*c*x*arccosh(c*x)-6*ln(1+(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)*x*c-10*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*x*c+
c*x)/d^3/(c^6*x^6-3*c^4*x^4+3*c^2*x^2-1)/x-5*b^2*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*c^2+9*b^2*(-d*(c^2*x^2-1))^(1/2)/d
^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/x*arccosh(c*x)^2-32/3*b^2*(-d*(c^2*
x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^9*c^10+40*b^2*(-d*
(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*c^8-160/3*b
^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*c^6+
29*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*
c^4-8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x
*(c*x-1)*(c*x+1)*arccosh(c*x)*c^2+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^
6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x
)*c^3+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2
-9)*x^7*(c*x-1)*(c*x+1)*arccosh(c*x)*c^8+10/3*b^2*(-d*(c^2*x^2-1))^(1/2...
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxi
ma")
```

output
```
1/3*a^2*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3
/2)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + integrate(b^2*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x^2) + 2*a*b*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^2 (d - c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^2}{(-c^2dx^2 + d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^2 (d - c^2dx^2)^{5/2}}\,dx = \int \frac{(a + b\,\text{acosh}(c\,x))^2}{x^2\,(d - c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arccosh}(cx))^2}{x^2 (d - c^2dx^2)^{5/2}}\, dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,c^4x^6-2\sqrt{-c^2x^2+1}\,c^2x^4+\sqrt{-c^2x^2+1}\,x^2}dx\right)ab\,c^2x^3 - 6\sqrt{-c^2}}{}$$

input
```
int((a+b*acosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**6
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1)*x**2),x)*a*
b*c**2*x**3 - 6*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)/(sqrt( - c**2*x**2 +
 1)*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1
)*x**2),x)*a*b*x + 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x)**2/(sqrt( - c**
2*x**2 + 1)*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*
x**2 + 1)*x**2),x)*b**2*c**2*x**3 - 3*sqrt( - c**2*x**2 + 1)*int(acosh(c*x
)**2/(sqrt( - c**2*x**2 + 1)*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**
4 + sqrt( - c**2*x**2 + 1)*x**2),x)*b**2*x + 8*a**2*c**4*x**4 - 12*a**2*c*
*2*x**2 + 3*a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*x*(c**2*x**2 - 1)
)
```

3.208 $\int \frac{x^4 \operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 243

$$\begin{aligned}\int \frac{x^4\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx =& -\frac{15x\sqrt{1-ax}\sqrt{1+ax}}{64a^4}-\frac{x^3\sqrt{1-ax}\sqrt{1+ax}}{32a^2}\\ &-\frac{15\sqrt{1-ax}\operatorname{arccosh}(ax)}{64a^5\sqrt{-1+ax}}+\frac{3x^2\sqrt{1-ax}\operatorname{arccosh}(ax)}{8a^3\sqrt{-1+ax}}\\ &+\frac{x^4\sqrt{1-ax}\operatorname{arccosh}(ax)}{8a\sqrt{-1+ax}}-\frac{3x\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^2}{8a^4}\\ &-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^2}{4a^2}-\frac{\sqrt{1-ax}\operatorname{arccosh}(ax)^3}{8a^5\sqrt{-1+ax}}\end{aligned}$$

output
```
-15/64*x*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^4-1/32*x^3*(-a*x+1)^(1/2)*(a*x+1)^
(1/2)/a^2-15/64*(-a*x+1)^(1/2)*arccosh(a*x)/a^5/(a*x-1)^(1/2)+3/8*x^2*(-a*
x+1)^(1/2)*arccosh(a*x)/a^3/(a*x-1)^(1/2)+1/8*x^4*(-a*x+1)^(1/2)*arccosh(a
*x)/a/(a*x-1)^(1/2)-3/8*x*(-a^2*x^2+1)^(1/2)*arccosh(a*x)^2/a^4-1/4*x^3*(-
a^2*x^2+1)^(1/2)*arccosh(a*x)^2/a^2-1/8*(-a*x+1)^(1/2)*arccosh(a*x)^3/a^5/
(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.48

$$\int \frac{x^4\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(32\text{arccosh}(ax)^3-4\text{arccosh}(ax)(16\cosh(2\text{arccosh}(ax))+\cosh(4\text{arccosh}(ax)))+32\,\text{si}\right.}{256a^5\sqrt{1}}$$

input
```
Integrate[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(32*ArcCosh[a*x]^3 - 4*ArcCosh[a*x]*
(16*Cosh[2*ArcCosh[a*x]] + Cosh[4*ArcCosh[a*x]]) + 32*Sinh[2*ArcCosh[a*x]]
 + Sinh[4*ArcCosh[a*x]] + 8*ArcCosh[a*x]^2*(8*Sinh[2*ArcCosh[a*x]] + Sinh[
4*ArcCosh[a*x]])))/(256*a^5*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6353, 6298, 111, 27, 101, 43, 6353, 6298, 101, 43, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$
$$\Big\downarrow \text{6353}$$
$$\frac{3\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\sqrt{ax-1}\int x^3\text{arccosh}(ax)dx}{2a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}$$
$$\Big\downarrow \text{6298}$$
$$\frac{3\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\int \frac{x^4}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{2a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{111}$$

$$\frac{3\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{\int \frac{3x^2}{\sqrt{ax-1}\sqrt{ax+1}}dx}{4a^2}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}\right)\right)}{2a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{3\int \frac{x^2}{\sqrt{ax-1}\sqrt{ax+1}}dx}{4a^2}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}\right)\right)}{2a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{101}$$

$$\frac{3\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}\right)\right)}{2a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{43}$$

$$\frac{3\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}+\frac{3\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)}{4a^2}\right)\right)}{2a\sqrt{1-ax}}$$

$$\Big\downarrow \text{6353}$$

$$\frac{3\left(\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\sqrt{ax-1}\int x\text{arccosh}(ax)dx}{a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}+\frac{3\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)}{4a^2}\right)\right)}{2a\sqrt{1-ax}}$$

$\downarrow$ 6298

$$\frac{3\left(\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\int \frac{x^2}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}+\frac{3\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)}{4a^2}\right)\right)}{2a\sqrt{1-ax}}$$

$\downarrow$ 101

$$\frac{3\left(-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)\right)}{a\sqrt{1-ax}}+\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}+\frac{3\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)}{4a^2}\right)\right)}{2a\sqrt{1-ax}}$$

$\downarrow$ 43

$$\frac{3\left(\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)\right)}{a\sqrt{1-ax}}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}+\frac{3\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)}{4a^2}\right)\right)}{2a\sqrt{1-ax}}$$

$$\Big\downarrow \; 6307$$

$$-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{4a^2}+\frac{3\left(\frac{\sqrt{ax-1}\text{arccosh}(ax)^3}{6a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)\right)}{a\sqrt{1-ax}}\right)}{4a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{4a^2}+\frac{3\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)}{4a^2}\right)\right)}{2a\sqrt{1-ax}}$$

input
```
Int[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2 - (Sqrt[-1 + a*x]*((x^4*Ar
cCosh[a*x])/4 - (a*((x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(4*a^2) + (3*((x*Sq
rt[-1 + a*x]*Sqrt[1 + a*x])/(2*a^2) + ArcCosh[a*x]/(2*a^3)))/(4*a^2)))/4))
/(2*a*Sqrt[1 - a*x]) + (3*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2 +
 (Sqrt[-1 + a*x]*ArcCosh[a*x]^3)/(6*a^3*Sqrt[1 - a*x]) - (Sqrt[-1 + a*x]*(
(x^2*ArcCosh[a*x])/2 - (a*((x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a^2) + ArcC
osh[a*x]/(2*a^3)))/2))/(a*Sqrt[1 - a*x])))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(199) = 398$.

Time = 0.39 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.01

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^3}{8a^5(a^2x^2-1)}-\frac{\sqrt{-a^2x^2+1}\,(8a^5x^5-12a^3x^3+8\sqrt{ax-1}\,\sqrt{ax+1}\,a^4x^4+4ax-8a^2x^2\sqrt{ax-1}\,\sqrt{ax+1}}{512a^5(a^2x^2-1)}$

input
```
int(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccos
h(a*x)^3-1/512*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x-1)^(1/2)*(a
*x+1)^(1/2)*a^4*x^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1
/2)*(a*x+1)^(1/2))*(8*arccosh(a*x)^2-4*arccosh(a*x)+1)/a^5/(a^2*x^2-1)-1/1
6*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2
)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x)+1)/a^5/(a^
2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2
*a^3*x^3+(a*x-1)^(1/2)*(a*x+1)^(1/2)-2*a*x)*(2*arccosh(a*x)^2+2*arccosh(a*
x)+1)/a^5/(a^2*x^2-1)-1/512*(-a^2*x^2+1)^(1/2)*(-8*(a*x-1)^(1/2)*(a*x+1)^(
1/2)*a^4*x^4+8*a^5*x^5+8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-12*a^3*x^3-(a
*x-1)^(1/2)*(a*x+1)^(1/2)+4*a*x)*(8*arccosh(a*x)^2+4*arccosh(a*x)+1)/a^5/(
a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^4 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^4 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**4*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^4\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{arcosh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^4*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^4*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2\,x^4}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^4*acosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**2*x**4)/sqrt( - a**2*x**2 + 1),x)
```

3.209 $\int \frac{x^3\textbf{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \frac{x^3\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{4\sqrt{1-ax}\sqrt{1+ax}}{27a^4} - \frac{2x^2\sqrt{1-ax}\sqrt{1+ax}}{27a^2} - \frac{4\sqrt{1-a^2x^2}}{3a^4} + \frac{4x\sqrt{1-ax}\text{arccosh}(ax)}{3a^3\sqrt{-1+ax}} + \frac{2x^3\sqrt{1-ax}\text{arccosh}(ax)}{9a\sqrt{-1+ax}} - \frac{2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}$$

output

```
-4/27*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^4-2/27*x^2*(-a*x+1)^(1/2)*(a*x+1)^(1/
2)/a^2-4/3*(-a^2*x^2+1)^(1/2)/a^4+4/3*x*(-a*x+1)^(1/2)*arccosh(a*x)/a^3/(a
*x-1)^(1/2)+2/9*x^3*(-a*x+1)^(1/2)*arccosh(a*x)/a/(a*x-1)^(1/2)-2/3*(-a^2*
x^2+1)^(1/2)*arccosh(a*x)^2/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arccosh(a*x)^2/
a^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.62

$$\int \frac{x^3\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \left(-\frac{40}{27a^4}-\frac{2x^2}{27a^2}\right)\sqrt{1-a^2x^2} + \frac{2x\sqrt{1-a^2x^2}(6+a^2x^2)\,\text{arccosh}(ax)}{9a^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\sqrt{1-a^2x^2}(2+a^2x^2)\,\text{arccosh}(ax)^2}{3a^4}$$

input
```
Integrate[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
(-40/(27*a^4) - (2*x^2)/(27*a^2))*Sqrt[1 - a^2*x^2] + (2*x*Sqrt[1 - a^2*x^
2]*(6 + a^2*x^2)*ArcCosh[a*x])/(9*a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (Sqr
t[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCosh[a*x]^2)/(3*a^4)
```

Rubi [A] (warning: unable to verify)

Time = 1.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6353, 6298, 111, 27, 83, 6329, 6294, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2} - \frac{2\sqrt{ax-1}\int x^2\text{arccosh}(ax)dx}{3a\sqrt{1-ax}} - \frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)-\frac{1}{3}a\int \frac{x^3}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{3a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{ 111}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)-\frac{1}{3}a\left(\frac{\int \frac{2x}{\sqrt{ax-1}\sqrt{ax+1}}dx}{3a^2}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{3a^2}\right)\right)}{3a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)-\frac{1}{3}a\left(\frac{2\int \frac{x}{\sqrt{ax-1}\sqrt{ax+1}}dx}{3a^2}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{3a^2}\right)\right)}{3a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}-\frac{2\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)-\frac{1}{3}a\left(\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a^4}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{3a^2}\right)\right)}{3a\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 6329}$$

$$\frac{2\left(-\frac{2\sqrt{ax-1}\int \text{arccosh}(ax)dx}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}-\frac{2\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)-\frac{1}{3}a\left(\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a^4}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{3a^2}\right)\right)}{3a\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 6294}$$

$$\frac{2\left(-\frac{2\sqrt{ax-1}\left(x\text{arccosh}(ax)-a\int\frac{x}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}-\frac{2\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)-\frac{1}{3}a\left(\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a^4}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{3a^2}\right)\right)}{3a\sqrt{1-ax}}$$

$$\Big\downarrow \text{83}$$

$$-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{a^2}-\frac{2\sqrt{ax-1}\left(x\text{arccosh}(ax)-\frac{\sqrt{ax-1}\sqrt{ax+1}}{a}\right)}{a\sqrt{1-ax}}\right)}{3a^2}-\frac{2\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)-\frac{1}{3}a\left(\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a^4}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{3a^2}\right)\right)}{3a\sqrt{1-ax}}$$

input
```
Int[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2 - (2*Sqrt[-1 + a*x]*(-1/3*
(a*((2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a^4) + (x^2*Sqrt[-1 + a*x]*Sqrt[1
+ a*x])/(3*a^2))) + (x^3*ArcCosh[a*x])/3))/(3*a*Sqrt[1 - a*x]) + (2*(-((Sq
rt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2) - (2*Sqrt[-1 + a*x]*(-((Sqrt[-1 + a*x
]*Sqrt[1 + a*x])/a) + x*ArcCosh[a*x]))/(a*Sqrt[1 - a*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(162) = 324$.

Time = 0.46 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,(4a^4x^4-5a^2x^2+4a^3x^3\sqrt{ax-1}\,\sqrt{ax+1}-3\sqrt{ax-1}\,\sqrt{ax+1}\,ax+1)\left(9\operatorname{arccosh}(ax)^2-6\operatorname{arccosh}(ax)+2\right)}{216a^4(a^2x^2-1)}-\frac{3\sqrt{-a^2x^2\cdots}}{}$
orering	$\frac{(19a^6x^6+100a^4x^4-380a^2x^2+240)\operatorname{arccosh}(ax)^2}{27a^6x^2\sqrt{-a^2x^2+1}}-\frac{2(ax-1)(ax+1)(a^4x^4+12a^2x^2-20)\left(\frac{3x^2\operatorname{arccosh}(ax)^2}{\sqrt{-a^2x^2+1}}+\frac{2x^3\operatorname{arccosh}(ax)a\cdots}{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{\cdots}}\cdots\right.}{9a^6x^4}$

input
```
int(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*
x+1)^(1/2)-3*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x+1)*(9*arccosh(a*x)^2-6*arccos
h(a*x)+2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x-1)^(1/2)*(a*x+1)^(1
/2)*a*x+a^2*x^2-1)*(arccosh(a*x)^2-2*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-3/8*(
-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-1)*(arccosh(a*x
)^2+2*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-
5*a^2*x^2-4*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*(a*x-1)^(1/2)*(a*x+1)^(1
/2)*a*x+1)*(9*arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.76

$$\int \frac{x^3\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx =$$
$$-\frac{9\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2-6\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(\cdots\right.}{27\left(a^6x^2-a^4\right)}$$

input
```
integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 - 1))^2 - 6*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 - 1)) + 2*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1))
/(a^6*x^2 - a^4)
```

Sympy [F]

$$\int \frac{x^3 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\, dx = \int \frac{x^3 \operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

input
```
integrate(x**3*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{x^3 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\, dx = -\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)^2 + \frac{2\left(-i\sqrt{a^2x^2-1}x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)}{27\,a^2} + \frac{2\,(i\,a^2x^3+6i\,x)\operatorname{arcosh}(ax)}{9\,a^3}$$

input
```
integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^
2 + 2/27*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)/a^2 + 2/9
*(I*a^2*x^3 + 6*I*x)*arccosh(a*x)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\,\mathrm{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*acosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**2*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.210 $\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-ax}\sqrt{1+ax}}{4a^2} - \frac{\sqrt{1-ax}\text{arccosh}(ax)}{4a^3\sqrt{-1+ax}} + \frac{x^2\sqrt{1-ax}\text{arccosh}(ax)}{2a\sqrt{-1+ax}} - \frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2} - \frac{\sqrt{1-ax}\text{arccosh}(ax)^3}{6a^3\sqrt{-1+ax}}$$

output
```
-1/4*x*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-1/4*(-a*x+1)^(1/2)*arccosh(a*x)/a^
3/(a*x-1)^(1/2)+1/2*x^2*(-a*x+1)^(1/2)*arccosh(a*x)/a/(a*x-1)^(1/2)-1/2*x*
(-a^2*x^2+1)^(1/2)*arccosh(a*x)^2/a^2-1/6*(-a*x+1)^(1/2)*arccosh(a*x)^3/a^
3/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-((-1+ax)(1+ax))}(4\text{arccosh}(ax)^3-6\text{arccosh}(ax)\cosh(2\text{arccosh}(ax))+(3+6\text{arccosh}(ax)^2)\text{si}}{24a^3\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

input
```
Integrate[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/24*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(4*ArcCosh[a*x]^3 - 6*ArcCosh[a*x]*Co
sh[2*ArcCosh[a*x]] + (3 + 6*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]]))/(a^3*Sq
rt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6353, 6298, 101, 43, 6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\sqrt{ax-1}\int x\text{arccosh}(ax)dx}{a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\int \frac{x^2}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}$$

$$\Big\downarrow \; 101$$

$$-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)\right)}{a\sqrt{1-ax}}+\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}$$

$$\Big\downarrow \; 43$$

$$\frac{\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)\right)}{a\sqrt{1-ax}}$$

$$\Big\downarrow \; 6307$$

$$\frac{\sqrt{ax-1}\text{arccosh}(ax)^3}{6a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)-\frac{1}{2}a\left(\frac{\text{arccosh}(ax)}{2a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a^2}\right)\right)}{a\sqrt{1-ax}}$$

input
```
Int[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2 + (Sqrt[-1 + a*x]*ArcCosh[a*
x]^3)/(6*a^3*Sqrt[1 - a*x]) - (Sqrt[-1 + a*x]*((x^2*ArcCosh[a*x])/2 - (a*(
(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a^2) + ArcCosh[a*x]/(2*a^3)))/2))/(a*S
qrt[1 - a*x])
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.58

method	result
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^{3}}{6a^{3}(a^{2}x^{2}-1)}-\frac{\sqrt{-a^{2}x^{2}+1}\,(2a^{3}x^{3}-2ax+2a^{2}x^{2}\sqrt{ax-1}\,\sqrt{ax+1}-\sqrt{ax-1}\,\sqrt{ax+1})\left(2\operatorname{arccosh}(ax\right.}{16a^{3}(a^{2}x^{2}-1)}$

input
```
int(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccos
h(a*x)^3-1/16*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*
(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x
)+1)/a^3/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(-2*a^2*x^2*(a*x-1)^(1/2)*(a*
x+1)^(1/2)+2*a^3*x^3+(a*x-1)^(1/2)*(a*x+1)^(1/2)-2*a*x)*(2*arccosh(a*x)^2+
2*arccosh(a*x)+1)/a^3/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^2\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\operatorname{acosh}^2\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(x**2*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{arcosh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*acosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**2*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.211 $\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{a^2}+\frac{2x\sqrt{1-ax}\text{arccosh}(ax)}{a\sqrt{-1+ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{a^2}$$

output
```
-2*(-a^2*x^2+1)^(1/2)/a^2+2*x*(-a*x+1)^(1/2)*arccosh(a*x)/a/(a*x-1)^(1/2)-
(-a^2*x^2+1)^(1/2)*arccosh(a*x)^2/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}\left(-2+\frac{2ax\text{arccosh}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}-\text{arccosh}(ax)^2\right)}{a^2}$$

input
```
Integrate[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(-2 + (2*a*x*ArcCosh[a*x])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x
]) - ArcCosh[a*x]^2))/a^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6329, 6294, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{2\sqrt{ax-1}\int \text{arccosh}(ax)dx}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{a^2}$$

$$\Big\downarrow \text{6294}$$

$$-\frac{2\sqrt{ax-1}\left(x\text{arccosh}(ax)-a\int \frac{x}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{a^2}$$

$$\Big\downarrow \text{83}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{a^2}-\frac{2\sqrt{ax-1}\left(x\text{arccosh}(ax)-\frac{\sqrt{ax-1}\sqrt{ax+1}}{a}\right)}{a\sqrt{1-ax}}$$

input
```
Int[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2) - (2*Sqrt[-1 + a*x]*(-((Sqrt[-1
+ a*x]*Sqrt[1 + a*x])/a) + x*ArcCosh[a*x]))/(a*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\left(\operatorname{arccosh}(ax)^2 a^2 x^2-2 \operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}\,ax+2a^2x^2-\operatorname{arccosh}(ax)^2-2\right)\sqrt{-a^2x^2+1}}{(a^2x^2-1)a^2}$
orering	$\frac{(a^4x^4-4a^2x^2+2)\operatorname{arccosh}(ax)^2}{a^4x^2\sqrt{-a^2x^2+1}}+\frac{2(ax-1)(ax+1)\left(\frac{\operatorname{arccosh}(ax)^2}{\sqrt{-a^2x^2+1}}+\frac{2x\operatorname{arccosh}(ax)a}{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{x^2\operatorname{arccosh}(ax)^2a^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\right)}{a^4x^2}+\frac{(ax-1)^2(a}{}$

input
```
int(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(arccosh(a*x)^2*a^2*x^2-2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x+2*
a^2*x^2-arccosh(a*x)^2-2)*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{2\,\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)+\left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^2-2\,(a^2x^2-1}{a^4x^2-a^2}$$

input
```
integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(2*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1)) +
 (-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 2*(a^2*x^2 - 1)*sqr
t(-a^2*x^2 + 1))/(a^4*x^2 - a^2)
```

Sympy [F]

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{acosh}^2\,(ax)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(x*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{2i\,x\,\text{arcosh}\,(ax)}{a}-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)^2}{a^2}-\frac{2i\,\sqrt{a^2x^2-1}}{a^2}$$

input
```
integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
2*I*x*arccosh(a*x)/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/a^2 - 2*I*sqrt(a^
2*x^2 - 1)/a^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{a^2} + \frac{2i\left(x\log\left(ax+i\sqrt{-a^2x^2+1}\right)-\frac{i\sqrt{-a^2x^2+1}}{a}\right)}{a}$$

input
```
integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^2/a^2 + 2*I*(x*log(a*x +
I*sqrt(-a^2*x^2 + 1)) - I*sqrt(-a^2*x^2 + 1)/a)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\mathrm{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2\,x}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*acosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**2*x)/sqrt( - a**2*x**2 + 1),x)
```

3.212 $\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-ax}\,\text{arccosh}(ax)^3}{3a\sqrt{-1+ax}}$$

output
```
-1/3*(-a*x+1)^(1/2)*arccosh(a*x)^3/a/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{arccosh}(ax)^3}{3a\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)^2}{\sqrt{1-a^2x^2}} \, dx$$

$$\Big\downarrow \text{6307}$$

$$\frac{\sqrt{ax-1}\text{arccosh}(ax)^3}{3a\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[-1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{arccosh}(ax)^3}{3(a^2x^2-1)a}$	51

input
```
int(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)/a*ar
ccosh(a*x)^3
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}^2\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^2/(1 - a^2*x^2)^(1/2),x)
```

output
```
int(acosh(a*x)^2/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(acosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**2/sqrt( - a**2*x**2 + 1),x)
```

3.213 $\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\begin{aligned}\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = & -\frac{2\sqrt{1-ax}\operatorname{arccosh}(ax)^2\arctan\left(e^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} \\ & + \frac{2i\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} \\ & - \frac{2i\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} \\ & - \frac{2i\sqrt{1-ax}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} \\ & + \frac{2i\sqrt{1-ax}\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\end{aligned}$$

output
```
-2*(-a*x+1)^(1/2)*arccosh(a*x)^2*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/(
a*x-1)^(1/2)+2*I*(-a*x+1)^(1/2)*arccosh(a*x)*polylog(2,-I*(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-2*I*(-a*x+1)^(1/2)*arccosh(a*x)*polylog(2
,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-2*I*(-a*x+1)^(1/2)*pol
ylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)+2*I*(-a*x+1)^(1
/2)*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{i\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(-\operatorname{arccosh}(ax)^2\left(\log\left(1-ie^{-\operatorname{arccosh}(ax)}\right)-\log\left(1+ie^{-\operatorname{arccosh}(ax)}\right)\right)-2\operatorname{arccosh}(ax)\left(\operatorname{Po}\right.\right.}{}$$

input
```
Integrate[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(I*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-(ArcCosh[a*x]^2*(Log[1 - I/E^Arc
Cosh[a*x]] - Log[1 + I/E^ArcCosh[a*x]])) - 2*ArcCosh[a*x]*(PolyLog[2, (-I)
/E^ArcCosh[a*x]] - PolyLog[2, I/E^ArcCosh[a*x]]) - 2*PolyLog[3, (-I)/E^Arc
Cosh[a*x]] + 2*PolyLog[3, I/E^ArcCosh[a*x]]))/Sqrt[1 - a^2*x^2]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6361, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{ 6361}$$
$$\frac{\sqrt{ax-1}\int \frac{\operatorname{arccosh}(ax)^2}{ax}d\operatorname{arccosh}(ax)}{\sqrt{1-ax}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{ax-1}\int \operatorname{arccosh}(ax)^2\csc\left(i\operatorname{arccosh}(ax)+\frac{\pi}{2}\right)d\operatorname{arccosh}(ax)}{\sqrt{1-ax}}$$
$$\Big\downarrow \text{ 4668}$$

$$\frac{\sqrt{ax-1}\left(-2i\int \text{arccosh}(ax)\log\left(1-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+2i\int \text{arccosh}(ax)\log\left(1+ie^{\text{arccosh}(ax)}\right)d\text{arccosl}\right.}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\sqrt{ax-1}\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)-\text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)\right)-2i\left(\int \text{PolyLo}\right.\right.}{\sqrt{1-}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{ax-1}\left(2i\left(\int e^{-\text{arccosh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)de^{\text{arccosh}(ax)}-\text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)\right)-2\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\sqrt{ax-1}\left(2\text{arccosh}(ax)^2\arctan\left(e^{\text{arccosh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arccosh}(ax)}\right)-\text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccos}}\right.\right.\right.}{\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(Sqrt[-1 + a*x]*(2*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]] + (2*I)*(-(ArcCos
h[a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]]) + PolyLog[3, (-I)*E^ArcCosh[a*x]])
 - (2*I)*(-(ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]]) + PolyLog[3, I*E^Ar
cCosh[a*x]])))/Sqrt[1 - a*x]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{-a^2x^2+1}}dx$$

input
```
int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}^2\left(ax\right)}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acosh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(acosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**2/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.214 $\int \frac{\text{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{\text{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{a\sqrt{1-ax}\,\text{arccosh}(ax)^2}{\sqrt{-1+ax}} - \frac{\sqrt{1-a^2x^2}\,\text{arccosh}(ax)^2}{x} + \frac{2a\sqrt{1-ax}\,\text{arccosh}(ax)\log\left(1+e^{2\text{arccosh}(ax)}\right)}{\sqrt{-1+ax}} + \frac{a\sqrt{1-ax}\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax)}\right)}{\sqrt{-1+ax}}$$

output
```
-a*(-a*x+1)^(1/2)*arccosh(a*x)^2/(a*x-1)^(1/2)-(-a^2*x^2+1)^(1/2)*arccosh(
a*x)^2/x+2*a*(-a*x+1)^(1/2)*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2))^2)/(a*x-1)^(1/2)+a*(-a*x+1)^(1/2)*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x
+1)^(1/2))^2)/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{a\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(\operatorname{arccosh}(ax)\left(-\operatorname{arccosh}(ax)+\frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\operatorname{arccosh}(ax)}{ax}-2\log\left(1+e^{-2\operatorname{arccosh}(ax)}\right)\right)\right.}{\sqrt{-((-1+ax)(1+ax))}}$$

input
```
Integrate[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(ArcCosh[a*x]*(-ArcCosh[a*x] + (Sq
rt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 2*Log[1 + E^(-2*A
rcCosh[a*x])]) + PolyLog[2, -E^(-2*ArcCosh[a*x])]))/Sqrt[-((-1 + a*x)*(1 +
 a*x))]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6332, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{6332}$$
$$-\frac{2a\sqrt{ax-1}\int \frac{\operatorname{arccosh}(ax)}{x}dx}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^2}{x}$$
$$\Big\downarrow \text{6297}$$

$$-\frac{2a\sqrt{ax-1}\int \frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\text{arccosh}(ax)}{ax}d\text{arccosh}(ax)}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{x}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{x}-\frac{2a\sqrt{ax-1}\int -i\text{arccosh}(ax)\tan(i\text{arccosh}(ax))d\text{arccosh}(ax)}{\sqrt{1-ax}}$$

$$\Big\downarrow \ 26$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{x}+\frac{2ia\sqrt{ax-1}\int \text{arccosh}(ax)\tan(i\text{arccosh}(ax))d\text{arccosh}(ax)}{\sqrt{1-ax}}$$

$$\Big\downarrow \ 4201$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{x}+\\ \frac{2ia\sqrt{ax-1}\left(2i\int \frac{e^{2\text{arccosh}(ax)}\text{arccosh}(ax)}{1+e^{2\text{arccosh}(ax)}}d\text{arccosh}(ax)-\frac{1}{2}i\text{arccosh}(ax)^2\right)}{\sqrt{1-ax}}$$

$$\Big\downarrow \ 2620$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{x}+\\ \frac{2ia\sqrt{ax-1}\left(2i\left(\frac{1}{2}\text{arccosh}(ax)\log\left(e^{2\text{arccosh}(ax)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{arccosh}(ax)}\right)d\text{arccosh}(ax)\right)-\frac{1}{2}i\text{arccosh}(ax)^2\right.}{\sqrt{1-ax}}$$

$$\Big\downarrow \ 2715$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{x}+\\ \frac{2ia\sqrt{ax-1}\left(2i\left(\frac{1}{2}\text{arccosh}(ax)\log\left(e^{2\text{arccosh}(ax)}+1\right)-\frac{1}{4}\int e^{-2\text{arccosh}(ax)}\log\left(1+e^{2\text{arccosh}(ax)}\right)de^{2\text{arccosh}(ax)}\right)-\frac{1}{2}ia\right.}{\sqrt{1-ax}}$$

$$\Big\downarrow \ 2838$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{x}+\\ \frac{2ia\sqrt{ax-1}\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax)}\right)+\frac{1}{2}\text{arccosh}(ax)\log\left(e^{2\text{arccosh}(ax)}+1\right)\right)-\frac{1}{2}i\text{arccosh}(ax)^2\right)}{\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/x) + ((2*I)*a*Sqrt[-1 + a*x]*((-1/2*I
)*ArcCosh[a*x]^2 + (2*I)*((ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])])/2 + P
olyLog[2, -E^(2*ArcCosh[a*x])]/4)))/Sqrt[1 - a*x]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,(a^2x^2-\sqrt{ax-1}\,\sqrt{ax+1}\,ax-1)\operatorname{arccosh}(ax)^2}{x(a^2x^2-1)}-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^2a}{a^2x^2-1}+\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax}}{}$

input
```
int(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-1)*arccosh(a*
x)^2/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x
^2-1)*arccosh(a*x)^2*a+2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a
^2*x^2-1)*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+(-a^2*x
^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(2,-(a*x+(a*x-1
)^(1/2)*(a*x+1)^(1/2))^2)*a
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}^2\left(ax\right)}{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acosh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
(a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(sqrt(a*x + 1)*sqrt
(-a*x + 1)*x) - integrate(2*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x -
 a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((sqrt(a*x + 1)*a*x^2 + (a*x +
1)*sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(acosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**2/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.215 $\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 297

$$\begin{aligned}\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx =& -\frac{a\sqrt{1-ax}\operatorname{arccosh}(ax)}{x\sqrt{-1+ax}}-\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^2}{2x^2}\\
&-\frac{a^2\sqrt{1-ax}\operatorname{arccosh}(ax)^2\arctan\left(e^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\
&+\frac{a^2\sqrt{1-ax}\arctan\left(\sqrt{-1+ax}\sqrt{1+ax}\right)}{\sqrt{-1+ax}}\\
&+\frac{ia^2\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\
&-\frac{ia^2\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\
&-\frac{ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\
&+\frac{ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\end{aligned}$$

output
```
-a*(-a*x+1)^(1/2)*arccosh(a*x)/x/(a*x-1)^(1/2)-1/2*(-a^2*x^2+1)^(1/2)*arcc
osh(a*x)^2/x^2-a^2*(-a*x+1)^(1/2)*arccosh(a*x)^2*arctan(a*x+(a*x-1)^(1/2)*
(a*x+1)^(1/2))/(a*x-1)^(1/2)+a^2*(-a*x+1)^(1/2)*arctan((a*x-1)^(1/2)*(a*x+
1)^(1/2))/(a*x-1)^(1/2)+I*a^2*(-a*x+1)^(1/2)*arccosh(a*x)*polylog(2,-I*(a*
x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-I*a^2*(-a*x+1)^(1/2)*arccosh
(a*x)*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-I*a^2*(
-a*x+1)^(1/2)*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2
)+I*a^2*(-a*x+1)^(1/2)*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x
-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{ia^2\sqrt{-((-1+ax)(1+ax))}\left(\frac{2i\operatorname{arccosh}(ax)}{ax}+\frac{i\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\operatorname{arccosh}(ax)^2}{a^2x^2}-4i\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arccosh}(ax)\right)\right)\right.}{}$$

input
```
Integrate[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
((I/2)*a^2*Sqrt[-((-1 + a*x)*(1 + a*x))]*(((2*I)*ArcCosh[a*x])/(a*x) + (I*
Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^2)/(a^2*x^2) - (4*I)*Arc
Tan[Tanh[ArcCosh[a*x]/2]] + ArcCosh[a*x]^2*Log[1 - I/E^ArcCosh[a*x]] - Arc
Cosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, (-I)/E^A
rcCosh[a*x]] - 2*ArcCosh[a*x]*PolyLog[2, I/E^ArcCosh[a*x]] + 2*PolyLog[3,
(-I)/E^ArcCosh[a*x]] - 2*PolyLog[3, I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(
1 + a*x)]*(1 + a*x))
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6347, 6298, 103, 218, 6361, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6347}$$

$$\frac{1}{2}a^2\int \frac{\text{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}dx-\frac{a\sqrt{ax-1}\int \frac{\text{arccosh}(ax)}{x^2}dx}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{1}{2}a^2\int \frac{\text{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}dx-\frac{a\sqrt{ax-1}\left(a\int \frac{1}{x\sqrt{ax-1}\sqrt{ax+1}}dx-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{2}a^2\int \frac{\text{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}dx-\frac{a\sqrt{ax-1}\left(a^2\int \frac{1}{(ax-1)(ax+1)a+a}d\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}a^2\int \frac{\text{arccosh}(ax)^2}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}-\frac{a\sqrt{ax-1}\left(a\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{6361}$$

$$\frac{a^2\sqrt{ax-1}\int \frac{\text{arccosh}(ax)^2}{ax}d\text{arccosh}(ax)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}-\frac{a\sqrt{ax-1}\left(a\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}$$

$\big\downarrow$ 3042

$$\frac{a^2\sqrt{ax-1}\int \text{arccosh}(ax)^2\csc\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)d\text{arccosh}(ax)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}-\frac{a\sqrt{ax-1}\left(a\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}$$

$\big\downarrow$ 4668

$$\frac{a^2\sqrt{ax-1}\left(-2i\int \text{arccosh}(ax)\log\left(1-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+2i\int \text{arccosh}(ax)\log\left(1+ie^{\text{arccosh}(ax)}\right)d\text{arcc}\right.}{2\sqrt{1-ax}}$$
$$\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}-\frac{a\sqrt{ax-1}\left(a\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}$$

$\big\downarrow$ 3011

$$\frac{a^2\sqrt{ax-1}\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)-\text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)\right)-2i\left(\int \text{Poly}\right.\right.}{2\sqrt{1}}$$
$$\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}-\frac{a\sqrt{ax-1}\left(a\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}$$

$\big\downarrow$ 2720

$$\frac{a^2\sqrt{ax-1}\left(2i\left(\int e^{-\text{arccosh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)de^{\text{arccosh}(ax)}-\text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)\right)-\right.}{}$$
$$\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}-\frac{a\sqrt{ax-1}\left(a\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}$$

$\big\downarrow$ 7143

$$\frac{a^2\sqrt{ax-1}\left(2\text{arccosh}(ax)^2\arctan\left(e^{\text{arccosh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arccosh}(ax)}\right)-\text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arc}}\right.\right.\right.}{2\sqrt{1-ax}}$$
$$\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^2}{2x^2}-\frac{a\sqrt{ax-1}\left(a\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\text{arccosh}(ax)}{x}\right)}{\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/x^2 - (a*Sqrt[-1 + a*x]*(-(ArcCosh
[a*x]/x) + a*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]]))/Sqrt[1 - a*x] + (a^2*S
qrt[-1 + a*x]*(2*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]] + (2*I)*(-(ArcCosh[
a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]]) + PolyLog[3, (-I)*E^ArcCosh[a*x]]) -
 (2*I)*(-(ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]]) + PolyLog[3, I*E^ArcC
osh[a*x]])))/(2*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{-a^2x^2+1}}dx$$

input
```
int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}^2(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^2}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^2}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(acosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**2/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.216 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2 \, dx = \text{Int}\left((fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2,x\right)$$

output
```
Defer(Int)((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2 \, dx = \int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2 \, dx$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 5.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(fx)^m(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{6345}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int (fx)^{m+1}(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{5d\int (fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$$\Big\downarrow \text{6327}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int (fx)^{m+1}\left(1-c^2x^2\right)^2(a+b\text{arccosh}(cx))dx}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{5d\int (fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$$\Big\downarrow \text{6336}$$

$$\frac{5d\int (fx)^m \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-bc\int \frac{(fx)^{m+2}\left(\frac{c^4x^4}{m+6}-\frac{2c^2x^2}{m+4}+\frac{1}{m+2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\right.}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$$\Big\downarrow \; 27$$

$$\frac{5d\int (fx)^m \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\int \frac{(fx)^{m+2}\left(\frac{c^4x^4}{m+6}-\frac{2c^2x^2}{m+4}+\frac{1}{m+2}\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f}+\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(f\right.}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$$\Big\downarrow \; 1905$$

$$\frac{5d\int (fx)^m \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\int \frac{(fx)^{m+2}\left(\frac{c^4x^4}{m+6}-\frac{2c^2x^2}{m+4}+\frac{1}{m+2}\right)}{\sqrt{c^2x^2-1}}dx}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(c\right.}{f^3(m+4)}}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$$\Big\downarrow \; 1590$$

$$\frac{5d\int(fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int\frac{c^2(fx)^{m+2}\left(\frac{m+6}{m+2}-\frac{c^2(m^2+15m+52)x^2}{(m+4)(m+6)}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+6)}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+5}}{f^3(m+6)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx)}{f^5(m+6)}\right.}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$\downarrow$ 27

$$\frac{5d\int(fx)^m\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int\frac{(fx)^{m+2}\left(\frac{m+6}{m+2}-\frac{c^2(m^2+15m+52)x^2}{(m+4)(m+6)}\right)}{\sqrt{c^2x^2-1}}dx}{m+6}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+5}}{f^3(m+6)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}\right.}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$\downarrow$ 363

$$\frac{5d\int (fx)^m \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\left(15m^2+130m+264\right)\int \frac{(fx)^{m+2}}{\sqrt{c^2x^2-1}}dx}{(m+2)(m+4)^2(m+6)}-\frac{\left(m^2+15m+52\right)\sqrt{c^2x^2-1}(fx)^{m+3}}{f(m+4)^2(m+6)}}{m+6}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+5}}{f^3(m+6)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^4(fx)^{\cdots}}{\cdots}\right.}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$$\Big\downarrow \text{279}$$

$$\frac{5d\int (fx)^m \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\left(15m^2+130m+264\right)\sqrt{1-c^2x^2}\int \frac{(fx)^{m+2}}{\sqrt{1-c^2x^2}}dx}{(m+2)(m+4)^2(m+6)\sqrt{c^2x^2-1}}-\frac{\left(m^2+15m+52\right)\sqrt{c^2x^2-1}(fx)^{m+3}}{f(m+4)^2(m+6)}}{m+6}+\frac{c^2\sqrt{c^2x^2-1}(fx)^{m+5}}{f^3(m+6)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\cdots\right.}{f(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}$$

$$\Big\downarrow \text{278}$$

$$\frac{5d\int (fx)^m \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2dx}{m+6}+$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\cdots\right.}{\cdots}\right.}{f(m+6)\sqrt{cx-\cdots}}$$

$$\Big\downarrow \text{6345}$$

$$\frac{5d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int -(fx)^{m+1}(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)}{f}\right.}{m+6}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 25

$$\frac{5d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int (fx)^{m+1}(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)}{f}\right.}{m+6}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 6327

$$\frac{5d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int (fx)^{m+1}(1-c^2x^2)(a+b\text{arccosh}(cx))dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}}{f(m+}\right.}{m+6}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 6336

$$\frac{5d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(-bc\int\frac{(fx)^{m+2}\left(\frac{1}{m+2}-\frac{c^2x^2}{m+4}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int(fx)^m\sqrt{d-c^2}}{}\right.}{m+6}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 27

$$\frac{5d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\int\frac{(fx)^{m+2}\left(\frac{1}{m+2}-\frac{c^2x^2}{m+4}\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int(fx)^m\sqrt{d-c^2}}{}\right.}{m+6}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 960

$$5d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\left(\frac{(3m+10)\int\frac{(fx)^{m+2}}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)(m+4)^2}-\frac{\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+3}}{f(m+4)^2}\right)}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$m+6$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 136

$$5d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\left(\frac{(3m+10)\sqrt{c^2x^2-1}\int\frac{(fx)^{m+2}}{\sqrt{c^2x^2-1}}dx}{(m+2)(m+4)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+3}}{f(m+4)^2}\right)}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}}{f(m+2)}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$m+6$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 279

$$5d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}\int\frac{(fx)^{m+2}}{\sqrt{1-c^2x^2}}dx}{(m+2)(m+4)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+3}}{f(m+4)^2}\right)}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}}{f(m+2)}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

$$m+6$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 278

$$5d\left(\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m}{}\right.}{}\right.}{f(m+4)\sqrt{cx}}\right.$$

$$m+$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4(fx)^{m+6}(a+b\text{arccosh}(cx))}{f^5(m+6)}-\frac{2c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$\downarrow$ 6345

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{2c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}+\frac{c^4(a+b\text{arccosh}(cx))(fx)^{m+6}}{f^5(m+6)}-\frac{bc\sqrt{c^2x^2-1}\left(\cdots\right.}{\cdots}\right.}{f(m+6)\sqrt{cx-\cdots}}$$

$$5d\left(\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+4)}-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}-\frac{bc\left(\frac{(3m+1\cdots}{\cdots}\right.}{\cdots}\right.}{f(m+4)\sqrt{cx-\cdots}}\right.$$

$\downarrow$ 6298

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{2c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}+\frac{c^4(a+b\text{arccosh}(cx))(fx)^{m+6}}{f^5(m+6)}-\frac{bc\sqrt{c^2x^2-1}\left(\cdots\right.}{\cdots}\right.}{f(m+6)\sqrt{cx-\cdots}}$$

$$5d\left(\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+4)}-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}-\frac{bc\left(\frac{(3m+1\cdots}{\cdots}\right.}{\cdots}\right.}{f(m+4)\sqrt{cx-\cdots}}\right.$$

$\downarrow$ 136

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{2c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}+\frac{c^4(a+b\text{arccosh}(cx))(fx)^{m+6}}{f^5(m+6)}-\frac{bc\sqrt{c^2x^2-1}\left(\cdots\right.}{\cdots}\right.}{f(m+6)\sqrt{cx-\cdots}}$$

$$5d\left(\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+4)}-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}-\frac{bc\left(\frac{(3m+1\cdots}{\cdots}\right.}{\cdots}\right.}{f(m+4)\sqrt{cx-\cdots}}\right.$$

$\downarrow$ 279

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{2c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}+\frac{c^4(a+b\text{arccosh}(cx))(fx)^{m+6}}{f^5(m+6)}-\frac{bc\sqrt{c^2x^2-1}\left(\cdots\right.}{\cdots}\right.}{f(m+6)\sqrt{cx-\cdots}}$$

$$5d\left(\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^2(fx)^{m+1}}{f(m+4)}-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{c^2(a+b\text{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}-\frac{bc\left(\frac{(3m+1\cdots}{\cdots}\right.}{\cdots}\right.}{f(m+4)\sqrt{cx-\cdots}}\right.$$

$\downarrow$ 278

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))^2(fx)^{m+1}}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{(a+b\operatorname{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{2c^2(a+b\operatorname{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}+\frac{c^4(a+b\operatorname{arccosh}(cx))(fx)^{m+6}}{f^5(m+6)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$$5d\left(\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))^2(fx)^{m+1}}{f(m+4)}-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{(a+b\operatorname{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{c^2(a+b\operatorname{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}-\frac{bc\left(\frac{(3m+1}{}\right.}{}\right.}{f(m+4)\sqrt{cx-}}\right.$$

$\downarrow$ 6375

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))^2(fx)^{m+1}}{f(m+6)}-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{(a+b\operatorname{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{2c^2(a+b\operatorname{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}+\frac{c^4(a+b\operatorname{arccosh}(cx))(fx)^{m+6}}{f^5(m+6)}-\frac{bc\sqrt{c^2x^2-1}\left(\right.}{}\right.}{f(m+6)\sqrt{cx-}}$$

$$5d\left(\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\operatorname{arccosh}(cx))^2(fx)^{m+1}}{f(m+4)}-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{(a+b\operatorname{arccosh}(cx))(fx)^{m+2}}{f(m+2)}-\frac{c^2(a+b\operatorname{arccosh}(cx))(fx)^{m+4}}{f^3(m+4)}-\frac{bc\left(\frac{(3m+1}{}\right.}{}\right.}{f(m+4)\sqrt{cx-}}\right.$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (fx)^m \left(-c^2 d\, x^2 + d\right)^{\frac{5}{2}} (a + b \operatorname{arccosh}(cx))^2 \, dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.39

$$\int (fx)^m (d - c^2 dx^2)^{5/2} (a + b\text{arccosh}(cx))^2 \, dx = \int \left(-c^2 dx^2 + d\right)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a)^2 (fx)^m \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="
fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="
maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d -c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2\,dx = \int (a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2)*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 267, normalized size of antiderivative = 8.61

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^2\,dx = f^m\sqrt{d}\,d^2\left(2\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,x^4dx\right)ab\,c^4 - 4\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,x^2dx\right)ab\,c^2 + 2\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)ab + \left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)^2\,x^4dx\right)b^2c^4 - 2\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)^2\,x^2dx\right)b^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)^2\,dx\right)b^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,x^4dx\right)a^2c^4 - 2\left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}dx\right)a^2\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^2,x)
```

output
```
f**m*sqrt(d)*d**2*(2*int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**4,x)*a*
b*c**4 - 4*int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*a*b*c**2 + 2
*int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*a*b + int(x**m*sqrt( - c**2
*x**2 + 1)*acosh(c*x)**2*x**4,x)*b**2*c**4 - 2*int(x**m*sqrt( - c**2*x**2
+ 1)*acosh(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*sqrt( - c**2*x**2 + 1)*aco
sh(c*x)**2,x)*b**2 + int(x**m*sqrt( - c**2*x**2 + 1)*x**4,x)*a**2*c**4 - 2
*int(x**m*sqrt( - c**2*x**2 + 1)*x**2,x)*a**2*c**2 + int(x**m*sqrt( - c**2
*x**2 + 1),x)*a**2)
```

3.217 $\int (fx)^m (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2 dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int (fx)^m (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2 dx = \text{Int}\left((fx)^m (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2, x\right)$$

output
```
Defer(Int)((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (fx)^m (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2 dx = \int (fx)^m (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^2 dx$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(fx)^m(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6345}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int -(fx)^{m+1}(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\int (fx)^{m+1}(1-cx)(cx+1)(a+b\text{arccosh}(cx))dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$$\Big\downarrow \text{ 6327}$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\int (fx)^{m+1}\left(1-c^2x^2\right)(a+b\text{arccosh}(cx))dx}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$$\Big\downarrow \text{ 6336}$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(-bc\int\frac{(fx)^{m+2}\left(\frac{1}{m+2}-\frac{c^2x^2}{m+4}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\Big\downarrow$ 27

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\int\frac{(fx)^{m+2}\left(\frac{1}{m+2}-\frac{c^2x^2}{m+4}\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\Big\downarrow$ 960

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\left(\frac{(3m+10)\int\frac{(fx)^{m+2}}{\sqrt{cx-1}\sqrt{cx+1}}dx}{(m+2)(m+4)^2}-\frac{\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+3}}{f(m+4)^2}\right)}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arcc}}{f(m+2)}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\Big\downarrow$ 136

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\left(\frac{(3m+10)\sqrt{c^2x^2-1}\int\frac{(fx)^{m+2}}{\sqrt{c^2x^2-1}}dx}{(m+2)(m+4)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+3}}{f(m+4)^2}\right)}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+ba}{f(m}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\Big\downarrow$ 279

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}\int\frac{(fx)^{m+2}}{\sqrt{1-c^2x^2}}dx}{(m+2)(m+4)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+3}}{f(m+4)^2}\right)}{f}-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\ldots}{f(m\ldots}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\downarrow$ 278

$$\frac{3d\int(fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2dx}{m+4}-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}(\ldots}{f(m+2)(m+3)(m+4)^2\sqrt{cx-1}\sqrt{c\ldots}}\right.}{f}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\downarrow$ 6345

$$\frac{3d\left(-\frac{2bc\sqrt{d-c^2dx^2}\int(fx)^{m+1}(a+b\text{arccosh}(cx))dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+2)}\right)}{m+4}-$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}(\ldots}{f(m+2)(m+3)(m+4)^2\sqrt{cx-1}\sqrt{c\ldots}}\right.}{f}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\downarrow$ 6298

$$\frac{3d\left(-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\int\frac{(fx)^{m+2}}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f(m+2)}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{a}}{f(m+2)}\right.}{m+4}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\right.}{f(m+2)(m+3)(m+4)^2\sqrt{cx-1}\sqrt{c}}\right.}{f}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\downarrow$ 136

$$\frac{3d\left(-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\int\frac{(fx)^{m+2}}{\sqrt{c^2x^2-1}}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(}{f(m-}\right.}{m+4}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\right.}{f(m+2)(m+3)(m+4)^2\sqrt{cx-1}\sqrt{c}}\right.}{f}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\downarrow$ 279

$$\frac{3d\left(-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{1-c^2x^2}\int\frac{(fx)^{m+2}}{\sqrt{1-c^2x^2}}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(}{f(m-}\right.}{m+4}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\right.}{f(m+2)(m+3)(m+4)^2\sqrt{cx-1}\sqrt{c}}\right.}{f}\right.}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\Big\downarrow$ 278

$$\frac{3d\left(\frac{d\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{f^2(m+2)(m+3)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+4}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\ldots\right)}{f(m+2)(m+3)(m+4)^2\sqrt{cx-1}\sqrt{c\ldots}}\ldots\right)}{f}\ldots\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

$\Big\downarrow$ 6375

$$\frac{3d\left(\frac{d\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{f^2(m+2)(m+3)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+4}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2(fx)^{m+4}(a+b\text{arccosh}(cx))}{f^3(m+4)}+\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\left(\frac{(3m+10)\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\ldots\right)}{f(m+2)(m+3)(m+4)^2\sqrt{cx-1}\sqrt{c\ldots}}\ldots\right)}{f}\ldots\right)}{f(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+4)}$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (fx)^m \left(-c^2 d\, x^2+d\right)^{\frac{3}{2}} (a+b \operatorname{arccosh}(cx))^2\, dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.81

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2\, dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (b \operatorname{arcosh}(cx)+a)^2 (fx)^m\, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="
fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2
+ 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="
maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\operatorname{arccosh}(cx))^2\,dx = \int (a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 5.61

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\operatorname{arccosh}(cx))^2\,dx = f^m\sqrt{d}\,d\left(-2\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,x^2dx\right)ab\,c^2 + 2\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)ab - \left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)^2\,x^2dx\right)b^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)^2\,dx\right)b^2 - \left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}dx\right)a^2\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^2,x)
```

output
```
f**m*sqrt(d)*d*( - 2*int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x)*x**2,x)*a*
b*c**2 + 2*int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*a*b - int(x**m*sq
rt( - c**2*x**2 + 1)*acosh(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*sqrt( - c*
*2*x**2 + 1)*acosh(c*x)**2,x)*b**2 - int(x**m*sqrt( - c**2*x**2 + 1)*x**2,
x)*a**2*c**2 + int(x**m*sqrt( - c**2*x**2 + 1),x)*a**2)
```

3.218 $\int (fx)^m \sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2\,dx = \mathrm{Int}\left((fx)^m \sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2,x\right)$$

output
```
Defer(Int)((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2\,dx = \int (fx)^m \sqrt{d-c^2dx^2}(a+b\,\mathrm{arccosh}(cx))^2\,dx$$

input
```
Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(fx)^m(a+b\text{arccosh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6345}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\int (fx)^{m+1}(a+b\text{arccosh}(cx))dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+2)}$$

$$\Big\downarrow \text{ 6298}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\int \frac{(fx)^{m+2}}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f(m+2)}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+2)}$$

$$\Big\downarrow \text{ 136}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{c^2x^2-1}\int \frac{(fx)^{m+2}}{\sqrt{c^2x^2-1}}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+2)}$$

$$\Big\downarrow \text{ 279}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{1-c^2x^2}\int\frac{(fx)^{m+2}}{\sqrt{1-c^2x^2}}dx}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d\int\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+2)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{d\int\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{f^2(m+2)(m+3)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+2)}$$

$$\Big\downarrow \text{ 6375}$$

$$\frac{d\int\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}-\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{(fx)^{m+2}(a+b\text{arccosh}(cx))}{f(m+2)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{f^2(m+2)(m+3)\sqrt{cx-1}\sqrt{cx+1}}\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}(a+b\text{arccosh}(cx))^2}{f(m+2)}$$

input
```
Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (fx)^m \sqrt{-c^2d\,x^2+d}\,(a+b\,\text{arccosh}\,(cx))^2\,dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2(fx)^m\;dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="
fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 56.70 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int (fx)^m\,\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^2\;dx$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

output
```
Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx$$
$$=\int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^2(fx)^m\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx=\int (a+b\,\text{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\begin{aligned}&\int (fx)^m\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^2\,dx\\&=f^m\sqrt{d}\left(2\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)\,dx\right)ab\right.\\&\quad\left.+\left(\int x^m\sqrt{-c^2x^2+1}\,acosh(cx)^2\,dx\right)b^2+\left(\int x^m\sqrt{-c^2x^2+1}dx\right)a^2\right)\end{aligned}$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^2,x)
```

output
```
f**m*sqrt(d)*(2*int(x**m*sqrt( - c**2*x**2 + 1)*acosh(c*x),x)*a*b + int(x*
*m*sqrt( - c**2*x**2 + 1)*acosh(c*x)**2,x)*b**2 + int(x**m*sqrt( - c**2*x*
*2 + 1),x)*a**2)
```

3.219 $\int \frac{(fx)^m(a+b\,\textbf{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(fx)^m (a+b\ \text{arccosh}\,(cx))^2}{\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^2(fx)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="
fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Sympy [N/A]

Not integrable

Time = 17.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(fx)^m\,(a+b\operatorname{acosh}(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2(fx)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2(fx)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="
giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^2\,(f\,x)^m}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{f^m\left(\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}}dx\right)a^2+2\left(\int \frac{x^m acosh(cx)}{\sqrt{-c^2x^2+1}}dx\right)ab+\left(\int \frac{x^m acosh(cx)^2}{\sqrt{-c^2x^2+1}}dx\right)b^2\right)}{\sqrt{d}}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(f**m*(int(x**m/sqrt( - c**2*x**2 + 1),x)*a**2 + 2*int((x**m*acosh(c*x))/s
qrt( - c**2*x**2 + 1),x)*a*b + int((x**m*acosh(c*x)**2)/sqrt( - c**2*x**2
+ 1),x)*b**2))/sqrt(d)
```

3.220 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(fx)^m (a+b\operatorname{arccosh}(cx))^2}{(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^2(fx)^m}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 21.93 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(fx)^m\,(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x
)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2(fx)^m}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="
maxima")
```

output
```
integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^2(fx)^m}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="
giac")
```

output
```
integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{\left(a+b\,\mathrm{acosh}(c\,x)\right)^2\,(f\,x)^m}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 5.03

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{f^m\left(-\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)a^2-2\left(\int \frac{x^m\,acosh(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right.}{\sqrt{d}\,d}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
(f**m*( - int(x**m/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 +
 1)),x)*a**2 - 2*int((x**m*acosh(c*x))/(sqrt( - c**2*x**2 + 1)*c**2*x**2 -
 sqrt( - c**2*x**2 + 1)),x)*a*b - int((x**m*acosh(c*x)**2)/(sqrt( - c**2*x
**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2))/(sqrt(d)*d)
```

3.221 $\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{1-c^2x^2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{1-c^2x^2}} \, dx = \operatorname{Int}\left(\frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{1-c^2x^2}}, x\right)$$

output
```
Defer(Int)((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{1-c^2x^2}} \, dx = \int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{1-c^2x^2}} \, dx$$

input
```
Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
```

output
```
Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(cx)^2(fx)^m}{\sqrt{1-c^2x^2}}\,dx$$
$$\Big\downarrow \; 6375$$
$$\int \frac{\text{arccosh}(cx)^2(fx)^m}{\sqrt{1-c^2x^2}}dx$$

input
```
Int[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{-c^2x^2+1}}dx$$

input
```
int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{(fx)^m \text{arccosh}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m \,\text{arcosh}\,(cx)^2}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*arccosh(c*x)^2/(c^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 9.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(fx)^m \text{arccosh}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m \,\text{acosh}^2\,(cx)}{\sqrt{-\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((f*x)**m*acosh(c*x)**2/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((f*x)**m*acosh(c*x)**2/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \text{arccosh}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m \,\text{arcosh}\,(cx)^2}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m \operatorname{arcosh}(cx)^2}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\operatorname{acosh}(c\,x)^2\,(f\,x)^m}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((acosh(c*x)^2*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((acosh(c*x)^2*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{(fx)^m \text{arccosh}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = f^m \left(\int \frac{x^m acosh(cx)^2}{\sqrt{-c^2x^2+1}}dx\right)$$

input
```
int((f*x)^m*acosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

output
```
f**m*int((x**m*acosh(c*x)**2)/sqrt( - c**2*x**2 + 1),x)
```

3.222 $\int \frac{x^4 \operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 315

$$\begin{aligned}\int \frac{x^4 \operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}} dx &= \frac{45x^2\sqrt{1-ax}}{128a^3\sqrt{-1+ax}} + \frac{3x^4\sqrt{1-ax}}{128a\sqrt{-1+ax}} \\ &\quad - \frac{45x\sqrt{1-ax}\sqrt{1+ax}\operatorname{arccosh}(ax)}{64a^4} \\ &\quad - \frac{3x^3\sqrt{1-ax}\sqrt{1+ax}\operatorname{arccosh}(ax)}{32a^2} \\ &\quad - \frac{45\sqrt{1-ax}\operatorname{arccosh}(ax)^2}{128a^5\sqrt{-1+ax}} + \frac{9x^2\sqrt{1-ax}\operatorname{arccosh}(ax)^2}{16a^3\sqrt{-1+ax}} \\ &\quad + \frac{3x^4\sqrt{1-ax}\operatorname{arccosh}(ax)^2}{16a\sqrt{-1+ax}} - \frac{3x\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{8a^4} \\ &\quad - \frac{x^3\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{4a^2} - \frac{3\sqrt{1-ax}\operatorname{arccosh}(ax)^4}{32a^5\sqrt{-1+ax}}\end{aligned}$$

output
```
45/128*x^2*(-a*x+1)^(1/2)/a^3/(a*x-1)^(1/2)+3/128*x^4*(-a*x+1)^(1/2)/a/(a*
x-1)^(1/2)-45/64*x*(-a*x+1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)/a^4-3/32*x^3*
(-a*x+1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)/a^2-45/128*(-a*x+1)^(1/2)*arccos
h(a*x)^2/a^5/(a*x-1)^(1/2)+9/16*x^2*(-a*x+1)^(1/2)*arccosh(a*x)^2/a^3/(a*x
-1)^(1/2)+3/16*x^4*(-a*x+1)^(1/2)*arccosh(a*x)^2/a/(a*x-1)^(1/2)-3/8*x*(-a
^2*x^2+1)^(1/2)*arccosh(a*x)^3/a^4-1/4*x^3*(-a^2*x^2+1)^(1/2)*arccosh(a*x)
^3/a^2-3/32*(-a*x+1)^(1/2)*arccosh(a*x)^4/a^5/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.43

$$\int \frac{x^4 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(-192(1+2\text{arccosh}(ax)^2)\cosh(2\text{arccosh}(ax)) - 3(1+8\text{arccosh}(ax)^2)\cosh(4\text{arccosh}(\right.}{1024}$$

input
```
Integrate[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-192*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*
ArcCosh[a*x]] - 3*(1 + 8*ArcCosh[a*x]^2)*Cosh[4*ArcCosh[a*x]] + 4*ArcCosh[
a*x]*(24*ArcCosh[a*x]^3 + 32*(3 + 2*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]] +
 (3 + 8*ArcCosh[a*x]^2)*Sinh[4*ArcCosh[a*x]])))/(1024*a^5*Sqrt[-((-1 + a*x
)*(1 + a*x))])
```

Rubi [A] (verified)

Time = 4.45 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6353, 6298, 6353, 6298, 6307, 6354, 15, 6308, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{3\int \frac{x^2 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2} - \frac{3\sqrt{ax-1}\int x^3 \text{arccosh}(ax)^2dx}{4a\sqrt{1-ax}} - \frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{3\int \frac{x^2\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\int \frac{x^4\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{4a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6353}$$

$$\frac{3\left(\frac{\int \frac{\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\sqrt{ax-1}\int x\text{arccosh}(ax)^2dx}{2a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\int \frac{x^4\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{4a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{3\left(\frac{\int \frac{\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\int \frac{x^2\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{2a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\int \frac{x^4\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{4a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6307}$$

$$\frac{3\left(-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\int \frac{x^2\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{2a\sqrt{1-ax}}+\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\int \frac{x^4\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{4a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6354}$$

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\left(\frac{3\int\frac{x^2\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{4a^2}-\frac{\int x^3dx}{4a}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{4a^2}\right)\right)}{4a\sqrt{1-ax}}+\frac{3\left(-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\int\frac{\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}\right)\right)}{2a\sqrt{1-ax}}+\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\ldots}{\ldots}\right.}{4a^2}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}$$

$\downarrow$ 15

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\left(\frac{3\int\frac{x^2\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{4a^2}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a\sqrt{1-ax}}+\frac{3\left(-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\int\frac{\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a\sqrt{1-ax}}+\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\ldots}{2\ldots}\right.}{4a^2}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}$$

$\downarrow$ 6308

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\left(\frac{3\int\frac{x^2\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{4a^2}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}+\frac{3\left(\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\text{arccosh}(ax)^2}{4a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}\ldots\right.\right.}{2a\sqrt{1-ax}}\right.}{4a^2}$$

$\downarrow$ 6354

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\left(\frac{3\left(\frac{\int\frac{\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(\right.\right.}{4a\sqrt{1-ax}}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}+$$

$$\frac{3\left(\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\text{arccosh}(ax)^2}{4a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\right.\right.}{2a\sqrt{1-ax}}\right.}{4a^2}$$

$\downarrow$ 15

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\left(\frac{3\left(\frac{\int\frac{\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}+\frac{x^3\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax\right.\right.}{4a\sqrt{1-ax}}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}+$$

$$\frac{3\left(\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\text{arccosh}(ax)^2}{4a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\right.\right.}{2a\sqrt{1-ax}}\right.}{4a^2}$$

$\downarrow$ 6308

$$-\frac{x^3\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{4a^2}+$$

$$\frac{3\left(\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\text{arccosh}(ax)^2}{4a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\right.\right.}{2a\sqrt{1-ax}}\right.}{4a^2}$$

$$\frac{3\sqrt{ax-1}\left(\frac{1}{4}x^4\text{arccosh}(ax)^2-\frac{1}{2}a\left(\frac{x^3\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{4a^2}+\frac{3\left(\frac{\text{arccosh}(ax)^2}{4a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\right.\right.}{4a\sqrt{1-ax}}$$

input
```
Int[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/a^2 - (3*Sqrt[-1 + a*x]*((x^4*
ArcCosh[a*x]^2)/4 - (a*(-1/16*x^4/a + (x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x])/(4*a^2) + (3*(-1/4*x^2/a + (x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Arc
Cosh[a*x])/(2*a^2) + ArcCosh[a*x]^2/(4*a^3)))/(4*a^2)))/2))/(4*a*Sqrt[1 -
a*x]) + (3*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/a^2 + (Sqrt[-1 + a*x
]*ArcCosh[a*x]^4)/(8*a^3*Sqrt[1 - a*x]) - (3*Sqrt[-1 + a*x]*((x^2*ArcCosh[
a*x]^2)/2 - a*(-1/4*x^2/a + (x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/
(2*a^2) + ArcCosh[a*x]^2/(4*a^3))))/(2*a*Sqrt[1 - a*x])))/(4*a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(259) = 518$.

Time = 0.44 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.65

method	result
default	$-\frac{3\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^4}{32a^5(a^2x^2-1)}-\frac{\sqrt{-a^2x^2+1}\,(8a^5x^5-12a^3x^3+8\sqrt{ax-1}\,\sqrt{ax+1}\,a^4x^4+4ax-8a^2x^2\sqrt{ax-1}\,\sqrt{ax+}}{2048a^5(a^2}$

input
```
int(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/32*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arcco
sh(a*x)^4-1/2048*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x-1)^(1/2)*
(a*x+1)^(1/2)*a^4*x^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^
(1/2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-
3)/a^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*a^2*x^2*(a*x
-1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*a
rccosh(a*x)^2+6*arccosh(a*x)-3)/a^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(-
2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2*a^3*x^3+(a*x-1)^(1/2)*(a*x+1)^(1/2
)-2*a*x)*(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^5/(a^2*x^2
-1)-1/2048*(-a^2*x^2+1)^(1/2)*(-8*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^4*x^4+8*a^
5*x^5+8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-12*a^3*x^3-(a*x-1)^(1/2)*(a*x+
1)^(1/2)+4*a*x)*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccosh(a*x)+3)/a^
5/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^4 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^4 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{acosh}^3\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(x**4*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^4 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \operatorname{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^4*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^4*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^3\,x^4}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^4*acosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**3*x**4)/sqrt( - a**2*x**2 + 1),x)
```

3.223 $\int \frac{x^3\textbf{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 243

$$\begin{aligned}\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = {} & \frac{40x\sqrt{1-ax}}{9a^3\sqrt{-1+ax}}+\frac{2x^3\sqrt{1-ax}}{27a\sqrt{-1+ax}} \\ & -\frac{40\sqrt{1-ax}\sqrt{1+ax}\text{arccosh}(ax)}{9a^4} \\ & -\frac{2x^2\sqrt{1-ax}\sqrt{1+ax}\text{arccosh}(ax)}{9a^2} \\ & +\frac{2x\sqrt{1-ax}\text{arccosh}(ax)^2}{a^3\sqrt{-1+ax}}+\frac{x^3\sqrt{1-ax}\text{arccosh}(ax)^2}{3a\sqrt{-1+ax}} \\ & -\frac{2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}\end{aligned}$$

output

```
40/9*x*(-a*x+1)^(1/2)/a^3/(a*x-1)^(1/2)+2/27*x^3*(-a*x+1)^(1/2)/a/(a*x-1)^
(1/2)-40/9*(-a*x+1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)/a^4-2/9*x^2*(-a*x+1)^
(1/2)*(a*x+1)^(1/2)*arccosh(a*x)/a^2+2*x*(-a*x+1)^(1/2)*arccosh(a*x)^2/a^3
/(a*x-1)^(1/2)+1/3*x^3*(-a*x+1)^(1/2)*arccosh(a*x)^2/a/(a*x-1)^(1/2)-2/3*(
-a^2*x^2+1)^(1/2)*arccosh(a*x)^3/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arccosh(a*
x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.58

$$\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$=\frac{\sqrt{1-a^2x^2}\left(2ax(60+a^2x^2)-6\sqrt{-1+ax}\sqrt{1+ax}(20+a^2x^2)\,\text{arccosh}(ax)+9ax(6+a^2x^2)\,\text{arccosh}(ax\right.}{27a^4\sqrt{-1+ax}\sqrt{1+ax}}$$

input
```
Integrate[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(2*a*x*(60 + a^2*x^2) - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
(20 + a^2*x^2)*ArcCosh[a*x] + 9*a*x*(6 + a^2*x^2)*ArcCosh[a*x]^2 - 9*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^3))/(27*a^4*Sqrt[-1 + a
*x]*Sqrt[1 + a*x])
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6353, 6298, 6329, 6294, 6330, 24, 6354, 15, 6330, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\sqrt{ax-1}\int x^2\text{arccosh}(ax)^2dx}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{2\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6329}$$

$$\frac{2\left(-\frac{3\sqrt{ax-1}\int \text{arccosh}(ax)^2dx}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}\right)}{3a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6294}$$

$$\frac{2\left(-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\int \frac{x\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}\right)}{3a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6330}$$

$$\frac{2\left(-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)\right)}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}\right)}{3a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a\sqrt{1-ax}}\right)}{3a^2}$$

$$\Big\downarrow \text{6354}$$

$$-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\left(\frac{2\int \frac{x\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{3a^2}-\frac{\int x^2dx}{3a}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{3a^2}\right)\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a\sqrt{1-ax}}\right)}{3a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\left(\frac{2\int\frac{x\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{3a^2}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{3a^2}-\frac{x^3}{9a}\right)\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a\sqrt{1-ax}}\right)}{3a^2}$$

$$\Big\downarrow \text{6330}$$

$$-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\left(\frac{2\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{3a^2}-\frac{x^3}{9a}\right)\right)}{a\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a\sqrt{1-ax}}\right)}{3a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a\sqrt{1-ax}}\right)}{3a^2}-\frac{\sqrt{ax-1}\left(\frac{1}{3}x^3\text{arccosh}(ax)^2-\frac{2}{3}a\left(\frac{x^2\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{3a^2}+\frac{2\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}\right)\right)}{a\sqrt{1-ax}}$$

input
```
Int[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/a^2 - (Sqrt[-1 + a*x]*((x^3*Ar
cCosh[a*x]^2)/3 - (2*a*(-1/9*x^3/a + (x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Arc
Cosh[a*x])/(3*a^2) + (2*(-(x/a) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*
x])/a^2))/(3*a^2)))/3))/(a*Sqrt[1 - a*x]) + (2*(-((Sqrt[1 - a^2*x^2]*ArcCo
sh[a*x]^3)/a^2) - (3*Sqrt[-1 + a*x]*(x*ArcCosh[a*x]^2 - 2*a*(-(x/a) + (Sqr
t[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a^2)))/(a*Sqrt[1 - a*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(4a^4x^4-5a^2x^2+4a^3x^3\sqrt{ax-1}\,\sqrt{ax+1}-3\sqrt{ax-1}\,\sqrt{ax+1}\,ax+1\right)\left(9\,\operatorname{arccosh}(ax)^3-9\,\operatorname{arccosh}(ax)^2+6\,\operatorname{arccosh}(ax)-2\right.}{216a^4(a^2x^2-1)}$
orering	Expression too large to display

input
```
int(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*
x+1)^(1/2)-3*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x+1)*(9*arccosh(a*x)^3-9*arccos
h(a*x)^2+6*arccosh(a*x)-2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x-1)
^(1/2)*(a*x+1)^(1/2)*a*x+a^2*x^2-1)*(arccosh(a*x)^3-3*arccosh(a*x)^2+6*arc
cosh(a*x)-6)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x-1)^(1/2)
*(a*x+1)^(1/2)*a*x-1)*(arccosh(a*x)^3+3*arccosh(a*x)^2+6*arccosh(a*x)+6)/a
^4/(a^2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*x^2-4*a^3*x^3*(a*
x-1)^(1/2)*(a*x+1)^(1/2)+3*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x+1)*(9*arccosh(a
*x)^3+9*arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.84

$$\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{9\,(a^4x^4+a^2x^2-2)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3 - 9\,(a^3x^3+6\,ax)\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(\right.}{}$$

input
```
integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 - 1))^3 - 9*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 - 1))^2 + 6*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1
)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(a^3*x^3 + 60*a*x)*sqrt(a^2*x^2 - 1)*sq
rt(-a^2*x^2 + 1))/(a^6*x^2 - a^4)
```

Sympy [F]

$$\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**3*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.54

$$\begin{aligned}&\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)^3\\ &\quad+\frac{2}{27}a\left(\frac{3\left(-i\sqrt{a^2x^2-1}x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)\operatorname{arcosh}(ax)}{a^3}+\frac{i\,a^2x^3+60i\,x}{a^4}\right)\\ &\quad+\frac{(i\,a^2x^3+6i\,x)\operatorname{arcosh}(ax)^2}{3\,a^3}\end{aligned}$$

input
```
integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^
3 + 2/27*a*(3*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)*arcc
osh(a*x)/a^3 + (I*a^2*x^3 + 60*I*x)/a^4) + 1/3*(I*a^2*x^3 + 6*I*x)*arccosh
(a*x)^2/a^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\,\text{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^3\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*acosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**3*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.224 $\int \frac{x^2\mathbf{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\int \frac{x^2\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{3x^2\sqrt{1-ax}}{8a\sqrt{-1+ax}} - \frac{3x\sqrt{1-ax}\sqrt{1+ax}\text{arccosh}(ax)}{4a^2} - \frac{3\sqrt{1-ax}\text{arccosh}(ax)^2}{8a^3\sqrt{-1+ax}} + \frac{3x^2\sqrt{1-ax}\text{arccosh}(ax)^2}{4a\sqrt{-1+ax}} - \frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2} - \frac{\sqrt{1-ax}\text{arccosh}(ax)^4}{8a^3\sqrt{-1+ax}}$$

output
```
3/8*x^2*(-a*x+1)^(1/2)/a/(a*x-1)^(1/2)-3/4*x*(-a*x+1)^(1/2)*(a*x+1)^(1/2)*
arccosh(a*x)/a^2-3/8*(-a*x+1)^(1/2)*arccosh(a*x)^2/a^3/(a*x-1)^(1/2)+3/4*x
^2*(-a*x+1)^(1/2)*arccosh(a*x)^2/a/(a*x-1)^(1/2)-1/2*x*(-a^2*x^2+1)^(1/2)*
arccosh(a*x)^3/a^2-1/8*(-a*x+1)^(1/2)*arccosh(a*x)^4/a^3/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int \frac{x^2\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-((-1+ax)(1+ax))}(-3(1+2\text{arccosh}(ax)^2)\cosh(2\text{arccosh}(ax))+2\text{arccosh}(ax)\,(\text{arccosh}(ax)^3+}{16a^3\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

input
```
Integrate[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/16*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-3*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*Arc
Cosh[a*x]] + 2*ArcCosh[a*x]*(ArcCosh[a*x]^3 + (3 + 2*ArcCosh[a*x]^2)*Sinh[
2*ArcCosh[a*x]])))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6353, 6298, 6307, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6353}$$

$$\frac{\int \frac{\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\sqrt{ax-1}\int x\text{arccosh}(ax)^2dx}{2a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\int \frac{\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\int \frac{x^2\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{2a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6307}$$

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\int\frac{x^2\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{2a\sqrt{1-ax}}+\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6354}$$

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\int\frac{\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}\right)\right)}{2a\sqrt{1-ax}}+\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\int\frac{\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx}{2a^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a\sqrt{1-ax}}+\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6308}$$

$$\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2a^2}-\frac{3\sqrt{ax-1}\left(\frac{1}{2}x^2\text{arccosh}(ax)^2-a\left(\frac{\text{arccosh}(ax)^2}{4a^3}+\frac{x\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a\sqrt{1-ax}}$$

input
```
Int[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/a^2 + (Sqrt[-1 + a*x]*ArcCosh[a*
x]^4)/(8*a^3*Sqrt[1 - a*x]) - (3*Sqrt[-1 + a*x]*((x^2*ArcCosh[a*x]^2)/2 -
a*(-1/4*x^2/a + (x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(2*a^2) + Ar
cCosh[a*x]^2/(4*a^3))))/(2*a*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6353
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2
*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]
- Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)
^p)]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^{4}}{8a^{3}(a^{2}x^{2}-1)}-\frac{\sqrt{-a^{2}x^{2}+1}\,(2a^{3}x^{3}-2ax+2a^{2}x^{2}\sqrt{ax-1}\,\sqrt{ax+1}-\sqrt{ax-1}\,\sqrt{ax+1})\left(4\operatorname{arccosh}(ax\right.}{32a^{3}(a^{2}x^{2}-1)}$

input
```
int(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccos
h(a*x)^4-1/32*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*
(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh(a*x
)^2+6*arccosh(a*x)-3)/a^3/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(-2*a^2*x^2*
(a*x-1)^(1/2)*(a*x+1)^(1/2)+2*a^3*x^3+(a*x-1)^(1/2)*(a*x+1)^(1/2)-2*a*x)*(
4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^3/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^2\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2 \operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**2*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^2 \operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\mathrm{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^3\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*acosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**3*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.225 $\int \frac{x\,\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{x\,\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{6x\sqrt{1-ax}}{a\sqrt{-1+ax}} - \frac{6\sqrt{1-a^2x^2}\,\text{arccosh}(ax)}{a^2} + \frac{3x\sqrt{1-ax}\,\text{arccosh}(ax)^2}{a\sqrt{-1+ax}} - \frac{\sqrt{1-a^2x^2}\,\text{arccosh}(ax)^3}{a^2}$$

output
```
6*x*(-a*x+1)^(1/2)/a/(a*x-1)^(1/2)-6*(-a^2*x^2+1)^(1/2)*arccosh(a*x)/a^2+3
*x*(-a*x+1)^(1/2)*arccosh(a*x)^2/a/(a*x-1)^(1/2)-(-a^2*x^2+1)^(1/2)*arccos
h(a*x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

$$\int \frac{x\,\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}\left(6ax - 6\sqrt{-1+ax}\sqrt{1+ax}\,\text{arccosh}(ax) + 3ax\,\text{arccosh}(ax)^2 - \sqrt{-1+ax}\sqrt{1+ax}\,\text{arccosh}(ax\right.}{a^2\sqrt{-1+ax}\sqrt{1+ax}}$$

input
```
Integrate[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(6*a*x - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] +
3*a*x*ArcCosh[a*x]^2 - Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3))/(a^2*
Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6329, 6294, 6330, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6329}$$

$$-\frac{3\sqrt{ax-1}\int \text{arccosh}(ax)^2dx}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}$$

$$\Big\downarrow \text{6294}$$

$$-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\int \frac{x\text{arccosh}(ax)}{\sqrt{ax-1}\sqrt{ax+1}}dx\right)}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}$$

$$\Big\downarrow \text{6330}$$

$$-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)\right)}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{a^2}-\frac{3\sqrt{ax-1}\left(x\text{arccosh}(ax)^2-2a\left(\frac{\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a\sqrt{1-ax}}$$

input
```
Int[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/a^2) - (3*Sqrt[-1 + a*x]*(x*ArcCosh[a
*x]^2 - 2*a*(-(x/a) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a^2)))/(
a*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6329
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && NeQ[p, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

method	result
default	$-\frac{\left(x^{2} \operatorname{arccosh}\left(a x\right)^{3} a^{2}-3 a x \operatorname{arccosh}\left(a x\right)^{2} \sqrt{a x -1}\, \sqrt{a x +1}+6 a^{2} x^{2} \operatorname{arccosh}\left(a x\right)-6 \sqrt{a x -1}\, \sqrt{a x +1}\, a x -\operatorname{arccosh}\left(a x\right)^{3}-6 \operatorname{arccosh}\left(a x\right)\right.}{\left(a^{2} x^{2}-1\right) a^{2}}$
orering	Expression too large to display

input
```
int(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x^2*arccosh(a*x)^3*a^2-3*a*x*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+
6*a^2*x^2*arccosh(a*x)-6*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-arccosh(a*x)^3-6*
arccosh(a*x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.51

$$\int \frac{x\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{3\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2+(-a^2x^2+1)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^3+6\sqrt{a^2x^2-}}{a^4x^2-a^2}$$

input
```
integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(3*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1))^2
 + (-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 6*sqrt(a^2*x^2 -
1)*sqrt(-a^2*x^2 + 1)*a*x - 6*(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x + s
qrt(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

Sympy [F]

$$\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{acosh}^3\,(ax)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(x*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

$$\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{3i\,x\,\text{arcosh}\,(ax)^2}{a} - \frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)^3}{a^2} + \frac{6\left(i\,x - \frac{i\,\sqrt{a^2x^2-1}\,\text{arcosh}(ax)}{a}\right)}{a}$$

input
```
integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
3*I*x*arccosh(a*x)^2/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/a^2 + 6*(I*x -
I*sqrt(a^2*x^2 - 1)*arccosh(a*x)/a)/a
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3}{a^2} + \frac{3i\left(x\log\left(ax+i\sqrt{-a^2x^2+1}\right)^2+2x-\frac{2i\sqrt{-a^2x^2+1}\log\left(ax+i\sqrt{-a^2x^2+1}\right)}{a}\right)}{a}$$

input
```
integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^3/a^2 + 3*I*(x*log(a*x +
I*sqrt(-a^2*x^2 + 1))^2 + 2*x - 2*I*sqrt(-a^2*x^2 + 1)*log(a*x + I*sqrt(-a
^2*x^2 + 1))/a)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{acosh}(ax)^3 x}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*acosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(a*x)**3*x)/sqrt( - a**2*x**2 + 1),x)
```

3.226 $\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx=-\frac{\sqrt{1-ax}\operatorname{arccosh}(ax)^4}{4a\sqrt{-1+ax}}$$

output
```
-1/4*(-a*x+1)^(1/2)*arccosh(a*x)^4/a/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx=\frac{\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)^4}{4a\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \; 6307$$

$$\frac{\sqrt{ax-1}\text{arccosh}(ax)^4}{4a\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[-1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{arccosh}(ax)^4}{4(a^2x^2-1)a}$	51

input
```
int(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)/a*ar
ccosh(a*x)^4
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}^3\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^3/(1 - a^2*x^2)^(1/2),x)
```

output
```
int(acosh(a*x)^3/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(acosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**3/sqrt( - a**2*x**2 + 1),x)
```

3.227 $\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 265

$$\begin{aligned}\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = &-\frac{2\sqrt{1-ax}\operatorname{arccosh}(ax)^3\arctan\left(e^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&+\frac{3i\sqrt{1-ax}\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&-\frac{3i\sqrt{1-ax}\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&-\frac{6i\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&+\frac{6i\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&+\frac{6i\sqrt{1-ax}\operatorname{PolyLog}\left(4,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&-\frac{6i\sqrt{1-ax}\operatorname{PolyLog}\left(4,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\end{aligned}$$

output
```
-2*(-a*x+1)^(1/2)*arccosh(a*x)^3*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/(
a*x-1)^(1/2)+3*I*(-a*x+1)^(1/2)*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-3*I*(-a*x+1)^(1/2)*arccosh(a*x)^2*polyl
og(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-6*I*(-a*x+1)^(1/2)
*arccosh(a*x)*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2
)+6*I*(-a*x+1)^(1/2)*arccosh(a*x)*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2)))/(a*x-1)^(1/2)+6*I*(-a*x+1)^(1/2)*polylog(4,-I*(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2)))/(a*x-1)^(1/2)-6*I*(-a*x+1)^(1/2)*polylog(4,I*(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.84

$$\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \frac{i\sqrt{-((-1+ax)(1+ax))}\left(7\pi^4+8i\pi^3\operatorname{arccosh}(ax)+24\pi^2\operatorname{arccosh}(ax)^2-32i\pi\operatorname{arccosh}(ax)^3-16\operatorname{arccosh}\right.}{}$$

input
```
Integrate[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
((I/64)*Sqrt[-((-1 + a*x)*(1 + a*x))]*(7*Pi^4 + (8*I)*Pi^3*ArcCosh[a*x] +
24*Pi^2*ArcCosh[a*x]^2 - (32*I)*Pi*ArcCosh[a*x]^3 - 16*ArcCosh[a*x]^4 + (8
*I)*Pi^3*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*ArcCosh[a*x]*Log[1 + I/E^ArcC
osh[a*x]] - (96*I)*Pi*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*ArcCos
h[a*x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*ArcCosh[a*x]*Log[1 - I*E^ArcC
osh[a*x]] + (96*I)*Pi*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I)*Pi^
3*Log[1 + I*E^ArcCosh[a*x]] + 64*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]]
 + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] - 48*(Pi - (2*I)*ArcCos
h[a*x])^2*PolyLog[2, (-I)/E^ArcCosh[a*x]] + 192*ArcCosh[a*x]^2*PolyLog[2,
(-I)*E^ArcCosh[a*x]] - 48*Pi^2*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*A
rcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^Ar
cCosh[a*x]] + 384*ArcCosh[a*x]*PolyLog[3, (-I)/E^ArcCosh[a*x]] - 384*ArcCo
sh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcCos
h[a*x]] + 384*PolyLog[4, (-I)/E^ArcCosh[a*x]] + 384*PolyLog[4, (-I)*E^ArcC
osh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6361, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6361}$$

$$\frac{\sqrt{ax-1}\int \frac{\text{arccosh}(ax)^3}{ax}d\text{arccosh}(ax)}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{ax-1}\int \text{arccosh}(ax)^3\csc\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)d\text{arccosh}(ax)}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{\sqrt{ax-1}\left(-3i\int \text{arccosh}(ax)^2\log\left(1-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+3i\int \text{arccosh}(ax)^2\log\left(1+ie^{\text{arccosh}(ax)}\right)d\text{arcco}\right.}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 3011}$$

$$\sqrt{ax-1}\left(3i\left(2\int \text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)-\text{arccosh}(ax)^2\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)\right)\right.$$

$$\Big\downarrow \text{ 7163}$$

$$\sqrt{ax-1}\left(3i\left(2\left(\text{arccosh}(ax)\text{PolyLog}\left(3,-ie^{\text{arccosh}(ax)}\right)-\int \text{PolyLog}\left(3,-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)\right)-\text{arccosh}(\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$\sqrt{ax-1}\left(3i\left(2\left(\text{arccosh}(ax)\text{PolyLog}\left(3,-ie^{\text{arccosh}(ax)}\right)-\int e^{-\text{arccosh}(ax)}\text{PolyLog}\left(3,-ie^{\text{arccosh}(ax)}\right)de^{\text{arccosh}(ax)}\right)\right.\right.$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\sqrt{ax-1}\left(2\text{arccosh}(ax)^3\arctan\left(e^{\text{arccosh}(ax)}\right)+3i\left(2\left(\text{arccosh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(ax)}\right)-\text{PolyLog}\left(4,-ie^{\text{arc}}\right.\right.\right.\right.}{}$$

input
```
Int[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(Sqrt[-1 + a*x]*(2*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]] + (3*I)*(-(ArcCos
h[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]]) + 2*(ArcCosh[a*x]*PolyLog[3, (-I
)*E^ArcCosh[a*x]] - PolyLog[4, (-I)*E^ArcCosh[a*x]])) - (3*I)*(-(ArcCosh[a
*x]^2*PolyLog[2, I*E^ArcCosh[a*x]]) + 2*(ArcCosh[a*x]*PolyLog[3, I*E^ArcCo
sh[a*x]] - PolyLog[4, I*E^ArcCosh[a*x]]))))/Sqrt[1 - a*x]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{-a^2x^2+1}}dx$$

input
```
int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\text{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\text{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{acosh}^3\,(ax)}{x\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\text{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^3}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(acosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**3/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.228 $\int \frac{\text{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\begin{aligned}\int \frac{\text{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = &-\frac{a\sqrt{1-ax}\,\text{arccosh}(ax)^3}{\sqrt{-1+ax}}-\frac{\sqrt{1-a^2x^2}\,\text{arccosh}(ax)^3}{x}\\&+\frac{3a\sqrt{1-ax}\,\text{arccosh}(ax)^2\log\left(1+e^{2\text{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&+\frac{3a\sqrt{1-ax}\,\text{arccosh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax)}\right)}{\sqrt{-1+ax}}\\&-\frac{3a\sqrt{1-ax}\,\text{PolyLog}\left(3,-e^{2\text{arccosh}(ax)}\right)}{2\sqrt{-1+ax}}\end{aligned}$$

output

```
-a*(-a*x+1)^(1/2)*arccosh(a*x)^3/(a*x-1)^(1/2)-(-a^2*x^2+1)^(1/2)*arccosh(
a*x)^3/x+3*a*(-a*x+1)^(1/2)*arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))^2)/(a*x-1)^(1/2)+3*a*(-a*x+1)^(1/2)*arccosh(a*x)*polylog(2,-(a*x+(
a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x-1)^(1/2)-3/2*a*(-a*x+1)^(1/2)*polylog(
3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{a\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(2\operatorname{arccosh}(ax)^2\left(-\operatorname{arccosh}(ax)+\frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\operatorname{arccosh}(ax)}{ax}-3\log\left(1+e^{-2\operatorname{arccosh}(ax)}\right)\right.\right.}{2\sqrt{-((-1+ax)(1+ax)}}$$

input
```
Integrate[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(2*ArcCosh[a*x]^2*(-ArcCosh[a*x] +
 (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 3*Log[1 + E^(
-2*ArcCosh[a*x])]) + 6*ArcCosh[a*x]*PolyLog[2, -E^(-2*ArcCosh[a*x])] + 3*P
olyLog[3, -E^(-2*ArcCosh[a*x])]))/(2*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.75, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6332, 6297, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \ 6332$$

$$-\frac{3a\sqrt{ax-1}\int \frac{\operatorname{arccosh}(ax)^2}{x}dx}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{x}$$

$$\Big\downarrow \ 6297$$

$$-\frac{3a\sqrt{ax-1}\int \frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\text{arccosh}(ax)^2}{ax}d\text{arccosh}(ax)}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}$$

$\Big\downarrow$ 3042

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}-\frac{3a\sqrt{ax-1}\int -i\text{arccosh}(ax)^2\tan(i\text{arccosh}(ax))d\text{arccosh}(ax)}{\sqrt{1-ax}}$$

$\Big\downarrow$ 26

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}+\frac{3ia\sqrt{ax-1}\int \text{arccosh}(ax)^2\tan(i\text{arccosh}(ax))d\text{arccosh}(ax)}{\sqrt{1-ax}}$$

$\Big\downarrow$ 4201

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}+\frac{3ia\sqrt{ax-1}\left(2i\int \frac{e^{2\text{arccosh}(ax)}\text{arccosh}(ax)^2}{1+e^{2\text{arccosh}(ax)}}d\text{arccosh}(ax)-\frac{1}{3}i\text{arccosh}(ax)^3\right)}{\sqrt{1-ax}}$$

$\Big\downarrow$ 2620

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}+\frac{3ia\sqrt{ax-1}\left(2i\left(\frac{1}{2}\text{arccosh}(ax)^2\log\left(e^{2\text{arccosh}(ax)}+1\right)-\int \text{arccosh}(ax)\log\left(1+e^{2\text{arccosh}(ax)}\right)d\text{arccosh}(ax)\right)-\frac{1}{3}i\text{ar}\ldots}{\sqrt{1-ax}}$$

$\Big\downarrow$ 3011

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}+\frac{3ia\sqrt{ax-1}\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+\frac{1}{2}\text{arccosh}(ax)\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax)}\right)+\frac{1}{2}\text{arc}\ldots}{\sqrt{1-ax}}$$

$\Big\downarrow$ 2720

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}+\frac{3ia\sqrt{ax-1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arccosh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax)}\right)de^{2\text{arccosh}(ax)}+\frac{1}{2}\text{arccosh}(ax)\text{PolyLog}\left(2,-e^{2\text{arccos}\ldots}\right.\right.\right.}{\sqrt{1-ax}}$$

$\Big\downarrow$ 7143

$$-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{x}+\frac{3ia\sqrt{ax-1}\left(2i\left(\frac{1}{2}\text{arccosh}(ax)\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax)}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2\text{arccosh}(ax)}\right)+\frac{1}{2}\text{arccosh}(ax)^2\log\left(e^{2}\ldots\right.\right.\right.}{\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/x) + ((3*I)*a*Sqrt[-1 + a*x]*((-1/3*I
)*ArcCosh[a*x]^3 + (2*I)*((ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])])/2 +
 (ArcCosh[a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])])/2 - PolyLog[3, -E^(2*ArcCo
sh[a*x])]/4)))/Sqrt[1 - a*x]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6332
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2
)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3
, 0] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.87

method	result
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,\left(a^{2}x^{2}-\sqrt{ax-1}\,\sqrt{ax+1}\,ax-1\right)\operatorname{arccosh}\left(ax\right)^{3}}{x\left(a^{2}x^{2}-1\right)}-\frac{2\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}\left(ax\right)^{3}a}{a^{2}x^{2}-1}+\frac{3\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax}}{}$

input
```
int(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-1)*arccosh(a*
x)^3/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x
^2-1)*arccosh(a*x)^3*a+3*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a
^2*x^2-1)*arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+3*(-a
^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*polyl
og(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a-3/2*(-a^2*x^2+1)^(1/2)*(a*x-1
)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1
/2))^2)*a
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}^3\left(ax\right)}{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acosh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
(a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(sqrt(a*x + 1)*sqrt
(-a*x + 1)*x) - integrate(3*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x -
 a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/((sqrt(a*x + 1)*a*x^2 + (a*x
+ 1)*sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^3}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^3}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(acosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**3/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.229 $\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 461

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{3a\sqrt{1-ax}\operatorname{arccosh}(ax)^2}{2x\sqrt{-1+ax}} - \frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2} + \frac{6a^2\sqrt{1-ax}\operatorname{arccosh}(ax)\arctan\left(e^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} - \frac{a^2\sqrt{1-ax}\operatorname{arccosh}(ax)^3\arctan\left(e^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} - \frac{3ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} + \frac{3ia^2\sqrt{1-ax}\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)}{2\sqrt{-1+ax}} + \frac{3ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} - \frac{3ia^2\sqrt{1-ax}\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(ax)}\right)}{2\sqrt{-1+ax}} - \frac{3ia^2\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} + \frac{3ia^2\sqrt{1-ax}\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(3,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} + \frac{3ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(4,-ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}} - \frac{3ia^2\sqrt{1-ax}\operatorname{PolyLog}\left(4,ie^{\operatorname{arccosh}(ax)}\right)}{\sqrt{-1+ax}}$$

output
```
-3/2*a*(-a*x+1)^(1/2)*arccosh(a*x)^2/x/(a*x-1)^(1/2)-1/2*(-a^2*x^2+1)^(1/2
)*arccosh(a*x)^3/x^2+6*a^2*(-a*x+1)^(1/2)*arccosh(a*x)*arctan(a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))/(a*x-1)^(1/2)-a^2*(-a*x+1)^(1/2)*arccosh(a*x)^3*arcta
n(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/(a*x-1)^(1/2)-3*I*a^2*(-a*x+1)^(1/2)*po
lylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)+3/2*I*a^2*(-a*
x+1)^(1/2)*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/
(a*x-1)^(1/2)+3*I*a^2*(-a*x+1)^(1/2)*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2)))/(a*x-1)^(1/2)-3/2*I*a^2*(-a*x+1)^(1/2)*arccosh(a*x)^2*polylog(2,
I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-3*I*a^2*(-a*x+1)^(1/2)*
arccosh(a*x)*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)
+3*I*a^2*(-a*x+1)^(1/2)*arccosh(a*x)*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2)))/(a*x-1)^(1/2)+3*I*a^2*(-a*x+1)^(1/2)*polylog(4,-I*(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)-3*I*a^2*(-a*x+1)^(1/2)*polylog(4,I*(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/(a*x-1)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1051 vs. $2(461) = 922$.

Time = 3.95 (sec) , antiderivative size = 1051, normalized size of antiderivative = 2.28

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
((-1/128*I)*a^2*(1 + a*x)*(7*Pi^4*Sqrt[(-1 + a*x)/(1 + a*x)] + (8*I)*Pi^3*
Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + 24*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x
)]*ArcCosh[a*x]^2 + ((192*I)*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2)/(a
*x) + ((64*I)*(-1 + a*x)*ArcCosh[a*x]^3)/(a^2*x^2) - (32*I)*Pi*Sqrt[(-1 +
a*x)/(1 + a*x)]*ArcCosh[a*x]^3 - 16*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x
]^4 - 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]
] + (8*I)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I/E^ArcCosh[a*x]] + 384*
Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^
2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] - (96*
I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]]
- 64*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I/E^ArcCosh[a*x]] -
 48*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a*x]]
 + (96*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh
[a*x]] - (8*I)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I*E^ArcCosh[a*x]] +
 64*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] +
(8*I)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]
] - 48*Sqrt[(-1 + a*x)/(1 + a*x)]*(8 + Pi^2 - (4*I)*Pi*ArcCosh[a*x] - 4*Ar
cCosh[a*x]^2)*PolyLog[2, (-I)/E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a
*x)]*PolyLog[2, I/E^ArcCosh[a*x]] + 192*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh
[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]] - 48*Pi^2*Sqrt[(-1 + a*x)/(1 + ...
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.59, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6347, 6298, 6361, 3042, 4668, 3011, 6362, 3042, 4668, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6347}$$

$$\frac{1}{2}a^2\int \frac{\text{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{3a\sqrt{ax-1}\int \frac{\text{arccosh}(ax)^2}{x^2}dx}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{1}{2}a^2\int \frac{\text{arccosh}(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{3a\sqrt{ax-1}\left(2a\int \frac{\text{arccosh}(ax)}{x\sqrt{ax-1}\sqrt{ax+1}}dx-\frac{\text{arccosh}(ax)^2}{x}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 6361

$$\frac{a^2\sqrt{ax-1}\int \frac{\text{arccosh}(ax)^3}{ax}d\text{arccosh}(ax)}{2\sqrt{1-ax}}-\frac{3a\sqrt{ax-1}\left(2a\int \frac{\text{arccosh}(ax)}{x\sqrt{ax-1}\sqrt{ax+1}}dx-\frac{\text{arccosh}(ax)^2}{x}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 3042

$$\frac{a^2\sqrt{ax-1}\int \text{arccosh}(ax)^3\csc\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)d\text{arccosh}(ax)}{2\sqrt{1-ax}}-\frac{3a\sqrt{ax-1}\left(2a\int \frac{\text{arccosh}(ax)}{x\sqrt{ax-1}\sqrt{ax+1}}dx-\frac{\text{arccosh}(ax)^2}{x}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 4668

$$\frac{a^2\sqrt{ax-1}\left(-3i\int \text{arccosh}(ax)^2\log\left(1-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)+3i\int \text{arccosh}(ax)^2\log\left(1+ie^{\text{arccosh}(ax)}\right)d\text{arc}\right.}{2\sqrt{1-ax}}$$
$$\frac{3a\sqrt{ax-1}\left(2a\int \frac{\text{arccosh}(ax)}{x\sqrt{ax-1}\sqrt{ax+1}}dx-\frac{\text{arccosh}(ax)^2}{x}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 3011

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\int \text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)-\text{arccosh}(ax)^2\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax}\right.\right.\right.}{}$$
$$\frac{3a\sqrt{ax-1}\left(2a\int \frac{\text{arccosh}(ax)}{x\sqrt{ax-1}\sqrt{ax+1}}dx-\frac{\text{arccosh}(ax)^2}{x}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 6362

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\int \text{arccosh}(ax)\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax)}\right)d\text{arccosh}(ax)-\text{arccosh}(ax)^2\text{PolyLog}\left(2,-ie^{\text{arccosh}(ax}\right.\right.\right.}{}$$
$$\frac{3a\sqrt{ax-1}\left(2a\int \frac{\text{arccosh}(ax)}{ax}d\text{arccosh}(ax)-\frac{\text{arccosh}(ax)^2}{x}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\text{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 3042

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\int \operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)d\operatorname{arccosh}(ax)-\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax}\right.\right.\right.}{}$$

$$\frac{3a\sqrt{ax-1}\left(-\frac{\operatorname{arccosh}(ax)^2}{x}+2a\int \operatorname{arccosh}(ax)\csc\left(i\operatorname{arccosh}(ax)+\frac{\pi}{2}\right)d\operatorname{arccosh}(ax)\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 4668

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\int \operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)d\operatorname{arccosh}(ax)-\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax}\right.\right.\right.}{}$$

$$\frac{3a\sqrt{ax-1}\left(-\frac{\operatorname{arccosh}(ax)^2}{x}+2a\left(-i\int \log\left(1-ie^{\operatorname{arccosh}(ax)}\right)d\operatorname{arccosh}(ax)+i\int \log\left(1+ie^{\operatorname{arccosh}(ax)}\right)d\operatorname{arccosh}(ax\right.\right.}{2\sqrt{1-ax}}$$

$$\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 2715

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\int \operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)d\operatorname{arccosh}(ax)-\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax}\right.\right.\right.}{}$$

$$\frac{3a\sqrt{ax-1}\left(-\frac{\operatorname{arccosh}(ax)^2}{x}+2a\left(-i\int e^{-\operatorname{arccosh}(ax)}\log\left(1-ie^{\operatorname{arccosh}(ax)}\right)de^{\operatorname{arccosh}(ax)}+i\int e^{-\operatorname{arccosh}(ax)}\log\left(1+ie\right.\right.\right.}{2\sqrt{1-ax}}$$

$$\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2}$$

$\downarrow$ 2838

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\int \operatorname{arccosh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)d\operatorname{arccosh}(ax)-\operatorname{arccosh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax}\right.\right.\right.}{}$$

$$\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2}-$$

$$\frac{3a\sqrt{ax-1}\left(-\frac{\operatorname{arccosh}(ax)^2}{x}+2a\left(2\operatorname{arccosh}(ax)\arctan\left(e^{\operatorname{arccosh}(ax)}\right)-i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)+i\operatorname{PolyLog}\left(2\right.\right.\right.}{2\sqrt{1-ax}}$$

$\downarrow$ 7163

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\left(\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)-\int \operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)d\operatorname{arccosh}(ax)\right)-\operatorname{arccosh}\right.}{}$$

$$\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2}-$$

$$\frac{3a\sqrt{ax-1}\left(-\frac{\operatorname{arccosh}(ax)^2}{x}+2a\left(2\operatorname{arccosh}(ax)\arctan\left(e^{\operatorname{arccosh}(ax)}\right)-i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)+i\operatorname{PolyLog}\left(2\right.\right.\right.}{2\sqrt{1-ax}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{a^2\sqrt{ax-1}\left(3i\left(2\left(\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)-\int e^{-\operatorname{arccosh}(ax)}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)de^{\operatorname{arccosh}(ax)}\right.\right.\right.}{}$$

$$\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2}-$$

$$\frac{3a\sqrt{ax-1}\left(-\frac{\operatorname{arccosh}(ax)^2}{x}+2a\left(2\operatorname{arccosh}(ax)\arctan\left(e^{\operatorname{arccosh}(ax)}\right)-i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)+i\operatorname{PolyLog}\left(2\right.\right.\right.}{2\sqrt{1-ax}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{a^2\sqrt{ax-1}\left(2\operatorname{arccosh}(ax)^3\arctan\left(e^{\operatorname{arccosh}(ax)}\right)+3i\left(2\left(\operatorname{arccosh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(ax)}\right)-\operatorname{PolyLog}\left(4,-ie\right.\right.\right.\right.}{}$$

$$\frac{\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)^3}{2x^2}-$$

$$\frac{3a\sqrt{ax-1}\left(-\frac{\operatorname{arccosh}(ax)^2}{x}+2a\left(2\operatorname{arccosh}(ax)\arctan\left(e^{\operatorname{arccosh}(ax)}\right)-i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)+i\operatorname{PolyLog}\left(2\right.\right.\right.}{2\sqrt{1-ax}}$$

input
```
Int[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/x^2 - (3*a*Sqrt[-1 + a*x]*(-(ArcCo
sh[a*x]^2/x) + 2*a*(2*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]] - I*PolyLog[2, (
-I)*E^ArcCosh[a*x]] + I*PolyLog[2, I*E^ArcCosh[a*x]])))/(2*Sqrt[1 - a*x])
 + (a^2*Sqrt[-1 + a*x]*(2*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]] + (3*I)*(-(
ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]]) + 2*(ArcCosh[a*x]*PolyLog[
3, (-I)*E^ArcCosh[a*x]] - PolyLog[4, (-I)*E^ArcCosh[a*x]])) - (3*I)*(-(Arc
Cosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]]) + 2*(ArcCosh[a*x]*PolyLog[3, I*E
^ArcCosh[a*x]] - PolyLog[4, I*E^ArcCosh[a*x]]))))/(2*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6347
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1
)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(
f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^
(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && ILtQ[m, -1]
```

rule 6361
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])]   Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Int
egerQ[m]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{-a^2x^2+1}}dx$$

input
```
int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acosh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acosh}(a\,x)^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acosh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acosh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{acosh(ax)^3}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(acosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acosh(a*x)**3/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.230 $\int \frac{(fx)^m(a+b\,\mathbf{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))^3}{\sqrt{1-c^2x^2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^3}{\sqrt{1-c^2x^2}} \, dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m (a+b \operatorname{arccosh}(cx))^3}{\sqrt{-c^2x^2+1}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)^3(fx)^m}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-(b^3*arccosh(c*x)^3 + 3*a*b^2*arccosh(c*x)^2 + 3*a^2*b*arccosh(c
*x) + a^3)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(c^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 58.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m\,(a+b\operatorname{acosh}(cx))^3}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**3/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))**3/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^3(fx)^m}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^3(fx)^m}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\operatorname{acosh}(c\,x))^3\,(f\,x)^m}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(((a + b*acosh(c*x))^3*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int(((a + b*acosh(c*x))^3*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.90

$$\int \frac{(fx)^m(a+b\,\text{arccosh}(cx))^3}{\sqrt{1-c^2x^2}}\,dx = f^m\left(\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}}dx\right)a^3 + 3\left(\int \frac{x^m\,acosh(cx)}{\sqrt{-c^2x^2+1}}dx\right)a^2b + \left(\int \frac{x^m\,acosh(cx)^3}{\sqrt{-c^2x^2+1}}dx\right)b^3 + 3\left(\int \frac{x^m\,acosh(cx)^2}{\sqrt{-c^2x^2+1}}dx\right)a\,b^2\right)$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

output
```
f**m*(int(x**m/sqrt( - c**2*x**2 + 1),x)*a**3 + 3*int((x**m*acosh(c*x))/sq
rt( - c**2*x**2 + 1),x)*a**2*b + int((x**m*acosh(c*x)**3)/sqrt( - c**2*x**
2 + 1),x)*b**3 + 3*int((x**m*acosh(c*x)**2)/sqrt( - c**2*x**2 + 1),x)*a*b*
*2)
```

3.231 $\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 339

$$
\begin{aligned}
\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = & -\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32bc^5\sqrt{-1+cx}} \\
& +\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16bc^5\sqrt{-1+cx}} \\
& +\frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32bc^5\sqrt{-1+cx}} \\
& -\frac{\sqrt{1-cx}\log(a+b\text{arccosh}(cx))}{16bc^5\sqrt{-1+cx}} \\
& +\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32bc^5\sqrt{-1+cx}} \\
& -\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16bc^5\sqrt{-1+cx}} \\
& -\frac{\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32bc^5\sqrt{-1+cx}}
\end{aligned}
$$

output
```
-1/32*(-c*x+1)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arccosh(c*x))/b)/b/c^5/(c*x-1)
^(1/2)+1/16*(-c*x+1)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arccosh(c*x))/b)/b/c^5/(
c*x-1)^(1/2)+1/32*(-c*x+1)^(1/2)*cosh(6*a/b)*Chi(6*(a+b*arccosh(c*x))/b)/b
/c^5/(c*x-1)^(1/2)-1/16*(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c^5/(c*x-1)^
(1/2)+1/32*(-c*x+1)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b/c^5/(c
*x-1)^(1/2)-1/16*(-c*x+1)^(1/2)*sinh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b/
c^5/(c*x-1)^(1/2)-1/32*(-c*x+1)^(1/2)*sinh(6*a/b)*Shi(6*(a+b*arccosh(c*x))
/b)/b/c^5/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.55

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Ch}\right.}{}$$

input
```
Integrate[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])])
+ 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*Cos
hIntegral[6*(a/b + ArcCosh[c*x])] - 2*Log[a + b*ArcCosh[c*x]] + Sinh[(2*a)
/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*
(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]
))/(32*c^5*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{\sqrt{1-cx}\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^5\sqrt{cx-1}}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}+\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}-\frac{1}{16(a+b\text{arccos}}\right.}{bc^5\sqrt{cx-1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\sqrt{1-cx}\left(-\frac{1}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\right.\right.$$

input
```
Int[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*(-1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))
/b]) + (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/16 + (Cosh
[(6*a)/b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32 - Log[a + b*ArcCosh
[c*x]]/16 + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/32 -
(Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/16 - (Sinh[(6*a)/
b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32))/(b*c^5*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(4\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\operatorname{arccosh}(cx))+4\ln(a+b\,\operatorname{arccosh}(cx))cx+\operatorname{expIntegral}_1(6\,\operatorname{arcc}\right.}{}$

input
```
int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/64*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(4*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))+4*ln(a+b*arccosh(c*x))*c*x+
Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)+Ei(1,-6*arccosh(c*x
)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)+2*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b
*arccosh(c*x)+4*a)/b)-Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/
b)-Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)+2*Ei(1,-4*arc
cosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b))/(c*x+1)/c^5/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^4}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^4\sqrt{-(cx-1)\,(cx+1)}}{a+b\,\text{acosh}\,(cx)}\,dx$$

input
```
integrate(x**4*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**4*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^4}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^4}{b\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\operatorname{acosh}(c\,x)}\,dx$$

input
```
int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acosh(cx)\,b+a}dx$$

input
```
int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**4)/(acosh(c*x)*b + a),x)
```

3.232 $\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\,\mathrm{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 297

$$\begin{aligned}\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\,\mathrm{arccosh}(cx)}\,dx =& -\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)}{8bc^4\sqrt{-1+cx}} \\ &+ \frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{16bc^4\sqrt{-1+cx}} \\ &+ \frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{16bc^4\sqrt{-1+cx}} \\ &+ \frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)}{8bc^4\sqrt{-1+cx}} \\ &- \frac{\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{16bc^4\sqrt{-1+cx}} \\ &- \frac{\sqrt{1-cx}\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{16bc^4\sqrt{-1+cx}}\end{aligned}$$

output
```
-1/8*(-c*x+1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^4/(c*x-1)^(1/2
)+1/16*(-c*x+1)^(1/2)*cosh(3*a/b)*Chi(3*(a+b*arccosh(c*x))/b)/b/c^4/(c*x-1
)^(1/2)+1/16*(-c*x+1)^(1/2)*cosh(5*a/b)*Chi(5*(a+b*arccosh(c*x))/b)/b/c^4/
(c*x-1)^(1/2)+1/8*(-c*x+1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^4
/(c*x-1)^(1/2)-1/16*(-c*x+1)^(1/2)*sinh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)
/b/c^4/(c*x-1)^(1/2)-1/16*(-c*x+1)^(1/2)*sinh(5*a/b)*Shi(5*(a+b*arccosh(c*
x))/b)/b/c^4/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\right.\right.}{}$$

input
```
Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + Cosh[(
3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[
5*(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - S
inh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcCosh[c*x])]))/(16*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c
*x))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}dx$$

$\Big\downarrow$ 6367

$$\frac{\sqrt{1-cx}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^4\sqrt{cx-1}}$$

$\Big\downarrow$ 5971

$$\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(}{bc^4\sqrt{cx-1}}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{1-cx}\left(-\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)+\frac{1}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{b}$$

input
```
Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*(-1/8*(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b]) + (C
osh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b])/16 + (Cosh[(5*a)/b]
*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/16 + (Sinh[a/b]*SinhIntegral[(a
 + b*ArcCosh[c*x])/b])/8 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c
*x]))/b])/16 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/16
))/(b*c^4*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.86

method	result
default	$-\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^{2}x^{2}-1\right)\left(2\,\mathrm{expIntegral}_{1}\left(\operatorname{arccosh}(cx)+\frac{a}{b}\right)\mathrm{e}^{\frac{a+b\,\operatorname{arccosh}(cx)}{b}}-\mathrm{expIntegral}_{1}\left(5\,\operatorname{arccosh}(cx)+\frac{5a}{b}\right.\right.}{}$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(2*E
i(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)-Ei(1,5*arccosh(c*x)+5*a/b)
*exp((b*arccosh(c*x)+5*a)/b)-Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x
)+3*a)/b)-Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(-b*arccosh(c*x)+3*a)/b)-Ei(1,-
5*arccosh(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+5*a)/b)+2*Ei(1,-arccosh(c*x)-a
/b)*exp(-(-b*arccosh(c*x)+a)/b))/(c*x+1)/c^4/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^3\sqrt{-(cx-1)(cx+1)}}{a+b\,\text{acosh}\,(cx)}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x^3\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\left(cx\right)b+a}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**3)/(acosh(c*x)*b + a),x)
```

3.233 $\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\textbf{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\log(a+b\text{arccosh}(cx))}{8bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8bc^3\sqrt{-1+cx}}$$

output
```
1/8*(-c*x+1)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arccosh(c*x))/b)/b/c^3/(c*x-1)^(
1/2)-1/8*(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c^3/(c*x-1)^(1/2)-1/8*(-c*x
+1)^(1/2)*sinh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b/c^3/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\operatorname{arccosh}(cx)}\,dx = -\frac{\sqrt{-((-1+cx)(1+cx))}\left(-\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(4\left(\frac{a}{b}+\operatorname{arccosh}(cx)\right)\right)+\log(a+b\operatorname{arccosh}(cx))+\sinh\left(\frac{4a}{b}\right)\operatorname{S}\right.}{8bc^3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

input
```
Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
-1/8*(Sqrt[-((-1 + c*x)*(1 + c*x))]*(-(Cosh[(4*a)/b]*CoshIntegral[4*(a/b +
 ArcCosh[c*x])]) + Log[a + b*ArcCosh[c*x]] + Sinh[(4*a)/b]*SinhIntegral[4*
(a/b + ArcCosh[c*x])]))/(b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\operatorname{arccosh}(cx)}\,dx$$

$$\Big\downarrow \ 6367$$

$$\frac{\sqrt{1-cx}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\operatorname{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\operatorname{arccosh}(cx)}{b}\right)}{a+b\operatorname{arccosh}(cx)}d(a+b\operatorname{arccosh}(cx))}{bc^3\sqrt{cx-1}}$$

$$\Big\downarrow \ 5971$$

$$\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{8(a+b\operatorname{arccosh}(cx))}-\frac{1}{8(a+b\operatorname{arccosh}(cx))}\right)d(a+b\operatorname{arccosh}(cx))}{bc^3\sqrt{cx-1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-cx}\left(\frac{1}{8}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\log(a+b\text{arccosh}(cx))\right)}{bc^3\sqrt{cx-1}}$$

input
```
Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*((Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8
 - Log[a + b*ArcCosh[c*x]]/8 - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCo
sh[c*x]))/b])/8))/(b*c^3*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^{2}x^{2}-1\right)\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\operatorname{arccosh}(cx))+2\ln(a+b\,\operatorname{arccosh}(cx))cx+\operatorname{expIntegral}_{1}(4\,\operatorname{arcc}\right.}{16(cx+1)c^{3}(cx-1)b}$

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(2*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))+2*ln(a+b*arccosh(c*x))*c*x+
Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)+Ei(1,-4*arccosh(c*x
)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b))/(c*x+1)/c^3/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^2}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^2\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\operatorname{acosh}\left(cx\right)}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x^2\,\sqrt{1-c^2\,x^2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)\,b+a}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c*x)*b + a),x)
```

3.234 $\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 197

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = -\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{4bc^2\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4bc^2\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{4bc^2\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4bc^2\sqrt{-1+cx}}$$

output
```
-1/4*(-c*x+1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^2/(c*x-1)^(1/2
)+1/4*(-c*x+1)^(1/2)*cosh(3*a/b)*Chi(3*(a+b*arccosh(c*x))/b)/b/c^2/(c*x-1)
^(1/2)+1/4*(-c*x+1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^2/(c*x-1
)^(1/2)-1/4*(-c*x+1)^(1/2)*sinh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)/b/c^2/(
c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.64

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{a}\right.\right.}{4c^2\sqrt{\frac{-1+cx}{1+cx}}(b+bcx)}$$

input

```
Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output

```
(Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Cosh[(
3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*
c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^2\sqrt{cx-1}}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{cx-1}}$$

$\downarrow$ 2009

$$\frac{\sqrt{1-cx}\left(-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c}{b}\right.\right.}{bc^2\sqrt{cx-1}}$$

input
```
Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*(-1/4*(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b]) + (C
osh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b])/4 + (Sinh[a/b]*Sinh
Integral[(a + b*ArcCosh[c*x])/b])/4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a +
b*ArcCosh[c*x]))/b])/4))/(b*c^2*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x\sqrt{-c^2x^2+1}}{a+b\ \mathrm{arccosh}\left(cx\right)}dx$$

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\mathrm{arccosh}(cx)}\, dx = \int \frac{\sqrt{-c^2x^2+1}x}{b\,\mathrm{arcosh}\left(cx\right)+a}\, dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\mathrm{arccosh}(cx)}\, dx = \int \frac{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\,\mathrm{acosh}\left(cx\right)}\, dx$$

input
```
integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x\,\sqrt{1-c^2\,x^2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)\,b+a}dx$$

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x)/(acosh(c*x)*b + a),x)
```

3.235 $\int \frac{\sqrt{1-c^2x^2}}{a+b\mathbf{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2bc\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\log(a+b\text{arccosh}(cx))}{2bc\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2bc\sqrt{-1+cx}}$$

output
```
1/2*(-c*x+1)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^(1/
2)-1/2*(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c/(c*x-1)^(1/2)-1/2*(-c*x+1)^
(1/2)*sinh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{-((-1+cx)(1+cx))}\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)-\log(a+b\text{arccosh}(cx))-\sinh\left(\frac{2a}{b}\right)\text{Shi}(2}{2bc\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[-((-1 + c*x)*(1 + c*x))]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCos
h[c*x])] - Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + A
rcCosh[c*x])]))/(2*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6321, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx$$

$$\Big\downarrow \; 6321$$

$$\frac{\sqrt{1-cx}\int \frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{1-cx}\int -\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\sqrt{1-cx}\int \left(\frac{1}{2(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-cx}\left(\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{2}\log(a+b\text{arccosh}(cx))\right)}{bc\sqrt{cx-1}}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/2
 - Log[a + b*ArcCosh[c*x]]/2 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCo
sh[c*x]))/b])/2))/(b*c*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\operatorname{arccosh}(cx))+2\ln(a+b\,\operatorname{arccosh}(cx))cx+\operatorname{expIntegral}_1(2\,\operatorname{arcc}\right.}{4(cx-1)(cx+1)cb}$

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(2*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))+2*ln(a+b*arccosh(c*x))*c*x+E
i(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)+Ei(1,-2*arccosh(c*x)
-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b))/(c*x-1)/(c*x+1)/c/b
```

Fricas [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{b\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{b\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x)),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acosh\left(cx\right)b+a}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b + a),x)
```

3.236 $\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))} \, dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))} \, dx = \int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))} \, dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))}dx$$

$$\Big\downarrow \text{ 6369}$$

$$\int \left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}-\frac{c^2x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx-\frac{\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{b\sqrt{1-cx}}+\frac{\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{b\sqrt{1-cx}}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x\left(a+b\ \text{arccosh}\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-(cx-1)\,(cx+1)}}{x\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x\,\left(a+b\,\text{acosh}\,(c\,x)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acosh\left(cx\right)bx+ax}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b*x + a*x),x)
```

3.237 $\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\textbf{arccosh}(cx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))} dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}, x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))} dx = \int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))} dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}dx$$
$$\Big\downarrow \text{6369}$$
$$\int \left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}-\frac{c^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx-\frac{c\sqrt{cx-1}\log(a+b\text{arccosh}(cx))}{b\sqrt{1-cx}}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\,(a+b\,\text{arccosh}\,(cx))}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^2\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{1-c^2\,x^2}}{x^2\,\left(a+b\,\text{acosh}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)\,b\,x^2+a\,x^2}dx$$

input

```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*acosh(c*x)),x)
```

output

```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b*x**2 + a*x**2),x)
```

3.238 $\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\mathbf{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 397

$$\begin{aligned}
\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx &= -\frac{3\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{64bc^4\sqrt{-1+cx}} \\
&+ \frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64bc^4\sqrt{-1+cx}} \\
&+ \frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64bc^4\sqrt{-1+cx}} \\
&- \frac{\sqrt{1-cx}\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64bc^4\sqrt{-1+cx}} \\
&+ \frac{3\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{64bc^4\sqrt{-1+cx}} \\
&- \frac{3\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64bc^4\sqrt{-1+cx}} \\
&- \frac{\sqrt{1-cx}\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64bc^4\sqrt{-1+cx}} \\
&+ \frac{\sqrt{1-cx}\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64bc^4\sqrt{-1+cx}}
\end{aligned}$$

output
```
-3/64*(-c*x+1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^4/(c*x-1)^(1/
2)+3/64*(-c*x+1)^(1/2)*cosh(3*a/b)*Chi(3*(a+b*arccosh(c*x))/b)/b/c^4/(c*x-
1)^(1/2)+1/64*(-c*x+1)^(1/2)*cosh(5*a/b)*Chi(5*(a+b*arccosh(c*x))/b)/b/c^4
/(c*x-1)^(1/2)-1/64*(-c*x+1)^(1/2)*cosh(7*a/b)*Chi(7*(a+b*arccosh(c*x))/b)
/b/c^4/(c*x-1)^(1/2)+3/64*(-c*x+1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*x))/
b)/b/c^4/(c*x-1)^(1/2)-3/64*(-c*x+1)^(1/2)*sinh(3*a/b)*Shi(3*(a+b*arccosh(
c*x))/b)/b/c^4/(c*x-1)^(1/2)-1/64*(-c*x+1)^(1/2)*sinh(5*a/b)*Shi(5*(a+b*ar
ccosh(c*x))/b)/b/c^4/(c*x-1)^(1/2)+1/64*(-c*x+1)^(1/2)*sinh(7*a/b)*Shi(7*(
a+b*arccosh(c*x))/b)/b/c^4/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.54

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\left(-3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(\right.\right.\right.}{}$$

input
```
Integrate[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh
[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegra
l[5*(a/b + ArcCosh[c*x])] - Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[c*
x])] + 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Sinh[(3*a)/b]*Sinh
Integral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
Cosh[c*x])] + Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64*c^4
*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(1-c^2x^2\right)^{3/2}}{a+b\text{arccosh}(cx)}\,dx$$

$\Big\downarrow$ 6367

$$-\frac{\sqrt{1-cx}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^4\sqrt{cx-1}}$$

$\Big\downarrow$ 5971

$$-\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}+\frac{3\cosh\left(\frac{a}{b}-\right.}{64(a+b}\right.}{bc^4\sqrt{cx-1}}$$

$\Big\downarrow$ 2009

$$-\frac{\sqrt{1-cx}\left(\frac{3}{64}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{3}{64}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{64}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arc}}{b}\right.\right.}{}$$

input
```
Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
-((Sqrt[1 - c*x]*((3*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/64 -
(3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b])/64 - (Cosh[(5*a
)/b]*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/64 + (Cosh[(7*a)/b]*CoshInt
egral[(7*(a + b*ArcCosh[c*x]))/b])/64 - (3*Sinh[a/b]*SinhIntegral[(a + b*A
rcCosh[c*x])/b])/64 + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]
))/b])/64 + (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/64 -
(Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/64))/(b*c^4*Sqrt[
-1 + c*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(\text{expIntegral}_1\left(7\,\text{arccosh}(cx)+\frac{7a}{b}\right)e^{\frac{b\,\text{arccosh}(cx)+7a}{b}}+\text{expIntegral}_1\left(-7\,\text{arccosh}(cx)\right.\right.}{}$

input
```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/128*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(Ei
(1,7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)+Ei(1,-7*arccosh(c*x)-
7*a/b)*exp(-(-b*arccosh(c*x)+7*a)/b)-Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*arc
cosh(c*x)+5*a)/b)-3*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)
+3*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)+3*Ei(1,-arccosh(c*x)-a
/b)*exp(-(-b*arccosh(c*x)+a)/b)-3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(-b*arc
cosh(c*x)+3*a)/b)-Ei(1,-5*arccosh(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+5*a)/b
))/(c*x+1)/c^4/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^3(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{a+b\,\text{acosh}\,(cx)}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**3*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^3\,(1-c^2\,x^2)^{3/2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=-\left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acosh\,(cx)\,b+a}dx\right)c^2+\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)\,b+a}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
 - int((sqrt( - c**2*x**2 + 1)*x**5)/(acosh(c*x)*b + a),x)*c**2 + int((sqr
t( - c**2*x**2 + 1)*x**3)/(acosh(c*x)*b + a),x)
```

3.239 $\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\mathbf{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 339

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\operatorname{arccosh}(cx)}\,dx = \frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\log(a+b\operatorname{arccosh}(cx))}{16bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}}$$

output
```
1/32*(-c*x+1)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arccosh(c*x))/b)/b/c^3/(c*x-1)^
(1/2)+1/16*(-c*x+1)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arccosh(c*x))/b)/b/c^3/(c
*x-1)^(1/2)-1/32*(-c*x+1)^(1/2)*cosh(6*a/b)*Chi(6*(a+b*arccosh(c*x))/b)/b/
c^3/(c*x-1)^(1/2)-1/16*(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c^3/(c*x-1)^(
1/2)-1/32*(-c*x+1)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b/c^3/(c*
x-1)^(1/2)-1/16*(-c*x+1)^(1/2)*sinh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b/c
^3/(c*x-1)^(1/2)+1/32*(-c*x+1)^(1/2)*sinh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/
b)/b/c^3/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.55

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx =$$
$$-\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)-2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Ch}}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
-1/32*(Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*
x])]) - 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/
b]*CoshIntegral[6*(a/b + ArcCosh[c*x])] + 2*Log[a + b*ArcCosh[c*x]] + Sinh
[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 2*Sinh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[
c*x])]))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}dx$$

$\downarrow$ 6367

$$-\frac{\sqrt{1-cx}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^3\sqrt{cx-1}}$$

$\downarrow$ 5971

$$-\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}+\frac{1}{16(a+b\text{arcc}}\right.}{bc^3\sqrt{cx-1}}$$

$\downarrow$ 2009

$$-\frac{\sqrt{1-cx}\left(-\frac{1}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{16}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a\cdot}{}\right.\right.}{}$$

input
```
Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
-((Sqrt[1 - c*x]*(-1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]
))/b]) - (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/16 + (Co
sh[(6*a)/b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32 + Log[a + b*ArcCo
sh[c*x]]/16 + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/32
+ (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/16 - (Sinh[(6*a
)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32))/(b*c^3*Sqrt[-1 + c*x])
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(-4\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\operatorname{arccosh}(cx))-4\ln(a+b\,\operatorname{arccosh}(cx))cx+\operatorname{expIntegral}_1(6\right.}{}$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-4*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))-4*ln(a+b*arccosh(c*x))*c*
x+Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)+Ei(1,-6*arccosh(c
*x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)-2*Ei(1,4*arccosh(c*x)+4*a/b)*exp(
(b*arccosh(c*x)+4*a)/b)-Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a
)/b)-Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-2*Ei(1,-4*a
rccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b))/(c*x+1)/c^3/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^2(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{a+b\,\text{acosh}\,(cx)}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^2\,(1-c^2\,x^2)^{3/2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=-\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acosh\,(cx)\,b+a}dx\right)c^2+\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)\,b+a}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
- int((sqrt( - c**2*x**2 + 1)*x**4)/(acosh(c*x)*b + a),x)*c**2 + int((sqr
t( - c**2*x**2 + 1)*x**2)/(acosh(c*x)*b + a),x)
```

3.240 $\int \frac{x(1-c^2x^2)^{3/2}}{a+b\mathbf{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\operatorname{arccosh}(cx)}\,dx = -\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)}{8bc^2\sqrt{-1+cx}} + \frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc^2\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc^2\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)}{8bc^2\sqrt{-1+cx}} - \frac{3\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc^2\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc^2\sqrt{-1+cx}}$$

output
```
-1/8*(-c*x+1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^2/(c*x-1)^(1/2
)+3/16*(-c*x+1)^(1/2)*cosh(3*a/b)*Chi(3*(a+b*arccosh(c*x))/b)/b/c^2/(c*x-1
)^(1/2)-1/16*(-c*x+1)^(1/2)*cosh(5*a/b)*Chi(5*(a+b*arccosh(c*x))/b)/b/c^2/
(c*x-1)^(1/2)+1/8*(-c*x+1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^2
/(c*x-1)^(1/2)-3/16*(-c*x+1)^(1/2)*sinh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)
/b/c^2/(c*x-1)^(1/2)+1/16*(-c*x+1)^(1/2)*sinh(5*a/b)*Shi(5*(a+b*arccosh(c*
x))/b)/b/c^2/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.58

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(\right.\right.\right.}{}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh
[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - Cosh[(5*a)/b]*CoshIntegra
l[5*(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] -
 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[(5*a)/b]*Sinh
Integral[5*(a/b + ArcCosh[c*x])]))/(16*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b +
 b*c*x))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx$$

$\downarrow$ 6367

$$-\frac{\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^2\sqrt{cx-1}}$$

$\downarrow$ 5971

$$-\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8(a+b\text{arccosh}(cx))}\right)d(a+b\text{arcco}}{bc^2\sqrt{cx-1}}$$

$\downarrow$ 2009

$$-\frac{\sqrt{1-cx}\left(\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{3}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{b}$$

input
```
Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
-((Sqrt[1 - c*x]*((Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/8 - (3*
Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b])/16 + (Cosh[(5*a)/b
]*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/16 - (Sinh[a/b]*SinhIntegral[(
a + b*ArcCosh[c*x])/b])/8 + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCos
h[c*x]))/b])/16 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])
/16))/(b*c^2*Sqrt[-1 + c*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^{2}x^{2}-1\right)\left(\operatorname{expIntegral}_{1}\left(5\,\operatorname{arccosh}(cx)+\frac{5a}{b}\right)\mathrm{e}^{\frac{b\,\operatorname{arccosh}(cx)+5a}{b}}+\operatorname{expIntegral}_{1}\left(-5\,\operatorname{arccosh}(cx)\right.\right.}{}$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(Ei(
1,5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)+Ei(1,-5*arccosh(c*x)-5
*a/b)*exp(-(-b*arccosh(c*x)+5*a)/b)-3*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*ar
ccosh(c*x)+3*a)/b)+2*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)+2*Ei
(1,-arccosh(c*x)-a/b)*exp(-(-b*arccosh(c*x)+a)/b)-3*Ei(1,-3*arccosh(c*x)-3
*a/b)*exp(-(-b*arccosh(c*x)+3*a)/b))/(c*x+1)/c^2/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\operatorname{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\operatorname{arccosh}(cx)}\,dx = \int \frac{x(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\operatorname{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{b\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\operatorname{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{b\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x\,(1-c^2\,x^2)^{3/2}}{a+b\,\mathrm{acosh}\,(c\,x)}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=-\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)\,b+a}dx\right)c^2+\int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)\,b+a}dx$$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
- int((sqrt( - c**2*x**2 + 1)*x**3)/(acosh(c*x)*b + a),x)*c**2 + int((sqr
t( - c**2*x**2 + 1)*x)/(acosh(c*x)*b + a),x)
```

3.241 $\int \frac{(1-c^2x^2)^{3/2}}{a+b\,\mathrm{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\,\mathrm{arccosh}(cx)}\,dx = \frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{2bc\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{8bc\sqrt{-1+cx}} - \frac{3\sqrt{1-cx}\log(a+b\,\mathrm{arccosh}(cx))}{8bc\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{2bc\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{8bc\sqrt{-1+cx}}$$

output
```
1/2*(-c*x+1)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^(1/
2)-1/8*(-c*x+1)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^
(1/2)-3/8*(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c/(c*x-1)^(1/2)-1/2*(-c*x+
1)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^(1/2)+1/8*(-c
*x+1)^(1/2)*sinh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.62

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx =$$
$$-\frac{\sqrt{1-c^2x^2}\left(-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+3\log(a+b\text{ar}}{8bc\sqrt{\frac{-1+cx}{1+cx}}(1+c}$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

output
```
-1/8*(Sqrt[1 - c^2*x^2]*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*
x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + 3*Log[a + b*Ar
cCosh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Sinh[
(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(b*c*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.59, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6321, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6321}$$

$$-\frac{\sqrt{1-cx}\int \frac{\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^4}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2(a+b\text{arccosh}(cx))}+\frac{3}{8(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{1-cx}\left(-\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{8}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+bar\right.\right.}{bc\sqrt{cx-1}}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

output
```
-((Sqrt[1 - c*x]*(-1/2*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x])
)/b]) + (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8 + (3*Lo
g[a + b*ArcCosh[c*x]])/8 + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c
*x]))/b])/2 - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8))
/(b*c*Sqrt[-1 + c*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(-6\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\operatorname{arccosh}(cx))-6\ln(a+b\,\operatorname{arccosh}(cx))cx+\operatorname{expIntegral}_1(4\right.}{}$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-6*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))-6*ln(a+b*arccosh(c*x))*c*
x+Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)+Ei(1,-4*arccosh(c
*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)-4*Ei(1,2*arccosh(c*x)+2*a/b)*exp(
(b*arccosh(c*x)+2*a)/b)-4*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x
)+2*a)/b))/(c*x-1)/(c*x+1)/c/b
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{b\,\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{a+b\,\text{acosh}\,(cx)}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x)),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)\,b+a}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)\,b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b + a),x) - int((sqrt( - c**2*x**2
+ 1)*x**2)/(acosh(c*x)*b + a),x)*c**2
```

3.242 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))} dx = \text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))}, x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))} dx = \int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))} dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))}dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{2c^2x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{c^4x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx-\frac{5\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{4b\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4b\sqrt{1-cx}}+\frac{5\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{4b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4b\sqrt{1-cx}}$$

input

```
Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\ \mathrm{arccosh}\,(cx))}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\mathrm{arccosh}(cx))}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\mathrm{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*x*arccosh(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\mathrm{arccosh}(cx))}\,dx=\int \frac{(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{x\,(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\operatorname{arccosh}(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{x\,\left(a+b\,\operatorname{acosh}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\operatorname{arccosh}(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\left(cx\right)bx+ax}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\left(cx\right)b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b*x + a*x),x) - int((sqrt( - c**2*x
**2 + 1)*x)/(acosh(c*x)*b + a),x)*c**2
```

3.243 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\,\mathrm{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\,\mathrm{arccosh}(cx))}\,dx = \mathrm{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^2(a+b\,\mathrm{arccosh}(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\,\mathrm{arccosh}(cx))}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\,\mathrm{arccosh}(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))}dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{2c^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{c^4x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx+\frac{c\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2b\sqrt{1-cx}}-\frac{c\sqrt{cx-1}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2b\sqrt{1-cx}}-\frac{3c\sqrt{cx-1}\log(a+b\text{arccosh}(cx))}{2b\sqrt{1-cx}}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\,\text{arccosh}\,(cx))}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{x^2\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{x^2\,\left(a+b\,\text{acosh}\,(c\,x)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\operatorname{arccosh}(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)\,b\,x^2+a\,x^2}dx-\left(\int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)\,b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b*x**2 + a*x**2),x) - int(sqrt( - c
**2*x**2 + 1)/(acosh(c*x)*b + a),x)*c**2
```

3.244 $\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\,\mathrm{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 397

$$\begin{aligned}\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\,\mathrm{arccosh}(cx)}\,dx &= -\frac{3\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)}{128bc^4\sqrt{-1+cx}} \\
&+\frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^4\sqrt{-1+cx}} \\
&-\frac{3\sqrt{1-cx}\cosh\left(\frac{7a}{b}\right)\mathrm{Chi}\left(\frac{7(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{256bc^4\sqrt{-1+cx}} \\
&+\frac{\sqrt{1-cx}\cosh\left(\frac{9a}{b}\right)\mathrm{Chi}\left(\frac{9(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{256bc^4\sqrt{-1+cx}} \\
&+\frac{3\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)}{128bc^4\sqrt{-1+cx}} \\
&-\frac{\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^4\sqrt{-1+cx}} \\
&+\frac{3\sqrt{1-cx}\sinh\left(\frac{7a}{b}\right)\mathrm{Shi}\left(\frac{7(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{256bc^4\sqrt{-1+cx}} \\
&-\frac{\sqrt{1-cx}\sinh\left(\frac{9a}{b}\right)\mathrm{Shi}\left(\frac{9(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{256bc^4\sqrt{-1+cx}}\end{aligned}$$

output
```
-3/128*(-c*x+1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^4/(c*x-1)^(1
/2)+1/32*(-c*x+1)^(1/2)*cosh(3*a/b)*Chi(3*(a+b*arccosh(c*x))/b)/b/c^4/(c*x
-1)^(1/2)-3/256*(-c*x+1)^(1/2)*cosh(7*a/b)*Chi(7*(a+b*arccosh(c*x))/b)/b/c
^4/(c*x-1)^(1/2)+1/256*(-c*x+1)^(1/2)*cosh(9*a/b)*Chi(9*(a+b*arccosh(c*x))
/b)/b/c^4/(c*x-1)^(1/2)+3/128*(-c*x+1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*
x))/b)/b/c^4/(c*x-1)^(1/2)-1/32*(-c*x+1)^(1/2)*sinh(3*a/b)*Shi(3*(a+b*arcc
osh(c*x))/b)/b/c^4/(c*x-1)^(1/2)+3/256*(-c*x+1)^(1/2)*sinh(7*a/b)*Shi(7*(a
+b*arccosh(c*x))/b)/b/c^4/(c*x-1)^(1/2)-1/256*(-c*x+1)^(1/2)*sinh(9*a/b)*S
hi(9*(a+b*arccosh(c*x))/b)/b/c^4/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.54

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\left(-6\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)+8\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(\right.\right.\right.}{}$$

input
```
Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-6*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 8*Cosh
[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Cosh[(7*a)/b]*CoshInteg
ral[7*(a/b + ArcCosh[c*x])] + Cosh[(9*a)/b]*CoshIntegral[9*(a/b + ArcCosh[
c*x])] + 6*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 8*Sinh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcCosh[c*x])] + 3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b +
 ArcCosh[c*x])] - Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcCosh[c*x])]))/(25
6*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}dx$$

$\big\downarrow$ 6367

$$\frac{\sqrt{1-cx}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^6\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^4\sqrt{cx-1}}$$

$\big\downarrow$ 5971

$$\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{9a}{b}-\frac{9(a+b\text{arccosh}(cx))}{b}\right)}{256(a+b\text{arccosh}(cx))}-\frac{3\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{256(a+b\text{arccosh}(cx))}+\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}-\frac{3\cosh\left(\frac{a}{b}-\frac{a+}{}\right.}{128(a+ba}\right.}{bc^4\sqrt{cx-1}}$$

$\big\downarrow$ 2009

$$\sqrt{1-cx}\left(-\frac{3}{128}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)+\frac{1}{32}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{256}\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+ba}{}\right.\right.$$

input
```
Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*((-3*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/128 +
(Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b])/32 - (3*Cosh[(7*a
)/b]*CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b])/256 + (Cosh[(9*a)/b]*CoshIn
tegral[(9*(a + b*ArcCosh[c*x]))/b])/256 + (3*Sinh[a/b]*SinhIntegral[(a + b
*ArcCosh[c*x])/b])/128 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x
]))/b])/32 + (3*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/25
6 - (Sinh[(9*a)/b]*SinhIntegral[(9*(a + b*ArcCosh[c*x]))/b])/256))/(b*c^4*
Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.80

method	result
default	$\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(\text{expIntegral}_1\left(9\,\text{arccosh}(cx)+\frac{9a}{b}\right)e^{\frac{b\,\text{arccosh}(cx)+9a}{b}}+\text{expIntegral}_1\left(-9\,\text{arccosh}(cx)-\frac{9}{\cdots}\right.\right.$

input
```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/512*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(Ei(
1,9*arccosh(c*x)+9*a/b)*exp((b*arccosh(c*x)+9*a)/b)+Ei(1,-9*arccosh(c*x)-9
*a/b)*exp(-(-b*arccosh(c*x)+9*a)/b)-3*Ei(1,7*arccosh(c*x)+7*a/b)*exp((b*ar
ccosh(c*x)+7*a)/b)+8*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b
)-6*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)-6*Ei(1,-arccosh(c*x)-
a/b)*exp(-(-b*arccosh(c*x)+a)/b)+8*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(-b*ar
ccosh(c*x)+3*a)/b)-3*Ei(1,-7*arccosh(c*x)-7*a/b)*exp(-(-b*arccosh(c*x)+7*a
)/b))/(c*x+1)/c^4/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) +
a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^3\,(1-c^2\,x^2)^{5/2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\left(\int \frac{\sqrt{-c^2x^2+1}\,x^7}{acosh\,(cx)\,b+a}dx\right)c^4 -2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acosh\,(cx)\,b+a}dx\right)c^2+\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)\,b+a}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**7)/(acosh(c*x)*b + a),x)*c**4 - 2*int((sqrt
( - c**2*x**2 + 1)*x**5)/(acosh(c*x)*b + a),x)*c**2 + int((sqrt( - c**2*x*
*2 + 1)*x**3)/(acosh(c*x)*b + a),x)
```

3.245 $\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\,\mathrm{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 439

$$\begin{aligned}\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\,\mathrm{arccosh}(cx)}\,dx &= \frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} \\ &+ \frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} \\ &- \frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} \\ &+ \frac{\sqrt{1-cx}\cosh\left(\frac{8a}{b}\right)\mathrm{Chi}\left(\frac{8(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{128bc^3\sqrt{-1+cx}} \\ &- \frac{5\sqrt{1-cx}\log(a+b\,\mathrm{arccosh}(cx))}{128bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} \\ &- \frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} \\ &+ \frac{\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\mathrm{Shi}\left(\frac{6(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}} \\ &- \frac{\sqrt{1-cx}\sinh\left(\frac{8a}{b}\right)\mathrm{Shi}\left(\frac{8(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{128bc^3\sqrt{-1+cx}}\end{aligned}$$

output
```
1/32*(-c*x+1)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arccosh(c*x))/b)/b/c^3/(c*x-1)^
(1/2)+1/32*(-c*x+1)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arccosh(c*x))/b)/b/c^3/(c
*x-1)^(1/2)-1/32*(-c*x+1)^(1/2)*cosh(6*a/b)*Chi(6*(a+b*arccosh(c*x))/b)/b/
c^3/(c*x-1)^(1/2)+1/128*(-c*x+1)^(1/2)*cosh(8*a/b)*Chi(8*(a+b*arccosh(c*x)
)/b)/b/c^3/(c*x-1)^(1/2)-5/128*(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c^3/(
c*x-1)^(1/2)-1/32*(-c*x+1)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b
/c^3/(c*x-1)^(1/2)-1/32*(-c*x+1)^(1/2)*sinh(4*a/b)*Shi(4*(a+b*arccosh(c*x)
)/b)/b/c^3/(c*x-1)^(1/2)+1/32*(-c*x+1)^(1/2)*sinh(6*a/b)*Shi(6*(a+b*arccos
h(c*x))/b)/b/c^3/(c*x-1)^(1/2)-1/128*(-c*x+1)^(1/2)*sinh(8*a/b)*Shi(8*(a+b
*arccosh(c*x))/b)/b/c^3/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.53

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\left(4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arccos}\right.\right.\right.}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] +
 4*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] - 4*Cosh[(6*a)/b]*Co
shIntegral[6*(a/b + ArcCosh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + A
rcCosh[c*x])] - 5*Log[a + b*ArcCosh[c*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcCosh[c*x])] - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x
])] + 4*Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - Sinh[(8*a)/b]
*SinhIntegral[8*(a/b + ArcCosh[c*x])]))/(128*c^3*Sqrt[(-1 + c*x)/(1 + c*x)
]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}dx$$

$$\Big\downarrow \text{6367}$$

$$\frac{\sqrt{1-cx}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^6\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^3\sqrt{cx-1}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{8a}{b}-\frac{8(a+b\text{arccosh}(cx))}{b}\right)}{128(a+b\text{arccosh}(cx))}-\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}+\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+}{32(a+bar}\right.}{bc^3\sqrt{cx-1}}$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{1-cx}\left(\frac{1}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+bar\right.\right.$$

input
```
Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/3
2 + (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/32 - (Cosh[(6
*a)/b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32 + (Cosh[(8*a)/b]*CoshI
ntegral[(8*(a + b*ArcCosh[c*x]))/b])/128 - (5*Log[a + b*ArcCosh[c*x]])/128
 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/32 - (Sinh[(4*
a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/32 + (Sinh[(6*a)/b]*SinhIn
tegral[(6*(a + b*ArcCosh[c*x]))/b])/32 - (Sinh[(8*a)/b]*SinhIntegral[(8*(a
 + b*ArcCosh[c*x]))/b])/128))/(b*c^3*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.83

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(10\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\mathrm{arccosh}(cx))+10\ln(a+b\,\mathrm{arccosh}(cx))cx-4\,\mathrm{expIntegral}_1(6\right.}{}$

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/256*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(10*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))+10*ln(a+b*arccosh(c*x))*c
*x-4*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)+Ei(1,8*arccosh
(c*x)+8*a/b)*exp((b*arccosh(c*x)+8*a)/b)+Ei(1,-8*arccosh(c*x)-8*a/b)*exp(-
(-b*arccosh(c*x)+8*a)/b)+4*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+
4*a)/b)+4*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)+4*Ei(1,-2
*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)+4*Ei(1,-4*arccosh(c*x)-
4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)-4*Ei(1,-6*arccosh(c*x)-6*a/b)*exp(-(-
b*arccosh(c*x)+6*a)/b))/(c*x+1)/c^3/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) +
a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x^2\,(1-c^2\,x^2)^{5/2}}{a+b\,\operatorname{acosh}(c\,x)}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^6}{acosh\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acosh\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)\,b+a}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**6)/(acosh(c*x)*b + a),x)*c**4 - 2*int((sqrt
( - c**2*x**2 + 1)*x**4)/(acosh(c*x)*b + a),x)*c**2 + int((sqrt( - c**2*x*
*2 + 1)*x**2)/(acosh(c*x)*b + a),x)
```

3.246 $\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 397

$$\begin{aligned}\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = &-\frac{5\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{64bc^2\sqrt{-1+cx}}\\
&+\frac{9\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64bc^2\sqrt{-1+cx}}\\
&-\frac{5\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64bc^2\sqrt{-1+cx}}\\
&+\frac{\sqrt{1-cx}\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64bc^2\sqrt{-1+cx}}\\
&+\frac{5\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{64bc^2\sqrt{-1+cx}}\\
&-\frac{9\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64bc^2\sqrt{-1+cx}}\\
&+\frac{5\sqrt{1-cx}\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64bc^2\sqrt{-1+cx}}\\
&-\frac{\sqrt{1-cx}\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64bc^2\sqrt{-1+cx}}\end{aligned}$$

output
```
-5/64*(-c*x+1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^2/(c*x-1)^(1/
2)+9/64*(-c*x+1)^(1/2)*cosh(3*a/b)*Chi(3*(a+b*arccosh(c*x))/b)/b/c^2/(c*x-
1)^(1/2)-5/64*(-c*x+1)^(1/2)*cosh(5*a/b)*Chi(5*(a+b*arccosh(c*x))/b)/b/c^2
/(c*x-1)^(1/2)+1/64*(-c*x+1)^(1/2)*cosh(7*a/b)*Chi(7*(a+b*arccosh(c*x))/b)
/b/c^2/(c*x-1)^(1/2)+5/64*(-c*x+1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*x))/
b)/b/c^2/(c*x-1)^(1/2)-9/64*(-c*x+1)^(1/2)*sinh(3*a/b)*Shi(3*(a+b*arccosh(
c*x))/b)/b/c^2/(c*x-1)^(1/2)+5/64*(-c*x+1)^(1/2)*sinh(5*a/b)*Shi(5*(a+b*ar
ccosh(c*x))/b)/b/c^2/(c*x-1)^(1/2)-1/64*(-c*x+1)^(1/2)*sinh(7*a/b)*Shi(7*(
a+b*arccosh(c*x))/b)/b/c^2/(c*x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.54

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\left(-5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)+9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(\right.\right.\right.}{}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-5*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 9*Cosh
[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 5*Cosh[(5*a)/b]*CoshInteg
ral[5*(a/b + ArcCosh[c*x])] + Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[
c*x])] + 5*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 9*Sinh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcCosh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a/b +
 ArcCosh[c*x])] - Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64
*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(1-c^2x^2\right)^{5/2}}{a+b\text{arccosh}(cx)}\,dx$$

$\Big\downarrow$ 6367

$$\frac{\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^6\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^2\sqrt{cx-1}}$$

$\Big\downarrow$ 5971

$$\frac{\sqrt{1-cx}\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}-\frac{5\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}+\frac{9\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}-\frac{5\cosh\left(\frac{a}{b}-\frac{c}{}\right.}{64(a+ba}\right.}{bc^2\sqrt{cx-1}}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{1-cx}\left(-\frac{5}{64}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)+\frac{9}{64}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{5}{64}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{}$$

input
```
Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*((-5*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/64 + (
9*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b])/64 - (5*Cosh[(5*
a)/b]*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/64 + (Cosh[(7*a)/b]*CoshIn
tegral[(7*(a + b*ArcCosh[c*x]))/b])/64 + (5*Sinh[a/b]*SinhIntegral[(a + b*
ArcCosh[c*x])/b])/64 - (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x
]))/b])/64 + (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/64
 - (Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/64))/(b*c^2*Sq
rt[-1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.80

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(\mathrm{expIntegral}_1\left(7\,\mathrm{arccosh}(cx)+\frac{7a}{b}\right)e^{\frac{b\,\mathrm{arccosh}(cx)+7a}{b}}+\mathrm{expIntegral}_1\left(-7\,\mathrm{arccosh}(cx)-\frac{7}{}\right.\right.}{}$

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/128*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(Ei(
1,7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)+Ei(1,-7*arccosh(c*x)-7
*a/b)*exp(-(-b*arccosh(c*x)+7*a)/b)-5*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*ar
ccosh(c*x)+5*a)/b)+9*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b
)-5*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)-5*Ei(1,-arccosh(c*x)-
a/b)*exp(-(-b*arccosh(c*x)+a)/b)+9*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(-b*ar
ccosh(c*x)+3*a)/b)-5*Ei(1,-5*arccosh(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+5*a
)/b))/(c*x+1)/c^2/(c*x-1)/b
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\text{Timed out}$$

input
```
integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x\,(1-c^2\,x^2)^{5/2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acosh\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)\,b+a}dx$$

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**5)/(acosh(c*x)*b + a),x)*c**4 - 2*int((sqrt
( - c**2*x**2 + 1)*x**3)/(acosh(c*x)*b + a),x)*c**2 + int((sqrt( - c**2*x*
*2 + 1)*x)/(acosh(c*x)*b + a),x)
```

3.247 $\int \frac{(1-c^2x^2)^{5/2}}{a+b\,\mathbf{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 339

$$\begin{aligned}\int \frac{(1-c^2x^2)^{5/2}}{a+b\operatorname{arccosh}(cx)}\,dx &= \frac{15\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc\sqrt{-1+cx}} \\ &\quad - \frac{3\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc\sqrt{-1+cx}} \\ &\quad + \frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc\sqrt{-1+cx}} - \frac{5\sqrt{1-cx}\log(a+b\operatorname{arccosh}(cx))}{16bc\sqrt{-1+cx}} \\ &\quad - \frac{15\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc\sqrt{-1+cx}} \\ &\quad + \frac{3\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{16bc\sqrt{-1+cx}} \\ &\quad - \frac{\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)}{32bc\sqrt{-1+cx}}\end{aligned}$$

output
```
15/32*(-c*x+1)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^(
1/2)-3/16*(-c*x+1)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arccosh(c*x))/b)/b/c/(c*x-
1)^(1/2)+1/32*(-c*x+1)^(1/2)*cosh(6*a/b)*Chi(6*(a+b*arccosh(c*x))/b)/b/c/(
c*x-1)^(1/2)-5/16*(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c/(c*x-1)^(1/2)-15
/32*(-c*x+1)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)^(1/
2)+3/16*(-c*x+1)^(1/2)*sinh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b/c/(c*x-1)
^(1/2)-1/32*(-c*x+1)^(1/2)*sinh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/b)/b/c/(c*
x-1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.56

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\left(15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)-6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arccc}\right.\right.\right.}{}$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])]
- 6*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*Cos
hIntegral[6*(a/b + ArcCosh[c*x])] - 10*Log[a + b*ArcCosh[c*x]] - 15*Sinh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 6*Sinh[(4*a)/b]*SinhIntegra
l[4*(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*
x])]))/(32*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6321, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}dx$$

$\Big\downarrow$ 6321

$$\frac{\sqrt{1-cx}\int \frac{\sinh^6\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{1-cx}\int -\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^6}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$\Big\downarrow$ 25

$$-\frac{\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^6}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}}$$

$\Big\downarrow$ 3793

$$-\frac{\sqrt{1-cx}\int \left(-\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}+\frac{3\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{15\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}+\frac{}{16(a+}\right.}{bc\sqrt{cx-1}}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{1-cx}\left(\frac{15}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{16}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{ar}\right.\right.}{}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

output
```
(Sqrt[1 - c*x]*((15*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]
)/32 - (3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/16 + (Co
sh[(6*a)/b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32 - (5*Log[a + b*Ar
cCosh[c*x]])/16 - (15*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/
b])/32 + (3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/16 - (
Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32))/(b*c*Sqrt[-1
+ c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(20\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\operatorname{arccosh}(cx))+20\ln(a+b\,\operatorname{arccosh}(cx))cx+\operatorname{expIntegral}_1(6\,\mathrm{a}\right.}{}$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/64*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(20*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))+20*ln(a+b*arccosh(c*x))*c*
x+Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)+Ei(1,-6*arccosh(c
*x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)-6*Ei(1,4*arccosh(c*x)+4*a/b)*exp(
(b*arccosh(c*x)+4*a)/b)+15*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+
2*a)/b)+15*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-6*Ei(
1,-4*arccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b))/(c*x-1)/(c*x+1)/c/
b
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x)),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x)), x)
```

Reduce [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)\,b+a}dx + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acosh\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)\,b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b + a),x) + int((sqrt( - c**2*x**2
+ 1)*x**4)/(acosh(c*x)*b + a),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**2
)/(acosh(c*x)*b + a),x)*c**2
```

3.248 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{3c^2x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}-\frac{c^6x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{}{\sqrt{1-c}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx-\frac{11\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{8b\sqrt{1-cx}}+\frac{7\sqrt{cx-1}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16b\sqrt{1-cx}}+\frac{11\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{8b\sqrt{1-cx}}-\frac{7\sqrt{cx-1}\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16b\sqrt{1-cx}}+\frac{\sqrt{cx-1}\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16b\sqrt{1-cx}}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\,\mathrm{arccosh}\,(cx))}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\mathrm{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\mathrm{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) +
a*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\mathrm{arccosh}(cx))}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x\,(a+b\,\text{acosh}\,(c\,x))}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.18

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)\,bx+ax}dx + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)\,b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b*x + a*x),x) + int((sqrt( - c**2*x
**2 + 1)*x**3)/(acosh(c*x)*b + a),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*
x)/(acosh(c*x)*b + a),x)*c**2
```

3.249 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\,\text{arccosh}(cx))} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))} \, dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))}, x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))} \, dx = \int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))} \, dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))}\,dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{3c^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}-\frac{c^6x^4}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}+\frac{}{\sqrt{1-}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx+\frac{c\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{b\sqrt{1-cx}}-\frac{c\sqrt{cx-1}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8b\sqrt{1-cx}}-\frac{c\sqrt{cx-1}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{b\sqrt{1-cx}}+\frac{c\sqrt{cx-1}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8b\sqrt{1-cx}}-\frac{15c\sqrt{cx-1}\log(a+b\text{arccosh}(cx))}{8b\sqrt{1-cx}}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^2 (a+b \operatorname{arccosh}(cx))}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b \operatorname{arcosh}(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x)
+ a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^2\,\left(a+b\,\text{acosh}\,(c\,x)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.29

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\operatorname{arccosh}(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh(cx)\,b\,x^2+a\,x^2}dx - 2\left(\int \frac{\sqrt{-c^2x^2+1}}{acosh(cx)\,b+a}dx\right)c^2 + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh(cx)\,b+a}dx\right)c^4$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*acosh(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b*x**2 + a*x**2),x) - 2*int(sqrt( -
 c**2*x**2 + 1)/(acosh(c*x)*b + a),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x
**2)/(acosh(c*x)*b + a),x)*c**4
```

3.250 $\int \frac{x^4}{\sqrt{1-a^2x^2}\,\mathbf{arccosh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\,\mathrm{arccosh}(ax)}\,dx = \frac{\sqrt{-1+ax}\,\mathrm{Chi}(2\mathrm{arccosh}(ax))}{2a^5\sqrt{1-ax}} + \frac{\sqrt{-1+ax}\,\mathrm{Chi}(4\mathrm{arccosh}(ax))}{8a^5\sqrt{1-ax}} + \frac{3\sqrt{-1+ax}\log(\mathrm{arccosh}(ax))}{8a^5\sqrt{1-ax}}$$

output
```
1/2*(a*x-1)^(1/2)*Chi(2*arccosh(a*x))/a^5/(-a*x+1)^(1/2)+1/8*(a*x-1)^(1/2)
*Chi(4*arccosh(a*x))/a^5/(-a*x+1)^(1/2)+3/8*(a*x-1)^(1/2)*ln(arccosh(a*x))
/a^5/(-a*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\,\mathrm{arccosh}(ax)}\,dx = \frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)(4\mathrm{Chi}(2\mathrm{arccosh}(ax))+\mathrm{Chi}(4\mathrm{arccosh}(ax))+3\log(\mathrm{arccosh}(ax)))}{8a^5\sqrt{-((-1+ax)(1+ax))}}$$

input
```
Integrate[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(4*CoshIntegral[2*ArcCosh[a*x]] + Co
shIntegral[4*ArcCosh[a*x]] + 3*Log[ArcCosh[a*x]]))/(8*a^5*Sqrt[-((-1 + a*x
)*(1 + a*x))])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{\sqrt{ax-1}\int \frac{a^4x^4}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^5\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{ax-1}\int \frac{\sin\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)^4}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^5\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\sqrt{ax-1}\int \left(\frac{\cosh(2\text{arccosh}(ax))}{2\text{arccosh}(ax)}+\frac{\cosh(4\text{arccosh}(ax))}{8\text{arccosh}(ax)}+\frac{3}{8\text{arccosh}(ax)}\right)d\text{arccosh}(ax)}{a^5\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{ax-1}\left(\frac{1}{2}\text{Chi}(2\text{arccosh}(ax))+\frac{1}{8}\text{Chi}(4\text{arccosh}(ax))+\frac{3}{8}\log(\text{arccosh}(ax))\right)}{a^5\sqrt{1-ax}}$$

input
```
Int[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[-1 + a*x]*(CoshIntegral[2*ArcCosh[a*x]]/2 + CoshIntegral[4*ArcCosh[a
*x]]/8 + (3*Log[ArcCosh[a*x]])/8))/(a^5*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,(\text{expIntegral}_1(4\,\text{arccosh}(ax))+\text{expIntegral}_1(-4\,\text{arccosh}(ax))-6\ln(\text{arccosh}(ax))+4\,\text{expIntegral}_1(2\,\text{ar}}{16a^5(a^2x^2-1)}$

input
```
int(x^4/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(Ei(1,4*arccosh(a*x))+
Ei(1,-4*arccosh(a*x))-6*ln(arccosh(a*x))+4*Ei(1,2*arccosh(a*x))+4*Ei(1,-2*
arccosh(a*x)))/a^5/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^4}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^4/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^4}{\sqrt{-(ax-1)(ax+1)}\,\text{acosh}\,(ax)}\,dx$$

input
```
integrate(x**4/(-a**2*x**2+1)**(1/2)/acosh(a*x),x)
```

output
```
Integral(x**4/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^4}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^4}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^4}{\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^4/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^4/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^4}{\sqrt{-a^2x^2+1}\,acosh\,(ax)}dx$$

input
```
int(x^4/(-a^2*x^2+1)^(1/2)/acosh(a*x),x)
```

output
```
int(x**4/(sqrt( - a**2*x**2 + 1)*acosh(a*x)),x)
```

3.251 $\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \frac{3\sqrt{-1+ax}\text{Chi}(\text{arccosh}(ax))}{4a^4\sqrt{1-ax}} + \frac{\sqrt{-1+ax}\text{Chi}(3\text{arccosh}(ax))}{4a^4\sqrt{1-ax}}$$

output
```
3/4*(a*x-1)^(1/2)*Chi(arccosh(a*x))/a^4/(-a*x+1)^(1/2)+1/4*(a*x-1)^(1/2)*C
hi(3*arccosh(a*x))/a^4/(-a*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)(3\text{Chi}(\text{arccosh}(ax))+\text{Chi}(3\text{arccosh}(ax)))}{4a^4\sqrt{-((-1+ax)(1+ax))}}$$

input
```
Integrate[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(3*CoshIntegral[ArcCosh[a*x]] + Cosh
Integral[3*ArcCosh[a*x]]))/(4*a^4*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$

$$\Big\downarrow \text{6367}$$

$$\frac{\sqrt{ax-1}\int \frac{a^3x^3}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^4\sqrt{1-ax}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{ax-1}\int \frac{\sin\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)^3}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^4\sqrt{1-ax}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{ax-1}\int \left(\frac{3ax}{4\text{arccosh}(ax)}+\frac{\cosh(3\text{arccosh}(ax))}{4\text{arccosh}(ax)}\right)d\text{arccosh}(ax)}{a^4\sqrt{1-ax}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{ax-1}\left(\frac{3}{4}\text{Chi}(\text{arccosh}(ax))+\frac{1}{4}\text{Chi}(3\text{arccosh}(ax))\right)}{a^4\sqrt{1-ax}}$$

input
```
Int[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[-1 + a*x]*((3*CoshIntegral[ArcCosh[a*x]])/4 + CoshIntegral[3*ArcCosh
[a*x]]/4))/(a^4*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{-a^{2} x^{2}+1}\,\sqrt{a x -1}\,\sqrt{a x +1}\,\left(\operatorname{expIntegral}_{1}\left(3\,\operatorname{arccosh}\left(a x \right)\right)+\operatorname{expIntegral}_{1}\left(-3\,\operatorname{arccosh}\left(a x \right)\right)+3\,\operatorname{expIntegral}_{1}\left(\operatorname{arccosh}\left(a x \right)\right)+3\,\operatorname{expInt}\right.}{8 a^{4} \left(a^{2} x^{2}-1\right)}$

input
```
int(x^3/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(Ei(1,3*arccosh(a*x))+E
i(1,-3*arccosh(a*x))+3*Ei(1,arccosh(a*x))+3*Ei(1,-arccosh(a*x)))/a^4/(a^2*
x^2-1)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^3}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^3/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\,\text{acosh}\,(ax)}\,dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**(1/2)/acosh(a*x),x)
```

output
```
Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^3}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^3}{\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^3/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^3/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^3}{\sqrt{-a^2x^2+1}\,acosh\,(ax)}dx$$

input
```
int(x^3/(-a^2*x^2+1)^(1/2)/acosh(a*x),x)
```

output
```
int(x**3/(sqrt( - a**2*x**2 + 1)*acosh(a*x)),x)
```

3.252 $\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx=\frac{\sqrt{-1+ax}\text{Chi}(2\text{arccosh}(ax))}{2a^3\sqrt{1-ax}}+\frac{\sqrt{-1+ax}\log(\text{arccosh}(ax))}{2a^3\sqrt{1-ax}}$$

output
```
1/2*(a*x-1)^(1/2)*Chi(2*arccosh(a*x))/a^3/(-a*x+1)^(1/2)+1/2*(a*x-1)^(1/2)
*ln(arccosh(a*x))/a^3/(-a*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = -\frac{\sqrt{-((-1+ax)(1+ax))}(\text{Chi}(2\text{arccosh}(ax))+\log(\text{arccosh}(ax)))}{2a^3\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

input
```
Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
-1/2*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(CoshIntegral[2*ArcCosh[a*x]] + Log[Ar
cCosh[a*x]]))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$

$$\Big\downarrow \text{6367}$$

$$\frac{\sqrt{ax-1}\int \frac{a^2x^2}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^3\sqrt{1-ax}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{ax-1}\int \frac{\sin\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)^2}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^3\sqrt{1-ax}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{ax-1}\int \left(\frac{\cosh(2\text{arccosh}(ax))}{2\text{arccosh}(ax)}+\frac{1}{2\text{arccosh}(ax)}\right)d\text{arccosh}(ax)}{a^3\sqrt{1-ax}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{ax-1}\left(\frac{1}{2}\text{Chi}(2\text{arccosh}(ax))+\frac{1}{2}\log(\text{arccosh}(ax))\right)}{a^3\sqrt{1-ax}}$$

input
```
Int[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[-1 + a*x]*(CoshIntegral[2*ArcCosh[a*x]]/2 + Log[ArcCosh[a*x]]/2))/(a
^3*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{a x -1}\, \sqrt{a x +1}\, \left(\operatorname{expIntegral}_{1}\left(2 \,\operatorname{arccosh}\left(a x \right)\right)+\operatorname{expIntegral}_{1}\left(-2 \,\operatorname{arccosh}\left(a x \right)\right)-2 \ln \left(\operatorname{arccosh}\left(a x \right)\right)\right)}{4 a^{3} \left(a^{2} x^{2}-1\right)}$	67

input
```
int(x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*(Ei(1,2
*arccosh(a*x))+Ei(1,-2*arccosh(a*x))-2*ln(arccosh(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\,\text{acosh}\,(ax)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**(1/2)/acosh(a*x),x)
```

output
```
Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^2}{\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^2/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^2/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\,acosh\,(ax)}dx$$

input
```
int(x^2/(-a^2*x^2+1)^(1/2)/acosh(a*x),x)
```

output
```
int(x**2/(sqrt( - a**2*x**2 + 1)*acosh(a*x)),x)
```

3.253 $\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \frac{\sqrt{-1+ax}\,\text{Chi}(\text{arccosh}(ax))}{a^2\sqrt{1-ax}}$$

output
```
(a*x-1)^(1/2)*Chi(arccosh(a*x))/a^2/(-a*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = -\frac{\sqrt{-((-1+ax)(1+ax))}\,\text{Chi}(\text{arccosh}(ax))}{a^2\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

input
```
Integrate[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
-((Sqrt[-((-1 + a*x)*(1 + a*x))]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[(-1
 + a*x)/(1 + a*x)]*(1 + a*x)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6367, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{\sqrt{ax-1}\int \frac{ax}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^2\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{ax-1}\int \frac{\sin\left(i\text{arccosh}(ax)+\frac{\pi}{2}\right)}{\text{arccosh}(ax)}d\text{arccosh}(ax)}{a^2\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 3782}$$

$$\frac{\sqrt{ax-1}\text{Chi}(\text{arccosh}(ax))}{a^2\sqrt{1-ax}}$$

input
```
Int[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[-1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{-a^2x^2+1}\,\operatorname{arccosh}(ax)}dx$$

input
```
int(x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

output
```
int(x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x}{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x}{\sqrt{-(ax-1)(ax+1)}\,\text{acosh}(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**(1/2)/acosh(a*x),x)
```

output
```
Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x}{\sqrt{-a^2x^2+1}\,\text{arcosh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x}{\sqrt{-a^2x^2+1}\,\text{arcosh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x}{\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{x}{\sqrt{-a^2x^2+1}\,acosh\,(ax)}dx$$

input
```
int(x/(-a^2*x^2+1)^(1/2)/acosh(a*x),x)
```

output
```
int(x/(sqrt( - a**2*x**2 + 1)*acosh(a*x)),x)
```

3.254 $\int \frac{1}{\sqrt{1-a^2x^2}\textbf{arccosh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = -\frac{\sqrt{1-ax}\log(\text{arccosh}(ax))}{a\sqrt{-1+ax}}$$

output
```
-(-a*x+1)^(1/2)*ln(arccosh(a*x))/a/(a*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\log(\text{arccosh}(ax))}{a\sqrt{-((-1+ax)(1+ax))}}$$

input
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Log[ArcCosh[a*x]])/(a*Sqrt[-((-1 + a
*x)*(1 + a*x))])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx$$
$$\Big\downarrow \text{ 6305}$$
$$\frac{\sqrt{ax-1}\log(\text{arccosh}(ax))}{a\sqrt{1-ax}}$$

input
```
Int[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
(Sqrt[-1 + a*x]*Log[ArcCosh[a*x]])/(a*Sqrt[1 - a*x])
```

Defintions of rubi rules used

rule 6305
```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])
]*Log[a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\ln(\text{arccosh}(ax))}{a(a^2x^2-1)}$	48

input
```
int(1/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*ln(arccosh(a
*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = -\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(\log\left(ax+\sqrt{a^2x^2-1}\right)\right)}{a^3x^2-a}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="fricas")
```

output
```
-sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(log(a*x + sqrt(a^2*x^2 - 1)))/(a
^3*x^2 - a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(1/2)/acosh(a*x),x)
```

output
```
Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,acosh\,(ax)}dx$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/acosh(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*acosh(a*x)),x)
```

3.255 $\int \frac{1}{x\sqrt{1-a^2x^2}\textbf{arccosh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx=\text{Int}\left(\frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)},x\right)$$

output
```
Defer(Int)(1/x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx=\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$

input
```
Int[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{-a^2x^2+1}\,\text{arccosh}\,(ax)}dx$$

input
```
int(1/x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

output
```
int(1/x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx=\int \frac{1}{\sqrt{-a^2x^2+1}x\,\text{arcosh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^3 - x)*arccosh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx=\int \frac{1}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\text{acosh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**(1/2)/acosh(a*x),x)
```

output
```
Integral(1/(x*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx=\int \frac{1}{\sqrt{-a^2x^2+1}x\,\text{arcosh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{x\,\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(x*acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(x*acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,acosh\,(ax)\,x}dx$$

input
```
int(1/x/(-a^2*x^2+1)^(1/2)/acosh(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*acosh(a*x)*x),x)
```

3.256 $\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)},x\right)$$

output
```
Defer(Int)(1/x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$

$$\Big\downarrow \text{6375}$$

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}dx$$

input
```
Int[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{-a^2x^2+1}\,\text{arccosh}\left(ax\right)}dx$$

input
```
int(1/x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

output
```
int(1/x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\mathrm{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x^2\operatorname{arcosh}(ax)}\,dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^4 - x^2)*arccosh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\mathrm{arccosh}(ax)}\,dx = \int \frac{1}{x^2\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}(ax)}\,dx$$

input
```
integrate(1/x**2/(-a**2*x**2+1)**(1/2)/acosh(a*x),x)
```

output
```
Integral(1/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\mathrm{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x^2\operatorname{arcosh}(ax)}\,dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x^2\,\text{arcosh}\,(ax)}\,dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arccosh(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arccosh}(ax)}\,dx = \int \frac{1}{x^2\,\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(x^2*acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\operatorname{arccosh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,acosh\,(ax)\,x^2}dx$$

input
```
int(1/x^2/(-a^2*x^2+1)^(1/2)/acosh(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*acosh(a*x)*x**2),x)
```

3.257 $\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 197

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \frac{3\sqrt{-1+cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}} + \frac{\sqrt{-1+cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4bc^4\sqrt{1-cx}} - \frac{3\sqrt{-1+cx}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}} - \frac{\sqrt{-1+cx}\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4bc^4\sqrt{1-cx}}$$

output
```
3/4*(c*x-1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^4/(-c*x+1)^(1/2)
+1/4*(c*x-1)^(1/2)*cosh(3*a/b)*Chi(3*(a+b*arccosh(c*x))/b)/b/c^4/(-c*x+1)^
(1/2)-3/4*(c*x-1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^4/(-c*x+1)
^(1/2)-1/4*(c*x-1)^(1/2)*sinh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)/b/c^4/(-c
*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$$
$$=\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\text{Sh}}{4bc^4\sqrt{-((-1+cx)(1+cx))}}$$

input
```
Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*Cosh[a/b]*CoshIntegral[a/b + ArcC
osh[c*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Sinh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
ArcCosh[c*x])]))/(4*b*c^4*Sqrt[-((-1 + c*x)*(1 + c*x))])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$$

$$\Big\downarrow \text{6367}$$

$$\frac{\sqrt{cx-1}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^4\sqrt{1-cx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{cx-1}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)^3}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^4\sqrt{1-cx}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{cx-1}\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{bc^4\sqrt{1-cx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx-1}\left(\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right.\right.}{bc^4\sqrt{1-cx}}$$

input
```
Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
(Sqrt[-1 + c*x]*((3*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/4 + (C
osh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b])/4 - (3*Sinh[a/b]*Si
nhIntegral[(a + b*ArcCosh[c*x])/b])/4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a
+ b*ArcCosh[c*x]))/b])/4))/(b*c^4*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(\text{expIntegral}_1(3\,\text{arccosh}(cx)+\frac{3a}{b})\,\mathrm{e}^{\frac{-b\,\text{arccosh}(cx)+3a}{b}}+\text{expIntegral}_1(-3\,\text{arccosh}(cx)-\frac{3}{b}\right.}{8b(c^2x^2}$

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-c^2*x^2+1)^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(Ei(1,3
*arccosh(c*x)+3*a/b)*exp((-b*arccosh(c*x)+3*a)/b)+Ei(1,-3*arccosh(c*x)-3*a
/b)*exp(-(b*arccosh(c*x)+3*a)/b)+3*Ei(1,arccosh(c*x)+a/b)*exp((-b*arccosh(
c*x)+a)/b)+3*Ei(1,-arccosh(c*x)-a/b)*exp(-(a+b*arccosh(c*x))/b))/b/(c^2*x^
2-1)/c^4
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{x^3}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
 - a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

input
```
integrate(x**3/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^3}{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^3/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^3/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int(x**3/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a),
x)
```

3.258 $\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\frac{\sqrt{-1+cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2bc^3\sqrt{1-cx}}+\frac{\sqrt{-1+cx}\log(a+b\text{arccosh}(cx))}{2bc^3\sqrt{1-cx}}-\frac{\sqrt{-1+cx}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2bc^3\sqrt{1-cx}}$$

output
```
1/2*(c*x-1)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arccosh(c*x))/b)/b/c^3/(-c*x+1)^(
1/2)+1/2*(c*x-1)^(1/2)*ln(a+b*arccosh(c*x))/b/c^3/(-c*x+1)^(1/2)-1/2*(c*x-
1)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b/c^3/(-c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = -\frac{\sqrt{1-c^2x^2}\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)+\log(a+b\text{arccosh}(cx))-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arccos}\right.\right.\right.}{2c^3\sqrt{\frac{-1+cx}{1+cx}}(b+bcx)}$$

input
```
Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
-1/2*(Sqrt[1 - c^2*x^2]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])
] + Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[
c*x])]))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$$

$$\Big\downarrow \; 6367$$

$$\frac{\sqrt{cx-1}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^3\sqrt{1-cx}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{cx-1}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)^2}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^3\sqrt{1-cx}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{cx-1}\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2(a+b\text{arccosh}(cx))}+\frac{1}{2(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{bc^3\sqrt{1-cx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx-1}\left(\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{2}\log(a+b\text{arccosh}(cx))\right)}{bc^3\sqrt{1-cx}}$$

input
```
Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
(Sqrt[-1 + c*x]*((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/
2 + Log[a + b*ArcCosh[c*x]]/2 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcC
osh[c*x]))/b])/2))/(b*c^3*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\mathrm{arccosh}(cx))-2\ln(a+b\,\mathrm{arccosh}(cx))cx+\mathrm{expIntegral}_1(2\,\mathrm{arcco}\right.}{4b(c^2x^2-1)c^3}$

input
```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c^2*x^2+1)^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(2*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*ln(a+b*arccosh(c*x))-2*ln(a+b*arccosh(c*x))*c*x+Ei
(1,2*arccosh(c*x)+2*a/b)*exp((-b*arccosh(c*x)+2*a)/b)+Ei(1,-2*arccosh(c*x)
-2*a/b)*exp(-(b*arccosh(c*x)+2*a)/b))/b/(c^2*x^2-1)/c^3
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\mathrm{arccosh}(cx))}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
 - a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\mathrm{arccosh}(cx))}\,dx = \int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{x^2}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{x^2}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{x^2}{(a+b\,\text{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int(x**2/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a),
x)
```

3.259 $\int \frac{x}{\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))}\,dx = \frac{\sqrt{-1+cx}\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)}{bc^2\sqrt{1-cx}} - \frac{\sqrt{-1+cx}\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}$$

output
```
(c*x-1)^(1/2)*cosh(a/b)*Chi((a+b*arccosh(c*x))/b)/b/c^2/(-c*x+1)^(1/2)-(c*
x-1)^(1/2)*sinh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^2/(-c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))}\,dx = \frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arccosh}(cx)\right)+\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{arccosh}(cx)\right)\right)}{c^2\sqrt{\frac{-1+cx}{1+cx}}(b+bcx)}$$

input
```
Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Sinh[a
/b]*SinhIntegral[a/b + ArcCosh[c*x]]))/(c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b
+ b*c*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6367, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{\sqrt{cx-1}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{cx-1}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{\sqrt{cx-1}\left(\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arcco\right.}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\sqrt{cx-1}\left(\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(c\right.}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{cx-1}\left(\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\sinh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+bar\right.}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \; 26$$

$$\frac{\sqrt{cx-1}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+barc\right.}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \; 3779$$

$$\frac{\sqrt{cx-1}\left(-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))\right)}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \; 3782$$

$$\frac{\sqrt{cx-1}\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{bc^2\sqrt{1-cx}}$$

input
```
Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
(Sqrt[-1 + c*x]*(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b] - Sinh[a/b
]*SinhIntegral[(a + b*ArcCosh[c*x])/b]))/(b*c^2*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{-c^2x^2+1}\,(a+b\,\mathrm{arccosh}\,(cx))}dx$$

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x) -
 a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x}{(a+b\,\text{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx \\ = -\left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)\,b\,c^2x^2-acosh\,(cx)\,b+a\,c^2x^2-a}dx\right)\end{aligned}$$

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
- int((sqrt( - c**2*x**2 + 1)*x)/(acosh(c*x)*b*c**2*x**2 - acosh(c*x)*b + 
a*c**2*x**2 - a),x)
```

3.260 $\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))} \, dx = -\frac{\sqrt{1-cx}\log(a+b\text{arccosh}(cx))}{bc\sqrt{-1+cx}}$$

output
```
-(-c*x+1)^(1/2)*ln(a+b*arccosh(c*x))/b/c/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))} \, dx = \frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\log(a+b\text{arccosh}(cx))}{bc\sqrt{-((-1+cx)(1+cx))}}$$

input
```
Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[-
((-1 + c*x)*(1 + c*x))])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$

$$\Big\downarrow \text{6305}$$

$$\frac{\sqrt{cx-1}\log(a+b\text{arccosh}(cx))}{bc\sqrt{1-cx}}$$

input
```
Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
(Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 6305
```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])
]*Log[a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\sqrt{-c^{2}x^{2}+1}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\ln(a+b\,\text{arccosh}(cx))}{c(c^{2}x^{2}-1)b}$	55

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(c^2*x^2-1)*ln(a+b*arcco
sh(c*x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = -\frac{\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}\log\left(\frac{b\log\left(cx+\sqrt{c^2x^2-1}\right)+a}{b}\right)}{bc^3x^2-bc}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
-sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*log((b*log(c*x + sqrt(c^2*x^2 - 1))
+ a)/b)/(b*c^3*x^2 - b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$$
$$= -\left(\int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)\,b\,c^2x^2 - acosh\,(cx)\,b + a\,c^2x^2 - a}dx\right)$$

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
 - int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)*b*c**2*x**2 - acosh(c*x)*b + a*c
**2*x**2 - a),x)
```

3.261 $\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\,\textbf{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))}\,dx = \mathrm{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))}\,dx = \int \frac{1}{x\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$

input
```
Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{-c^2x^2+1}\,(a+b\,\text{arccosh}\,(cx))}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^3 - a*x + (b*c^2*x^3 - b*x)*arccosh(
c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{x\sqrt{-(cx-1)\,(cx+1)}\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,bx+\sqrt{-c^2x^2+1}\,ax}dx$$

input

```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output

```
int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*x + sqrt( - c**2*x**2 + 1)*a*x)
,x)
```

3.262 $\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$

input
```
Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{-c^2x^2+1}\,(a+b\,\text{arccosh}\,(cx))}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^4 - a*x^2 + (b*c^2*x^4 - b*x^2)*arcc
osh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{x^2\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{x^2\,\left(a+b\,\text{acosh}\,(c\,x)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b\,x^2+\sqrt{-c^2x^2+1}\,a\,x^2}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a
*x**2),x)
```

3.263 $\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\,\text{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \text{Int}\left(\frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx$$

input
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}dx$$

input
```
Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\text{arccosh}\,(cx))}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*
b*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 7.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^2}{(-(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^2}{(a+b\,\text{acosh}\,(c\,x))\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 9.89

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \frac{-asin(cx) - \left(\int \frac{acosh(cx)}{\sqrt{-c^2x^2+1}\,acosh(cx)b\,c^2x^2-\sqrt{-c^2x^2+1}\,acosh(cx)b+\sqrt{-c^2x^2+1}}$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
( - asin(c*x) - int(acosh(c*x)/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**2*x
**2 - sqrt( - c**2*x**2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*
x**2 - sqrt( - c**2*x**2 + 1)*a),x)*b*c + int((acosh(c*x)*x**2)/(sqrt( - c
**2*x**2 + 1)*acosh(c*x)*b*c**2*x**2 - sqrt( - c**2*x**2 + 1)*acosh(c*x)*b
 + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a),x)*b*c**
3 - int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**2*x**2 - sqrt( - c**2*x*
*2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x
**2 + 1)*a),x)*a*c)/(a*c**3)
```

3.264 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\,\text{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\,\text{arccosh}(cx))}\,dx = \text{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}(a+b\,\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 6.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\,\text{arccosh}(cx))}\,dx = \int \frac{x}{(1-c^2x^2)^{3/2}(a+b\,\text{arccosh}(cx))}\,dx$$

input
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{x}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))}dx$$

input
```
Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}} (a+b \operatorname{arccosh}(cx))}dx$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*
c^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 6.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{x}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}\,dx = \int \frac{x}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx =$$

$$-\left(\int \frac{x}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b\,c^2x^2-\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a}dx\right)$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
 - int(x/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**2*x**2 - sqrt( - c**2*x**
2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x*
*2 + 1)*a),x)
```

3.265 $\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\,\textbf{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\operatorname{arccosh}(cx))}\,dx=\operatorname{Int}\left(\frac{1}{(1-c^2x^2)^{3/2}(a+b\operatorname{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\operatorname{arccosh}(cx))}\,dx=\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\operatorname{arccosh}(cx))}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx$$
$$\Big\downarrow \text{ 6325}$$
$$\int \frac{1}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))}dx$$

input
```
Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}} (a+b \operatorname{arccosh}(cx))}dx$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^
2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 5.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{acosh}\,(cx)\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{acosh}\,(c\,x))\,(1-c^2\,x^2)^{3/2}}\,dx$$

input
```
int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx =$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b\,c^2x^2-\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a}dx\right)$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
 - int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**2*x**2 - sqrt( - c**2*x**
2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x*
*2 + 1)*a),x)
```

3.266 $\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\mathrm{arccosh}(cx))}\,dx=\mathrm{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\mathrm{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\mathrm{arccosh}(cx))}\,dx=\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\mathrm{arccosh}(cx))}\,dx$$

input
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}dx$$

input
```
Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\operatorname{arccosh}\left(cx\right)\right)}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^5 - 2*a*c^2*x^3 + a*x + (b*c^4*x^5 -
2*b*c^2*x^3 + b*x)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 17.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{acosh}\,(cx)\right)}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}\,dx=\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.96

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\text{arccosh}(cx)\right)}\,dx = -\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\left(cx\right)b\,c^2x^3-\sqrt{-c^2x^2+1}\,acosh\left(cx\right)bx+\sqrt{-c^2x^2+1}\,a\,c^2x^3-\sqrt{-c^2x^2+1}\,ax}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
- int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**2*x**3 - sqrt( - c**2*x**
2 + 1)*acosh(c*x)*b*x + sqrt( - c**2*x**2 + 1)*a*c**2*x**3 - sqrt( - c**2*
x**2 + 1)*a*x),x)
```

3.267 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \text{Int}\left(\frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.58 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx$$

input
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))}dx$$

input
```
Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 \left(-c^2x^2+1\right)^{\frac{3}{2}} \left(a+b \,\text{arccosh}\left(cx\right)\right)}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b \operatorname{arcosh}(cx)+a)x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^6 - 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6
- 2*b*c^2*x^4 + b*x^2)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 38.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx = \int \frac{1}{x^2 (-(cx-1)(cx+1))^{\frac{3}{2}} (a+b \operatorname{acosh}(cx))} \, dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima
")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

input
```
int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))}\,dx =$$

$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\, acosh\,(cx)\, b\, c^2x^4 - \sqrt{-c^2x^2+1}\, acosh\,(cx)\, b\, x^2 + \sqrt{-c^2x^2+1}\, a\, c^2x^4 - \sqrt{-c^2x^2+1}\, a\, x^2}$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
- int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**2*x**4 - sqrt( - c**2*x**
2 + 1)*acosh(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a*c**2*x**4 - sqrt( - c*
*2*x**2 + 1)*a*x**2),x)
```

3.268 $\int \frac{x^m (1-c^2x^2)^{3/2}}{a+b\mathbf{arccosh}(cx)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m (1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)} \, dx = \text{Int}\left(\frac{x^m (1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}, x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m (1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)} \, dx = \int \frac{x^m (1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)} \, dx$$

input
```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}x^m}{a+b\text{arccosh}(cx)}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{\left(1-c^2x^2\right)^{3/2}x^m}{a+b\text{arccosh}(cx)}dx$$

input
```
Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m(-c^2x^2+1)^{\frac{3}{2}}}{a+b\ \text{arccosh}\left(cx\right)}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^m/(b*arccosh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x^m\,{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\text{arccosh}(cx)}\,dx = -\left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^2}{acosh\left(cx\right)b+a}dx\right)c^2+\int \frac{x^m\sqrt{-c^2x^2+1}}{acosh\left(cx\right)b+a}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output

```
 - int((x**m*sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c*x)*b + a),x)*c**2 + int
((x**m*sqrt( - c**2*x**2 + 1))/(acosh(c*x)*b + a),x)
```

3.269 $\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\textbf{arccosh}(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\text{Int}\left(\frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)},x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx$$

input
```
Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}x^m}{a+b\text{arccosh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{\sqrt{1-c^2x^2}x^m}{a+b\text{arccosh}(cx)}dx$$

input
```
Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m\sqrt{-c^2x^2+1}}{a+b\,\text{arccosh}\,(cx)}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^m}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{x^m\sqrt{-(cx-1)(cx+1)}}{a+b\,\text{acosh}\,(cx)}\,dx$$

input
```
integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^m}{b\,\text{arcosh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x^m\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

output
```
int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\text{arccosh}(cx)}\,dx = \int \frac{x^m\sqrt{-c^2x^2+1}}{acosh\,(cx)\,b+a}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int((x**m*sqrt( - c**2*x**2 + 1))/(acosh(c*x)*b + a),x)
```

3.270 $\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\,\mathbf{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx$$

input
```
Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$
$$\downarrow \text{6375}$$
$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}dx$$

input
```
Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}\,(a+b\,\text{arccosh}\,(cx))}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
 - a), x)
```

Sympy [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(a+b\,\text{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x)),x)
```

output
```
int(x**m/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a),
x)
```

3.271 $\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx=\text{Int}\left(\frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx=\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx$$

input
```
Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))} \, dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{x^m}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))}dx$$

input
```
Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}} (a+b \,\text{arccosh}\,(cx))}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^m/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*
b*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 65.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\text{acosh}\,(cx))}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

output
```
Integral(x**m/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(a+b\,\text{acosh}\,(c\,x))\,(1-c^2\,x^2)^{3/2}}\,dx$$

input
```
int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))}\,dx =$$
$$-\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b\,c^2x^2-\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a}dx\right)$$

input
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x)),x)
```

output
```
 - int(x**m/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**2*x**2 - sqrt( - c**2*
x**2 + 1)*acosh(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2
*x**2 + 1)*a),x)
```

3.272 $\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))}\,dx = \operatorname{Int}\left(\frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))}\,dx = \int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))}\,dx$$

input
```
Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))}dx$$

$$\Big\downarrow \text{6375}$$

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))}dx$$

input
```
Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(a+b\,\text{arccosh}\,(cx))}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

output
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 +
(b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccosh(c*x) - a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{x^m}{(a+b\,\text{acosh}\,(c\,x))\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

input
```
int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.46

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b\,c^4x^4-2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,b\,c^2x^2}$$

input
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x)),x)
```

output
```
int(x**m/(sqrt( - c**2*x**2 + 1)*acosh(c*x)*b*c**4*x**4 - 2*sqrt( - c**2*x
**2 + 1)*acosh(c*x)*b*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acosh(c*x)*b + sq
rt( - c**2*x**2 + 1)*a*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 +
sqrt( - c**2*x**2 + 1)*a),x)
```

3.273 $\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 350

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\operatorname{arccosh}(cx))^2}\,dx = -\frac{x^3\sqrt{-1+cx}\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc(a+b\operatorname{arccosh}(cx))} + \frac{\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^4\sqrt{-1+cx}} - \frac{3\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{3(a+b\operatorname{arccosh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c^4\sqrt{-1+cx}} - \frac{5\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{5(a+b\operatorname{arccosh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c^4\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)}{8b^2c^4\sqrt{-1+cx}} + \frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\operatorname{arccosh}(cx))}{b}\right)}{16b^2c^4\sqrt{-1+cx}} + \frac{5\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\operatorname{arccosh}(cx))}{b}\right)}{16b^2c^4\sqrt{-1+cx}}$$

output
```
-x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccosh(c*x))
+1/8*(-c*x+1)^(1/2)*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^4/(c*x-1)^(1
/2)-3/16*(-c*x+1)^(1/2)*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^4/(c
*x-1)^(1/2)-5/16*(-c*x+1)^(1/2)*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)/b^
2/c^4/(c*x-1)^(1/2)-1/8*(-c*x+1)^(1/2)*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)
/b^2/c^4/(c*x-1)^(1/2)+3/16*(-c*x+1)^(1/2)*cosh(3*a/b)*Shi(3*(a+b*arccosh(
c*x))/b)/b^2/c^4/(c*x-1)^(1/2)+5/16*(-c*x+1)^(1/2)*cosh(5*a/b)*Shi(5*(a+b*
arccosh(c*x))/b)/b^2/c^4/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(16bc^3x^3-16bc^5x^5+2(a+b\text{arccosh}(cx))\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)\sinh\left(\frac{a}{b}\right)-3(a+b\text{arccosh}(cx\right.}{}$$

input
```
Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[1 - c^2*x^2]*(16*b*c^3*x^3 - 16*b*c^5*x^5 + 2*(a + b*ArcCosh[c*x])*C
oshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshInt
egral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*CoshIntegral[5*(a/b + Ar
cCosh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshIntegral[5*(a/b + ArcCos
h[c*x])]*Sinh[(5*a)/b] - 2*a*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] -
2*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*Cosh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(5*a)/b]*SinhIntegral[5*
(a/b + ArcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)/b]*SinhIntegral[5*(a/b
 + ArcCosh[c*x])]))/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCos
h[c*x]))
```

Rubi [A] (verified)

Time = 1.04 (sec), antiderivative size = 352, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {6357, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{5c\sqrt{1-cx}\int \frac{x^4}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6302}$$

$$\frac{5\sqrt{1-cx}\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{cx-1}}-\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{5\sqrt{1-cx}\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{cx-1}}-\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{5\sqrt{1-cx}\int\left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8(a+b\text{arccosh}(cx))}\right)d(a+b\text{arcco}}{b^2c^4\sqrt{cx-1}}$$

$$\frac{3\sqrt{1-cx}\int\left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{cx-1}}-$$

$$\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \; 2009$$

$$-\frac{3\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccos}}{b}\right.\right.}{b^2c^4\sqrt{cx-1}}$$

$$\frac{5\sqrt{1-cx}\left(-\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{3}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{b^2}$$

$$\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh
[c*x]))) - (3*Sqrt[1 - c*x]*(-1/4*(CoshIntegral[(a + b*ArcCosh[c*x])/b]*Si
nh[a/b]) - (CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/4 + (C
osh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/4 + (Cosh[(3*a)/b]*SinhInte
gral[(3*(a + b*ArcCosh[c*x]))/b])/4))/(b^2*c^4*Sqrt[-1 + c*x]) + (5*Sqrt[1
 - c*x]*(-1/8*(CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b]) - (3*CoshIn
tegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/16 - (CoshIntegral[(5*(a
 + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*SinhIntegral[(a + b*
ArcCosh[c*x])/b])/8 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]
))/b])/16 + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/16))/
(b^2*c^4*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.59

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(32\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^5x^5+32b\,c^6x^6-32\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^3x^3-32b\,c^4x^4+5\ \mathrm{arccosh}(cx)\right.}{}$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(32*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^5*x^5+32*b*c^6*x^6-32*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*b*c^3*x^3-32*b*c^4*x^4+5*arccosh(c*x)*b*Ei(1,-5*arccosh(c*x)-5*a/b)
*exp(-(-b*arccosh(c*x)+5*a)/b)+3*arccosh(c*x)*b*Ei(1,-3*arccosh(c*x)-3*a/b
)*exp(-(-b*arccosh(c*x)+3*a)/b)-2*arccosh(c*x)*b*Ei(1,-arccosh(c*x)-a/b)*e
xp(-(-b*arccosh(c*x)+a)/b)-5*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x
)+5*a)/b)*b*arccosh(c*x)-3*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+
3*a)/b)*b*arccosh(c*x)+2*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)*
b*arccosh(c*x)+5*a*Ei(1,-5*arccosh(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+5*a)/
b)+3*a*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(-b*arccosh(c*x)+3*a)/b)-2*a*Ei(1,
-arccosh(c*x)-a/b)*exp(-(-b*arccosh(c*x)+a)/b)-5*Ei(1,5*arccosh(c*x)+5*a/b
)*exp((b*arccosh(c*x)+5*a)/b)*a-3*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccos
h(c*x)+3*a)/b)*a+2*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)*a)/(c*
x+1)/c^4/(c*x-1)/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^3}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^3/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
 a^2), x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^3\sqrt{-(cx-1)(cx+1)}}{(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^3}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-((c^2*x^5 - x^3)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^6 - c*x^4)*sqrt(c*x + 1
))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a
*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((5*c^3*x^5 - 2*c*x^3)*(c*x
 + 1)^(3/2)*(c*x - 1) + (10*c^4*x^6 - 11*c^2*x^4 + 3*x^2)*(c*x + 1)*sqrt(c
*x - 1) + (5*c^5*x^7 - 9*c^3*x^5 + 4*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/
(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2
*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c
*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b
^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^3\,\sqrt{1-c^2\,x^2}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\left(cx\right)^2b^2+2acosh\left(cx\right)ab+a^2}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**3)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b +
 a**2),x)
```

3.274 $\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))} - \frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{2b^2c^3\sqrt{-1+cx}}$$

output

```
-x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccosh(c*x))
-1/2*(-c*x+1)^(1/2)*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)/b^2/c^3/(c*x-1
)^(1/2)+1/2*(-c*x+1)^(1/2)*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b^2/c^3
/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\operatorname{arccosh}(cx))^2}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(-2bc^2x^2(-1+c^2x^2)-(a+b\operatorname{arccosh}(cx))\operatorname{Chi}\left(4\left(\frac{a}{b}+\operatorname{arccosh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)+(a+b\operatorname{arccos}\right.}{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{arccosh}(cx))}$$

input
```
Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-2*b*c^2*x^2*(-1 + c^2*x^2) - (a + b*ArcCosh[c*x])*Cos
hIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] + (a + b*ArcCosh[c*x])*Cos
h[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^3*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.70, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6357, 6302, 25, 5971, 27, 2009, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\operatorname{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{4c\sqrt{1-cx}\int \frac{x^3}{a+b\operatorname{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{2\sqrt{1-cx}\int \frac{x}{a+b\operatorname{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\operatorname{arccosh}(cx))}$$

$$\Big\downarrow \text{6302}$$

$$\frac{4\sqrt{1-cx}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{2\sqrt{1-cx}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 25$$

$$-\frac{4\sqrt{1-cx}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 5971$$

$$\frac{2\sqrt{1-cx}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2(a+b\text{arccosh}(cx))}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{4\sqrt{1-cx}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{1-cx}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{4\sqrt{1-cx}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{1-cx}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}+\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}}{}\right.\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3042

$$\frac{\sqrt{1-cx}\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}+\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}}{}\right.\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 26

$$-\frac{i\sqrt{1-cx}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}+\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}}{}\right.\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3784

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}}{}\right.\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 26

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\right.}{b^2c^3\sqrt{cx-1}}$$
$$\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}\right.\right.}{b^2c^3\sqrt{cx-1}}$$
$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3042

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d\right.}{b^2c^3\sqrt{cx-1}}$$
$$\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}\right.\right.}{b^2c^3\sqrt{cx-1}}$$
$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 26

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\right.}{b^2c^3\sqrt{cx-1}}$$
$$\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}\right.\right.}{b^2c^3\sqrt{cx-1}}$$
$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3779

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\right)}{b^2c^3\sqrt{cx-1}}+$$
$$\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}\right.\right.}{b^2c^3\sqrt{cx-1}}$$
$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 3782

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\right)}{b^2c^3\sqrt{cx-1}}+$$

$$\frac{4\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}}{}\right.\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh
[c*x]))) - (I*Sqrt[1 - c*x]*(I*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Si
nh[(2*a)/b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b]))/(
b^2*c^3*Sqrt[-1 + c*x]) + (4*Sqrt[1 - c*x]*(-1/4*(CoshIntegral[(2*(a + b*A
rcCosh[c*x]))/b]*Sinh[(2*a)/b]) - (CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b
]*Sinh[(4*a)/b])/8 + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/
b])/4 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8))/(b^2*
c^3*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(136) = 272$.

Time = 0.23 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(4\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^4x^4+4b\,c^5x^5-4\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^2x^2-4b\,c^3x^3+\operatorname{arccosh}(cx)b\ \exp\mathrm{I}\right.}{}$

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(4*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*b*c^4*x^4+4*b*c^5*x^5-4*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*b*c^2*x^2-4*b*c^3*x^3+arccosh(c*x)*b*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-(
-b*arccosh(c*x)+4*a)/b)-Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a
)/b)*b*arccosh(c*x)+a*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*
a)/b)-Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)*a)/(c*x+1)/c^
3/(c*x-1)/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
 a^2), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2\sqrt{-(cx-1)(cx+1)}}{(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-((c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^5 - c*x^3)*sqrt(c*x + 1
))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a
*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((4*c^3*x^4 - c*x^2)*(c*x +
 1)^(3/2)*(c*x - 1) + 2*(4*c^4*x^5 - 4*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1
) + (4*c^5*x^6 - 7*c^3*x^4 + 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c
^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*
c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1
)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2
*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))),
x)
```

Giac [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2\,\sqrt{1-c^2\,x^2}}{(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^2}{a cosh\left(cx\right)^2 b^2+2a cosh\left(cx\right)ab+a^2}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b +
 a**2),x)
```

3.275 $\int \frac{x\sqrt{1-c^2x^2}}{(a+b\,\mathrm{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\,\mathrm{arccosh}(cx))^2}\,dx = -\frac{x\sqrt{-1+cx}\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc(a+b\,\mathrm{arccosh}(cx))} + \frac{\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^2\sqrt{-1+cx}} - \frac{3\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{3(a+b\,\mathrm{arccosh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^2\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\,\mathrm{arccosh}(cx)}{b}\right)}{4b^2c^2\sqrt{-1+cx}} + \frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{4b^2c^2\sqrt{-1+cx}}$$

output
```
-x*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccosh(c*x))+1
/4*(-c*x+1)^(1/2)*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^2/(c*x-1)^(1/2
)-3/4*(-c*x+1)^(1/2)*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^2/(c*x-
1)^(1/2)-1/4*(-c*x+1)^(1/2)*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b^2/c^2/(c
*x-1)^(1/2)+3/4*(-c*x+1)^(1/2)*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)/b^2
/c^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.88

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(4bcx-4bc^3x^3+(a+b\text{arccosh}(cx))\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)\sinh\left(\frac{a}{b}\right)-3(a+b\text{arccosh}(cx))\text{Ch}\right.}{}$$

input
```
Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[1 - c^2*x^2]*(4*b*c*x - 4*b*c^3*x^3 + (a + b*ArcCosh[c*x])*CoshInteg
ral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshIntegral[3*
(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - a*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]] - b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a
*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {6357, 6296, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{3c\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{1-cx}\int \frac{1}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6296}$$

$$-\frac{\sqrt{1-cx}\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{3c\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{1-cx}\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{3c\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-cx}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{3c\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{3c\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arcc}\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{3c\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arcc}\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{3c\sqrt{1-cx}\int\frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 3042

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{3c\sqrt{1-cx}\int\frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 26

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+ba\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{3c\sqrt{1-cx}\int\frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 3779

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{cx-1}}+$$

$$\frac{3c\sqrt{1-cx}\int\frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 3782

$$\frac{3c\sqrt{1-cx}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 6302$$

$$\frac{3\sqrt{1-cx}\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 25$$

$$-\frac{3\sqrt{1-cx}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 5971$$

$$-\frac{3\sqrt{1-cx}\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 2009$$

$$-\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{cx-1}}+$$
$$\frac{3\sqrt{1-cx}\left(-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arccosh}(\right.\right.}{b^2c^2\sqrt{cx-1}}$$
$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\operatorname{arccosh}(cx))}$$

input
```
Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[c
*x]))) - (I*Sqrt[1 - c*x]*(I*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b
] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b]))/(b^2*c^2*Sqrt[-1 +
c*x]) + (3*Sqrt[1 - c*x]*(-1/4*(CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[
a/b]) - (CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/4 + (Cosh
[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/4 + (Cosh[(3*a)/b]*SinhIntegra
l[(3*(a + b*ArcCosh[c*x]))/b])/4))/(b^2*c^2*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

Maple [F]

$$\int \frac{x\sqrt{-c^2x^2+1}}{(a+b\,\mathrm{arccosh}\,(cx))^2}dx$$

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\mathrm{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\mathrm{arccosh}(cx))^2}\,dx = \int \frac{x\sqrt{-(cx-1)(cx+1)}}{(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^4 - c*x^2)*sqrt(c*x + 1))
*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b
*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*(c*x - 1)*
c^3*x^3 + (6*c^4*x^4 - 5*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (3*c^5*x^5
 - 5*c^3*x^3 + 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x +
1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^
2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*
c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1
)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{x\,\sqrt{1-c^2\,x^2}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\left(cx\right)^2b^2+2acosh\left(cx\right)ab+a^2}dx$$

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a*
*2),x)
```

3.276 $\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\frac{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))} - \frac{\sqrt{1-cx}\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{b^2c\sqrt{-1+cx}}$$

output
```
-(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccosh(c*x))-(-c
*x+1)^(1/2)*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c/(c*x-1)^(1/2)+(-
c*x+1)^(1/2)*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b^2/c/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\frac{\sqrt{1-c^2x^2}\left(b(-1+c^2x^2)+(a+b\text{arccosh}(cx))\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-(a+b\text{arccosh}(cx)}{b^2c\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))}$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((Sqrt[1 - c^2*x^2]*(b*(-1 + c^2*x^2) + (a + b*ArcCosh[c*x])*CoshIntegral
[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b
]*SinhIntegral[2*(a/b + ArcCosh[c*x])]))/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x])))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {6319, 6302, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6319}$$

$$\frac{2c\sqrt{1-cx}\int \frac{x}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6302}$$

$$\frac{2\sqrt{1-cx}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{2\sqrt{1-cx}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2(a+b\text{arccosh}(cx))}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{1-cx}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}-\frac{\sqrt{1-cx}\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}+\frac{i\sqrt{1-cx}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}+$$

$$\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\right.}{b^2c\sqrt{cx-1}}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}+\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b}{b^2c\sqrt{cx-1}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}+\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(}{b^2c\sqrt{cx-1}}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}+\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+}{b^2c\sqrt{cx-1}}$$

$$\Big\downarrow \; 3779$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}+\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\right)}{b^2c\sqrt{cx-1}}$$

$$\Big\downarrow \; 3782$$

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc(a+b\text{arccosh}(cx))}+\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\right)}{b^2c\sqrt{cx-1}}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[c*x
]))) + (I*Sqrt[1 - c*x]*(I*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(
2*a)/b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b]))/(b^2*
c*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6319
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p - 1/2)*(-
1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(132) = 264$.

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^2x^2+2b\,c^3x^3+\operatorname{arccosh}(cx)b\ \operatorname{expIntegral}_1\left(-2\ \operatorname{arccosh}(cx)-\frac{2a}{b}\right)\mathrm{e}^{-}\right.}{}$

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(2*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*b*c^2*x^2+2*b*c^3*x^3+arccosh(c*x)*b*Ei(1,-2*arcc
osh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-Ei(1,2*arccosh(c*x)+2*a/b)*e
xp((b*arccosh(c*x)+2*a)/b)*b*arccosh(c*x)-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b+
a*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-Ei(1,2*arccosh
(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)*a-2*b*c*x)/(c*x-1)/(c*x+1)/c/b^2/
(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2
), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*s
qrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c
 + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((2*c^2*x^2 + 1)*(c*x + 1)^(3/2
)*(c*x - 1) + 2*(2*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*x - 1) + (2*c^4*x^4 - 3
*c^2*x^2 + 1)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x
- 1)*a*b*c^2*x^2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)
*sqrt(c*x - 1) + a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*
b^2*c^2*x^2 + 2*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x))^2,x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),
x)
```

3.277 $\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx=\text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 9.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{c\sqrt{1-cx}\int \frac{1}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6296}$$

$$\frac{\sqrt{1-cx}\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{1-cx}\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{1-cx}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3784

$$\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arcc}\right.}{b^2\sqrt{cx-1}}$$
$$\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$\downarrow$ 26

$$\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccos}\right.}{b^2\sqrt{cx-1}}$$
$$\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3042

$$\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\right.}{b^2\sqrt{cx-1}}$$
$$\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$\downarrow$ 26

$$\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arc}\right.}{b^2\sqrt{cx-1}}$$
$$\frac{\sqrt{1-cx}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3779

$$\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int\frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \; 3782$$

$$\frac{\sqrt{1-cx}\int\frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \; 6303$$

$$\frac{\sqrt{1-cx}\int\frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{i\sqrt{1-cx}\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx(a+b\text{arccosh}(cx))}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x\left(a+b\,\text{arccosh}\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\left(b\,\text{arcosh}\,(cx)+a\right)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x) +
 a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x\left(a+b\,\text{acosh}\,(cx)\right)^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 442, normalized size of antiderivative = 15.79

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*s
qrt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b
*c*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*x^3 + 2*c*x)*(c*x
 + 1)^(3/2)*(c*x - 1) + (2*c^4*x^4 + c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1)
+ (c^5*x^5 - c^3*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x +
1)*(c*x - 1)*a*b*c^3*x^4 - 2*a*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*
b*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1
)*b^2*c^3*x^4 - 2*b^2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*
sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{1-c^2\,x^2}}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acosh\left(cx\right)^2b^2x+2acosh\left(cx\right)abx+a^2x}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2*x + 2*acosh(c*x)*a*b*x + a*
*2*x),x)
```

3.278 $\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\,\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6355}$$

$$\frac{2\sqrt{1-cx}\int \frac{1}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx^2(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6303}$$

$$\frac{2\sqrt{1-cx}\int \frac{1}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx^2(a+b\text{arccosh}(cx))}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*
x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^2\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 427, normalized size of antiderivative = 15.25

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*s
qrt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b
*c*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^
2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*
(c*x - 1)*c*x + 2*(2*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x
)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*c^3
*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*x
+ 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b^
2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

input

```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))^2),x)
```

output

```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2x^2+2acosh\,(cx)\,ab\,x^2+a^2x^2}dx$$

input

```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*acosh(c*x))^2,x)
```

output

```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2*x**2 + 2*acosh(c*x)*a*b*x**
2 + a**2*x**2),x)
```

3.279 $\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 354

$$\begin{aligned}\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\operatorname{arccosh}(cx))^2}\,dx = &-\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{bc(a+b\operatorname{arccosh}(cx))} \\ &-\frac{\sqrt{1-cx}\operatorname{Chi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}} \\ &-\frac{\sqrt{1-cx}\operatorname{Chi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c^3\sqrt{-1+cx}} \\ &+\frac{3\sqrt{1-cx}\operatorname{Chi}\left(\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3\sqrt{-1+cx}} \\ &+\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)}{16b^2c^3\sqrt{-1+cx}} \\ &+\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{4b^2c^3\sqrt{-1+cx}} \\ &-\frac{3\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)}{16b^2c^3\sqrt{-1+cx}}\end{aligned}$$

output
```
-x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccosh(c*x))
-1/16*(-c*x+1)^(1/2)*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^3/(c*x-
1)^(1/2)-1/4*(-c*x+1)^(1/2)*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)/b^2/c^
3/(c*x-1)^(1/2)+3/16*(-c*x+1)^(1/2)*Chi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b
)/b^2/c^3/(c*x-1)^(1/2)+1/16*(-c*x+1)^(1/2)*cosh(2*a/b)*Shi(2*(a+b*arccosh
(c*x))/b)/b^2/c^3/(c*x-1)^(1/2)+1/4*(-c*x+1)^(1/2)*cosh(4*a/b)*Shi(4*(a+b*
arccosh(c*x))/b)/b^2/c^3/(c*x-1)^(1/2)-3/16*(-c*x+1)^(1/2)*cosh(6*a/b)*Shi
(6*(a+b*arccosh(c*x))/b)/b^2/c^3/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\frac{\sqrt{-1+cx}\sqrt{1+cx}\left(16bc^2x^2-32bc^4x^4+16bc^6x^6-(a+b\text{arccosh}(cx))\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right.\right.}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-1/16*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(16*b*c^2*x^2 - 32*b*c^4*x^4 + 16*b*c^
6*x^6 - (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*
a)/b] - 4*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(
4*a)/b] + 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3*b*Arc
Cosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(2*a)/b]*S
inhIntegral[2*(a/b + ArcCosh[c*x])] + 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/
b + ArcCosh[c*x])] + 4*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b +
ArcCosh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] -
3*b*ArcCosh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(b^2
*c^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6357, 25, 6327, 6367, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6357}$$

$$-\frac{6c\sqrt{1-cx}\int -\frac{x^3(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int -\frac{x(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{6c\sqrt{1-cx}\int \frac{x^3(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{2\sqrt{1-cx}\int \frac{x(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6327}$$

$$-\frac{2\sqrt{1-cx}\int \frac{x(1-c^2x^2)}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}+\frac{6c\sqrt{1-cx}\int \frac{x^3(1-c^2x^2)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6367}$$

$$-\frac{6\sqrt{1-cx}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 25

$$\frac{6\sqrt{1-cx}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{2\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 5971

$$-\frac{2\sqrt{1-cx}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}+\frac{6\sqrt{1-cx}\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}-\frac{3\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 2009

$$\frac{2\sqrt{1-cx}\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcc}}{b}\right.\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{6\sqrt{1-cx}\left(\frac{3}{32}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{32}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{32}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{ar}}{}\right.\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCo
sh[c*x]))) + (2*Sqrt[1 - c*x]*((CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*S
inh[(2*a)/b])/4 - (CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])
/8 - (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/4 + (Cosh[(4
*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8))/(b^2*c^3*Sqrt[-1 + c*
x]) - (6*Sqrt[1 - c*x]*((3*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(
2*a)/b])/32 - (CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/32
- (3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/32 + (Cosh[(6
*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/32))/(b^2*c^3*Sqrt[-1 + c
*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(-32\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^6x^6-32b\,c^7x^7+64\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^4x^4+64b\,c^5x^5-32\sqrt{cx-1}\,\sqrt{\right.}{}$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-32*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^6*x^6-32*b*c^7*x^7+64*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*b*c^4*x^4+64*b*c^5*x^5-32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^2*x^2-32
*b*c^3*x^3+4*arccosh(c*x)*b*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c
*x)+4*a)/b)+arccosh(c*x)*b*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*
x)+2*a)/b)-3*arccosh(c*x)*b*Ei(1,-6*arccosh(c*x)-6*a/b)*exp(-(-b*arccosh(c
*x)+6*a)/b)+3*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)*b*arc
cosh(c*x)-4*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)*b*arcco
sh(c*x)-Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)*b*arccosh(c
*x)+4*a*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)+a*Ei(1,-
2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-3*a*Ei(1,-6*arccosh(c*
x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)+3*Ei(1,6*arccosh(c*x)+6*a/b)*exp((
b*arccosh(c*x)+6*a)/b)*a-4*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+
4*a)/b)*a-Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)*a)/(c*x+1
)/c^3/(c*x-1)/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*a
rccosh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
((c^4*x^6 - 2*c^2*x^4 + x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^7 - 2*c^3*x^
5 + c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt
(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b
^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((6*
c^5*x^6 - 7*c^3*x^4 + c*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(6*c^6*x^7 - 11
*c^4*x^5 + 6*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^7*x^8 - 5*c^5*x
^6 + 4*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x
+ 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*
c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^
2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x +
 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx=-\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acosh\left(cx\right)^2b^2+2acosh\left(cx\right)ab+a^2}dx\right)c^2+\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\left(cx\right)^2b^2+2acosh\left(cx\right)ab+a^2}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
 - int((sqrt( - c**2*x**2 + 1)*x**4)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*
b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c*x)**2*b**2
+ 2*acosh(c*x)*a*b + a**2),x)
```

3.280 $\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\textbf{arccosh}(cx))^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 348

$$\begin{aligned}\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2} \, dx &= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))} \\ &+ \frac{\sqrt{1-cx}\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^2\sqrt{-1+cx}} \\ &- \frac{9\sqrt{1-cx}\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c^2\sqrt{-1+cx}} \\ &+ \frac{5\sqrt{1-cx}\text{Chi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c^2\sqrt{-1+cx}} \\ &- \frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{8b^2c^2\sqrt{-1+cx}} \\ &+ \frac{9\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c^2\sqrt{-1+cx}} \\ &- \frac{5\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c^2\sqrt{-1+cx}}\end{aligned}$$

output
```
-x*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccosh(c*x))+1
/8*(-c*x+1)^(1/2)*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^2/(c*x-1)^(1/2
)-9/16*(-c*x+1)^(1/2)*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^2/(c*x
-1)^(1/2)+5/16*(-c*x+1)^(1/2)*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)/b^2/
c^2/(c*x-1)^(1/2)-1/8*(-c*x+1)^(1/2)*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b
^2/c^2/(c*x-1)^(1/2)+9/16*(-c*x+1)^(1/2)*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*
x))/b)/b^2/c^2/(c*x-1)^(1/2)-5/16*(-c*x+1)^(1/2)*cosh(5*a/b)*Shi(5*(a+b*ar
ccosh(c*x))/b)/b^2/c^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.94

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\operatorname{arccosh}(cx))^2}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(-16bcx+32bc^3x^3-16bc^5x^5-2(a+b\operatorname{arccosh}(cx))\operatorname{Chi}\left(\frac{a}{b}\right.\right.}{}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c*x + 32*b*c^3*x^3 - 16*b*c^5*x^5 - 2
*(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] + 9*(a +
b*ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*C
oshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshI
ntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 2*a*Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c*x]] + 2*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]] - 9*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 9*b*Ar
cCosh[c*x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(
5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)
/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])]))/(16*b^2*c^2*Sqrt[1 - c^2*x^2]*(
a + b*ArcCosh[c*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {6357, 25, 6304, 6321, 25, 3042, 26, 3793, 2009, 6327, 6367, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(1-c^2x^2\right)^{3/2}}{(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6357}$$

$$-\frac{5c\sqrt{1-cx}\int -\frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int -\frac{(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6304}$$

$$-\frac{\sqrt{1-cx}\int \frac{1-c^2x^2}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}+\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6321}$$

$$\frac{\sqrt{1-cx}\int -\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{1-cx}\int \frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sqrt{1-cx}\int \frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^3}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^3}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{i\sqrt{1-cx}\int \left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4(a+b\text{arccosh}(cx))}-\frac{i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5c\sqrt{1-cx}\int \frac{x^2(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(\right.\right.}{b}}{b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6327}$$

$$\frac{5c\sqrt{1-cx}\int\frac{x^2(1-c^2x^2)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(}{b}\right.\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 6367

$$-\frac{5\sqrt{1-cx}\int-\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(}{b}\right.\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 25

$$\frac{5\sqrt{1-cx}\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}-\frac{i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(}{b}\right.\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 5971

$$\frac{5\sqrt{1-cx}\int\left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}}{b^2c^2\sqrt{cx-1}}$$

$$\frac{i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(}{b}\right.\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 2009

$$-\frac{i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccos}}{b}\right.\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{5\sqrt{1-cx}\left(\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)+\frac{1}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arccos}}{b}\right.\right.}{b^2}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCosh
[c*x]))) - (I*Sqrt[1 - c*x]*(((3*I)/4)*CoshIntegral[(a + b*ArcCosh[c*x])/b
]*Sinh[a/b] - (I/4)*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b]
 - ((3*I)/4)*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b] + (I/4)*Cosh[(
3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b]))/(b^2*c^2*Sqrt[-1 + c*x]
) - (5*Sqrt[1 - c*x]*((CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/8 +
 (CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/16 - (CoshIntegr
al[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/16 - (Cosh[a/b]*SinhIntegral
[(a + b*ArcCosh[c*x])/b])/8 - (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCos
h[c*x]))/b])/16 + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])
/16))/(b^2*c^2*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.68

method	result
default	$-\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^{2}x^{2}-1\right)\left(32\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^{5}x^{5}+32b\,c^{6}x^{6}-64\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^{3}x^{3}-64b\,c^{4}x^{4}+32\sqrt{cx-1}\right.}{}$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(32*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^5*x^5+32*b*c^6*x^6-64*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*b*c^3*x^3-64*b*c^4*x^4+32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c*x+32*b*c
^2*x^2+5*arccosh(c*x)*b*Ei(1,-5*arccosh(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+
5*a)/b)+2*arccosh(c*x)*b*Ei(1,-arccosh(c*x)-a/b)*exp(-(-b*arccosh(c*x)+a)/
b)-9*arccosh(c*x)*b*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(-b*arccosh(c*x)+3*a)
/b)-5*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)*b*arccosh(c*x
)+9*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)*b*arccosh(c*x)-
2*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)*b*arccosh(c*x)+5*a*Ei(1
,-5*arccosh(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+5*a)/b)+2*a*Ei(1,-arccosh(c*
x)-a/b)*exp(-(-b*arccosh(c*x)+a)/b)-9*a*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(
-b*arccosh(c*x)+3*a)/b)-5*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5
*a)/b)*a+9*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)*a-2*Ei(1
,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)*a)/(c*x+1)/c^2/(c*x-1)/b^2/(a
+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arc
cosh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
((c^4*x^5 - 2*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^6 - 2*c^3*x^4
+ c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c
*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2
*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate((5*(c^
5*x^5 - c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + (10*c^6*x^6 - 17*c^4*x^4 + 8*
c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (5*c^7*x^7 - 12*c^5*x^5 + 9*c^3*x^3
 - 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)
*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*
x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*
b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x -
 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x\,(1-c^2\,x^2)^{3/2}}{(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx$$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
 - int((sqrt( - c**2*x**2 + 1)*x**3)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*
b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x)/(acosh(c*x)**2*b**2 + 2
*acosh(c*x)*a*b + a**2),x)
```

3.281 $\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\frac{\sqrt{-1+cx}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))} - \frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{b^2c\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{2b^2c\sqrt{-1+cx}}$$

output
```
-(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccosh(c*x))-(-c
*x+1)^(1/2)*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c/(c*x-1)^(1/2)+1/
2*(-c*x+1)^(1/2)*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)/b^2/c/(c*x-1)^(1/
2)+(-c*x+1)^(1/2)*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b^2/c/(c*x-1)^(1
/2)-1/2*(-c*x+1)^(1/2)*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b^2/c/(c*x-
1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.94

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(-2b+4bc^2x^2-2bc^4x^4+2(a+b\text{arccosh}(cx))\text{Chi}\left(2\left(\frac{a}{b}+\right.\right.\right.}{}$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-2*b + 4*b*c^2*x^2 - 2*b*c^4*x^4 + 2*(a + b
*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b
*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] - 2*a*Co
sh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Cosh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + a*Cosh[(4*a)/b]*SinhIntegra
l[4*(a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a
/b + ArcCosh[c*x])]))/(2*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6319, 25, 6327, 6367, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6319}$$

$$-\frac{4c\sqrt{1-cx}\int -\frac{x(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4c\sqrt{1-cx}\int \frac{x(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6327}$$

$$\frac{4c\sqrt{1-cx}\int \frac{x(1-c^2x^2)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6367}$$

$$-\frac{4\sqrt{1-cx}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$\frac{4\sqrt{1-cx}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\sqrt{1-cx}\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}\right.\right.}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCosh[c
*x]))) - (4*Sqrt[1 - c*x]*((CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[
(2*a)/b])/4 - (CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/8 -
 (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/4 + (Cosh[(4*a)/
b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8))/(b^2*c*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6319
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p - 1/2)*(-
1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(220) = 440.

Time = 0.26 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(-4\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^4x^4-4b\,c^5x^5+8\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^2x^2+8b\,c^3x^3+2\,\operatorname{arccosh}(cx)b\,\mathrm{e}\right.}{}$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-4*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^4*x^4-4*b*c^5*x^5+8*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*b*c^2*x^2+8*b*c^3*x^3+2*arccosh(c*x)*b*Ei(1,-2*arccosh(c*x)-2*a/b)*exp
(-(-b*arccosh(c*x)+2*a)/b)-arccosh(c*x)*b*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(
-(-b*arccosh(c*x)+4*a)/b)+Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4
*a)/b)*b*arccosh(c*x)-2*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a
)/b)*b*arccosh(c*x)-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b+2*a*Ei(1,-2*arccosh(c*
x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-a*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(
-(-b*arccosh(c*x)+4*a)/b)+Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4
*a)/b)*a-2*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)*a-4*b*c*
x)/(c*x-1)/(c*x+1)/c/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3
+ c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x
 - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c
^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((4*c^4*
x^4 - 3*c^2*x^2 - 1)*(c*x + 1)^(3/2)*(c*x - 1) + 4*(2*c^5*x^5 - 3*c^3*x^3
+ c*x)*(c*x + 1)*sqrt(c*x - 1) + (4*c^6*x^6 - 9*c^4*x^4 + 6*c^2*x^2 - 1)*s
qrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^
2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
+ a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2
*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(1-c^2\,x^2)^{3/2}}{(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x))^2,x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),
x) - int((sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*
a*b + a**2),x)*c**2
```

3.282 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 18.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6357}$$

$$-\frac{\sqrt{1-cx}\int -\frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{3c\sqrt{1-cx}\int -\frac{(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{3c\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6304}$$

$$\frac{3c\sqrt{1-cx}\int \frac{1-c^2x^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6321}$$

$$-\frac{3\sqrt{1-cx}\int -\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\sqrt{1-cx}\int \frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt{1-cx}\int \frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^3}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{26}$$

$$\frac{3i\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^3}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3i\sqrt{1-cx}\int \left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4(a+b\text{arccosh}(cx))}-\frac{i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{3i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}\ldots}{b}\right.\right.}{b^2\sqrt{cx-1}}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6327}$$

$$\frac{\sqrt{1-cx}\int \frac{1-c^2x^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{3i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right.}{b^2\sqrt{cx-1}}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \; 6375$$

$$\frac{\sqrt{1-cx}\int \frac{1-c^2x^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{3i\sqrt{1-cx}\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right.}{b^2\sqrt{cx-1}}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx(a+b\text{arccosh}(cx))}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x\left(a+b\operatorname{arccosh}(cx)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
 + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 16.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{x\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 476, normalized size of antiderivative = 17.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3
+ c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x
 - 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b
^2*c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
((3*c^5*x^5 - c^3*x^3 - 2*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + (6*c^6*x^6 - 7*
c^4*x^4 + 1)*(c*x + 1)*sqrt(c*x - 1) + 3*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*s
qrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^
4 - 2*a*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c
^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x
- 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2x+2acosh\,(cx)\,abx+a^2x}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2*x + 2*acosh(c*x)*a*b*x + a*
*2*x),x) - int((sqrt( - c**2*x**2 + 1)*x)/(acosh(c*x)**2*b**2 + 2*acosh(c*
x)*a*b + a**2),x)*c**2
```

3.283 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\operatorname{arccosh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^2(a+b\operatorname{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\operatorname{arccosh}(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6357}$$

$$-\frac{2\sqrt{1-cx}\int -\frac{(1-cx)(cx+1)}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{2c\sqrt{1-cx}\int -\frac{(1-cx)(cx+1)}{x(a+b\text{arccosh}(cx))}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx^2(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{2c\sqrt{1-cx}\int \frac{(1-cx)(cx+1)}{x(a+b\text{arccosh}(cx))}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx^2(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6327}$$

$$\frac{2c\sqrt{1-cx}\int \frac{1-c^2x^2}{x(a+b\text{arccosh}(cx))}dx}{b\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int \frac{1-c^2x^2}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx^2(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6375}$$

$$\frac{2c\sqrt{1-cx}\int \frac{1-c^2x^2}{x(a+b\text{arccosh}(cx))}dx}{b\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int \frac{1-c^2x^2}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{bcx^2(a+b\text{arccosh}(cx))}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^2 (a+b \operatorname{arccosh}(cx))^2}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\operatorname{arcosh}(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(
c*x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 56.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x^2\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 484, normalized size of antiderivative = 17.29

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3
+ c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x
 - 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)
*b^2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integr
ate(((2*c^5*x^5 + c^3*x^3 - 3*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(2*c^6*x^
6 - c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (2*c^7*x^7 - 3*c^5*
x^5 + c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1
)*a*b*c^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*
sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x
^5 - 2*b^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x +
1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(1-c^2\,x^2)^{3/2}}{x^2\,(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.29

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2x^2+2acosh\,(cx)\,ab\,x^2+a^2x^2}dx - \left(\int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2*x**2 + 2*acosh(c*x)*a*b*x**
2 + a**2*x**2),x) - int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2 + 2*aco
sh(c*x)*a*b + a**2),x)*c**2
```

3.284 $\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 454

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))} - \frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8b^2c^3\sqrt{-1+cx}} + \frac{3\sqrt{1-cx}\,\text{Chi}\left(\frac{6(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{8(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{8a}{b}\right)}{16b^2c^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8b^2c^3\sqrt{-1+cx}} - \frac{3\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\cosh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c^3\sqrt{-1+cx}}$$

output
```
-x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccosh(c*x))
-1/16*(-c*x+1)^(1/2)*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^3/(c*x-
1)^(1/2)-1/8*(-c*x+1)^(1/2)*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)/b^2/c^
3/(c*x-1)^(1/2)+3/16*(-c*x+1)^(1/2)*Chi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b
)/b^2/c^3/(c*x-1)^(1/2)-1/16*(-c*x+1)^(1/2)*Chi(8*(a+b*arccosh(c*x))/b)*si
nh(8*a/b)/b^2/c^3/(c*x-1)^(1/2)+1/16*(-c*x+1)^(1/2)*cosh(2*a/b)*Shi(2*(a+b
*arccosh(c*x))/b)/b^2/c^3/(c*x-1)^(1/2)+1/8*(-c*x+1)^(1/2)*cosh(4*a/b)*Shi
(4*(a+b*arccosh(c*x))/b)/b^2/c^3/(c*x-1)^(1/2)-3/16*(-c*x+1)^(1/2)*cosh(6*
a/b)*Shi(6*(a+b*arccosh(c*x))/b)/b^2/c^3/(c*x-1)^(1/2)+1/16*(-c*x+1)^(1/2)
*cosh(8*a/b)*Shi(8*(a+b*arccosh(c*x))/b)/b^2/c^3/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.98

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(-16bc^2x^2+48bc^4x^4-48bc^6x^6+16bc^8x^8+(a+b\text{arccosh}\right.}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c^2*x^2 + 48*b*c^4*x^4 - 48*b*c^6*x^6
 + 16*b*c^8*x^8 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])
]*Sinh[(2*a)/b] + 2*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x
])]*Sinh[(4*a)/b] - 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b]
 - 3*b*ArcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a
*CoshIntegral[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] + b*ArcCosh[c*x]*CoshI
ntegral[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] - a*Cosh[(2*a)/b]*SinhIntegr
al[2*(a/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(
a/b + ArcCosh[c*x])] - 2*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x
])] - 2*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]
+ 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x
]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - a*Cosh[(8*a)/b]*Sin
hIntegral[8*(a/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(8*a)/b]*SinhInteg
ral[8*(a/b + ArcCosh[c*x])]))/(16*b^2*c^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh
[c*x]))
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6357, 6327, 6367, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6357}$$

$$\frac{8c\sqrt{1-cx}\int \frac{x^3(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{2\sqrt{1-cx}\int \frac{x(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 6327$$

$$-\frac{2\sqrt{1-cx}\int \frac{x\left(1-c^2x^2\right)^2}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}+\frac{8c\sqrt{1-cx}\int \frac{x^3\left(1-c^2x^2\right)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 6367$$

$$\frac{8\sqrt{1-cx}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{2\sqrt{1-cx}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 25$$

$$-\frac{8\sqrt{1-cx}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 5971$$

$$-\frac{8\sqrt{1-cx}\int\left(\frac{\sinh\left(\frac{8a}{b}-\frac{8(a+b\text{arccosh}(cx))}{b}\right)}{128(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}+\frac{3\sinh\left(\frac{2a}{b}-\right.}{64(a+}\right.}{b^2c^3\sqrt{cx-1}}$$

$$\frac{2\sqrt{1-cx}\int\left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}\right)d(a+b\text{arcc}}{b^2c^3\sqrt{cx-1}}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{1-cx}\left(-\frac{5}{32}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{32}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+\right.\right.}{}$$

$$\frac{8\sqrt{1-cx}\left(-\frac{3}{64}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{64}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{64}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\right.\right.}{}$$

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCo
sh[c*x]))) - (2*Sqrt[1 - c*x]*((-5*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b
]*Sinh[(2*a)/b])/32 + (CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a)
/b])/8 - (CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/32 + (5*
Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/32 - (Cosh[(4*a)/b
]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8 + (Cosh[(6*a)/b]*SinhIntegra
l[(6*(a + b*ArcCosh[c*x]))/b])/32))/(b^2*c^3*Sqrt[-1 + c*x]) + (8*Sqrt[1 -
 c*x]*((-3*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b])/64 + (C
oshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/64 + (CoshIntegral[
(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/64 - (CoshIntegral[(8*(a + b*Ar
cCosh[c*x]))/b]*Sinh[(8*a)/b])/128 + (3*Cosh[(2*a)/b]*SinhIntegral[(2*(a +
 b*ArcCosh[c*x]))/b])/64 - (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c
*x]))/b])/64 - (Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/64
 + (Cosh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcCosh[c*x]))/b])/128))/(b^2*c^3
*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.70

method	result	size
default	Expression too large to display	773

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-96*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^6*x^6+96*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^4
*x^4-32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^2*x^2+32*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*b*c^8*x^8-96*b*c^7*x^7+96*b*c^5*x^5-32*b*c^3*x^3+2*arccosh(c*x)*b*Ei(1,
-4*arccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)+arccosh(c*x)*b*Ei(1,-
2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-3*arccosh(c*x)*b*Ei(1,
-6*arccosh(c*x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)+3*Ei(1,6*arccosh(c*x)
+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)*b*arccosh(c*x)-2*Ei(1,4*arccosh(c*x)+4
*a/b)*exp((b*arccosh(c*x)+4*a)/b)*b*arccosh(c*x)-Ei(1,2*arccosh(c*x)+2*a/b
)*exp((b*arccosh(c*x)+2*a)/b)*b*arccosh(c*x)-Ei(1,8*arccosh(c*x)+8*a/b)*ex
p((b*arccosh(c*x)+8*a)/b)*b*arccosh(c*x)+arccosh(c*x)*b*Ei(1,-8*arccosh(c*
x)-8*a/b)*exp(-(-b*arccosh(c*x)+8*a)/b)+32*b*c^9*x^9-Ei(1,8*arccosh(c*x)+8
*a/b)*exp((b*arccosh(c*x)+8*a)/b)*a+a*Ei(1,-8*arccosh(c*x)-8*a/b)*exp(-(-b
*arccosh(c*x)+8*a)/b)-2*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a
)/b)*a-Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)*a+2*a*Ei(1,-
4*arccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)+a*Ei(1,-2*arccosh(c*x)
-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)-3*a*Ei(1,-6*arccosh(c*x)-6*a/b)*exp(
-(-b*arccosh(c*x)+6*a)/b)+3*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)
+6*a)/b)*a)/(c*x+1)/c^3/(c*x-1)/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^
2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-((c^6*x^8 - 3*c^4*x^6 + 3*c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x
^9 - 3*c^5*x^7 + 3*c^3*x^5 - c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3
*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt
(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))) + integrate(((8*c^7*x^8 - 17*c^5*x^6 + 10*c^3*x^4 - c*x^2)*(c*x +
 1)^(3/2)*(c*x - 1) + 2*(8*c^8*x^9 - 22*c^6*x^7 + 21*c^4*x^5 - 8*c^2*x^3 +
 x)*(c*x + 1)*sqrt(c*x - 1) + (8*c^9*x^10 - 27*c^7*x^8 + 33*c^5*x^6 - 17*c
^3*x^4 + 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(
c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*
x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sq
rt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2\,(1-c^2\,x^2)^{5/2}}{(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx &= \left(\int \frac{\sqrt{-c^2x^2+1}\,x^6}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 \\ &\quad -2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 \\ &\quad +\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\end{aligned}$$

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**6)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b +
 a**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**4)/(acosh(c*x)**2*b**2 +
 2*acosh(c*x)*a*b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x**2)/(aco
sh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x)
```

3.285 $\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 448

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = -\frac{x\sqrt{-1+cx}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))} + \frac{5\sqrt{1-cx}\,\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64b^2c^2\sqrt{-1+cx}} - \frac{27\sqrt{1-cx}\,\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64b^2c^2\sqrt{-1+cx}} + \frac{25\sqrt{1-cx}\,\text{Chi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64b^2c^2\sqrt{-1+cx}} - \frac{7\sqrt{1-cx}\,\text{Chi}\left(\frac{7(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64b^2c^2\sqrt{-1+cx}} - \frac{5\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{64b^2c^2\sqrt{-1+cx}} + \frac{27\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64b^2c^2\sqrt{-1+cx}} - \frac{25\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64b^2c^2\sqrt{-1+cx}} + \frac{7\sqrt{1-cx}\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64b^2c^2\sqrt{-1+cx}}$$

output

```
-x*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccosh(c*x))+5
/64*(-c*x+1)^(1/2)*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^2/(c*x-1)^(1/
2)-27/64*(-c*x+1)^(1/2)*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^2/(c
*x-1)^(1/2)+25/64*(-c*x+1)^(1/2)*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)/b
^2/c^2/(c*x-1)^(1/2)-7/64*(-c*x+1)^(1/2)*Chi(7*(a+b*arccosh(c*x))/b)*sinh(
7*a/b)/b^2/c^2/(c*x-1)^(1/2)-5/64*(-c*x+1)^(1/2)*cosh(a/b)*Shi((a+b*arccos
h(c*x))/b)/b^2/c^2/(c*x-1)^(1/2)+27/64*(-c*x+1)^(1/2)*cosh(3*a/b)*Shi(3*(a
+b*arccosh(c*x))/b)/b^2/c^2/(c*x-1)^(1/2)-25/64*(-c*x+1)^(1/2)*cosh(5*a/b)
*Shi(5*(a+b*arccosh(c*x))/b)/b^2/c^2/(c*x-1)^(1/2)+7/64*(-c*x+1)^(1/2)*cos
h(7*a/b)*Shi(7*(a+b*arccosh(c*x))/b)/b^2/c^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.97

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}\big(-64bcx+192bc^3x^3-192bc^5x^5+64bc^7x^7-5(a+b\text{arcco}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-64*b*c*x + 192*b*c^3*x^3 - 192*b*c^5*x^5 +
 64*b*c^7*x^7 - 5*(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Si
nh[a/b] + 27*(a + b*ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sin
h[(3*a)/b] - 25*a*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 25*
b*ArcCosh[c*x]*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 7*a*Co
shIntegral[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 7*b*ArcCosh[c*x]*CoshIn
tegral[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 5*a*Cosh[a/b]*SinhIntegral[
a/b + ArcCosh[c*x]] + 5*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCos
h[c*x]] - 27*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 27*b*A
rcCosh[c*x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 25*a*Cosh
[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])] + 25*b*ArcCosh[c*x]*Cosh[(5
*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])] - 7*a*Cosh[(7*a)/b]*SinhIntegr
al[7*(a/b + ArcCosh[c*x])] - 7*b*ArcCosh[c*x]*Cosh[(7*a)/b]*SinhIntegral[7
*(a/b + ArcCosh[c*x])]))/(64*b^2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]
))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.81 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6357, 6304, 6321, 25, 3042, 26, 3793, 2009, 6327, 6367, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(1-c^2x^2\right)^{5/2}}{(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6304}$$

$$-\frac{\sqrt{1-cx}\int \frac{\left(1-c^2x^2\right)^2}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{cx-1}}+\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6321}$$

$$-\frac{\sqrt{1-cx}\int -\frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{1-cx}\int \frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-cx}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^5}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^5}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}+$$
$$\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 3793

$$-\frac{i\sqrt{1-cx}\int \left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{5i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{5i\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8(a+b\text{arccosh}(cx))}\right)d(a+b\text{ar}}{b^2c^2\sqrt{cx-1}}$$
$$\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 2009

$$\frac{7c\sqrt{1-cx}\int \frac{x^2(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-$$
$$\frac{i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arc}}{b}\right.\right.}{}$$
$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 6327

$$\frac{7c\sqrt{1-cx}\int \frac{x^2\left(1-c^2x^2\right)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-$$
$$\frac{i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arc}}{b}\right.\right.}{}$$
$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 6367

$$\frac{7\sqrt{1-cx}\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}-$$

$$\frac{i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{7\sqrt{1-cx}\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{cx-1}}-$$

$$\frac{i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{7\sqrt{1-cx}\int\left(\frac{\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}-\frac{3\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64(a+b\text{arccosh}(cx))}+\frac{5\sinh\left(\frac{a}{b}-\right.}{64(a+}\right.}{b^2c^2\sqrt{cx-1}}$$

$$\frac{i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{ar}}{}\right.\right.}{}$$

$$\frac{7\sqrt{1-cx}\left(-\frac{5}{64}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{64}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{3}{64}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arc}}{}\right.\right.}{}$$

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCosh
[c*x]))) - (I*Sqrt[1 - c*x]*(((5*I)/8)*CoshIntegral[(a + b*ArcCosh[c*x])/b
]*Sinh[a/b] - ((5*I)/16)*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*
a)/b] + (I/16)*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b] - ((
5*I)/8)*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b] + ((5*I)/16)*Cosh[(
3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b] - (I/16)*Cosh[(5*a)/b]*Si
nhIntegral[(5*(a + b*ArcCosh[c*x]))/b]))/(b^2*c^2*Sqrt[-1 + c*x]) + (7*Sqr
t[1 - c*x]*((-5*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/64 - (Cosh
Integral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/64 + (3*CoshIntegral[(
5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/64 - (CoshIntegral[(7*(a + b*Arc
Cosh[c*x]))/b]*Sinh[(7*a)/b])/64 + (5*Cosh[a/b]*SinhIntegral[(a + b*ArcCos
h[c*x])/b])/64 + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/
64 - (3*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/64 + (Cosh
[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/64))/(b^2*c^2*Sqrt[-1
+ c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6304
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.69

method	result	size
default	Expression too large to display	759

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/128*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-12
8*b*c^2*x^2+128*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^7*x^7-384*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*b*c^5*x^5+384*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^3*x^3-128*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*b*c*x+7*arccosh(c*x)*b*Ei(1,-7*arccosh(c*x)-7*a/b)*e
xp(-(-b*arccosh(c*x)+7*a)/b)+27*arccosh(c*x)*b*Ei(1,-3*arccosh(c*x)-3*a/b)
*exp(-(-b*arccosh(c*x)+3*a)/b)-7*Ei(1,7*arccosh(c*x)+7*a/b)*exp((b*arccosh
(c*x)+7*a)/b)*b*arccosh(c*x)-25*arccosh(c*x)*b*Ei(1,-5*arccosh(c*x)-5*a/b)
*exp(-(-b*arccosh(c*x)+5*a)/b)-5*arccosh(c*x)*b*Ei(1,-arccosh(c*x)-a/b)*ex
p(-(-b*arccosh(c*x)+a)/b)+25*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x
)+5*a)/b)*b*arccosh(c*x)-27*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)
+3*a)/b)*b*arccosh(c*x)+5*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)
*b*arccosh(c*x)-384*b*c^6*x^6+384*b*c^4*x^4+27*a*Ei(1,-3*arccosh(c*x)-3*a/
b)*exp(-(-b*arccosh(c*x)+3*a)/b)-25*a*Ei(1,-5*arccosh(c*x)-5*a/b)*exp(-(-b
*arccosh(c*x)+5*a)/b)-5*a*Ei(1,-arccosh(c*x)-a/b)*exp(-(-b*arccosh(c*x)+a)
/b)-7*Ei(1,7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)*a+25*Ei(1,5*a
rccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)*a-27*Ei(1,3*arccosh(c*x)+3*
a/b)*exp((b*arccosh(c*x)+3*a)/b)*a+5*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arcco
sh(c*x))/b)*a+128*b*c^8*x^8+7*a*Ei(1,-7*arccosh(c*x)-7*a/b)*exp(-(-b*arcco
sh(c*x)+7*a)/b))/(c*x+1)/c^2/(c*x-1)/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2
+ 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*x^7 - 3*c^4*x^5 + 3*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^8
 - 3*c^5*x^6 + 3*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c
*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1))) + integrate((7*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*(c*x + 1)^(3/2)*(c*x
 - 1) + (14*c^8*x^8 - 37*c^6*x^6 + 33*c^4*x^4 - 11*c^2*x^2 + 1)*(c*x + 1)*
sqrt(c*x - 1) + (7*c^9*x^9 - 23*c^7*x^7 + 27*c^5*x^5 - 13*c^3*x^3 + 2*c*x)
*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*
x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sq
rt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x
^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x\,(1-c^2\,x^2)^{5/2}}{(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx$$

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**5)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b +
 a**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**3)/(acosh(c*x)**2*b**2 +
 2*acosh(c*x)*a*b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x)/(acosh(
c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x)
```

3.286 $\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 351

$$\begin{aligned}\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2} \, dx =& -\frac{\sqrt{-1+cx}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))} \\ &- \frac{15\sqrt{1-cx}\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c\sqrt{-1+cx}} \\ &+ \frac{3\sqrt{1-cx}\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c\sqrt{-1+cx}} \\ &- \frac{3\sqrt{1-cx}\text{Chi}\left(\frac{6(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c\sqrt{-1+cx}} \\ &+ \frac{15\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c\sqrt{-1+cx}} \\ &- \frac{3\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{4b^2c\sqrt{-1+cx}} \\ &+ \frac{3\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c\sqrt{-1+cx}}\end{aligned}$$

output
```
-(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccosh(c*x))-15/
16*(-c*x+1)^(1/2)*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c/(c*x-1)^(1
/2)+3/4*(-c*x+1)^(1/2)*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)/b^2/c/(c*x-
1)^(1/2)-3/16*(-c*x+1)^(1/2)*Chi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)/b^2/c
/(c*x-1)^(1/2)+15/16*(-c*x+1)^(1/2)*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b
)/b^2/c/(c*x-1)^(1/2)-3/4*(-c*x+1)^(1/2)*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*
x))/b)/b^2/c/(c*x-1)^(1/2)+3/16*(-c*x+1)^(1/2)*cosh(6*a/b)*Shi(6*(a+b*arcc
osh(c*x))/b)/b^2/c/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.98

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(-16b+48bc^2x^2-48bc^4x^4+16bc^6x^6+15(a+b\text{arccosh}(c\right.}{}$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b + 48*b*c^2*x^2 - 48*b*c^4*x^4 + 16*b*
c^6*x^6 + 15*(a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sin
h[(2*a)/b] - 12*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*
Sinh[(4*a)/b] + 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3
*b*ArcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] - 15*a*
Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 15*b*ArcCosh[c*x]*Cos
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 12*a*Cosh[(4*a)/b]*SinhI
ntegral[4*(a/b + ArcCosh[c*x])] + 12*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhInte
gral[4*(a/b + ArcCosh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + Arc
Cosh[c*x])] - 3*b*ArcCosh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh
[c*x])]))/(16*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6319, 6327, 6367, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2} \, dx$$

$$\Big\downarrow \text{ 6319}$$

$$\frac{6c\sqrt{1-cx}\int \frac{x(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6327}$$

$$\frac{6c\sqrt{1-cx}\int \frac{x(1-c^2x^2)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{6\sqrt{1-cx}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{6\sqrt{1-cx}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{6\sqrt{1-cx}\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}-\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32(a+b\text{arccosh}(cx))}\right)d(a+bar}{b^2c\sqrt{cx-1}}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

$\downarrow$ 2009

$$\frac{6\sqrt{1-cx}\left(-\frac{5}{32}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{32}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b}{}\right.\right.}{}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bc(a+b\text{arccosh}(cx))}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

output
```
-((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCosh[c
*x]))) + (6*Sqrt[1 - c*x]*((-5*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Si
nh[(2*a)/b])/32 + (CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])
/8 - (CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/32 + (5*Cosh
[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/32 - (Cosh[(4*a)/b]*Si
nhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/8 + (Cosh[(6*a)/b]*SinhIntegral[(6
*(a + b*ArcCosh[c*x]))/b])/32))/(b^2*c*Sqrt[-1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6319
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p - 1/2)*(-
1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

rule 6327
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(309) = 618$.

Time = 0.31 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(-32\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^6x^6-32b\,c^7x^7+96\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^4x^4+96b\,c^5x^5-96\sqrt{cx-\right.}{}}$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-32
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^6*x^6-32*b*c^7*x^7+96*(c*x-1)^(1/2)*(c*x+
1)^(1/2)*b*c^4*x^4+96*b*c^5*x^5-96*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^2*x^2-9
6*b*c^3*x^3+12*arccosh(c*x)*b*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-(-b*arccosh
(c*x)+4*a)/b)-15*arccosh(c*x)*b*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arcco
sh(c*x)+2*a)/b)-3*arccosh(c*x)*b*Ei(1,-6*arccosh(c*x)-6*a/b)*exp(-(-b*arcc
osh(c*x)+6*a)/b)+3*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)*
b*arccosh(c*x)-12*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)*b
*arccosh(c*x)+15*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)*b*
arccosh(c*x)+32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b+12*a*Ei(1,-4*arccosh(c*x)-4*
a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)-15*a*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(
-b*arccosh(c*x)+2*a)/b)-3*a*Ei(1,-6*arccosh(c*x)-6*a/b)*exp(-(-b*arccosh(c
*x)+6*a)/b)+3*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)*a-12*
Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)*a+15*Ei(1,2*arccosh
(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)*a+32*b*c*x)/(c*x-1)/(c*x+1)/c/b^2
/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2
+ 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
 - 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2
 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x
 + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))) + integrate(((6*c^6*x^6 - 11*c^4*x^4 + 4*c^2*x^2 + 1)*(c*x + 1)^(3/2)
*(c*x - 1) + 6*(2*c^7*x^7 - 5*c^5*x^5 + 4*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*
x - 1) + (6*c^8*x^8 - 19*c^6*x^6 + 21*c^4*x^4 - 9*c^2*x^2 + 1)*sqrt(c*x +
1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^2 - 2*a*b*
c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + a*b + (b
^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2*(b^2*c^3*
x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))), x)
```

Giac [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{(a+b\,\mathrm{acosh}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x))^2,x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),
x) + int((sqrt( - c**2*x**2 + 1)*x**4)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*
a*b + a**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c*x)**2*b
**2 + 2*acosh(c*x)*a*b + a**2),x)*c**2
```

3.287 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\operatorname{arccosh}(cx))^2}\,dx=\operatorname{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x(a+b\operatorname{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\operatorname{arccosh}(cx))^2}\,dx=\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\operatorname{arccosh}(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{5c\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6304}$$

$$\frac{5c\sqrt{1-cx}\int \frac{(1-c^2x^2)^2}{a+b\text{arccosh}(cx)}dx}{b\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6321}$$

$$\frac{5\sqrt{1-cx}\int -\frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{5\sqrt{1-cx}\int \frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{5\sqrt{1-cx}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^5}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 26$$

$$\frac{5i\sqrt{1-cx}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^5}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 3793$$

$$\frac{5i\sqrt{1-cx}\int \left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}-\frac{5i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{5i\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8(a+b\text{arccosh}(cx))}\right)d(a+b\text{arc}}{b^2\sqrt{cx-1}}$$

$$\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+$$

$$\frac{5i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arc}}{}\right.\right.}{}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 6327$$

$$\frac{\sqrt{1-cx}\int \frac{\left(1-c^2x^2\right)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+$$

$$\frac{5i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arc}}{}\right.\right.}{}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6375}$$

$$\frac{\sqrt{1-cx}\int \frac{(1-c^2x^2)^2}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+$$

$$\frac{5i\sqrt{1-cx}\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arc}}{}\right.\right.}{}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}}{bcx(a+b\text{arccosh}(cx))}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{x\left(a+b\ \text{arccosh}\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\,\text{arcosh}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^
2 + 2*a*b*x*arccosh(c*x) + a^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 524, normalized size of antiderivative = 18.71

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
 - 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3
 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt
(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((5*c^7*x^7 - 8*c^5*x^5 + c^3*x^3 + 2*c*x)*(c*x +
 1)^(3/2)*(c*x - 1) + (10*c^8*x^8 - 23*c^6*x^6 + 15*c^4*x^4 - c^2*x^2 - 1)
*(c*x + 1)*sqrt(c*x - 1) + 5*(c^9*x^9 - 3*c^7*x^7 + 3*c^5*x^5 - c^3*x^3)*s
qrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^
4 - 2*a*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c
^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x
- 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x\,(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.71

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2x+2acosh\,(cx)\,abx+a^2x}dx + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2*x + 2*acosh(c*x)*a*b*x + a*
*2*x),x) + int((sqrt( - c**2*x**2 + 1)*x**3)/(acosh(c*x)**2*b**2 + 2*acosh
(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x)/(acosh(c*x)**
2*b**2 + 2*acosh(c*x)*a*b + a**2),x)*c**2
```

3.288 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 22.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \ 6357$$

$$\frac{2\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}+\frac{4c\sqrt{1-cx}\int \frac{(1-cx)^2(cx+1)^2}{x(a+b\text{arccosh}(cx))}dx}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bcx^2(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 6327$$

$$\frac{4c\sqrt{1-cx}\int \frac{(1-c^2x^2)^2}{x(a+b\text{arccosh}(cx))}dx}{b\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int \frac{(1-c^2x^2)^2}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bcx^2(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \ 6375$$

$$\frac{4c\sqrt{1-cx}\int \frac{(1-c^2x^2)^2}{x(a+b\text{arccosh}(cx))}dx}{b\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\int \frac{(1-c^2x^2)^2}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{bcx^2(a+b\text{arccosh}(cx))}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^2 (a+b \operatorname{arccosh}(cx))^2} dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\operatorname{arccosh}(cx))^2} dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b \operatorname{arcosh}(cx)+a)^2 x^2} dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x
)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\operatorname{arccosh}(cx))^2} dx = \text{Timed out}$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 534, normalized size of antiderivative = 19.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
 - 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4
 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sq
rt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))) + integrate(((4*c^7*x^7 - 5*c^5*x^5 - 2*c^3*x^3 + 3*c*x)*
(c*x + 1)^(3/2)*(c*x - 1) + 2*(4*c^8*x^8 - 8*c^6*x^6 + 3*c^4*x^4 + 2*c^2*x
^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (4*c^9*x^9 - 11*c^7*x^7 + 9*c^5*x^5 - c^
3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x -
 1)*a*b*c^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4
)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3
*x^5 - 2*b^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^2\,(a+b\,\text{acosh}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.89

$$\begin{aligned}\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\text{arccosh}(cx))^2}\,dx &= \int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2x^2+2acosh\,(cx)\,ab\,x^2+a^2x^2}dx \\ &- 2\left(\int \frac{\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 \\ &+ \left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4\end{aligned}$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*acosh(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2*x**2 + 2*acosh(c*x)*a*b*x**
2 + a**2*x**2),x) - 2*int(sqrt( - c**2*x**2 + 1)/(acosh(c*x)**2*b**2 + 2*a
cosh(c*x)*a*b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c
*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x)*c**4
```

3.289 $\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 337

$$\begin{aligned}\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = &-\frac{x^5\sqrt{-1+cx}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}\\
&-\frac{5\sqrt{-1+cx}\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^6\sqrt{1-cx}}\\
&-\frac{15\sqrt{-1+cx}\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c^6\sqrt{1-cx}}\\
&-\frac{5\sqrt{-1+cx}\text{Chi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c^6\sqrt{1-cx}}\\
&+\frac{5\sqrt{-1+cx}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{8b^2c^6\sqrt{1-cx}}\\
&+\frac{15\sqrt{-1+cx}\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c^6\sqrt{1-cx}}\\
&+\frac{5\sqrt{-1+cx}\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16b^2c^6\sqrt{1-cx}}\end{aligned}$$

output
```
-x^5*(c*x-1)^(1/2)/b/c/(-c*x+1)^(1/2)/(a+b*arccosh(c*x))-5/8*(c*x-1)^(1/2)
*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^6/(-c*x+1)^(1/2)-15/16*(c*x-1)^
(1/2)*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^6/(-c*x+1)^(1/2)-5/16*
(c*x-1)^(1/2)*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)/b^2/c^6/(-c*x+1)^(1/
2)+5/8*(c*x-1)^(1/2)*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b^2/c^6/(-c*x+1)^
(1/2)+15/16*(c*x-1)^(1/2)*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)/b^2/c^6/
(-c*x+1)^(1/2)+5/16*(c*x-1)^(1/2)*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)/
b^2/c^6/(-c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.56

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(\frac{16bc^5x^5}{a+b\text{arccosh}(cx)}+5\left(2\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)\sinh\left(\frac{a}{b}\right)+3\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\right.\right.}{}$$

input
```
Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
(Sqrt[1 - c^2*x^2]*((16*b*c^5*x^5)/(a + b*ArcCosh[c*x]) + 5*(2*CoshIntegra
l[a/b + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*S
inh[(3*a)/b] + CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 2*Cosh
[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a
/b + ArcCosh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])]))
)/(16*b^2*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {6365, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}dx$$

$\Big\downarrow$ 6365

$$\frac{5\sqrt{cx-1}\int \frac{x^4}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{1-cx}}-\frac{x^5\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 6302

$$\frac{5\sqrt{cx-1}\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^6\sqrt{1-cx}}-\frac{x^5\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 25

$$-\frac{5\sqrt{cx-1}\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^6\sqrt{1-cx}}-\frac{x^5\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 5971

$$-\frac{5\sqrt{cx-1}\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8(a+b\text{arccosh}(cx))}\right)d(a+\text{barcc}}{b^2c^6\sqrt{1-cx}}$$

$$\frac{x^5\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 2009

$$\frac{5\sqrt{cx-1}\left(-\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{3}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcc}}{b}\right.\right.}{b^2}$$

$$\frac{x^5\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

input
```
Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
-((x^5*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) + (5*Sqrt
[-1 + c*x]*(-1/8*(CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b]) - (3*Cos
hIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/16 - (CoshIntegral[(5
*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*SinhIntegral[(a +
 b*ArcCosh[c*x])/b])/8 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c
*x]))/b])/16 + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/16
))/(b^2*c^6*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6365
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Si
mp[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
   Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.54

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1)\left(-32\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^5x^5+32b\,c^6x^6+5\,\operatorname{arccosh}(cx)b\,\operatorname{expIntegral}_1\left(-5\,\operatorname{arccosh}(cx)-\frac{5a}{b}\right.\right.}{}$

input
```
int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-c^2*x^2+1)^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-32*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^5*x^5+32*b*c^6*x^6+5*arccosh(c*x)*b*Ei(1,-5
*arccosh(c*x)-5*a/b)*exp(-(b*arccosh(c*x)+5*a)/b)+15*arccosh(c*x)*b*Ei(1,-
3*arccosh(c*x)-3*a/b)*exp(-(b*arccosh(c*x)+3*a)/b)+10*arccosh(c*x)*b*Ei(1,
-arccosh(c*x)-a/b)*exp(-(a+b*arccosh(c*x))/b)-5*Ei(1,5*arccosh(c*x)+5*a/b)
*exp((-b*arccosh(c*x)+5*a)/b)*b*arccosh(c*x)-15*Ei(1,3*arccosh(c*x)+3*a/b)
*exp((-b*arccosh(c*x)+3*a)/b)*b*arccosh(c*x)-10*Ei(1,arccosh(c*x)+a/b)*exp
((-b*arccosh(c*x)+a)/b)*b*arccosh(c*x)+5*a*Ei(1,-5*arccosh(c*x)-5*a/b)*exp
(-(b*arccosh(c*x)+5*a)/b)+15*a*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(b*arccosh
(c*x)+3*a)/b)+10*a*Ei(1,-arccosh(c*x)-a/b)*exp(-(a+b*arccosh(c*x))/b)-5*Ei
(1,5*arccosh(c*x)+5*a/b)*exp((-b*arccosh(c*x)+5*a)/b)*a-15*Ei(1,3*arccosh(
c*x)+3*a/b)*exp((-b*arccosh(c*x)+3*a)/b)*a-10*Ei(1,arccosh(c*x)+a/b)*exp((
-b*arccosh(c*x)+a)/b)*a)/c^6/(c^2*x^2-1)/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\mathrm{arccosh}(cx))^2}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
h(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^5}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x**5/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-(c^3*x^8 - c*x^6 + (c^2*x^7 - x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrat
e((5*c^5*x^9 - 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 - 4*c*x^5)*(c*x + 1)*(c*x
 - 1) + 5*(2*c^4*x^8 - 3*c^2*x^6 + x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*
x + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)
*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt
(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x
- 1)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (
a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^5}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^5/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^5/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(x**5/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2
+ 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.290 $\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 236

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = -\frac{x^4\sqrt{-1+cx}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))} - \frac{\sqrt{-1+cx}\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5\sqrt{1-cx}} - \frac{\sqrt{-1+cx}\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^5\sqrt{1-cx}} + \frac{\sqrt{-1+cx}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{b^2c^5\sqrt{1-cx}} + \frac{\sqrt{-1+cx}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{2b^2c^5\sqrt{1-cx}}$$

output
```
-x^4*(c*x-1)^(1/2)/b/c/(-c*x+1)^(1/2)/(a+b*arccosh(c*x))-(c*x-1)^(1/2)*Chi
(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^5/(-c*x+1)^(1/2)-1/2*(c*x-1)^(1
/2)*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)/b^2/c^5/(-c*x+1)^(1/2)+(c*x-1)
^(1/2)*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b^2/c^5/(-c*x+1)^(1/2)+1/2*
(c*x-1)^(1/2)*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)/b^2/c^5/(-c*x+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(\frac{2bc^4x^4}{a+b\operatorname{arccosh}(cx)} + 2\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arccosh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) + \operatorname{Chi}\left(4\left(\frac{a}{b}+\operatorname{arccosh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)\right.}{2b^2c^5\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
(Sqrt[1 - c^2*x^2]*((2*b*c^4*x^4)/(a + b*ArcCosh[c*x]) + 2*CoshIntegral[2*
(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] + CoshIntegral[4*(a/b + ArcCosh[c*x])]
*Sinh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Co
sh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {6365, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6365}$$

$$\frac{4\sqrt{cx-1}\int \frac{x^3}{a+b\operatorname{arccosh}(cx)}dx}{bc\sqrt{1-cx}} - \frac{x^4\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\operatorname{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6302}$$

$$\frac{4\sqrt{cx-1}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^5\sqrt{1-cx}}-\frac{x^4\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4\sqrt{cx-1}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^5\sqrt{1-cx}}-\frac{x^4\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{4\sqrt{cx-1}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^5\sqrt{1-cx}}-\frac{x^4\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\sqrt{cx-1}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arc}\right.\right.}{b^2c^5\sqrt{1-cx}} \\ \frac{x^4\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

input
```
Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
-((x^4*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) + (4*Sqrt
[-1 + c*x]*(-1/4*(CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b])
- (CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/8 + (Cosh[(2*a)
/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/4 + (Cosh[(4*a)/b]*SinhInteg
ral[(4*(a + b*ArcCosh[c*x]))/b])/8))/(b^2*c^5*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6365
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Si
mp[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
   Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.64

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(-4\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^4x^4+4b\,c^5x^5+2\,\mathrm{arccosh}(cx)b\,\mathrm{expIntegral}_1\left(-2\,\mathrm{arccosh}(cx)-\frac{2a}{b}\right)e}{}$

input
```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c^2*x^2+1)^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-4*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*b*c^4*x^4+4*b*c^5*x^5+2*arccosh(c*x)*b*Ei(1,-2*ar
ccosh(c*x)-2*a/b)*exp(-(b*arccosh(c*x)+2*a)/b)+arccosh(c*x)*b*Ei(1,-4*arcc
osh(c*x)-4*a/b)*exp(-(b*arccosh(c*x)+4*a)/b)-Ei(1,4*arccosh(c*x)+4*a/b)*ex
p((-b*arccosh(c*x)+4*a)/b)*b*arccosh(c*x)-2*Ei(1,2*arccosh(c*x)+2*a/b)*exp
((-b*arccosh(c*x)+2*a)/b)*b*arccosh(c*x)+2*a*Ei(1,-2*arccosh(c*x)-2*a/b)*e
xp(-(b*arccosh(c*x)+2*a)/b)+a*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-(b*arccosh(
c*x)+4*a)/b)-Ei(1,4*arccosh(c*x)+4*a/b)*exp((-b*arccosh(c*x)+4*a)/b)*a-2*E
i(1,2*arccosh(c*x)+2*a/b)*exp((-b*arccosh(c*x)+2*a)/b)*a)/c^5/(c^2*x^2-1)/
b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
h(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-(cx-1)\,(cx+1)}\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x**4/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-(c^3*x^7 - c*x^5 + (c^2*x^6 - x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrat
e((4*c^5*x^8 - 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 - 3*c*x^4)*(c*x + 1)*(c*x
- 1) + 4*(2*c^4*x^7 - 3*c^2*x^5 + x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x
 + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(
-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x -
 1)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a
*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^4}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(x**4/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2
+ 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.291 $\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 237

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = -\frac{x^3\sqrt{-1+cx}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))} - \frac{3\sqrt{-1+cx}\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^4\sqrt{1-cx}} - \frac{3\sqrt{-1+cx}\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^4\sqrt{1-cx}} + \frac{3\sqrt{-1+cx}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{4b^2c^4\sqrt{1-cx}} + \frac{3\sqrt{-1+cx}\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4b^2c^4\sqrt{1-cx}}$$

output
```
-x^3*(c*x-1)^(1/2)/b/c/(-c*x+1)^(1/2)/(a+b*arccosh(c*x))-3/4*(c*x-1)^(1/2)
*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^4/(-c*x+1)^(1/2)-3/4*(c*x-1)^(1
/2)*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^4/(-c*x+1)^(1/2)+3/4*(c*
x-1)^(1/2)*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b^2/c^4/(-c*x+1)^(1/2)+3/4*
(c*x-1)^(1/2)*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)/b^2/c^4/(-c*x+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}\left(\frac{4bc^3x^3}{a+b\text{arccosh}(cx)}+3\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)\sinh\left(\frac{a}{b}\right)+3\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\right.}{4b^2c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
(Sqrt[1 - c^2*x^2]*((4*b*c^3*x^3)/(a + b*ArcCosh[c*x]) + 3*CoshIntegral[a/
b + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[
(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*
SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {6365, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6365}$$

$$\frac{3\sqrt{cx-1}\int \frac{x^2}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{1-cx}}-\frac{x^3\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6302}$$

$$\frac{3\sqrt{cx-1}\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{1-cx}}-\frac{x^3\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \; 25$$

$$-\frac{3\sqrt{cx-1}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{1-cx}}-\frac{x^3\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \; 5971$$

$$-\frac{3\sqrt{cx-1}\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4(a+b\text{arccosh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4(a+b\text{arccosh}(cx))}\right)d(a+b\text{arccosh}(cx))}{b^2c^4\sqrt{1-cx}}-\frac{x^3\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \; 2009$$

$$\frac{3\sqrt{cx-1}\left(-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(}{b}\right.\right.}{b^2c^4\sqrt{1-cx}}$$

$$\frac{x^3\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

input
```
Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
-((x^3*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) + (3*Sqrt
[-1 + c*x]*(-1/4*(CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b]) - (CoshI
ntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/4 + (Cosh[a/b]*SinhInte
gral[(a + b*ArcCosh[c*x])/b])/4 + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*Ar
cCosh[c*x]))/b])/4))/(b^2*c^4*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6365
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Si
mp[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
   Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.58

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(-8\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^3x^3+8b\,c^4x^4+3\ \operatorname{arccosh}(cx)b\ \operatorname{expIntegral}_1\left(-3\ \operatorname{arccosh}(cx)-\frac{3a}{b}\right)e\right.}{}$

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-c^2*x^2+1)^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-8*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*b*c^3*x^3+8*b*c^4*x^4+3*arccosh(c*x)*b*Ei(1,-3*ar
ccosh(c*x)-3*a/b)*exp(-(b*arccosh(c*x)+3*a)/b)+3*arccosh(c*x)*b*Ei(1,-arcc
osh(c*x)-a/b)*exp(-(a+b*arccosh(c*x))/b)-3*Ei(1,3*arccosh(c*x)+3*a/b)*exp(
(-b*arccosh(c*x)+3*a)/b)*b*arccosh(c*x)-3*Ei(1,arccosh(c*x)+a/b)*exp((-b*a
rccosh(c*x)+a)/b)*b*arccosh(c*x)+3*a*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-(b*a
rccosh(c*x)+3*a)/b)+3*a*Ei(1,-arccosh(c*x)-a/b)*exp(-(a+b*arccosh(c*x))/b)
-3*Ei(1,3*arccosh(c*x)+3*a/b)*exp((-b*arccosh(c*x)+3*a)/b)*a-3*Ei(1,arccos
h(c*x)+a/b)*exp((-b*arccosh(c*x)+a)/b)*a)/c^4/(c^2*x^2-1)/b^2/(a+b*arccosh
(c*x))
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
h(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^3}{\sqrt{-(cx-1)\,(cx+1)}\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x**3/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-(c^3*x^6 - c*x^4 + (c^2*x^5 - x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrat
e((3*c^5*x^7 - 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 - 2*c*x^3)*(c*x + 1)*(c*x
- 1) + 3*(2*c^4*x^6 - 3*c^2*x^4 + x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x
 + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(
-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x -
 1)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a
*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\text{acosh}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(x**3/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2
+ 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.292 $\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 136

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\,\mathrm{arccosh}(cx))^2}\,dx = -\frac{x^2\sqrt{-1+cx}}{bc\sqrt{1-cx}(a+b\,\mathrm{arccosh}(cx))} - \frac{\sqrt{-1+cx}\,\mathrm{Chi}\left(\frac{2(a+b\,\mathrm{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3\sqrt{1-cx}} + \frac{\sqrt{-1+cx}\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\,\mathrm{arccosh}(cx))}{b}\right)}{b^2c^3\sqrt{1-cx}}$$

output
```
-x^2*(c*x-1)^(1/2)/b/c/(-c*x+1)^(1/2)/(a+b*arccosh(c*x))-(c*x-1)^(1/2)*Chi
(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^3/(-c*x+1)^(1/2)+(c*x-1)^(1/2)*
cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)/b^2/c^3/(-c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$
$$= \frac{\sqrt{1-c^2x^2}\left(bc^2x^2+(a+b\text{arccosh}(cx))\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-(a+b\text{arccosh}(cx))\cosh\left(\frac{2a}{b}\right.\right.}{b^2c^3\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))}$$

input
```
Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(b*c^2*x^2 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b +
 ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcCosh[c*x])]))/(b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {6365, 6302, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6365}$$

$$\frac{2\sqrt{cx-1}\int \frac{x}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6302}$$

$$\frac{2\sqrt{cx-1}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 25

$$-\frac{2\sqrt{cx-1}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 5971

$$-\frac{2\sqrt{cx-1}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2(a+b\text{arccosh}(cx))}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{cx-1}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$\Big\downarrow$ 3042

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}-\frac{\sqrt{cx-1}\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{1-cx}}$$

$\Big\downarrow$ 26

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^3\sqrt{1-cx}}$$

$\Big\downarrow$ 3784

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+$$
$$\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\right.}{b^2c^3\sqrt{1-cx}}$$

$\big\downarrow$ 26

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\right.}{b^2c^3\sqrt{1-cx}}$$

$\big\downarrow$ 3042

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(\right.}{b^2c^3\sqrt{1-cx}}$$

$\big\downarrow$ 26

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\right.}{b^2c^3\sqrt{1-cx}}$$

$\big\downarrow$ 3779

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\right)}{b^2c^3\sqrt{1-cx}}$$

$\big\downarrow$ 3782

$$-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(cx))}{b}\right)\right)}{b^2c^3\sqrt{1-cx}}$$

input

```
Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
-((x^2*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) + (I*Sqrt
[-1 + c*x]*(I*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b] - I*C
osh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b]))/(b^2*c^3*Sqrt[1 -
c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6365
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Si
mp[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
   Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2-1\right)\left(-2\sqrt{cx-1}\,\sqrt{cx+1}\,b\,c^2x^2+2b\,c^3x^3+\operatorname{arccosh}(cx)b\ \mathrm{expIntegral}_1\left(-2\ \operatorname{arccosh}(cx)-\frac{2a}{b}\right)\mathrm{e}^{-}\right.}{}$

input
```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-c^2*x^2+1)^(1/2)*((c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2-1)*(-2*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*b*c^2*x^2+2*b*c^3*x^3+arccosh(c*x)*b*Ei(1,-2*arcc
osh(c*x)-2*a/b)*exp(-(b*arccosh(c*x)+2*a)/b)-Ei(1,2*arccosh(c*x)+2*a/b)*ex
p((-b*arccosh(c*x)+2*a)/b)*b*arccosh(c*x)+a*Ei(1,-2*arccosh(c*x)-2*a/b)*ex
p(-(b*arccosh(c*x)+2*a)/b)-Ei(1,2*arccosh(c*x)+2*a/b)*exp((-b*arccosh(c*x)
+2*a)/b)*a)/c^3/(c^2*x^2-1)/b^2/(a+b*arccosh(c*x))
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
h(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-(c^3*x^5 - c*x^3 + (c^2*x^4 - x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrat
e((2*c^5*x^6 - 5*c^3*x^4 + (2*c^3*x^4 - c*x^2)*(c*x + 1)*(c*x - 1) + 3*c*x
^2 + 2*(2*c^4*x^5 - 3*c^2*x^3 + x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt
(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
 + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*
a*b*c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c
^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{acosh}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$
$$=\int \frac{x^2}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(x**2/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2
+ 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.293 $\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = -\frac{x\sqrt{-1+cx}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))} - \frac{\sqrt{-1+cx}\,\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{1-cx}} + \frac{\sqrt{-1+cx}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}$$

output

```
-x*(c*x-1)^(1/2)/b/c/(-c*x+1)^(1/2)/(a+b*arccosh(c*x))-(c*x-1)^(1/2)*Chi((
a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^2/(-c*x+1)^(1/2)+(c*x-1)^(1/2)*cosh(a
/b)*Shi((a+b*arccosh(c*x))/b)/b^2/c^2/(-c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}\left(bcx+(a+b\text{arccosh}(cx))\text{Chi}\left(\frac{a}{b}+\text{arccosh}(cx)\right)\sinh\left(\frac{a}{b}\right)-(a+b\text{arccosh}(cx))\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.\right.}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))}$$

input
```
Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(b*c*x + (a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCos
h[c*x]]*Sinh[a/b] - (a + b*ArcCosh[c*x])*Cosh[a/b]*SinhIntegral[a/b + ArcC
osh[c*x]]))/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6365, 6296, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6365}$$

$$\frac{\sqrt{cx-1}\int \frac{1}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{1-cx}}-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6296}$$

$$\frac{\sqrt{cx-1}\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{1-cx}}-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{cx-1}\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{1-cx}}-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}-\frac{\sqrt{cx-1}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))}{b^2c^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+ \frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\text{barcc}\right.}{b^2c^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+ \frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arccosh}(cx)}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\text{barccos}\right.}{b^2c^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+ \frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+\right.}{b^2c^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arcc}$$

$$\Big\downarrow \textbf{3779}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(cx)}d(a+b\text{arccosh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{1-cx}}$$

$$\Big\downarrow \textbf{3782}$$

$$-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}+\frac{i\sqrt{cx-1}\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(cx)}{b}\right)\right)}{b^2c^2\sqrt{1-cx}}$$

input
```
Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
-((x*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) + (I*Sqrt[-
1 + c*x]*(I*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b] - I*Cosh[a/b]*S
inhIntegral[(a + b*ArcCosh[c*x])/b]))/(b^2*c^2*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6365
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Si
mp[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
   Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{x}{\sqrt{-c^2x^2+1}\,(a+b\,\mathrm{arccosh}\,(cx))^2}dx$$

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\mathrm{arccosh}(cx))^2}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(
c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\mathrm{arccosh}(cx))^2}\,dx = \int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^4 - c*x^2 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)
*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b
*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(
(c^5*x^5 + (c*x + 1)*(c*x - 1)*c^3*x^3 - 3*c^3*x^3 + (2*c^4*x^4 - 3*c^2*x^
2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*c*x)/(((c*x + 1)^(3/2)*(c*x - 1)*b^
2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5
*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^2 + 2*(a*b
*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x
^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(x/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1
)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.294 $\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \frac{\sqrt{1-cx}}{bc\sqrt{-1+cx}(a+b\text{arccosh}(cx))}$$

output
```
(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)/(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))}$$

input
```
Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
-((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x
])))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6307}$$

$$-\frac{\sqrt{cx-1}}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

input
```
Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
-(Sqrt[-1 + c*x]/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x])))
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{-(cx-1)(cx+1)}}{(c^2x^2-1)c(a+b\,\text{arccosh}(cx))b}$	57

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-(c*x-1)*(c*x+1))^(1/2)/(c^2*x^2-1)/c/(a+b*ar
ccosh(c*x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \frac{\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}}{abc^3x^2-abc+(b^2c^3x^2-b^2c)\log\left(cx+\sqrt{c^2x^2-1}\right)}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 -
 b^2*c)*log(c*x + sqrt(c^2*x^2 - 1)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*s
qrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x +
1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c
^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(-(
c^2*x^2 - (c*x + 1)*(c*x - 1) - 1)/(((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^2*x^2
 + 2*(b^2*c^3*x^3 - b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^4*x^4 - 2*b^
2*c^2*x^2 + b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*
b*c*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(
c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = -\frac{b\sqrt{1-c^2\,x^2}\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(b^2-b^2\,c^2\,x^2\right)}$$

input
```
int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
-(b*(1 - c^2*x^2)^(1/2)*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/(c*(a + b*acosh(c
*x))*(b^2 - b^2*c^2*x^2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\left(cx\right)^2b^2+2\sqrt{-c^2x^2+1}\,acosh\left(cx\right)ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1
)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.295 $\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx = \int \frac{1}{x\sqrt{1-c^2x^2}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6365}$$
$$-\frac{\sqrt{cx-1}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$
$$\Big\downarrow \text{ 6303}$$
$$-\frac{\sqrt{cx-1}\int \frac{1}{x^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

input
```
Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{-c^2x^2+1}\,(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^3 - a^2*x + (b^2*c^2*x^3 - b^2*x)*
arccosh(c*x)^2 + 2*(a*b*c^2*x^3 - a*b*x)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{x\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 481, normalized size of antiderivative = 17.18

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
-(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*s
qrt(c*x - 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x^2 + (a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - inte
grate((c^5*x^5 - c^3*x^3 + (c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (2*c^4*
x^4 - 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^(3/2)*(c*x -
 1)*b^2*c^3*x^4 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) +
(b^2*c^5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*
x^4 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^6
 - 2*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{1}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$
$$=\int \frac{1}{\sqrt{-c^2x^2+1}\,acosh(cx)^2\,b^2x+2\sqrt{-c^2x^2+1}\,acosh(cx)\,abx+\sqrt{-c^2x^2+1}\,a^2x}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*x + 2*sqrt( - c**2*x**2 +
 1)*acosh(c*x)*a*b*x + sqrt( - c**2*x**2 + 1)*a**2*x),x)
```

3.296 $\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6365}$$

$$-\frac{2\sqrt{cx-1}\int \frac{1}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx^2\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{ 6303}$$

$$-\frac{2\sqrt{cx-1}\int \frac{1}{x^3(a+b\text{arccosh}(cx))}dx}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx^2\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

input
```
Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{-c^2x^2+1}\,(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^4 - a^2*x^2 + (b^2*c^2*x^4 - b^2*x
^2)*arccosh(c*x)^2 + 2*(a*b*c^2*x^4 - a*b*x^2)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 17.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{x^2\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 491, normalized size of antiderivative = 17.54

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
-(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((((c*x + 1)*s
qrt(c*x - 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-
c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)
*a*b*c^2*x^3 + (a*b*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) -
integrate((2*c^5*x^5 - 3*c^3*x^3 + (2*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1)
 + 2*(2*c^4*x^4 - 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/((((c*x
 + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^5 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^7 - 2*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c*x + 1))
*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*
(c*x - 1)*a*b*c^3*x^5 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x -
 1) + (a*b*c^5*x^7 - 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x +
 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$
$$= \int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2x^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab\,x^2+\sqrt{-c^2x^2+1}\,a^2x^2}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**
2 + 1)*acosh(c*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)
```

3.297 $\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\,\mathrm{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\mathrm{arccosh}(cx))^2}\,dx = \mathrm{Int}\left(\frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\mathrm{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\mathrm{arccosh}(cx))^2}\,dx = \int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\mathrm{arccosh}(cx))^2}\,dx$$

input
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6355}$$

$$\frac{2\sqrt{cx-1}\int \frac{x}{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{bc(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6327}$$

$$\frac{2\sqrt{cx-1}\int \frac{x}{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{bc(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6375}$$

$$\frac{2\sqrt{cx-1}\int \frac{x}{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}dx}{bc\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{bc(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))}$$

input
```
Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}} (a+b \operatorname{arccosh}(cx))^2} dx$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{x^2}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}} (b \operatorname{arcosh}(cx)+a)^2} dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^
4 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2
*x^2 + a*b)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 56.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 503, normalized size of antiderivative = 17.96

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arcosh}\left(cx\right)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
(c*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^2)/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^
2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2
 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^3*x^4 + (c*x + 1
)*(c*x - 1)*c*x^2 - 3*c*x^2 + 2*(2*c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1
))/(((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^
5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^6 - 3*
b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^2)*(c*x + 1)^(
3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)*(c*x + 1)*sqr
t(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c)*sqrt(c*
x + 1))*sqrt(-c*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{acosh}\,(c\,x))^2\,(1-c^2\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.04

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx =$$

$$-\left(\int \frac{x^2}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2c^2x^2-\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab\,c^2x^2-2\sqrt{}\right.$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
 - int(x**2/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**2*x**2 - sqrt( -
 c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a
*b*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2
 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.298 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx = \text{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}dx$$

input
```
Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
- 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x
^2 + a*b)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 50.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{x}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))^2}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 511, normalized size of antiderivative = 19.65

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
(c*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*x)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*
x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 -
 a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((c^5*x^5 + (c*x + 1)*(c
*x - 1)*c^3*x^3 + c^3*x^3 + (2*c^4*x^4 + c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c
*x - 1) - 2*c*x)/(((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) +
 2*(b^2*c^6*x^5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^
2*c^7*x^6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^
2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)
*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 -
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{x}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 5.35

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx =$$

$$-\left(\int \frac{x}{\sqrt{-c^2x^2+1}\,acosh\left(cx\right)^2 b^2c^2x^2-\sqrt{-c^2x^2+1}\,acosh\left(cx\right)^2 b^2+2\sqrt{-c^2x^2+1}\,acosh\left(cx\right) ab\,c^2x^2-2\surd}\right.$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
 - int(x/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**2*x**2 - sqrt( - c*
*2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b*
c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2 +
1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.299 $\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\,\text{arccosh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)^{3/2}\,(a+b\,\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\,\text{arccosh}(cx))^2}\,dx = \int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\,\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} \, dx$$

$$\Big\downarrow \text{6319}$$

$$\frac{2c\sqrt{cx-1}\int \frac{x}{(1-cx)^2(cx+1)^2(a+b\text{arccosh}(cx))}dx}{b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6327}$$

$$\frac{2c\sqrt{cx-1}\int \frac{x}{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}dx}{b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6375}$$

$$\frac{2c\sqrt{cx-1}\int \frac{x}{(1-c^2x^2)^2(a+b\text{arccosh}(cx))}dx}{b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))}$$

input
```
Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{2}}d x$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.12

$$\int \frac{1}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} \, dx=\int { \frac {1}{{\left (-c^{2} x^{2} + 1\right)}^{\frac {3}{2}} {\left (b \operatorname {arcosh}\left (c x\right) + a\right)}^{2}} \,d x }$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 -
2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2
 + a*b)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 40.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} \, dx=\int \frac {1}{\left (- \left (c x - 1\right) \left (c x + 1\right)\right)^{\frac {3}{2}} \left (a + b \operatorname {acosh}{\left (c x \right)}\right)^{2}}\, dx$$

input
```
integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 502, normalized size of antiderivative = 20.08

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

output
```
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x +
(b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b
*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((2*c^4*x^4 - c^2*x^2 + (2*c
^2*x^2 - 1)*(c*x + 1)*(c*x - 1) + 2*(2*c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c
*x - 1) - 1)/(((b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(
b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^6*
x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x
+ 1)^(3/2)*(c*x - 1) + 2*(a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*(c*x + 1)
*sqrt(c*x - 1) + (a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*sqrt(
c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\mathrm{arccosh}(cx))^2}\,dx=\int \frac{1}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,(1-c^2\,x^2)^{3/2}}\,dx$$

input
```
int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\mathrm{arccosh}(cx))^2}\,dx=$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2c^2x^2-\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab\,c^2x^2-2\sqrt{}\right.$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
 - int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**2*x**2 - sqrt( - c*
*2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b*
c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2 +
1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.300 $\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 18.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}dx$$

input
```
Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\text{arccosh}\left(cx\right)\right)^2}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.86

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\operatorname{arccosh}(cx)\right)^2}\,dx=\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^5 - 2*a^2*c^2*x^3 + a^2*x + (b^2*c^
4*x^5 - 2*b^2*c^2*x^3 + b^2*x)*arccosh(c*x)^2 + 2*(a*b*c^4*x^5 - 2*a*b*c^2
*x^3 + a*b*x)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 108.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\operatorname{arccosh}(cx)\right)^2}\,dx=\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 530, normalized size of antiderivative = 18.93

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\,(cx)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima
")
```

output
```
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2
+ (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 + (a*b*c^3*x^3
 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^5*x^5 - 3*c^3*
x^3 + (3*c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (6*c^4*x^4 - 5*c^2*x^2 + 1
)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^5*x^6 - b^2*c^3*x^4)*(c*x + 1)^(3/
2)*(c*x - 1) + 2*(b^2*c^6*x^7 - 2*b^2*c^4*x^5 + b^2*c^2*x^3)*(c*x + 1)*sqr
t(c*x - 1) + (b^2*c^7*x^8 - 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 - b^2*c*x^2)*sqr
t(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*
c^5*x^6 - a*b*c^3*x^4)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^7 - 2*a*b*
c^4*x^5 + a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^8 - 3*a*b*c^5*
x^6 + 3*a*b*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{1}{x\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.00

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx =$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acosh(cx)^2\,b^2c^2x^3-\sqrt{-c^2x^2+1}\,acosh(cx)^2\,b^2x+2\sqrt{-c^2x^2+1}\,acosh(cx)\,ab\,c^2x^3-2\right.$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
 - int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**2*x**3 - sqrt( - c*
*2*x**2 + 1)*acosh(c*x)**2*b**2*x + 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*
b*c**2*x**3 - 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b*x + sqrt( - c**2*x**
2 + 1)*a**2*c**2*x**3 - sqrt( - c**2*x**2 + 1)*a**2*x),x)
```

3.301 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx = \text{Int}\left(\frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2}, x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 15.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx = \int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx$$

input
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx$$

input
```
Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (-c^2x^2+1)^{\frac{3}{2}} (a+b \operatorname{arccosh}(cx))^2} dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int \frac{1}{x^2 (1 - c^2x^2)^{3/2} (a + b\text{arccosh}(cx))^2} \, dx = \int \frac{1}{(-c^2x^2 + 1)^{\frac{3}{2}} (b\,\text{arcosh}\,(cx) + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^6 - 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*
c^4*x^6 - 2*b^2*c^2*x^4 + b^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*x^6 - 2*a*b
*c^2*x^4 + a*b*x^2)*arccosh(c*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1 - c^2x^2)^{3/2} (a + b\text{arccosh}(cx))^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 538, normalized size of antiderivative = 19.21

$$\int \frac{1}{x^2 (1 - c^2x^2)^{3/2} (a + b\text{arccosh}(cx))^2} \, dx = \int \frac{1}{(-c^2x^2 + 1)^{\frac{3}{2}} (b\,\text{arcosh}\,(cx) + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxi
ma")
```

output
```
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3
+ (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 + (a*b*c^3*x
^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((4*c^5*x^5 - 5*
c^3*x^3 + (4*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1) + 2*(4*c^4*x^4 - 4*c^2*x
^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/(((b^2*c^5*x^7 - b^2*c^3*x^5)*(
c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^8 - 2*b^2*c^4*x^6 + b^2*c^2*x^4)*(
c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^9 - 3*b^2*c^5*x^7 + 3*b^2*c^3*x^5 - b^
2*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1)) + ((a*b*c^5*x^7 - a*b*c^3*x^5)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*
x^8 - 2*a*b*c^4*x^6 + a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^9
- 3*a*b*c^5*x^7 + 3*a*b*c^3*x^5 - a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)
), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}} (b \operatorname{arcosh}(cx)+a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac
")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx = \int \frac{1}{x^2 \left(a+b\,\text{acosh}\,(c\,x)\right)^2 \left(1-c^2\,x^2\right)^{3/2}} dx$$

input
```
int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.21

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\text{arccosh}(cx))^2} dx =$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2c^2x^4-\sqrt{-c^2x^2+1}\,acosh\,(cx)^2\,b^2x^2+2\sqrt{-c^2x^2+1}\,acosh\,(cx)\,ab\,c^2x^4-}\right.$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
 - int(1/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**2*x**4 - sqrt( - c*
*2*x**2 + 1)*acosh(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)
*a*b*c**2*x**4 - 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b*x**2 + sqrt( - c*
*2*x**2 + 1)*a**2*c**2*x**4 - sqrt( - c**2*x**2 + 1)*a**2*x**2),x)
```

3.302 $\int \frac{(fx)^m(1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2} dx = \text{Int}\left(\frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}, x\right)$$

output
```
Defer(Int)((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2} dx = \int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2} dx$$

input
```
Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}(fx)^m}{(a+b\text{arccosh}(cx))^2}dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{\left(1-c^2x^2\right)^{3/2}(fx)^m}{(a+b\text{arccosh}(cx))^2}dx$$

input
```
Int[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m\left(-c^2x^2+1\right)^{\frac{3}{2}}}{(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}(fx)^m}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fr
icas")
```

output
```
integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2
*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 577, normalized size of antiderivative = 19.23

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}(fx)^m}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="ma
xima")
```

output
```
((c^4*f^m*x^4 - 2*c^2*f^m*x^2 + f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^
m*x^5 - 2*c^3*f^m*x^3 + c*f^m*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^
3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqr
t(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1))) - integrate((((c^5*f^m*(m + 4)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^
m*(m - 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^6*f^m*(m + 4)*x^6 - c^4*
f^m*(5*m + 12)*x^4 + 4*c^2*f^m*(m + 1)*x^2 - f^m*m)*(c*x + 1)*sqrt(c*x - 1
)*x^m + (c^7*f^m*(m + 4)*x^7 - 3*c^5*f^m*(m + 3)*x^5 + 3*c^3*f^m*(m + 2)*x
^3 - c*f^m*(m + 1)*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*
x + 1)*(c*x - 1)*a*b*c^3*x^3 - 2*a*b*c^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 -
a*b*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^5 + (c*x + 1)*(c*x -
 1)*b^2*c^3*x^3 - 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*
sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(f\,x)^m\,(1-c^2\,x^2)^{3/2}}{(a+b\,\mathrm{acosh}\,(c\,x))^2}\,dx$$

input
```
int(((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int(((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.17

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\text{arccosh}(cx))^2}\,dx = f^m\left(-\left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^2}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{x^m\sqrt{-c^2x^2+1}}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)$$

input
```
int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
f**m*( - int((x**m*sqrt( - c**2*x**2 + 1)*x**2)/(acosh(c*x)**2*b**2 + 2*ac
osh(c*x)*a*b + a**2),x)*c**2 + int((x**m*sqrt( - c**2*x**2 + 1))/(acosh(c*
x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x))
```

3.303 $\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \text{Int}\left(\frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}(fx)^m}{(a+b\text{arccosh}(cx))^2} \, dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{\sqrt{1-c^2x^2}(fx)^m}{(a+b\text{arccosh}(cx))^2}dx$$

input
```
Int[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m \sqrt{-c^2x^2+1}}{(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}(fx)^m}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fr
icas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*
x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

input
```
integrate((f*x)**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral((f*x)**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 514, normalized size of antiderivative = 17.13

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}(fx)^m}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="ma
xima")
```

output
```
-((c^2*f^m*x^2 - f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f^m*x
)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x
- 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^
2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*f^m
*(m + 2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^4*f^m
*(m + 2)*x^4 - c^2*f^m*(3*m + 2)*x^2 + f^m*m)*(c*x + 1)*sqrt(c*x - 1)*x^m
+ (c^5*f^m*(m + 2)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^m*(m + 1)*x)*sqrt(c*x
 + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^3 -
 2*a*b*c^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*sqrt(c*x + 1)*sqr
t(c*x - 1) + (b^2*c^5*x^5 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^3 - 2*b^2*c^3*x^
3 + b^2*c*x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(f\,x)^m\,\sqrt{1-c^2\,x^2}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int(((f*x)^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

output
```
int(((f*x)^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{(a+b\text{arccosh}(cx))^2}\,dx=f^m\left(\int \frac{x^m\sqrt{-c^2x^2+1}}{acosh\left(cx\right)^2b^2+2acosh\left(cx\right)ab+a^2}dx\right)$$

input
```
int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
f**m*int((x**m*sqrt( - c**2*x**2 + 1))/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*
a*b + a**2),x)
```

3.304 $\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\,\mathbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \text{Int}\left(\frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6365}$$

$$\frac{fm\sqrt{cx-1}\int \frac{(fx)^{m-1}}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}(fx)^m}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

$$\Big\downarrow \text{6303}$$

$$\frac{fm\sqrt{cx-1}\int \frac{(fx)^{m-1}}{a+b\text{arccosh}(cx)}dx}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}(fx)^m}{bc\sqrt{1-cx}(a+b\text{arccosh}(cx))}$$

input
```
Int[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m}{\sqrt{-c^2x^2+1}\,(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fr
icas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*ar
ccosh(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 22.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))^2}\,dx$$

input
```
integrate((f*x)**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Integral((f*x)**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 544, normalized size of antiderivative = 18.13

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(fx)^m}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="ma
xima")
```

output
```
-((c^2*f^m*x^2 - f^m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f
^m*x)*x^m)/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x
+ 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-
c*x + 1)) + integrate(((c^3*f^m*m*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x -
1)*x^m + (2*c^4*f^m*m*x^4 - 3*c^2*f^m*m*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*
x - 1)*x^m + (c^5*f^m*m*x^5 - c^3*f^m*(2*m + 1)*x^3 + c*f^m*(m + 1)*x)*x^m
)/((((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*
(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x
 + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^
(3/2)*(c*x - 1)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt
(c*x - 1) + (a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*sqrt(c*x + 1))*sqrt(-c
*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(fx)^m}{\sqrt{-c^2x^2+1}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gi
ac")
```

output
```
integrate((f*x)^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{{\left(f\,x\right)}^m}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}} \,d x$$

input
```
int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^2}\,dx = f^m \left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,acosh\left(cx\right)^2 b^2+2\sqrt{-c^2x^2+1}\,acosh\left(cx\right)ab+\sqrt{-c^2x^2+1}\,a^2}d x \right)$$

input
```
int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*acosh(c*x))^2,x)
```

output
```
f**m*int(x**m/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*
x**2 + 1)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.305 $\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\text{Int}\left(\frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\text{arccosh}(cx))^2}dx$$

input
```
Int[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.60

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fr
icas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^
4*x^4 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b
*c^2*x^2 + a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 596, normalized size of antiderivative = 19.87

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="ma
xima")
```

output
```
(c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/((((c*x + 1)*sqrt(c*x -
 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a
*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate(((c^3*f^m*(
m - 2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 2)
*x^4 - c^2*f^m*(3*m - 2)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c
^5*f^m*(m - 2)*x^5 - c^3*f^m*(2*m - 1)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*
c^5*x^5 - b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^6 - 2*b^2*
c^4*x^4 + b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^7 - 3*b^2*c^5*
x^5 + 3*b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^5 - a*b*c^3*x^3)*(c*x + 1)^(3/2)*(
c*x - 1) + 2*(a*b*c^6*x^6 - 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c*x + 1)*sqrt(c*
x - 1) + (a*b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x
+ 1))*sqrt(-c*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gi
ac")
```

output
```
integrate((f*x)^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{{(f\,x)}^m}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.80

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\,(a+b\text{arccosh}(cx))^2}\,dx=$$
$$-f^m\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,acosh(cx)^2 b^2c^2x^2-\sqrt{-c^2x^2+1}\,acosh(cx)^2 b^2+2\sqrt{-c^2x^2+1}\,acosh(cx)\,ab\,c^2x^2-\right.$$

input
```
int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*acosh(c*x))^2,x)
```

output
```
- f**m*int(x**m/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**2*x**2 - sq
rt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acosh(c
*x)*a*b*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b + sqrt( - c**2
*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.306 $\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}(a+b\textbf{arccosh}(cx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}\,dx=\text{Int}\left(\frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2},x\right)$$

output
```
Defer(Int)((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}\,dx$$

input
```
Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}dx$$

$$\Big\downarrow \text{6375}$$

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}dx$$

input
```
Int[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{5}{2}}(a+b\,\text{arccosh}\,(cx))^2}dx$$

input
```
int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

output
```
int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.87

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fr
icas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*
c^2*x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)
^2 - a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c
*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x)**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 700, normalized size of antiderivative = 23.33

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="ma
xima")
```

output
```
-(c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/(((b^2*c^4*x^3 - b^2*
c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*
c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^
2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(((c^3*f^m*(m - 4)*x^
3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 4)*x^4 - c^
2*f^m*(3*m - 4)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m*(m
 - 4)*x^5 - c^3*f^m*(2*m - 3)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*c^7*x^7 -
 2*b^2*c^5*x^5 + b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^8 -
 3*b^2*c^6*x^6 + 3*b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b
^2*c^9*x^9 - 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 - 4*b^2*c^3*x^3 + b^2*c*x)*sqrt
(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c
^7*x^7 - 2*a*b*c^5*x^5 + a*b*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c
^8*x^8 - 3*a*b*c^6*x^6 + 3*a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x -
 1) + (a*b*c^9*x^9 - 4*a*b*c^7*x^7 + 6*a*b*c^5*x^5 - 4*a*b*c^3*x^3 + a*b*c
*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\text{arccosh}(cx))^2}\,dx = \int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="gi
ac")
```

output
```
integrate((f*x)^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}\,dx=\int \frac{{\left(f\,x\right)}^m}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}}\,dx$$

input
```
int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 7.20

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}\,(a+b\text{arccosh}(cx))^2}\,dx=f^m\left(\int \frac{}{\sqrt{-c^2x^2+1}\,acosh\left(cx\right)^2 b^2c^4x^4-2\sqrt{-c^2x^2+1}\,acosh\left(cx\right.}\right.$$

input
```
int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*acosh(c*x))^2,x)
```

output
```
f**m*int(x**m/(sqrt( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**4*x**4 - 2*sqr
t( - c**2*x**2 + 1)*acosh(c*x)**2*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*
acosh(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b*c**4*x**4 - 4
*sqrt( - c**2*x**2 + 1)*acosh(c*x)*a*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1
)*acosh(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 2*sqrt( - c**2*
x**2 + 1)*a**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.307 $\int \frac{x^3(d-c^2dx^2)}{(a+b\,\mathbf{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 259

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = \frac{2dx^3(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\operatorname{arccosh}(cx)}} + \frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} - \frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{6}\sqrt{a+b\operatorname{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{3de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} - \frac{de^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\operatorname{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}$$

output

```
2*d*x^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)+3/32*d*ex
p(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^
(3/2)/c^4-1/32*d*exp(6*a/b)*6^(1/2)*Pi^(1/2)*erf(6^(1/2)*(a+b*arccosh(c*x)
)^(1/2)/b^(1/2))/b^(3/2)/c^4+3/32*d*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arc
cosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4/exp(2*a/b)-1/32*d*6^(1/2)*Pi^(1/2)*e
rfi(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4/exp(6*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.16

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \frac{de^{-\frac{6a}{b}}\left(-\sqrt{6}\sqrt{-\frac{a+b\text{arccosh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{6(a+b\text{arccosh}(cx))}{b}\right)+3\sqrt{2}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{ar}}{}}\right.}{}$$

input
```
Integrate[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(d*(-(Sqrt[6]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCos
h[c*x]))/b]) + 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma
[1/2, (-2*(a + b*ArcCosh[c*x]))/b] + E^((6*a)/b)*(-64*c^3*x^3*Sqrt[(-1 + c
*x)/(1 + c*x)] - 64*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*Sqrt[2]*E^((2*a
)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c*x]))/b] + Sqr
t[6]*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x
]))/b] + 10*Sinh[2*ArcCosh[c*x]] + 8*Sinh[4*ArcCosh[c*x]] + 2*Sinh[6*ArcCo
sh[c*x]])))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6357, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(d-c^2dx^2\right)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \; 6357$$

$$-\frac{12cd\int \frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{6d\int \frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{2dx^3(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 6368$$

$$-\frac{12d\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^4}+\frac{6d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^4}+\frac{2dx^3(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{6d\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8\sqrt{a+b\text{arccosh}(cx)}}-\frac{1}{8\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c^4}-\frac{12d\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{a+b\text{arccosh}(cx)}}+\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{a+b\text{arccosh}(cx)}}-\frac{1}{16\sqrt{a+b\text{arccosh}(cx}}\right.}{b^2c^4}$$

$$\frac{2dx^3(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6d\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2c^4}-$$

$$\frac{12d\left(\frac{1}{64}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+bar}}{\sqrt{}}\right.\right.}{}$$

$$\frac{2dx^3(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

input
```
Int[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(2*d*x^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]])
+ (6*d*(-1/4*Sqrt[a + b*ArcCosh[c*x]] + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[
(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt
[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b))))/(b^2*c^4) - (12*d*(-1/8
*Sqrt[a + b*ArcCosh[c*x]] + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/64 + (Sqrt[b]*E^((6*a)/b)*Sqrt[Pi/6
]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/64 + (Sqrt[b]*Sqrt[Pi]*
Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*E^((4*a)/b)) - (Sqrt[b]*Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*E^((2*a)/b)
) + (Sqrt[b]*Sqrt[Pi/6]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(64*E^((6*a)/b))))/(b^2*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3(-c^2d\,x^2+d)}{(a+b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\mathrm{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = -d\left(\int \left(-\frac{x^3}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\right)dx + \int \frac{c^2x^5}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx\right)$$

input
```
integrate(x**3*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

output
```
-d*(Integral(-x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a +
b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int -\frac{(c^2dx^2-d)x^3}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima
")
```

output
```
-integrate((c^2*d*x^2 - d)*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = \int \frac{x^3\,(d-c^2\,d\,x^2)}{(a+b\,\mathrm{acosh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^3*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((x^3*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = d\left(-\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^5}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x^3*(-c^2*d*x^2+d)/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d*( - int((sqrt(acosh(c*x)*b + a)*x**5)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)
*a*b + a**2),x)*c**2 + int((sqrt(acosh(c*x)*b + a)*x**3)/(acosh(c*x)**2*b*
*2 + 2*acosh(c*x)*a*b + a**2),x))
```

3.308 $\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 340

$$\begin{aligned}\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx &= \frac{2dx^2(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}} \\ &+ \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} + \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} \\ &- \frac{de^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} \\ &+ \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{de^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}\end{aligned}$$

output
```
2*d*x^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)+1/8*d*exp
(a/b)*Pi^(1/2)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+1/16*d*ex
p(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^
(3/2)/c^3-1/16*d*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arccosh(c*x)
)^(1/2)/b^(1/2))/b^(3/2)/c^3+1/8*d*Pi^(1/2)*erfi((a+b*arccosh(c*x))^(1/2)/
b^(1/2))/b^(3/2)/c^3/exp(a/b)+1/16*d*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*ar
ccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/exp(3*a/b)-1/16*d*5^(1/2)*Pi^(1/2)*
erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/exp(5*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.15 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.13

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \frac{de^{-\frac{5a}{b}}\left(-4e^{\frac{5a}{b}}\sqrt{\frac{-1+cx}{1+cx}}-4ce^{\frac{5a}{b}}x\sqrt{\frac{-1+cx}{1+cx}}-2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\right.\right.}{}$$

input
```
Integrate[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(d*(-4*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*c*E^((5*a)/b)*x*Sqrt[(-1
 + c*x)/(1 + c*x)] - 2*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b
 + ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-5*
(a + b*ArcCosh[c*x]))/b] + Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])
/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a + b
*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - Sqrt[3]*E^((8*a
)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + Sqr
t[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[c*
x]))/b] - 2*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/b)*Sinh[5*ArcCos
h[c*x]]))/(16*b*c^3*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6357, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6357}$$

$$-\frac{10cd\int \frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{4d\int \frac{x\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{2dx^2(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 6368$$

$$-\frac{10d\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^3}+\frac{4d\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^3}+\frac{2dx^2(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 5971$$

$$\frac{4d\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c^3}-\frac{10d\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}+\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c^3}+\frac{2dx^2(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{4d\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccos}}}{\sqrt{b}}\right.\right.}{b^2c^3}$$

$$10d\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+ba}}{}\right.\right.$$

$$\frac{2dx^2(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

input
```
Int[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(2*d*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]])
+ (4*d*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b
]]) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]
])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])
/Sqrt[b]])/(8*E^((3*a)/b))))/(b^2*c^3) - (10*d*(-1/16*(Sqrt[b]*E^(a/b)*Sqr
t[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]) + (Sqrt[b]*E^((3*a)/b)*Sqrt[P
i/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((5*
a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/32 - (Sq
rt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*E^(a/b)) + (Sqr
t[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*E^((
3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(32*E^((5*a)/b))))/(b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^{2}\left(-c^{2} d \,x^{2}+d \right)}{\left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

input
```
int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = -d\left(\int \left(-\frac{x^2}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\right)dx + \int \frac{c^2x^4}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx\right)$$

input
```
integrate(x**2*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

output
```
-d*(Integral(-x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a +
b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int -\frac{(c^2dx^2-d)x^2}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima
")
```

output
```
-integrate((c^2*d*x^2 - d)*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int -\frac{(c^2dx^2-d)x^2}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{x^2\,(d-c^2\,d\,x^2)}{(a+b\,\text{acosh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^2*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((x^2*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=d\left(-\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^4}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2+\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^2}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x^2*(-c^2*d*x^2+d)/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d*( - int((sqrt(acosh(c*x)*b + a)*x**4)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)
*a*b + a**2),x)*c**2 + int((sqrt(acosh(c*x)*b + a)*x**2)/(acosh(c*x)**2*b*
*2 + 2*acosh(c*x)*a*b + a**2),x))
```

3.309 $\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 241

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \frac{2dx(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}} - \frac{de^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2} - \frac{de^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}$$

output
```
2*d*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)-1/4*d*exp(4
*a/b)*Pi^(1/2)*erf(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2+1/4*d*e
xp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b
^(3/2)/c^2-1/4*d*Pi^(1/2)*erfi(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)
/c^2/exp(4*a/b)+1/4*d*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/
2)/b^(1/2))/b^(3/2)/c^2/exp(2*a/b)
```

Mathematica [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

input
```
Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 3.99 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6357, 6322, 3042, 25, 3793, 2009, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \; 6357$$

$$-\frac{8cd\int \frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{2d\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 6322$$

$$\frac{2d\int \frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^2}-\frac{8cd\int \frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2d\int -\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^2}-\frac{8cd\int \frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2d\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^2}-\frac{8cd\int \frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{2d\int \left(\frac{1}{2\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c^2}-\frac{8cd\int \frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{8cd\int \frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2c^2}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 6368$$

$$-\frac{8d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^2}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2c^2}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \textit{5971}$$

$$-\frac{8d\int\left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8\sqrt{a+b\text{arccosh}(cx)}}-\frac{1}{8\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c^2}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2c^2}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \textit{2009}$$

$$-\frac{8d\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2c^2}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2c^2}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

input

```
Int[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

output

```
(2*d*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) -
(8*d*(-1/4*Sqrt[a + b*ArcCosh[c*x]] + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2
*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a
 + b*ArcCosh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b))))/(b^2*c^2) + (2*d*(-Sqrt[a
 + b*ArcCosh[c*x]] + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]])/(4*E^((2*a)/b))))/(b^2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6322
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c))*Simp[(d1 + e1*x)^p/
(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[Int[x^n*Sinh[-a/b + x
/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[2*p, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x(-c^2 d\,x^2 + d)}{(a + b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}} dx$$

input
```
int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d - c^2dx^2)}{(a + b\mathrm{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = -d\left(\int \left(-\frac{x}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\right)dx + \int \frac{c^2x^3}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx\right)$$

input
```
integrate(x*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

output
```
-d*(Integral(-x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh
(c*x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*a
cosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int -\frac{(c^2dx^2-d)x}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)*x/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{x\,(d-c^2\,d\,x^2)}{(a+b\,\text{acosh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((x*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d\left(-\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acosh\,(cx)\,b+a}\,x}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x*(-c^2*d*x^2+d)/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d*( - int((sqrt(acosh(c*x)*b + a)*x**3)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)
*a*b + a**2),x)*c**2 + int((sqrt(acosh(c*x)*b + a)*x)/(acosh(c*x)**2*b**2
+ 2*acosh(c*x)*a*b + a**2),x))
```

3.310 $\int \frac{d-c^2dx^2}{(a+b\,\text{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 233

$$\int \frac{d-c^2dx^2}{(a+b\,\text{arccosh}(cx))^{3/2}}\,dx = \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\,\text{arccosh}(cx)}} + \frac{3de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\,\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{3de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\,\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

output
```
2*d*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)+3/4*d*exp(a/b
)*Pi^(1/2)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-1/4*d*exp(3*a/b
)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c
+3/4*d*Pi^(1/2)*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(a/b)-
1/4*d*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3
/2)/c/exp(3*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06

$$\int \frac{d-c^2dx^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{3a}{b}}\left(-3de^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arccosh}(cx)\right)-\sqrt{3}d\sqrt{-\frac{a+b\text{arccos}}{b}}\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(-3*d*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]]
- Sqrt[3]*d*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[
c*x]))/b] + d*E^((2*a)/b)*(3*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -(
(a + b*ArcCosh[c*x])/b)] + E^(a/b)*(-6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
) + Sqrt[3]*E^((3*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcC
osh[c*x]))/b] + 2*Sinh[3*ArcCosh[c*x]])))/(4*b*c*E^((3*a)/b)*Sqrt[a + b*Ar
cCosh[c*x]])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6319, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-c^2dx^2}{(a+b\text{arccosh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6319}$$

$$\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}-\frac{6cd\int \frac{x\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}$$

$$\Big\downarrow \text{ 6368}$$

$$\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}-\frac{6d\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c}$$

$$\Big\downarrow \text{5971}$$

$$\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}-\frac{6d\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{4\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}-\frac{6d\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccos}}}{\sqrt{b}}\right.\right.}{b^2c}$$

input
```
Int[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(2*d*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (6
*d*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
+ (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*
E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(8*E^((3*a)/b))))/(b^2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6319
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p - 1/2)*(-
1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{-c^2 d\,x^2 + d}{(a + b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}} dx$$

input
```
int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d - c^2 d x^2}{(a + b\operatorname{arccosh}(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d - c^2 d x^2}{(a + b\operatorname{arccosh}(cx))^{3/2}} dx = -d\left(\int \frac{c^2 x^2}{a\sqrt{a + b\operatorname{acosh}(cx)} + b\sqrt{a + b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)} dx + \int \left(-\frac{1}{a\sqrt{a + b\operatorname{acosh}(cx)} + b\sqrt{a + b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right) dx\right)$$

input
```
integrate((-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

output
```
-d*(Integral(c**2*x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x)
)*acosh(c*x)), x) + Integral(-1/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*a
cosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{d-c^2dx^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int -\frac{c^2dx^2-d}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d-c^2dx^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int -\frac{c^2dx^2-d}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-c^2dx^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{d-c^2\,d\,x^2}{(a+b\,\text{acosh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d - c^2*d*x^2)/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((d - c^2*d*x^2)/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{d - c^2dx^2}{(a + b\text{arccosh}(cx))^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((-c^2*d*x^2+d)/(a+b*acosh(c*x))^(3/2),x)
```

output
```
(d*( - 3*acosh(c*x)*int((sqrt(acosh(c*x)*b + a)*x**4)/(acosh(c*x)**2*b**2*
c**2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)
*a*b + a**2*c**2*x**2 - a**2),x)*b**2*c**5 + 3*acosh(c*x)*int((sqrt(acosh(
c*x)*b + a)*x**2)/(acosh(c*x)**2*b**2*c**2*x**2 - acosh(c*x)**2*b**2 + 2*a
cosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**
2*c**3 + 6*acosh(c*x)*int((sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(acosh(c*x)*b +
 a)*acosh(c*x)*x**3)/(acosh(c*x)**2*b**2*c**2*x**2 - acosh(c*x)**2*b**2 +
2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*
b**2*c**4 + 6*acosh(c*x)*int((sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(acosh(c*x)*
b + a)*x**3)/(acosh(c*x)**2*b**2*c**2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(
c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b*c**4
 - 2*sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(acosh(c*x)*b + a)*c**2*x**2 - 4*sqrt
(c*x + 1)*sqrt(c*x - 1)*sqrt(acosh(c*x)*b + a) - 3*int((sqrt(acosh(c*x)*b
+ a)*x**4)/(acosh(c*x)**2*b**2*c**2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*
x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b*c**5 +
 3*int((sqrt(acosh(c*x)*b + a)*x**2)/(acosh(c*x)**2*b**2*c**2*x**2 - acosh
(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*
x**2 - a**2),x)*a*b*c**3 + 6*int((sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(acosh(c
*x)*b + a)*acosh(c*x)*x**3)/(acosh(c*x)**2*b**2*c**2*x**2 - acosh(c*x)**2*
b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*x**2 -...
```

3.311 $\int \frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx=\text{Int}\left(\frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

input
```
Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

output
```
Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 3.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6357}$$

$$-\frac{2d\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{4cd\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{6322}$$

$$-\frac{4d\int \frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2}-\frac{2d\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d\int -\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2}-\frac{2d\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4d\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2}-\frac{2d\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{4d\int \left(\frac{1}{2\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2}-\frac{2d\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d\int \frac{\sqrt{cx-1}\sqrt{cx+1}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{4d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{6370}$$

$$-\frac{2d\int \left(\frac{c^2}{\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\text{arccosh}(cx)}}-\frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\text{arccosh}(cx)}}\right)dx}{bc}-\frac{4d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d\left(\frac{2c\sqrt{a+b\text{arccosh}(cx)}}{b}-\int \frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\text{arccosh}(cx)}}dx\right)}{bc}-\frac{4d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(cx)}\right)}{b^2}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

input
```
Int[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{-c^2 d\, x^2 + d}{x\, (a + b \operatorname{arccosh}(cx))^{\frac{3}{2}}} dx$$

input
```
int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d - c^2 dx^2}{x(a + b\operatorname{arccosh}(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 7.90 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx = -d\left(\int \frac{c^2x^2}{ax\sqrt{a+b\,\text{acosh}\,(cx)}+bx\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx + \int \left(-\frac{1}{ax\sqrt{a+b\,\text{acosh}\,(cx)}+bx\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)/x/(a+b*acosh(c*x))**(3/2),x)
```

output
```
-d*(Integral(c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*acosh(
c*x))*acosh(c*x)), x) + Integral(-1/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt
(a + b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{d-c^2dx^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int -\frac{c^2dx^2-d}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d-c^2dx^2}{x(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{d-c^2dx^2}{x(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = \int \frac{d-c^2\,d\,x^2}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

input
```
int((d - c^2*d*x^2)/(x*(a + b*acosh(c*x))^(3/2)),x)
```

output
```
int((d - c^2*d*x^2)/(x*(a + b*acosh(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{d-c^2dx^2}{x(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = d\left(\int \frac{\sqrt{acosh\left(cx\right)b+a}}{acosh\left(cx\right)^{2}b^{2}x+2acosh\left(cx\right)abx+a^{2}x}dx - \left(\int \frac{\sqrt{acosh\left(cx\right)b+a}\,x}{acosh\left(cx\right)^{2}b^{2}+2acosh\left(cx\right)ab+a^{2}}dx\right)c^{2}\right)$$

input
```
int((-c^2*d*x^2+d)/x/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d*(int(sqrt(acosh(c*x)*b + a)/(acosh(c*x)**2*b**2*x + 2*acosh(c*x)*a*b*x +
 a**2*x),x) - int((sqrt(acosh(c*x)*b + a)*x)/(acosh(c*x)**2*b**2 + 2*acosh
(c*x)*a*b + a**2),x)*c**2)
```

3.312 $\int \frac{x^3(d-c^2dx^2)^2}{(a+b\mathbf{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 479

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = -\frac{2d^2x^3(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{3d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{-\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$

output
```
-2*d^2*x^3*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)-1/32*d
^2*exp(4*a/b)*Pi^(1/2)*erf(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4
+3/64*d^2*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)
/b^(1/2))/b^(3/2)/c^4+1/64*d^2*exp(8*a/b)*2^(1/2)*Pi^(1/2)*erf(2*2^(1/2)*(
a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4-1/64*d^2*exp(6*a/b)*6^(1/2)*P
i^(1/2)*erf(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4-1/32*d^2
*Pi^(1/2)*erfi(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4/exp(4*a/b)+
3/64*d^2*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b
^(3/2)/c^4/exp(2*a/b)+1/64*d^2*2^(1/2)*Pi^(1/2)*erfi(2*2^(1/2)*(a+b*arccos
h(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4/exp(8*a/b)-1/64*d^2*6^(1/2)*Pi^(1/2)*er
fi(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4/exp(6*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 2.59 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.10

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx =$$

$$-\frac{d^2e^{-\frac{8a}{b}}\left(128c^3e^{\frac{8a}{b}}x^3\sqrt{\frac{-1+cx}{1+cx}}+128c^4e^{\frac{8a}{b}}x^4\sqrt{\frac{-1+cx}{1+cx}}-\sqrt{2}\sqrt{-\frac{a+b\text{arccosh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{8(a+b\text{arccosh}(cx))}{b}\right)+\sqrt{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
-1/64*(d^2*(128*c^3*E^((8*a)/b)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 128*c^4*E
^((8*a)/b)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - Sqrt[2]*Sqrt[-((a + b*ArcCosh[
c*x])/b)]*Gamma[1/2, (-8*(a + b*ArcCosh[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sq
rt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh[c*x]))/b] + 2*
E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[
c*x]))/b] - 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/
2, (-2*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[2]*E^((10*a)/b)*Sqrt[a/b + ArcCos
h[c*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c*x]))/b] - 2*E^((12*a)/b)*Sqrt[a/b +
 ArcCosh[c*x]]*Gamma[1/2, (4*(a + b*ArcCosh[c*x]))/b] - Sqrt[6]*E^((14*a)/
b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b] + Sqrt[
2]*E^((16*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (8*(a + b*ArcCosh[c*x]
))/b] - 26*E^((8*a)/b)*Sinh[2*ArcCosh[c*x]] - 18*E^((8*a)/b)*Sinh[4*ArcCos
h[c*x]] - 2*E^((8*a)/b)*Sinh[6*ArcCosh[c*x]] + E^((8*a)/b)*Sinh[8*ArcCosh[
c*x]]))/(b*c^4*E^((8*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {6357, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(d-c^2 d x^2\right)^2}{(a+b\text{arccosh}(cx))^{3/2}} \, dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{16 c d^2 \int \frac{x^4 (cx-1)^{3/2} (cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{6 d^2 \int \frac{x^2 (cx-1)^{3/2} (cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{2 d^2 x^3 (cx-1)^{5/2} (cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{6368}$$

$$\frac{16d^2\int\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^4}-\frac{6d^2\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^4}-\frac{2d^2x^3(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{16d^2\int\left(\frac{\cosh\left(\frac{8a}{b}-\frac{8(a+b\text{arccosh}(cx))}{b}\right)}{128\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{a+b\text{arccosh}(cx)}}+\frac{3}{128\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c^4}-\frac{6d^2\int\left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{a+b\text{arccosh}(cx)}}+\frac{1}{16\sqrt{a+b\text{arccosh}(cx}}\right.}{b^2c^4}\frac{2d^2x^3(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16d^2\left(-\frac{1}{128}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{512}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{8a}{b}}\text{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{128}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(2\right.\right.}{b^2c^4}$$

$$\frac{6d^2\left(-\frac{1}{64}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+ba}}{}\right.\right.}{}$$

$$\frac{2d^2x^3(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

input

```
Int[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(-2*d^2*x^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]
]) + (16*d^2*((3*Sqrt[a + b*ArcCosh[c*x]])/64 - (Sqrt[b]*E^((4*a)/b)*Sqrt[
Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/128 + (Sqrt[b]*E^((8*a)/b)*
Sqrt[Pi/2]*Erf[(2*Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/512 - (Sqrt[
b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(128*E^((4*a)/b))
+ (Sqrt[b]*Sqrt[Pi/2]*Erfi[(2*Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(512*E^((8*a)/b))))/(b^2*c^4) - (6*d^2*(Sqrt[a + b*ArcCosh[c*x]]/8 - (Sqrt
[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/64 - (
Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt
[b]])/64 + (Sqrt[b]*E^((6*a)/b)*Sqrt[Pi/6]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh
[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])
/Sqrt[b]])/(64*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcCosh[c*x]])/Sqrt[b]])/(64*E^((2*a)/b)) + (Sqrt[b]*Sqrt[Pi/6]*Erfi[(Sqr
t[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*E^((6*a)/b))))/(b^2*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3(-c^2d\,x^2+d)^2}{(a+b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\mathrm{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fric
as")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x^3}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx + \int \left(-\frac{2c^2x^5}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx + \int \frac{c^4x^7}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx\right)$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*a
cosh(c*x)), x) + Integral(-2*c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(
a + b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*acosh
(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2-d)^2x^3}{(b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((c^2*d*x^2 - d)^2*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{x^3\,(d-c^2\,d\,x^2)^2}{(a+b\,\text{acosh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^3*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((x^3*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^7}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^5}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x^3*(-c^2*d*x^2+d)^2/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d**2*(int((sqrt(acosh(c*x)*b + a)*x**7)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)
*a*b + a**2),x)*c**4 - 2*int((sqrt(acosh(c*x)*b + a)*x**5)/(acosh(c*x)**2*
b**2 + 2*acosh(c*x)*a*b + a**2),x)*c**2 + int((sqrt(acosh(c*x)*b + a)*x**3
)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x))
```

3.313 $\int \frac{x^2(d-c^2dx^2)^2}{(a+b\textbf{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 462

$$\begin{aligned}\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx &= -\frac{2d^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}} \\ &\quad+\frac{5d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} \\ &\quad-\frac{3d^2e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{d^2e^{\frac{7a}{b}}\sqrt{7\pi}\text{erf}\left(\frac{\sqrt{7}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} \\ &\quad+\frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} \\ &\quad-\frac{3d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{d^2e^{-\frac{7a}{b}}\sqrt{7\pi}\text{erfi}\left(\frac{\sqrt{7}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}\end{aligned}$$

output
```
-2*d^2*x^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)+5/64*d
^2*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+1/6
4*d^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(
1/2))/b^(3/2)/c^3-3/64*d^2*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*ar
ccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+1/64*d^2*exp(7*a/b)*7^(1/2)*Pi^(1/2
)*erf(7^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+5/64*d^2*Pi^(1
/2)*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/exp(a/b)+1/64*d^2*3
^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3
/exp(3*a/b)-3/64*d^2*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2
)/b^(1/2))/b^(3/2)/c^3/exp(5*a/b)+1/64*d^2*7^(1/2)*Pi^(1/2)*erfi(7^(1/2)*(
a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/exp(7*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.08

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx =$$

$$-\frac{d^2e^{-\frac{7a}{b}}\left(10e^{\frac{7a}{b}}\sqrt{\frac{-1+cx}{1+cx}}+10ce^{\frac{7a}{b}}x\sqrt{\frac{-1+cx}{1+cx}}+5e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\operatorname{arccosh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arccosh}(cx)\right)-\sqrt{7}\sqrt{-\frac{a+ba}{}}\right.}{}$$

input
```
Integrate[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
-1/64*(d^2*(10*E^((7*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 10*c*E^((7*a)/b)*x
*Sqrt[(-1 + c*x)/(1 + c*x)] + 5*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma
[1/2, a/b + ArcCosh[c*x]] - Sqrt[7]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[
1/2, (-7*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[5]*E^((2*a)/b)*Sqrt[-((a + b*Ar
cCosh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcCosh[c*x]))/b] - Sqrt[3]*E^((4*a)
/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b
] - 5*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcC
osh[c*x])/b)] + Sqrt[3]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (
3*(a + b*ArcCosh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b)*Sqrt[a/b + ArcCosh[c*x
]]*Gamma[1/2, (5*(a + b*ArcCosh[c*x]))/b] + Sqrt[7]*E^((14*a)/b)*Sqrt[a/b
+ ArcCosh[c*x]]*Gamma[1/2, (7*(a + b*ArcCosh[c*x]))/b] + 2*E^((7*a)/b)*Sin
h[3*ArcCosh[c*x]] - 6*E^((7*a)/b)*Sinh[5*ArcCosh[c*x]] + 2*E^((7*a)/b)*Sin
h[7*ArcCosh[c*x]]))/(b*c^3*E^((7*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {6357, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{14cd^2\int \frac{x^3(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{4d^2\int \frac{x(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{2d^2x^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{6368}$$

$$\frac{14d^2\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^3}-\frac{4d^2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^3}-\frac{2d^2x^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{14d^2\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{64\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{64\sqrt{a+b\text{arccosh}(cx)}}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{64\sqrt{a+b\text{arccosh}(cx)}}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arc}\cdots}{}\right)}{64\sqrt{a+b\text{arcc}\cdots}}\cdots}{b^2c^3}$$

$$\frac{4d^2\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}+\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx)\cdots}{b^2c^3}$$

$$\frac{2d^2x^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4d^2\left(\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cdots}}{\cdots}\right.\right.}{\cdots}$$

$$\frac{14d^2\left(\frac{3}{128}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{128}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{128}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a\cdots}}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2d^2x^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

input
```
Int[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(-2*d^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]
]) - (4*d^2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b
]])/16 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]])/(16*E^(a/b)) - (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(S
qrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/(b^2*c^3) +
(14*d^2*((3*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]
)/128 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*
x]])/Sqrt[b]])/128 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]])/128 + (Sqrt[b]*E^((7*a)/b)*Sqrt[Pi/7]*Erf[(Sqr
t[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/128 + (3*Sqrt[b]*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(128*E^(a/b)) - (Sqrt[b]*Sqrt[3*Pi]*Erfi[
(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(128*E^((3*a)/b)) - (Sqrt[b]*
Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(128*E^((5*a)
/b)) + (Sqrt[b]*Sqrt[Pi/7]*Erfi[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(128*E^((7*a)/b))))/(b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2(-c^2d\,x^2+d)^2}{(a+b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fric
as")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x^2}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx + \int \left(-\frac{2c^2x^4}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\right)dx + \int \frac{c^4x^6}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx\right)$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*a
cosh(c*x)), x) + Integral(-2*c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(
a + b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*acosh
(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2x^2}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((c^2*d*x^2 - d)^2*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2x^2}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac
")
```

output
```
integrate((c^2*d*x^2 - d)^2*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{x^2\,(d-c^2\,d\,x^2)^2}{(a+b\,\text{acosh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^2*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((x^2*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^6}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^4}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^2}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x^2*(-c^2*d*x^2+d)^2/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d**2*(int((sqrt(acosh(c*x)*b + a)*x**6)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)
*a*b + a**2),x)*c**4 - 2*int((sqrt(acosh(c*x)*b + a)*x**4)/(acosh(c*x)**2*
b**2 + 2*acosh(c*x)*a*b + a**2),x)*c**2 + int((sqrt(acosh(c*x)*b + a)*x**2
)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x))
```

3.314 $\int \frac{x(d-c^2dx^2)^2}{(a+b\,\mathrm{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 363

$$\begin{aligned}\int \frac{x(d-c^2dx^2)^2}{(a+b\,\mathrm{arccosh}(cx))^{3/2}}\,dx &= -\frac{2d^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\,\mathrm{arccosh}(cx)}} \\ &- \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\,\mathrm{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} \\ &+ \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{6}\sqrt{a+b\,\mathrm{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} - \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2\sqrt{a+b\,\mathrm{arccosh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} \\ &+ \frac{5d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} + \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\,\mathrm{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}\end{aligned}$$

output
```
-2*d^2*x*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)-1/4*d^2*
exp(4*a/b)*Pi^(1/2)*erf(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2+5/
32*d^2*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^
(1/2))/b^(3/2)/c^2+1/32*d^2*exp(6*a/b)*6^(1/2)*Pi^(1/2)*erf(6^(1/2)*(a+b*a
rccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2-1/4*d^2*Pi^(1/2)*erfi(2*(a+b*arcco
sh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2/exp(4*a/b)+5/32*d^2*2^(1/2)*Pi^(1/2)*e
rfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2/exp(2*a/b)+1/32*
d^2*6^(1/2)*Pi^(1/2)*erfi(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2
)/c^2/exp(6*a/b)
```

Mathematica [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

input
```
Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 4.06 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.56, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.296$, Rules used = {6357, 6322, 3042, 3793, 2009, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{12cd^2\int \frac{x^2(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{2d^2\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 6322$$

$$-\frac{2d^2\int \frac{\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2d^2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^4}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{2d^2\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(cx)}}+\frac{3}{8\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{12cd^2\int \frac{x^2(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcc}}}{\sqrt{b}}\right.\right.}{b^2c^2}$$

$$\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \; 6368$$

$$\frac{12d^2\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c^2}-$$

$$\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arc}}}{\sqrt{b}}\right.\right.}{b^2c^2}$$

$$\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$\Big\downarrow$ 5971

$$\frac{12d^2\int\left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{a+b\text{arccosh}(cx)}}+\frac{1}{16\sqrt{a+b\text{arccosh}(c}}\right.}{b^2c^2}$$

$$\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arc}}}{\sqrt{b}}\right.\right.}{b^2c^2}$$

$$\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$\Big\downarrow$ 2009

$$-\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{ar}}}{}\right.\right.}{b^2c^2}$$

$$\frac{12d^2\left(-\frac{1}{64}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+}}{}\right.\right.}{}$$

$$\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

input
```
Int[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(-2*d^2*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]])
 - (2*d^2*((3*Sqrt[a + b*ArcCosh[c*x]])/4 + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*
Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((2*a)/b)*Sqrt[
Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[P
i]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b)) - (Sqrt[b]
*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*E^((2*a)/
b))))/(b^2*c^2) + (12*d^2*(Sqrt[a + b*ArcCosh[c*x]]/8 - (Sqrt[b]*E^((4*a)/
b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/64 + (S
qrt[b]*E^((6*a)/b)*Sqrt[Pi/6]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/64 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(6
4*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]
])/Sqrt[b]])/(64*E^((2*a)/b)) + (Sqrt[b]*Sqrt[Pi/6]*Erfi[(Sqrt[6]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]])/(64*E^((6*a)/b))))/(b^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6322
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c))*Simp[(d1 + e1*x)^p/
(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[Int[x^n*Sinh[-a/b + x
/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[2*p, 0]
```

rule 6357
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[(f
*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((
1 + c*x)^p*(-1 + c*x)^p)]   Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x
)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m +
2*p + 1, 0] && IGtQ[m, -3]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x(-c^2 d\,x^2+d)^2}{(a+b\ \text{arccosh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx + \int \left(-\frac{2c^2x^3}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx + \int \frac{c^4x^5}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx\right)$$

input
```
integrate(x*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acos
h(c*x)), x) + Integral(-2*c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a +
 b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2-d)^2x}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c^2*d*x^2 - d)^2*x/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{x\,(d-c^2\,d\,x^2)^2}{(a+b\,\text{acosh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((x*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^5}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acosh\,(cx)\,b+a}\,x}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x*(-c^2*d*x^2+d)^2/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d**2*(int((sqrt(acosh(c*x)*b + a)*x**5)/(acosh(c*x)**2*b**2 + 2*acosh(c*x)
*a*b + a**2),x)*c**4 - 2*int((sqrt(acosh(c*x)*b + a)*x**3)/(acosh(c*x)**2*
b**2 + 2*acosh(c*x)*a*b + a**2),x)*c**2 + int((sqrt(acosh(c*x)*b + a)*x)/(
acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x))
```

3.315 $\int \frac{(d-c^2dx^2)^2}{(a+b\textbf{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 351

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{d^2e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$

output
```
-2*d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)+5/8*d^2*ex
p(a/b)*Pi^(1/2)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-5/16*d^2*e
xp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b
^(3/2)/c+1/16*d^2*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arccosh(c*x
))^(1/2)/b^(1/2))/b^(3/2)/c+5/8*d^2*Pi^(1/2)*erfi((a+b*arccosh(c*x))^(1/2)
/b^(1/2))/b^(3/2)/c/exp(a/b)-5/16*d^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*a
rccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(3*a/b)+1/16*d^2*5^(1/2)*Pi^(1/2)
*erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(5*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.45 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.10

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx =$$
$$-\frac{d^2e^{-\frac{5a}{b}}\left(20e^{\frac{5a}{b}}\sqrt{\frac{-1+cx}{1+cx}}+20ce^{\frac{5a}{b}}x\sqrt{\frac{-1+cx}{1+cx}}+10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arccosh}(cx)\right)-\sqrt{5}\sqrt{-\frac{a+b}{}}\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
-1/16*(d^2*(20*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 20*c*E^((5*a)/b)*x
*Sqrt[(-1 + c*x)/(1 + c*x)] + 10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, a/b + ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma
[1/2, (-5*(a + b*ArcCosh[c*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*A
rcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 10*E^((4*a)/b)*
Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - 5*
Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[
c*x]))/b] + Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a
 + b*ArcCosh[c*x]))/b] - 10*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/
b)*Sinh[5*ArcCosh[c*x]]))/(b*c*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6319, 6368, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx$$
$$\Big\downarrow \text{6319}$$

$$\frac{10cd^2\int \frac{x(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$\Big\downarrow$ 6368

$$\frac{10d^2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2c}-\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$\Big\downarrow$ 5971

$$\frac{10d^2\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(cx)}}+\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx}{b^2c}$$
$$\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

$\Big\downarrow$ 2009

$$\frac{10d^2\left(\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+ba}}{}\right.\right.}{}$$
$$\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\text{arccosh}(cx)}}$$

input
```
Int[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

output
```
(-2*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
 (10*d^2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/16 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]
])/Sqrt[b]])/32 + (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]])/(16*E^(a/b)) - (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt
[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/(b^2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6319
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]   Int[x*(1 + c*x)^(p - 1/2)*(-
1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(-c^2 d\,x^2 + d)^2}{(a + b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\int \left(-\frac{2c^2x^2}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\right)dx + \int \frac{c^4x^4}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx + \int \frac{1}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(-2*c**2*x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh
(c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*acosh(c*x)) + b*
sqrt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2}{(b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 - d)^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2}{(b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 - d)^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx=\int \frac{(d-c^2\,d\,x^2)^2}{(a+b\operatorname{acosh}(c\,x))^{3/2}}\,dx$$

input
```
int((d - c^2*d*x^2)^2/(a + b*acosh(c*x))^(3/2),x)
```

output
```
int((d - c^2*d*x^2)^2/(a + b*acosh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((-c^2*d*x^2+d)^2/(a+b*acosh(c*x))^(3/2),x)
```

output
```
(d**2*(2*acosh(c*x)*int((sqrt(acosh(c*x)*b + a)*x**6)/(acosh(c*x)**2*b**2*
c**2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)
*a*b + a**2*c**2*x**2 - a**2),x)*b**2*c**7 - 7*acosh(c*x)*int((sqrt(acosh(
c*x)*b + a)*x**4)/(acosh(c*x)**2*b**2*c**2*x**2 - acosh(c*x)**2*b**2 + 2*a
cosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**
2*c**5 + 5*acosh(c*x)*int((sqrt(acosh(c*x)*b + a)*x**2)/(acosh(c*x)**2*b**
2*c**2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*
x)*a*b + a**2*c**2*x**2 - a**2),x)*b**2*c**3 + 6*acosh(c*x)*int((sqrt(c*x
+ 1)*sqrt(c*x - 1)*sqrt(acosh(c*x)*b + a)*acosh(c*x)*x**3)/(acosh(c*x)**2*
b**2*c**2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh
(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**2*c**4 + 6*acosh(c*x)*int((sqrt(c
*x + 1)*sqrt(c*x - 1)*sqrt(acosh(c*x)*b + a)*x**3)/(acosh(c*x)**2*b**2*c**
2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*
b + a**2*c**2*x**2 - a**2),x)*a*b*c**4 - 2*sqrt(c*x + 1)*sqrt(c*x - 1)*sqr
t(acosh(c*x)*b + a)*c**2*x**2 - 4*sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(acosh(c
*x)*b + a) + 2*int((sqrt(acosh(c*x)*b + a)*x**6)/(acosh(c*x)**2*b**2*c**2*
x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**2*x**2 - 2*acosh(c*x)*a*b
+ a**2*c**2*x**2 - a**2),x)*a*b*c**7 - 7*int((sqrt(acosh(c*x)*b + a)*x**4)
/(acosh(c*x)**2*b**2*c**2*x**2 - acosh(c*x)**2*b**2 + 2*acosh(c*x)*a*b*c**
2*x**2 - 2*acosh(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b*c**5 + 5*int(...
```

3.316 $\int \frac{(d-c^2dx^2)^2}{x(a+b\,\textbf{arccosh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = \operatorname{Int}\left(\frac{(d-c^2dx^2)^2}{x(a+b\operatorname{arccosh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx = \int \frac{(d-c^2dx^2)^2}{x(a+b\operatorname{arccosh}(cx))^{3/2}}\,dx$$

input
```
Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

output
```
Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6357}$$

$$\frac{2d^2\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+\frac{8cd^2\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{\sqrt{a+b\text{arccosh}(cx)}}dx}{b}-\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{6322}$$

$$\frac{8d^2\int \frac{\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2}+\frac{2d^2\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8d^2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^4}{\sqrt{a+b\text{arccosh}(cx)}}d(a+b\text{arccosh}(cx))}{b^2}+\frac{2d^2\int \frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{8d^2\int\left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{8\sqrt{a+b\text{arccosh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(cx)}}+\frac{3}{8\sqrt{a+b\text{arccosh}(cx)}}\right)d(a+b\text{arccosh}(cx))}{b^2}+$$
$$\frac{2d^2\int\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}-\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2\int\frac{(cx-1)^{3/2}(cx+1)^{3/2}}{x^2\sqrt{a+b\text{arccosh}(cx)}}dx}{bc}+$$
$$\frac{8d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arc}}}{\sqrt{b}}\right.\right.}{b^2}$$
$$\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{6370}$$

$$\frac{2d^2\int\left(\frac{x^2c^4}{\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\text{arccosh}(cx)}}-\frac{2c^2}{\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\text{arccosh}(cx)}}+\frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\text{arccosh}(cx)}}\right)dx}{bc}+$$
$$\frac{8d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arc}}}{\sqrt{b}}\right.\right.}{b^2}$$
$$\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2\left(\int\frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\text{arccosh}(cx)}}dx+\frac{\sqrt{\frac{\pi}{2}}ce^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}ce^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}}-\right.}{bc}$$
$$\frac{8d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arc}}}{\sqrt{b}}\right.\right.}{b^2}$$
$$\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bcx\sqrt{a+b\text{arccosh}(cx)}}$$

input
```
Int[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\left(-c^{2} d \,x^{2}+d \right)^{2}}{x \left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

input
```
int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

output
```
int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 8.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\int \left(-\frac{2c^2x^2}{ax\sqrt{a+b\,\text{acosh}\,(cx)}+bx\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\right)dx + \int \frac{c^4x^4}{ax\sqrt{a+b\,\text{acosh}\,(cx)}+bx\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx + \int \frac{1}{ax\sqrt{a+b\,\text{acosh}\,(cx)}+bx\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2/x/(a+b*acosh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(-2*c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*a
cosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*acosh(c*x)
) + b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*x*sqrt(a +
b*acosh(c*x)) + b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2-d)^2}{(b\,\text{arcosh}\,(cx)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c^2*d*x^2 - d)^2/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 4.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx = \int \frac{\left(d-c^2\,d\,x^2\right)^2}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{3/2}}\,dx$$

input
```
int((d - c^2*d*x^2)^2/(x*(a + b*acosh(c*x))^(3/2)),x)
```

output
```
int((d - c^2*d*x^2)^2/(x*(a + b*acosh(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.48

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\text{arccosh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}}{acosh\,(cx)^2\,b^2x+2acosh\,(cx)\,abx+a^2x}dx + \left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x^3}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acosh\,(cx)\,b+a}\,x}{acosh\,(cx)^2\,b^2+2acosh\,(cx)\,ab+a^2}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^2/x/(a+b*acosh(c*x))^(3/2),x)
```

output
```
d**2*(int(sqrt(acosh(c*x)*b + a)/(acosh(c*x)**2*b**2*x + 2*acosh(c*x)*a*b*
x + a**2*x),x) + int((sqrt(acosh(c*x)*b + a)*x**3)/(acosh(c*x)**2*b**2 + 2
*acosh(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt(acosh(c*x)*b + a)*x)/(acosh(
c*x)**2*b**2 + 2*acosh(c*x)*a*b + a**2),x)*c**2)
```

3.317 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx=\frac{\sqrt{\pi}\sqrt{-1+x}\,\operatorname{erf}\left(\sqrt{\operatorname{arccosh}(x)}\right)}{2\sqrt{1-x}}+\frac{\sqrt{\pi}\sqrt{-1+x}\,\operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(x)}\right)}{2\sqrt{1-x}}$$

output
```
1/2*Pi^(1/2)*(-1+x)^(1/2)*erf(arccosh(x)^(1/2))/(1-x)^(1/2)+1/2*Pi^(1/2)*(
-1+x)^(1/2)*erfi(arccosh(x)^(1/2))/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx=-\frac{\sqrt{-((-1+x)(1+x))}\left(\sqrt{-\operatorname{arccosh}(x)}\,\Gamma\left(\frac{1}{2},-\operatorname{arccosh}(x)\right)-\sqrt{\operatorname{arccosh}(x)}\,\Gamma\left(\frac{1}{2},\operatorname{arccosh}(x)\right)\right)}{2\sqrt{\frac{-1+x}{1+x}}(1+x)\sqrt{\operatorname{arccosh}(x)}}$$

input
```
Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

output
```
-1/2*(Sqrt[-((-1 + x)*(1 + x))]*(Sqrt[-ArcCosh[x]]*Gamma[1/2, -ArcCosh[x]]
 - Sqrt[ArcCosh[x]]*Gamma[1/2, ArcCosh[x]]))/(Sqrt[(-1 + x)/(1 + x)]*(1 +
x)*Sqrt[ArcCosh[x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6367, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\text{arccosh}(x)}}dx$$

$$\Big\downarrow \text{6367}$$

$$\frac{\sqrt{x-1}\int \frac{x}{\sqrt{\text{arccosh}(x)}}d\text{arccosh}(x)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{x-1}\int \frac{\sin\left(i\text{arccosh}(x)+\frac{\pi}{2}\right)}{\sqrt{\text{arccosh}(x)}}d\text{arccosh}(x)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\sqrt{x-1}\left(\frac{1}{2}i\int -\frac{ie^{\text{arccosh}(x)}}{\sqrt{\text{arccosh}(x)}}d\text{arccosh}(x)-\frac{1}{2}i\int \frac{ie^{-\text{arccosh}(x)}}{\sqrt{\text{arccosh}(x)}}d\text{arccosh}(x)\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{x-1}\left(\frac{1}{2}\int \frac{e^{-\text{arccosh}(x)}}{\sqrt{\text{arccosh}(x)}}d\text{arccosh}(x)+\frac{1}{2}\int \frac{e^{\text{arccosh}(x)}}{\sqrt{\text{arccosh}(x)}}d\text{arccosh}(x)\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\sqrt{x-1}\left(\int e^{-\operatorname{arccosh}(x)}d\sqrt{\operatorname{arccosh}(x)}+\int e^{\operatorname{arccosh}(x)}d\sqrt{\operatorname{arccosh}(x)}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{x-1}\left(\int e^{-\operatorname{arccosh}(x)}d\sqrt{\operatorname{arccosh}(x)}+\frac{1}{2}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(x)}\right)\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt{x-1}\left(\frac{1}{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arccosh}(x)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(x)}\right)\right)}{\sqrt{1-x}}$$

input
```
Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

output
```
(Sqrt[-1 + x]*((Sqrt[Pi]*Erf[Sqrt[ArcCosh[x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[Ar
cCosh[x]]])/2))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{-x^2+1}\,\sqrt{\operatorname{arccosh}(x)}}dx$$

input
```
int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
```

output
```
int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx = \int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\operatorname{acosh}(x)}}\,dx$$

input
```
integrate(x/(-x**2+1)**(1/2)/acosh(x)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acosh(x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx = \int \frac{x}{\sqrt{-x^2+1}\sqrt{\operatorname{arcosh}(x)}}\,dx$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx = \int \frac{x}{\sqrt{-x^2+1}\sqrt{\operatorname{arcosh}(x)}}\,dx$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx = \int \frac{x}{\sqrt{\operatorname{acosh}(x)}\,\sqrt{1-x^2}}\,dx$$

input
```
int(x/(acosh(x)^(1/2)*(1 - x^2)^(1/2)),x)
```

output
```
int(x/(acosh(x)^(1/2)*(1 - x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\operatorname{arccosh}(x)}}\,dx = -\left(\int \frac{\sqrt{-x^2+1}\,\sqrt{acosh(x)}\,x}{acosh(x)\,x^2-acosh(x)}dx\right)$$

input
```
int(x/(-x^2+1)^(1/2)/acosh(x)^(1/2),x)
```

output
```
- int((sqrt( - x**2 + 1)*sqrt(acosh(x))*x)/(acosh(x)*x**2 - acosh(x)),x)
```

3.318 $\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 253

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=-\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2^{-2(3+n)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2^{-2(3+n)}e^{\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/8*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c^3/(1+n)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-4*a
-4*b*arccosh(c*x))/b)/(2^(6+2*n))/c^3/exp(4*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)/((-(a+b*arccosh(c*x))/b)^n)-exp(4*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)/(2^(6+2*n))/c^3/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.72

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx =$$
$$-\frac{d\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{8(a+b\text{arccosh}(cx))}{b(1+n)}+4^{-n}e^{-\frac{4a}{b}}\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\text{a}\right.\right.\right.}{64c^3\sqrt{-d(-1}}$$

input
```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
-1/64*(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((-8*
(a + b*ArcCosh[c*x]))/(b*(1 + n)) + ((a/b + ArcCosh[c*x])^n*Gamma[1 + n, (
-4*(a + b*ArcCosh[c*x]))/b] - E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*Ga
mma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcCosh
[c*x])^2/b^2))^n)))/(c^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{6367}$$

$$\frac{\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\sqrt{d-c^2dx^2}\int\left(\frac{1}{8}(a+b\text{arccosh}(cx))^n\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}(a+b\text{arccosh}(cx))^n\right)d(a+b\text{arccosh}(cx)}{bc^3\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{(a+b\text{arccosh}(cx))^{n+1}}{8(n+1)}+b2^{-2(n+3)}e^{-\frac{4a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+ba}{}\right.\right.}{bc^3\sqrt{cx-1}\sqrt{cx-}}$$

input
```
Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-1/8*(a + b*ArcCosh[c*x])^(1 + n)/(1 + n) + (b*(a +
b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n)
)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (b*E^((4*a)/b)*(a + b*ArcCo
sh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*((a +
b*ArcCosh[c*x])/b)^n)))/(b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2\sqrt{-c^2d\,x^2+d}\,(a+b\,\mathrm{arccosh}\,(cx))^n\,dx$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\mathrm{arccosh}(cx))^n\,dx=\int \sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)^nx^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\mathrm{arccosh}(cx))^n\,dx$$
$$=\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acosh}\,(cx))^n\,dx$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Maxima [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac
")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\int x^2\,(a+b\,\text{acosh}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\sqrt{d}\left(\int(a\cosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)
```

3.319 $\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 27, antiderivative size = 379

$$\begin{aligned}
&\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx\\
&=\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}\\
&-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}\\
&+\frac{e^{a/b}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arccosh}(cx)}{b}\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}\\
&-\frac{3^{-1-n}e^{\frac{3a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
1/8*3^(-1-n)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-3*a-3*b
*arccosh(c*x))/b)/c^2/exp(3*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arcco
sh(c*x))/b)^n)-1/8*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-(a
+b*arccosh(c*x))/b)/c^2/exp(a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arcco
sh(c*x))/b)^n)+1/8*exp(a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMM
A(1+n,(a+b*arccosh(c*x))/b)/c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh
(c*x))/b)^n)-1/8*3^(-1-n)*exp(3*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x
))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)/c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((
a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.64

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx =$$

$$-\frac{de^{-\frac{3a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(3e^{\frac{4a}{b}}\left(\frac{a}{b}+\text{arccosh}(cx)\right)^{-n}\Gamma\left(1+n,\frac{a}{b}+\text{arccosh}(cx)\right)+\left(\right.\right.}{}$$

input
```
Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
-1/24*(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((3*E
^((4*a)/b)*Gamma[1 + n, a/b + ArcCosh[c*x]])/(a/b + ArcCosh[c*x])^n + (Gam
ma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b]/3^n - 3*E^((2*a)/b)*Gamma[1 + n, -(
(a + b*ArcCosh[c*x])/b)] - (E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*
Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/(3^n*(-((a + b*ArcCosh[c*x])^2/b
^2))^n))/(-((a + b*ArcCosh[c*x])/b))^n))/(c^2*E^((3*a)/b)*Sqrt[-(d*(-1 + c
*x)*(1 + c*x))])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\sqrt{d-c^2dx^2}\int\left(\frac{1}{4}(a+b\text{arccosh}(cx))^n\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{4}(a+b\text{arccosh}(cx))^n\cosh\left(\frac{a}{b}-\frac{a+b\text{arccos}}{b}\right.\right.}{bc^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{1}{8}b3^{-n-1}e^{-\frac{3a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{8}be^{-\frac{a}{b}}(a+\right.}{}$$

input
```
Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*((3^(-1 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-
3*(a + b*ArcCosh[c*x]))/b])/(8*E^((3*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n)
- (b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*E^
(a/b)*(-((a + b*ArcCosh[c*x])/b))^n) + (b*E^(a/b)*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*((a + b*ArcCosh[c*x])/b)^n) - (3^(
-1 - n)*b*E^((3*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCos
h[c*x]))/b])/(8*((a + b*ArcCosh[c*x])/b)^n)))/(b*c^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\sqrt{-c^2d\,x^2+d}\,(a+b\,\mathrm{arccosh}\,(cx))^n\,dx$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\mathrm{arccosh}(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)^nx\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\int x\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^n\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Maxima [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima
")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\int x\,(a+b\,\text{acosh}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\sqrt{d}\left(\int(a\cosh(cx)\,b+a)^n\,\sqrt{-c^2x^2+1}\,xdx\right)$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x)
```

3.320 $\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = -\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^{1+n}}{2bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-3-n}e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c/(1+n)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+2^(-3-n)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+
n,(-2*a-2*b*arccosh(c*x))/b)/c/exp(2*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(
a+b*arccosh(c*x))/b)^n)-2^(-3-n)*exp(2*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2
)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.85

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\,dx$$
$$=\frac{2^{-3-n}de^{-\frac{2a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\operatorname{arccosh}(cx))^n\left(-\frac{(a+b\operatorname{arccosh}(cx))^2}{b^2}\right)^{-n}\left(2^{2+n}e^{\frac{2a}{b}}(a+b\operatorname{arccosh}(cx))\left(-\right.\right.}{}$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(2^(-3 - n)*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*
(2^(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2)
)^n - b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x
]))/b] + b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n,
(2*(a + b*ArcCosh[c*x]))/b]))/(b*c*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]
*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6321, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\,dx$$
$$\Big\downarrow \text{6321}$$
$$\frac{\sqrt{d-c^2dx^2}\int (a+b\operatorname{arccosh}(cx))^n\sinh^2\left(\frac{a}{b}-\frac{a+b\operatorname{arccosh}(cx)}{b}\right)d(a+b\operatorname{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{d-c^2dx^2}\int -(a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^2d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\sqrt{d-c^2dx^2}\int \left(\frac{1}{2}(a+b\text{arccosh}(cx))^n-\frac{1}{2}(a+b\text{arccosh}(cx))^n\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)\right)d(a+b\text{arccosh}(c}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{(a+b\text{arccosh}(cx))^{n+1}}{2(n+1)}+b2^{-n-3}e^{-\frac{2a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcc}}{b}\right.\right.}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-1/2*(a + b*ArcCosh[c*x])^(1 + n)/(1 + n) + (2^(-3 -
 n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E
^((2*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*b*E^((2*a)/b)*(a +
 b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/((a + b*ArcCo
sh[c*x])/b)^n))/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \sqrt{-c^2 d\,x^2+d}\,(a+b\,\mathrm{arccosh}\,(cx))^n\,dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b\mathrm{arccosh}(cx))^n\,dx=\int \sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = \int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^n\;dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = \int \sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\int (a+b\,\text{acosh}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx=\sqrt{d}\left(\int (a\cosh(cx)\,b+a)^n\,\sqrt{-c^2x^2+1}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x)
```

3.321 $\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \text{Int}\left(\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x},x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]
```

output
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x, x]
```

Rubi [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}dx$$
$$\Big\downarrow \text{6369}$$
$$\int \left(\frac{d(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}-\frac{c^2dx(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$d\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}dx-\frac{de^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{2\sqrt{d-c^2dx^2}}+\frac{de^{a/b}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arccosh}(cx)}{b}\right)}{2\sqrt{d-c^2dx^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2 d\,x^2+d}\,(a+b\operatorname{arccosh}(cx))^n}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 4.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acosh}(cx))^n}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n/x,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima
")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n}{x}\,dx=\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n}{x}\,dx=\sqrt{d}\left(\int \frac{(acosh(cx)\,b+a)^n\,\sqrt{-c^2x^2+1}}{x}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^n/x,x)
```

output
```
sqrt(d)*int(((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x,x)
```

3.322 $\int \frac{\sqrt{d-c^2dx^2}(a+b\,\mathbf{arccosh}(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx = \text{Int}\left(\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2},x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx = \int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

output
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}dx$$

$$\Big\downarrow \text{ 6369}$$

$$\int \left(\frac{d(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}-\frac{c^2d(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$d\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx-\frac{cd\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^{n+1}}{b(n+1)\sqrt{d-c^2dx^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2d\,x^2+d}\,(a+b\,\text{arccosh}\,(cx))^n}{x^2}dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 6.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acosh}\,(cx))^n}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n/x**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx = \int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2}}{x^2} \,d x$$

input
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\sqrt{d}\left(\int \frac{(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}}{x^2}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^n/x^2,x)
```

output
```
sqrt(d)*int(((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x**2,x)
```

3.323 $\int x^2(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 658

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=-\frac{d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-7-2n}de^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-7-n}de^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-7-n}de^{\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-7-2n}de^{\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-7-n}3^{-1-n}de^{\frac{6a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/16*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c^3/(1+n)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-2^(-7-n)*3^(-1-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(
c*x))^n*GAMMA(1+n,(-6*a-6*b*arccosh(c*x))/b)/c^3/exp(6*a/b)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+2^(-7-2*n)*d*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-4*a-4*b*arccosh(c*x))/b)/c^3/exp(4*a/b)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+2^(-7-n)*d*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-2*a-2*b*arccosh(c*x))/b)/c
^3/exp(2*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-2^(-
7-n)*d*exp(2*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a
+b*arccosh(c*x))/b)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b
)^n)-2^(-7-2*n)*d*exp(4*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAM
MA(1+n,4*(a+b*arccosh(c*x))/b)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arcc
osh(c*x))/b)^n)+2^(-7-n)*3^(-1-n)*d*exp(6*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccosh(c*x))^n*GAMMA(1+n,6*(a+b*arccosh(c*x))/b)/c^3/(c*x-1)^(1/2)/(c*x+1)
^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 2.07 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.67

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n\,dx = \frac{2^{-7-2n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-n}\left(2^n\right.$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*A
rcCosh[c*x])^n*(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a +
 b*ArcCosh[c*x]))/b] - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcCosh[c*x]
)^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^n*3^(1 + n)*b*E^((4*a)/b
)*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b]
 + 2^n*3^(1 + n)*b*E^((8*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma
[1 + n, (2*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-(
(a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] + 2^n*
E^((6*a)/b)*(2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*
x])^2/b^2))^n - b*E^((6*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[
1 + n, (6*(a + b*ArcCosh[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d -
c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^n \, dx$$

$$\Big\downarrow \text{6367}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx)}{bc^3\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \left(\frac{1}{32}\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n-\frac{1}{16}\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)(a+}{bc^3\sqrt{c}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\sqrt{d-c^2dx^2}\left(\frac{(a+b\text{arccosh}(cx))^{n+1}}{16(n+1)}+b2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{6(}{}\right.\right.}{}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
-((d*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])^(1 + n)/(16*(1 + n)) + (2^(
-7 - n)*3^(-1 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCos
h[c*x]))/b])/(E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7 - 2*n)*b
*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(E^((4*
a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*b*(a + b*ArcCosh[c*x])^
n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*(-((a + b*ArcCos
h[c*x])/b))^n) + (2^(-7 - n)*b*E^((2*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1
+ n, (2*(a + b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*x])/b)^n + (2^(-7 - 2*
n)*b*E^((4*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x
]))/b])/((a + b*ArcCosh[c*x])/b)^n - (2^(-7 - n)*3^(-1 - n)*b*E^((6*a)/b)*
(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b])/((a + b*A
rcCosh[c*x])/b)^n))/(b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \left(-c^2 d\, x^2+d\right)^{\frac{3}{2}} \left(a+b \operatorname{arccosh}\left(c x\right)\right)^{n}d x$$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^n\, dx = \int \left(-c^2 d x^2 + d\right)^{\frac{3}{2}} (b\,\text{arcosh}\,(cx) + a)^n x^2\, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fric
as")
```

output
```
integral(-(c^2*d*x^4 - d*x^2)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\text{arccosh}(cx))^n\, dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^n \, dx = \int (-c^2dx^2+d)^{\frac{3}{2}} (b \operatorname{arcosh}(cx)+a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxi
ma")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2 (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^n \, dx = \int (-c^2dx^2+d)^{\frac{3}{2}} (b \operatorname{arcosh}(cx)+a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac
")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d-c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^n \, dx = \int x^2 (a+b\,\mathrm{acosh}(c\,x))^n \left(d-c^2\,d\,x^2\right)^{3/2} \, dx$$

input
```
int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b \text{arccosh}(cx))^n \, dx = \sqrt{d}\, d\left(-\left(\int (acosh(cx)\, b + a)^n \sqrt{-c^2 x^2 + 1}\, x^4 dx\right) c^2 + \int (acosh(cx)\, b + a)^n \sqrt{-c^2 x^2 + 1}\, x^2 dx\right)$$

input

```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^n,x)
```

output

```
sqrt(d)*d*( - int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**4,x)*c**
2 + int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x))
```

3.324 $\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 27, antiderivative size = 578

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=-\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{32c^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3^{-n}de^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{32c^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{16c^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{de^{a/b}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arccosh}(cx)}{b}\right)}{16c^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3^{-n}de^{\frac{3a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{32c^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5^{-1-n}de^{\frac{5a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{32c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/32*5^(-1-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-5*a
-5*b*arccosh(c*x))/b)/c^2/exp(5*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*a
rccosh(c*x))/b)^n)+1/32*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(
1+n,(-3*a-3*b*arccosh(c*x))/b)/(3^n)/c^2/exp(3*a/b)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)/((-(a+b*arccosh(c*x))/b)^n)-1/16*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh
(c*x))^n*GAMMA(1+n,-(a+b*arccosh(c*x))/b)/c^2/exp(a/b)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+1/16*d*exp(a/b)*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)/c^2/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)-1/32*d*exp(3*a/b)*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)/(3^n)/c^2/(c*x-1
)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)+1/32*5^(-1-n)*d*exp(5*a/b
)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,5*(a+b*arccosh(c*x))
/b)/c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 500, normalized size of antiderivative = 0.87

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx =$$

$$-\frac{15^{-1-n}d^2e^{-\frac{5a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-3n}\left(2\ 15^{1+n}e^{\frac{6a}{b}}\left(-\frac{a+b\text{arccosh}(cx)}{b}\right.\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
-1/32*(15^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh
[c*x])^n*(2*15^(1 + n)*E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b
*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]] + (a/b + Arc
Cosh[c*x])^n*(-(3^(1 + n)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 +
n, (-5*(a + b*ArcCosh[c*x]))/b]) + 3*5^(1 + n)*E^((2*a)/b)*(-((a + b*ArcCo
sh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] - 2*15^(1
 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a
+ b*ArcCosh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a
 + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] - 4*
5^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*(-((a + b*ArcCosh[
c*x])^2/b^2))^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*E^((1
0*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 +
 n, (5*(a + b*ArcCosh[c*x]))/b])))/(c^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(-
((a + b*ArcCosh[c*x])^2/b^2))^(3*n))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{ 6367}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \left(\frac{1}{16}\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n-\frac{3}{16}\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)(a+}{bc^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d\sqrt{d-c^2dx^2}\left(\frac{1}{32}b5^{-n-1}e^{-\frac{5a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{32}b3^{-}\right.}{}$$

input
```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
-((d*Sqrt[d - c^2*d*x^2]*((5^(-1 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (-5*(a + b*ArcCosh[c*x]))/b])/(32*E^((5*a)/b)*(-((a + b*ArcCosh[c*x])/b)
)^n) - (b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b]
)/(32*3^n*E^((3*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) + (b*(a + b*ArcCosh[c
*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(16*E^(a/b)*(-((a + b*ArcC
osh[c*x])/b))^n) - (b*E^(a/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*A
rcCosh[c*x])/b])/(16*((a + b*ArcCosh[c*x])/b)^n) + (b*E^((3*a)/b)*(a + b*A
rcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*((a + b*A
rcCosh[c*x])/b)^n) - (5^(-1 - n)*b*E^((5*a)/b)*(a + b*ArcCosh[c*x])^n*Gamm
a[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*((a + b*ArcCosh[c*x])/b)^n)))/(b
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b \,\operatorname{arccosh}\left(c x\right)\right)^{n}d x$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\int\left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}}\left(b \operatorname{arcosh}\left(c x\right)+a\right)^{n} x \,d x$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas
")
```

output
```
integral(-(c^2*d*x^3 - d*x)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x
)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Timed out}$$

input
```
integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\int(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima
")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\int x\,(a+b\,\text{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^{3/2}(a+b\operatorname{arccosh}(cx))^n\,dx=\sqrt{d}\,d\left(-\left(\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^3dx\right)c^2+\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*d*( - int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**3,x)*c**
2 + int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x))
```

3.325 $\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 450

$$
\begin{aligned}
\int \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n\,dx &= -\frac{3d\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^{1+n}}{8bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{2^{-3-n}de^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{2^{-3-n}de^{\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{4^{-3-n}de^{\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}
$$

output
```
-3/8*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c/(1+n)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-4^(-3-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMM
A(1+n,(-4*a-4*b*arccosh(c*x))/b)/c/exp(4*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/
((-(a+b*arccosh(c*x))/b)^n)+2^(-3-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c
*x))^n*GAMMA(1+n,(-2*a-2*b*arccosh(c*x))/b)/c/exp(2*a/b)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-2^(-3-n)*d*exp(2*a/b)*(-c^2*d*x^2+d
)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)/c/(c*x-1)^(
1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)+4^(-3-n)*d*exp(4*a/b)*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)/c/(c
*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 1.39 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.85

$$\int (d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx = \frac{4^{-3-n}d^2e^{-\frac{4a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-2n}\left(3\;2^{3+2n}e\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(4^(-3 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^
n*(3*2^(3 + 2*n)*E^((4*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^
2/b^2))^(2*n) + b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x
])/b))^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*b*E^((2*a)/
b)*(1 + n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[
1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + 2^(3 + n)*b*E^((6*a)/b)*(1 + n)*(-((
a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (
2*(a + b*ArcCosh[c*x]))/b] - b*E^((8*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*
(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b]
))/(b*c*E^((4*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/
b^2))^(2*n))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6321, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \ 6321$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^4d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 3793$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \left(\frac{1}{8}\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n-\frac{1}{2}\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)(a+ba$$

$$bc\sqrt{cx-1}\sqrt{cx+1}$$

$$\Big\downarrow \ 2009$$

$$-\frac{d\sqrt{d-c^2dx^2}\left(\frac{3(a+b\text{arccosh}(cx))^{n+1}}{8(n+1)}+b2^{-2(n+3)}e^{-\frac{4a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+\right.\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
-((d*Sqrt[d - c^2*d*x^2]*((3*(a + b*ArcCosh[c*x])^(1 + n))/(8*(1 + n)) + (
b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*
(3 + n))*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*b*(a + b
*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*(
-((a + b*ArcCosh[c*x])/b))^n) + (2^(-3 - n)*b*E^((2*a)/b)*(a + b*ArcCosh[c
*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*x])/b)^
n - (b*E^((4*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c
*x]))/b])/(2^(2*(3 + n))*((a + b*ArcCosh[c*x])/b)^n)))/(b*c*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{n}d x$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\int (a+b\,\text{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n\,dx=\sqrt{d}\,d\left(-\left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)c^2+\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*d*( - int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)*c**
2 + int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x))
```

3.326 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x} dx = \text{Int}\left(\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x}, x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x} dx = \int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x} dx$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

Rubi [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\text{arccosh}(cx))^n}{x}dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{2c^2d^2x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^2(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}+\frac{c^4d^2x^3(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d^2\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}dx+\frac{d^2 3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}-\frac{5d^2e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8\sqrt{d-c^2dx^2}}+\frac{5d^2e^{a/b}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arccosh}(cx)}{b}\right)}{8\sqrt{d-c^2dx^2}}-\frac{d^2 3^{-n-1}e^{\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 d\, x^2+d)^{\frac{3}{2}}\, (a+b \operatorname{arccosh}(cx))^n}{x} dx$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}\, (a+b\text{arccosh}(cx))^n}{x}\, dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}} (b \operatorname{arcosh}(cx)+a)^n}{x}\, dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas
")
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\, (a+b\text{arccosh}(cx))^n}{x}\, dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima
")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^n\,(d-c^2\,d\,x^2)^{3/2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \sqrt{d}\,d\left(\int \frac{(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}}{x}dx - \left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^n/x,x)
```

output
```
sqrt(d)*d*(int(((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x,x) - int((
acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x)*c**2)
```

3.327 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\text{Int}\left(\frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2},x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2}dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{2c^2d^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^2(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}+\frac{c^4d^2x^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d^2\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx-\frac{3cd^2\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^{n+1}}{2b(n+1)\sqrt{d-c^2dx^2}}+\frac{cd^22^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}-\frac{cd^22^{-n-3}e^{\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{n}}{x^{2}}d x$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n}{x^2}\,dx=\int \frac{{\left(-c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n}}{x^{2}} \,d x$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="fric
as")
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxi
ma")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n\,(d-c^2\,d\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\text{arccosh}(cx))^n}{x^2}\,dx=\sqrt{d}\,d\left(\int \frac{(acosh(cx)\,b+a)^n\,\sqrt{-c^2x^2+1}}{x^2}dx-\left(\int (acosh(cx)\,b+a)^n\,\sqrt{-c^2x^2+1}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^n/x^2,x)
```

output
```
sqrt(d)*d*(int(((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x**2,x) - in
t((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x)*c**2)
```

3.328 $\int x^2(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 870

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\operatorname{arccosh}(cx))^n\,dx=-\frac{5d^2\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(-\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{8(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(-\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-2(4+n)}d^2e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(-\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(-\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-2(4+n)}d^2e^{\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$+\frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$-\frac{2^{-11-3n}d^2e^{\frac{8a}{b}}\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\left(\frac{a+b\operatorname{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{8(a+b\operatorname{arccosh}(cx))}{b}\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-5/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c^3/(1+n)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)+2^(-11-3*n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c
*x))^n*GAMMA(1+n,(-8*a-8*b*arccosh(c*x))/b)/c^3/exp(8*a/b)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-2^(-7-n)*3^(-1-n)*d^2*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-6*a-6*b*arccosh(c*x))/b)/c^3/ex
p(6*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+d^2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-4*a-4*b*arccosh(c*x))/b)/
(2^(8+2*n))/c^3/exp(4*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x
))/b)^n)+2^(-7-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,
(-2*a-2*b*arccosh(c*x))/b)/c^3/exp(2*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(
a+b*arccosh(c*x))/b)^n)-2^(-7-n)*d^2*exp(2*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)/c^3/(c*x-1)^(1/2)/(c*x+1
)^(1/2)/(((a+b*arccosh(c*x))/b)^n)-d^2*exp(4*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)/(2^(8+2*n))/c^3/(c*x-1
)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)+2^(-7-n)*3^(-1-n)*d^2*exp
(6*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,6*(a+b*arccosh
(c*x))/b)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)-2^(-1
1-3*n)*d^2*exp(8*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,
8*(a+b*arccosh(c*x))/b)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x
))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 4.89 (sec) , antiderivative size = 677, normalized size of antiderivative = 0.78

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^n\,dx = \frac{2^{-11-3n}3^{-1-n}d^3e^{-\frac{8a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-n}\left(-\right.}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*
ArcCosh[c*x])^n*(-(3^(1 + n)*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n,
 (-8*(a + b*ArcCosh[c*x]))/b]) + 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + Ar
cCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*3^(1 +
 n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*A
rcCosh[c*x]))/b] - 3^(1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcCos
h[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + E^((8*a)/b)*(5*2^(4
+ 3*n)*3^(1 + n)*a*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1
+ n)*b*ArcCosh[c*x]*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 3^(1 + n)*4^(2 + n
)*b*E^((2*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a +
 b*ArcCosh[c*x]))/b] + 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b
*ArcCosh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] - 4^(2 + n)*
b*E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcCosh
[c*x]))/b] - 4^(2 + n)*b*E^((6*a)/b)*n*(-((a + b*ArcCosh[c*x])/b))^n*Gamma
[1 + n, (6*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*Ar
cCosh[c*x])/b))^n*Gamma[1 + n, (8*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E
^((8*a)/b)*n*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (8*(a + b*ArcCosh[
c*x]))/b])))/(b*c^3*E^((8*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcC
osh[c*x])^2/b^2))^n)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 567, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} (a+b\text{arccosh}(cx))^n \, dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{d^2 \sqrt{d-c^2 d x^2} \int (a+b\text{arccosh}(cx))^n \cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right) \sinh^6\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right) d(a+b\text{arccosh}(cx))}{b c^3 \sqrt{cx-1} \sqrt{cx+1}}$$

$$\Big\downarrow \text{ 5971}$$

$$d^2\sqrt{d-c^2dx^2}\int\left(\frac{1}{128}\cosh\left(\frac{8a}{b}-\frac{8(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n-\frac{1}{32}\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)(a+$$

$$\Big\downarrow \text{2009}$$

$$d^2\sqrt{d-c^2dx^2}\left(-\frac{5(a+b\text{arccosh}(cx))^{n+1}}{128(n+1)}+b2^{-3n-11}e^{-\frac{8a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{8(a+}{}\right.\right.$$

input
```
Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*((-5*(a + b*ArcCosh[c*x])^(1 + n))/(128*(1 + n))
+ (2^(-11 - 3*n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcCosh[
c*x]))/b])/(E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*3^(-1
 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/
(E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) + (b*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(4 + n))*E^((4*a)/b)*(-((a
+ b*ArcCosh[c*x])/b))^n) + (2^(-7 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 +
n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*(-((a + b*ArcCosh[c*x])/b))^
n) - (2^(-7 - n)*b*E^((2*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a +
 b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*x])/b)^n - (b*E^((4*a)/b)*(a + b*A
rcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(4 + n))*((
a + b*ArcCosh[c*x])/b)^n) + (2^(-7 - n)*3^(-1 - n)*b*E^((6*a)/b)*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*
x])/b)^n - (2^(-11 - 3*n)*b*E^((8*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (8*(a + b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*x])/b)^n))/(b*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2(-c^2d\,x^2+d)^{\frac{5}{2}}\,(a+b\,\mathrm{arccosh}\,(cx))^n\,dx$$

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int x^2(d-c^2dx^2)^{5/2}\,(a+b\mathrm{arccosh}(cx))^n\,dx=\int(-c^2dx^2+d)^{\frac{5}{2}}(b\,\mathrm{arcosh}\,(cx)+a)^nx^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fric
as")
```

output
```
integral((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(-c^2*d*x^2 + d)*(b*a
rccosh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))^n \, dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))^n \, dx = \int (-c^2dx^2+d)^{\frac{5}{2}} (b\,\text{arcosh}\,(cx)+a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxi
ma")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2 (d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))^n \, dx = \int (-c^2dx^2+d)^{\frac{5}{2}} (b\,\text{arcosh}\,(cx)+a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac
")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\int x^2\,(a+b\,\text{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\sqrt{d}\,d^2\left(\left(\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^6dx\right)c^4-2\left(\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^4dx\right)c^2+\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)$$

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*d**2*(int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**6,x)*c**
4 - 2*int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**4,x)*c**2 + int(
(acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x))
```

3.329 $\int x(d-c^2dx^2)^{5/2}(a+b\,\mathbf{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 27, antiderivative size = 793

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7(a+b\text{ar}}{}\right.}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$-\frac{5^{-n}d^2e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$+\frac{3^{1-n}d^2e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$-\frac{5d^2e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$+\frac{5d^2e^{a/b}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arccosh}(cx)}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$-\frac{3^{1-n}d^2e^{\frac{3a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$+\frac{5^{-n}d^2e^{\frac{5a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$-\frac{7^{-1-n}d^2e^{\frac{7a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{7(a+b\text{arccosh}(cx))}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/128*7^(-1-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-7
*a-7*b*arccosh(c*x))/b)/c^2/exp(7*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b
*arccosh(c*x))/b)^n)-1/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*G
AMMA(1+n,(-5*a-5*b*arccosh(c*x))/b)/(5^n)/c^2/exp(5*a/b)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+1/128*3^(1-n)*d^2*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-3*a-3*b*arccosh(c*x))/b)/c^2/exp(3*a/
b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-5/128*d^2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-(a+b*arccosh(c*x))/b)/c^2/
exp(a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+5/128*d^2
*exp(a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh
(c*x))/b)/c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)-1/128
*3^(1-n)*d^2*exp(3*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+
n,3*(a+b*arccosh(c*x))/b)/c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh(c
*x))/b)^n)+1/128*d^2*exp(5*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*
GAMMA(1+n,5*(a+b*arccosh(c*x))/b)/(5^n)/c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((
(a+b*arccosh(c*x))/b)^n)-1/128*7^(-1-n)*d^2*exp(7*a/b)*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccosh(c*x))^n*GAMMA(1+n,7*(a+b*arccosh(c*x))/b)/c^2/(c*x-1)^(1/2)
/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 2.53 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.80

$$\int x(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^n\,dx = \frac{5^{-n}21^{-1-n}d^3e^{-\frac{7a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\operatorname{arccosh}(cx))^n\left(-\frac{(a+b\operatorname{arccosh}(cx))^2}{b^2}\right)^{-3n}\left(-1\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(21^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])
^n*(-(105^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcC
osh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + ArcCosh
[c*x])^n*(-(3^(1 + n)*5^n*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 +
n, (-7*(a + b*ArcCosh[c*x]))/b]) + E^((2*a)/b)*(21^(1 + n)*(-((a + b*ArcCo
sh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b] - 9*5^n*7
^(1 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-
3*(a + b*ArcCosh[c*x]))/b] + 105^(1 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x
])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)] - 5^n*7^(2 + n)*E
^((8*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[
1 + n, (3*(a + b*ArcCosh[c*x]))/b] + 16*5^n*7^(1 + n)*E^((8*a)/b)*(-((a +
b*ArcCosh[c*x])/b))^(2*n)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (
3*(a + b*ArcCosh[c*x]))/b] - 21^(1 + n)*E^((10*a)/b)*(a/b + ArcCosh[c*x])^
n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/
b] + 3^(1 + n)*5^n*E^((12*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c
*x])/b))^(3*n)*Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b]))))/(128*5^n*c^2*E
^((7*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(3*n))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6367, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)\sinh^6\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int \left(\frac{1}{64}\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n-\frac{5}{64}\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(cx))}{b}\right)(a+b\cdots}{}$$

$\downarrow$ 2009

$$\frac{d^2\sqrt{d-c^2dx^2}\left(\frac{1}{128}b7^{-n-1}e^{-\frac{7a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{7(a+b\text{arccosh}(cx))}{b}\right)-\frac{1}{128}b5^{-}\right.}{}$$

input
```
Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*((7^(-1 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (-7*(a + b*ArcCosh[c*x]))/b])/(128*E^((7*a)/b)*(-((a + b*ArcCosh[c*x])/b
))^n) - (b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b
])/(128*5^n*E^((5*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) + (3^(1 - n)*b*(a +
 b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(128*E^((3*a
)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (5*b*(a + b*ArcCosh[c*x])^n*Gamma[1
+ n, -((a + b*ArcCosh[c*x])/b)])/(128*E^(a/b)*(-((a + b*ArcCosh[c*x])/b))^
n) + (5*b*E^(a/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])
/b])/(128*((a + b*ArcCosh[c*x])/b)^n) - (3^(1 - n)*b*E^((3*a)/b)*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/(128*((a + b*ArcCo
sh[c*x])/b)^n) + (b*E^((5*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (5*(a
+ b*ArcCosh[c*x]))/b])/(128*5^n*((a + b*ArcCosh[c*x])/b)^n) - (7^(-1 - n)*
b*E^((7*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))
/b])/(128*((a + b*ArcCosh[c*x])/b)^n)))/(b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{n}d x$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\int \left(-c^{2} d x^{2} + d\right)^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x \,d x$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas
")
```

output
```
integral((c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(-c^2*d*x^2 + d)*(b*arc
cosh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Timed out}$$

input
```
integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\int\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^nx\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima
")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^n\,dx=\int x\,(a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^{5/2}(a+b\operatorname{arccosh}(cx))^n\,dx=\sqrt{d}\,d^2\left(\left(\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^5dx\right)c^4-2\left(\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^3dx\right)c^2+\int(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*d**2*(int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**5,x)*c**
4 - 2*int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**3,x)*c**2 + int(
(acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x))
```

3.330 $\int \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^n \, dx$

Optimal result

Integrand size = 26, antiderivative size = 674

$$\begin{aligned}
\int \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^n \, dx &= -\frac{5d^2\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^{1+n}}{16bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{3\ 2^{-7-2n}d^2e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{15\ 2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{15\ 2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{3\ 2^{-7-2n}d^2e^{\frac{4a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&- \frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arccosh}(cx))}{b}\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
-5/16*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c/(1+n)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
h(c*x))^n*GAMMA(1+n,(-6*a-6*b*arccosh(c*x))/b)/c/exp(6*a/b)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-3*2^(-7-2*n)*d^2*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-4*a-4*b*arccosh(c*x))/b)/c/exp(4*a/
b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+15*2^(-7-n)*d^2
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-2*a-2*b*arccosh(c*x
))/b)/c/exp(2*a/b)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)
-15*2^(-7-n)*d^2*exp(2*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n*GAMM
A(1+n,2*(a+b*arccosh(c*x))/b)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(((a+b*arccosh
(c*x))/b)^n)+3*2^(-7-2*n)*d^2*exp(4*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh
(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(
((a+b*arccosh(c*x))/b)^n)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,6*(a+b*arccosh(c*x))/b)/c/(c*x-1)^(1/
2)/(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 3.48 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.80

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx = \frac{2^{-7-2n}3^{-1-n}d^3e^{-\frac{6a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-2n}\left(-\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*A
rcCosh[c*x])^n*(-(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCo
sh[c*x])/b))^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b]) + 3^(2 + n)*b*E^
((2*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x])/b))^n
*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 5*2^n*3^(2 + n)*b*E^((4*a)/b)
*(1 + n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1
+ n, (-2*(a + b*ArcCosh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((8*a)/b)*(1 + n)*
(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 +
n, (2*(a + b*ArcCosh[c*x]))/b] - 3^(2 + n)*b*E^((10*a)/b)*(1 + n)*(a/b + A
rcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*Ar
cCosh[c*x]))/b] + 2^n*E^((6*a)/b)*(5*2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*
x])*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + A
rcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*Ar
cCosh[c*x]))/b])))/(b*c*E^((6*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*
ArcCosh[c*x])^2/b^2))^(2*n))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6321, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{6321}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int (a+b\text{arccosh}(cx))^n\sinh^6\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int -(a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^6d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int(a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}\right)^6d(a+b\text{arccosh}(cx))}{bc\sqrt{cx-1}\sqrt{cx+1}}$$

$\Big\downarrow$ 3793

$$-\frac{d^2\sqrt{d-c^2dx^2}\int\left(-\frac{1}{32}\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n+\frac{3}{16}\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(cx))}{b}\right)(a\right.}{bc\sqrt{}}$$

$\Big\downarrow$ 2009

$$\frac{d^2\sqrt{d-c^2dx^2}\left(-\frac{5(a+b\text{arccosh}(cx))^{n+1}}{16(n+1)}+b2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\right.\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*((-5*(a + b*ArcCosh[c*x])^(1 + n))/(16*(1 + n)) +
 (2^(-7 - n)*3^(-1 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*A
rcCosh[c*x]))/b])/(E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (3*2^(-7 -
 2*n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/
(E^((4*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) + (15*2^(-7 - n)*b*(a + b*ArcC
osh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*(-((a
+ b*ArcCosh[c*x])/b))^n) - (15*2^(-7 - n)*b*E^((2*a)/b)*(a + b*ArcCosh[c*x
])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*x])/b)^n
+ (3*2^(-7 - 2*n)*b*E^((4*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a
+ b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*x])/b)^n - (2^(-7 - n)*3^(-1 - n)
*b*E^((6*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x])
)/b])/((a + b*ArcCosh[c*x])/b)^n))/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6321
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)]
  Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\operatorname{arccosh}\left(c x \right)\right)^{n}d x$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{5/2} (a+b\text{arccosh}(cx))^n\,dx=\int \left(-c^{2} d x^{2} + d\right)^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} \,d x$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcco
sh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n\,dx=\int (a+b\,\text{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \left(d-c^2dx^2\right)^{5/2}(a\\&+b\text{arccosh}(cx))^n\,dx=\sqrt{d}\,d^2\left(\left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^4dx\right)c^4\right.\\&-2\left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)c^2\\&\left.+\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}dx\right)\end{aligned}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^n,x)
```

output
```
sqrt(d)*d**2*(int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**4,x)*c**
4 - 2*int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)*c**2 + int(
(acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x))
```

3.331 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x} dx = \text{Int}\left(\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x}, x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x} dx = \int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x} dx$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

Rubi [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\text{arccosh}(cx))^n}{x}dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{3c^2d^3x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^3(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}-\frac{c^6d^3x^5(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{3c^4d^3x^3(a+b\text{arccos}}{\sqrt{d-c^2dx^2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5^{-n-1}d^3e^{-\frac{5a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\Gamma\left(n+1,-\frac{5(a+b\text{arccosh}(cx))}{b}\right)\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}}{32\sqrt{d-c^2dx^2}}-\frac{5\ 3^{-n-1}d^3e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\Gamma\left(n+1,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}}{32\sqrt{d-c^2dx^2}}+\frac{3^{-n}d^3e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\Gamma\left(n+1,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}}{8\sqrt{d-c^2dx^2}}-\frac{11d^3e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\Gamma\left(n+1,-\frac{a+b\text{arccosh}(cx)}{b}\right)\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}}{16\sqrt{d-c^2dx^2}}+\frac{11d^3e^{a/b}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arccosh}(cx)}{b}\right)}{16\sqrt{d-c^2dx^2}}+\frac{5\ 3^{-n-1}d^3e^{\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}-\frac{3^{-n}d^3e^{\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}+\frac{5^{-n-1}d^3e^{\frac{5a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{5(a+b\text{arccosh}(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}+d^3\int\frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}dx$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int\frac{(-c^2d\,x^2+d)^{\frac{5}{2}}\,(a+b\,\text{arccosh}\,(cx))^n}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b\text{arccosh}(cx))^n}{x} \, dx = \int \frac{(-c^2 dx^2 + d)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a)^n}{x} \, dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas
")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcco
sh(c*x) + a)^n/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b\text{arccosh}(cx))^n}{x} \, dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima
")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n\,(d-c^2\,d\,x^2)^{5/2}}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x}\,dx = \sqrt{d}\,d^2\left(\int \frac{(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}}{x}dx + \left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^3dx\right)c^4 - 2\left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^n/x,x)
```

output
```
sqrt(d)*d**2*(int(((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x,x) + in
t((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**3,x)*c**4 - 2*int((acosh
(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x)*c**2)
```

3.332 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx = \text{Int}\left(\frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x^2},x\right)$$

output
```
Defer(Int)((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx = \int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x^2}\,dx$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\text{arccosh}(cx))^n}{x^2}dx$$

$$\Big\downarrow \text{6369}$$

$$\int \left(-\frac{3c^2d^3(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^3(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}-\frac{c^6d^3x^4(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{3c^4d^3x^2(a+b\text{arccosh}\right.$$

$$\Big\downarrow \text{2009}$$

$$d^3\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx-\frac{15cd^3\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^{n+1}}{8b(n+1)\sqrt{d-c^2dx^2}}-$$
$$\frac{cd^32^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}+$$
$$\frac{cd^32^{-n-2}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}-$$
$$\frac{cd^32^{-n-2}e^{\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}+$$
$$\frac{cd^32^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{4(a+b\text{arccosh}(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 d\,x^2 + d)^{\frac{5}{2}} (a + b \operatorname{arccosh}(cx))^n}{x^2} dx$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\text{arccosh}(cx))^n}{x^2} \, dx = \int \frac{(-c^2dx^2 + d)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a)^n}{x^2} \, dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="fric
as")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcco
sh(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\text{arccosh}(cx))^n}{x^2} \, dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))^n}{x^2} dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}} (b \operatorname{arcosh}(cx)+a)^n}{x^2} dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxi
ma")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2} (a+b\text{arccosh}(cx))^n}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))^n}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n\,(d-c^2\,d\,x^2)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.24

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\operatorname{arccosh}(cx))^n}{x^2}\,dx=\sqrt{d}\,d^2\left(\int \frac{(acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}}{x^2}dx+\left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)c^4-2\left(\int (acosh(cx)\,b+a)^n\sqrt{-c^2x^2+1}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acosh(c*x))^n/x^2,x)
```

output
```
sqrt(d)*d**2*(int(((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x**2,x) +
 int((acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)*c**4 - 2*int((ac
osh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x)*c**2)
```

3.333 $\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 323

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8c^4\sqrt{1-cx}} + \frac{3e^{-\frac{a}{b}}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8c^4\sqrt{1-cx}} - \frac{3e^{a/b}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arccosh}(cx)}{b}\right)}{8c^4\sqrt{1-cx}} - \frac{3^{-1-n}e^{\frac{3a}{b}}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8c^4\sqrt{1-cx}}$$

output
```
1/8*3^(-1-n)*(c*x-1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-3*a-3*b*arccos
h(c*x))/b)/c^4/exp(3*a/b)/(-c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)+3/8*(
c*x-1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-(a+b*arccosh(c*x))/b)/c^4/exp
(a/b)/(-c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-3/8*exp(a/b)*(c*x-1)^(1/2
)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)/c^4/(-c*x+1)^(1/2)/
(((a+b*arccosh(c*x))/b)^n)-1/8*3^(-1-n)*exp(3*a/b)*(c*x-1)^(1/2)*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)/c^4/(-c*x+1)^(1/2)/(((a+b*ar
ccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.86 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.90

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$$
$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{1-c^2x^2}(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-2n}\left(3^{2+n}e^{\frac{4a}{b}}\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^n\left(-\frac{(a+b\ldots\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
(3^(-1 - n)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*(3^(2 + n)*E^((4*a)/b
)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1
+ n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*((-((a + b*ArcCosh[c*x])
^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] + 3^(2 + n)*E^((2*a)/
b)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b
)] - E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*
ArcCosh[c*x]))/b])))/(8*c^4*E^((3*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$$

$$\Big\downarrow \ 6367$$

$$\frac{\sqrt{cx-1}\int (a+b\text{arccosh}(cx))^n\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^4\sqrt{1-cx}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{cx-1}\int (a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)^3d(a+b\text{arccosh}(cx))}{bc^4\sqrt{1-cx}}$$

$$\Big\downarrow \ 3793$$

$$\frac{\sqrt{cx-1}\int \left(\frac{1}{4}\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n+\frac{3}{4}\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)(a+b\text{arccosh}(c\ldots}{bc^4\sqrt{1-cx}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx-1}\left(\frac{1}{8}b3^{-n-1}e^{-\frac{3a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{3}{8}be^{-\frac{a}{b}}(a+ba\ldots}{}$$

input

```
Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*((3^(-1 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcCosh[c*x]))/b])/(8*E^((3*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) + (3*
b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*E^(a/
b)*(-((a + b*ArcCosh[c*x])/b))^n) - (3*b*E^(a/b)*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*((a + b*ArcCosh[c*x])/b)^n) - (3^(-
1 - n)*b*E^((3*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh
[c*x]))/b])/(8*((a + b*ArcCosh[c*x])/b)^n)))/(b*c^4*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3(a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2x^2 + 1}} dx$$

input
```
int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^3}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^3/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{x^3(a+b\,\text{acosh}\,(cx))^n}{\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^3}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{x^3\,(a+b\,\text{acosh}(c\,x))^n}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(acosh(cx)\,b+a)^n\,x^3}{\sqrt{-c^2x^2+1}}dx$$

input
```
int(x^3*(a+b*acosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(((acosh(c*x)*b + a)**n*x**3)/sqrt( - c**2*x**2 + 1),x)
```

3.334 $\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 211

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\sqrt{-1+cx}(a+b\text{arccosh}(cx))^{1+n}}{2bc^3(1+n)\sqrt{1-cx}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{1-cx}} - \frac{2^{-3-n}e^{\frac{2a}{b}}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{1-cx}}$$

output
```
1/2*(c*x-1)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c^3/(1+n)/(-c*x+1)^(1/2)+2^(-
3-n)*(c*x-1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-2*a-2*b*arccosh(c*x))/
b)/c^3/exp(2*a/b)/(-c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-2^(-3-n)*exp(
2*a/b)*(c*x-1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b
)/c^3/(-c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$$
$$=\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-n}\left(2^{2+n}e^{\frac{2a}{b}}(a+b\text{arccosh}(cx))\left(-\right.\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
(2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2
^(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^
n + b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x])
)/b] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2
*(a + b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(
-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$$
$$\Big\downarrow \text{ 6367}$$
$$\frac{\sqrt{cx-1}\int (a+b\text{arccosh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^3\sqrt{1-cx}}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{cx-1}\int (a+b\text{arccosh}(cx))^n \sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)^2 d(a+b\text{arccosh}(cx))}{bc^3\sqrt{1-cx}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{cx-1}\int \left(\frac{1}{2}\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n+\frac{1}{2}(a+b\text{arccosh}(cx))^n\right) d(a+b\text{arccosh}(cx))}{bc^3\sqrt{1-cx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx-1}\left(\frac{(a+b\text{arccosh}(cx))^{n+1}}{2(n+1)}+b2^{-n-3}e^{-\frac{2a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arccosh}(c}{b}\right.\right.}{bc^3\sqrt{1-cx}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(1 + n)/(2*(1 + n)) + (2^(-3 - n)*b*
(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*a
)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*b*E^((2*a)/b)*(a + b*Arc
Cosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/((a + b*ArcCosh[c*x
])/b)^n))/(b*c^3*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2(a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2x^2 + 1}} dx$$

input
```
int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))^n}{\sqrt{1 - c^2x^2}} dx = \int \frac{(b \operatorname{arcosh}(cx) + a)^n x^2}{\sqrt{-c^2x^2 + 1}} dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^2/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{x^2(a+b\,\text{acosh}\,(cx))^n}{\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^2}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^2}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acosh}(c\,x)\right)^n}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{\left(acosh(cx)\,b+a\right)^n x^2}{\sqrt{-c^2x^2+1}}dx$$

input
```
int(x^2*(a+b*acosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(((acosh(c*x)*b + a)**n*x**2)/sqrt( - c**2*x**2 + 1),x)
```

3.335 $\int \frac{x(a+b\mathbf{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 154

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{2c^2\sqrt{1-cx}} - \frac{e^{a/b}\sqrt{-1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arccosh}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}$$

output
```
1/2*(c*x-1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-(a+b*arccosh(c*x))/b)/c^
2/exp(a/b)/(-c*x+1)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-1/2*exp(a/b)*(c*x-1)
^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)/c^2/(-c*x+1)^(
1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = -\frac{e^{-\frac{a}{b}}\sqrt{-((-1+cx)(1+cx))}(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-n}\left(-e^{\frac{2a}{b}}\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^n\Gamma}{2c^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
-1/2*(Sqrt[-((-1 + c*x)*(1 + c*x))]*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b)*
(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + A
rcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(c^2*E^(a/b)*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6367, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$$

$$\Big\downarrow \text{6367}$$

$$\frac{\sqrt{cx-1}\int (a+b\text{arccosh}(cx))^n\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{1-cx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{cx-1}\int (a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{1-cx}}$$

$\big\downarrow$ 3788

$$\frac{\sqrt{cx-1}\left(\frac{1}{2}i\int -ie^{-\text{arccosh}(cx)}(a+b\text{arccosh}(cx))^nd(a+b\text{arccosh}(cx))-\frac{1}{2}i\int ie^{\text{arccosh}(cx)}(a+b\text{arccosh}(cx))^nd(a+\right.}{bc^2\sqrt{1-cx}}$$

$\big\downarrow$ 26

$$\frac{\sqrt{cx-1}\left(\frac{1}{2}\int e^{-\text{arccosh}(cx)}(a+b\text{arccosh}(cx))^nd(a+b\text{arccosh}(cx))+\frac{1}{2}\int e^{\text{arccosh}(cx)}(a+b\text{arccosh}(cx))^nd(a+b\text{arc}\right.}{bc^2\sqrt{1-cx}}$$

$\big\downarrow$ 2612

$$\frac{\sqrt{cx-1}\left(\frac{1}{2}be^{-\frac{a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{2}be^{a/b}(a+b\text{arccosh}(cx\right.}{bc^2\sqrt{1-cx}}$$

input
```
Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*((b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c
*x])/b)])/(2*E^(a/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (b*E^(a/b)*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*((a + b*ArcCosh[c*x
])/b)^n)))/(b*c^2*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x(a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2x^2 + 1}} dx$$

input
```
int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x(a + b\text{arccosh}(cx))^n}{\sqrt{1 - c^2x^2}} dx = \int \frac{(b \operatorname{arcosh}(cx) + a)^n x}{\sqrt{-c^2x^2 + 1}} dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{x(a+b\,\text{acosh}\,(cx))^n}{\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((x*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\operatorname{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{\left(acosh(cx)\,b+a\right)^n x}{\sqrt{-c^2x^2+1}}dx$$

input
```
int(x*(a+b*acosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(((acosh(c*x)*b + a)**n*x)/sqrt( - c**2*x**2 + 1),x)
```

3.336 $\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}} \, dx = -\frac{\sqrt{1-cx}(a+b\text{arccosh}(cx))^{1+n}}{bc(1+n)\sqrt{-1+cx}}$$

output
```
-(-c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c/(1+n)/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}} \, dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*S
qrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{6307}$$

$$\frac{\sqrt{cx-1}(a+b\text{arccosh}(cx))^{n+1}}{bc(n+1)\sqrt{1-cx}}$$

input
```
Int[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c*x])
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\,\text{arccosh}(cx))^{1+n}}{bc(1+n)\sqrt{-(cx-1)(cx+1)}}$	53

input
```
int((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/c/(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1+n)/(-(c*x-1)
*(c*x+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 213, normalized size of antiderivative = 4.84

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{1 - c^2x^2}} dx = \frac{\left(\sqrt{c^2x^2 - 1}\sqrt{-c^2x^2 + 1}b\log\left(cx + \sqrt{c^2x^2 - 1}\right) + \sqrt{c^2x^2 - 1}\sqrt{-c^2x^2 + 1}a\right)\cosh\left(n\log\left(b\log\left(cx + \sqrt{c^2x^2 - 1}\right)\right.\right.}{}$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
((sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) + sq
rt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*x^2
- 1)) + a)) + (sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x
^2 - 1)) + sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*sinh(n*log(b*log(c*x +
sqrt(c^2*x^2 - 1)) + a)))/(b*c*n - (b*c^3*n + b*c^3)*x^2 + b*c)
```

Sympy [F]

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{1 - c^2x^2}} dx = \int \frac{(a + b\,\text{acosh}\,(cx))^n}{\sqrt{-(cx - 1)(cx + 1)}} dx$$

input
```
integrate((a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{1 - c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^n}{\sqrt{-c^2x^2 + 1}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{1 - c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^n}{\sqrt{-c^2x^2 + 1}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{1 - c^2x^2}}\,dx = \int \frac{(a + b\,\text{acosh}(c\,x))^n}{\sqrt{1 - c^2\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))^n/(1 - c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(acosh(cx)\,b+a)^n}{\sqrt{-c^2x^2+1}}dx$$

input

```
int((a+b*acosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output

```
int((acosh(c*x)*b + a)**n/sqrt( - c**2*x**2 + 1),x)
```

3.337 $\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx=\text{Int}\left(\frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
```

output
```
Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}dx$$

input
```
Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^n}{x\sqrt{-c^2x^2+1}}dx$$

input
```
int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^3 - x), x)
```

Sympy [N/A]

Not integrable

Time = 6.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx = \int \frac{(a+b\,\text{acosh}\,(cx))^n}{x\sqrt{-\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**n/x/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**n/(x*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x), x)
```

Giac [N/A]

Not integrable

Time = 9.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n}{x\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(x*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^n/(x*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx = \int \frac{(acosh(cx)\,b+a)^n}{\sqrt{-c^2x^2+1}\,x}dx$$

input
```
int((a+b*acosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x),x)
```

3.338 $\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx = \text{Int}\left(\frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx = \int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
```

output
```
Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}dx$$

input
```
Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^n}{x^2\sqrt{-c^2x^2+1}}dx$$

input
```
int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^4 - x^2), x)
```

Sympy [N/A]

Not integrable

Time = 28.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx = \int \frac{(a+b\,\text{acosh}\,(cx))^n}{x^2\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**n/x**2/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 9.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n}{x^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(x^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^n/(x^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{1-c^2x^2}}\,dx = \int \frac{(acosh(cx)\,b+a)^n}{\sqrt{-c^2x^2+1}\,x^2}dx$$

input
```
int((a+b*acosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((acosh(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)
```

3.339 $\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 379

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8c^4\sqrt{d-c^2dx^2}} + \frac{3e^{-\frac{a}{b}}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}} - \frac{3e^{a/b}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arccosh}(cx)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}} - \frac{3^{-1-n}e^{\frac{3a}{b}}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arccosh}(cx))}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}$$

output
```
1/8*3^(-1-n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-
3*a-3*b*arccosh(c*x))/b)/c^4/exp(3*a/b)/(-c^2*d*x^2+d)^(1/2)/((-(a+b*arcco
sh(c*x))/b)^n)+3/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(
1+n,-(a+b*arccosh(c*x))/b)/c^4/exp(a/b)/(-c^2*d*x^2+d)^(1/2)/((-(a+b*arcco
sh(c*x))/b)^n)-3/8*exp(a/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))
^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)/c^4/(-c^2*d*x^2+d)^(1/2)/(((a+b*arccosh
(c*x))/b)^n)-1/8*3^(-1-n)*exp(3*a/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)/c^4/(-c^2*d*x^2+d)^(1/2)/(((
a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.77

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx =$$

$$-\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-2n}\left(3^{2+n}e^{\frac{4a}{b}}\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^n\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/8*(3^(-1 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])
^n*(3^(2 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[
c*x])^2/b^2))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*
((-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/
b] + 3^(2 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n,
-((a + b*ArcCosh[c*x])/b)] - E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)
*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])))/(c^4*E^((3*a)/b)*Sqrt[d - c^2
*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{ 6367}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^n\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^4\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)^3d(a+b\text{arccosh}(cx))}{bc^4\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int \left(\frac{1}{4}\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n+\frac{3}{4}\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)(a+bar}{bc^4\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{8}b3^{-n-1}e^{-\frac{3a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arccosh}(cx))}{b}\right)+\frac{3}{8}be^-\right.}{}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((3^(-1 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[
1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(8*E^((3*a)/b)*(-((a + b*ArcCosh[c*x]
)/b))^n) + (3*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])
/b)])/(8*E^(a/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (3*b*E^(a/b)*(a + b*ArcC
osh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*((a + b*ArcCosh[c*x])
/b)^n) - (3^(-1 - n)*b*E^((3*a)/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*
(a + b*ArcCosh[c*x]))/b])/(8*((a + b*ArcCosh[c*x])/b)^n)))/(b*c^4*Sqrt[d -
 c^2*d*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3(a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 d\, x^2 + d}}dx$$

input
```
int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^n x^3}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^3/(c^2*d*x^2 - d),
 x)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^3(a+b\operatorname{acosh}(cx))^n}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^n x^3}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x^3\,(a + b\,\text{acosh}(c\,x))^n}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\int \frac{(acosh(cx)b+a)^n x^3}{\sqrt{-c^2x^2+1}}dx}{\sqrt{d}}$$

input
```
int(x^3*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(((acosh(c*x)*b + a)**n*x**3)/sqrt( - c**2*x**2 + 1),x)/sqrt(d)
```

3.340 $\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 253

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{d-c^2dx^2}} - \frac{2^{-3-n}e^{\frac{2a}{b}}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arccosh}(cx))}{b}\right)}{c^3\sqrt{d-c^2dx^2}}$$

output
```
1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c^3/(1+n)/(-c^2
*d*x^2+d)^(1/2)+2^(-3-n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^n*
GAMMA(1+n,(-2*a-2*b*arccosh(c*x))/b)/c^3/exp(2*a/b)/(-c^2*d*x^2+d)^(1/2)/(
(-(a+b*arccosh(c*x))/b)^n)-2^(-3-n)*exp(2*a/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)/c^3/(-c^2*d*x^2+d)
^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-n}\left(2^{2+n}e^{\frac{2a}{b}}(a+b\text{arccosh}(cx))\left(-\right.\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2
^(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^
n + b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x])
)/b] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2
*(a + b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]
*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {6367, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \; 6367$$
$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^3\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^n \sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)^2 d(a+b\text{arccosh}(cx))}{bc^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 3793

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int \left(\frac{1}{2}\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(cx))}{b}\right)(a+b\text{arccosh}(cx))^n+\frac{1}{2}(a+b\text{arccosh}(cx))^n\right) d(a+b\text{arccos}$$
$$bc^3\sqrt{d-c^2dx^2}$$

$\Big\downarrow$ 2009

$$\sqrt{cx-1}\sqrt{cx+1}\left(\frac{(a+b\text{arccosh}(cx))^{n+1}}{2(n+1)}+b2^{-n-3}e^{-\frac{2a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b}{}\right.\right.$$
$$bc^3\sqrt{d-c^2dx^2}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])^(1 + n)/(2*(1 + n)) +
(2^(-3 - n)*b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x])
)/b])/(E^((2*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*b*E^((2*a)
/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/((a +
 b*ArcCosh[c*x])/b)^n))/(b*c^3*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arccosh}\,(cx))^n}{\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2(a+b\mathrm{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^nx^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^2*d*x^2 - d),
 x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{x^2(a + b\,\text{acosh}\,(cx))^n}{\sqrt{-d\,(cx - 1)\,(cx + 1)}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^n x^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)^n x^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\frac{\int \frac{(acosh(cx)b+a)^n x^2}{\sqrt{-c^2x^2+1}}dx}{\sqrt{d}}$$

input
```
int(x^2*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(((acosh(c*x)*b + a)**n*x**2)/sqrt( - c**2*x**2 + 1),x)/sqrt(d)
```

3.341 $\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arccosh}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}} - \frac{e^{a/b}\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^n\left(\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arccosh}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}$$

output

```
1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-(a+b*arcco
sh(c*x))/b)/c^2/exp(a/b)/(-c^2*d*x^2+d)^(1/2)/((-(a+b*arccosh(c*x))/b)^n)-
1/2*exp(a/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a
+b*arccosh(c*x))/b)/c^2/(-c^2*d*x^2+d)^(1/2)/(((a+b*arccosh(c*x))/b)^n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.84

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{e^{-\frac{a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(a+b\text{arccosh}(cx))^n\left(-\frac{(a+b\text{arccosh}(cx))^2}{b^2}\right)^{-n}\left(-e^{\frac{2a}{b}}\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\right.\right.}{2c^2\sqrt{-d(-1+cx)(1+cx)}}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b
)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b +
 ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(2*c^2*E^(a/b)*
Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6367, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \text{6367}$$
$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^n\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(cx)}{b}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{cx-1}\sqrt{cx+1}\int (a+b\text{arccosh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arccosh}(cx))}{bc^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}i\int -ie^{-\text{arccosh}(cx)}(a+b\text{arccosh}(cx))^n d(a+b\text{arccosh}(cx))-\frac{1}{2}i\int ie^{\text{arccosh}(cx)}(a+b\text{arccosh}(cx$$
$$bc^2\sqrt{d-c^2dx^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}\int e^{-\text{arccosh}(cx)}(a+b\text{arccosh}(cx))^n d(a+b\text{arccosh}(cx))+\frac{1}{2}\int e^{\text{arccosh}(cx)}(a+b\text{arccosh}(cx))^n d($$
$$bc^2\sqrt{d-c^2dx^2}$$

$$\Big\downarrow \text{2612}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{1}{2}be^{-\frac{a}{b}}(a+b\text{arccosh}(cx))^n\left(-\frac{a+b\text{arccosh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arccosh}(cx)}{b}\right)-\frac{1}{2}be^{a/b}(a+bar$$
$$bc^2\sqrt{d-c^2dx^2}$$

input
```
Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((b*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a
 + b*ArcCosh[c*x])/b)])/(2*E^(a/b)*(-((a + b*ArcCosh[c*x])/b))^n) - (b*E^(
a/b)*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*((a +
 b*ArcCosh[c*x])/b)^n)))/(b*c^2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6367
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)]   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
 + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && Eq
Q[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x(a+b\,\mathrm{arccosh}\,(cx))^n}{\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x(a+b\mathrm{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^n x}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^2*d*x^2 - d), x
)
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x(a+b\,\text{acosh}\,(cx))^n}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x\,(a+b\,\text{acosh}(c\,x))^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\int \frac{(acosh(cx)b+a)^n x}{\sqrt{-c^2x^2+1}}dx}{\sqrt{d}}$$

input
```
int(x*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(((acosh(c*x)*b + a)**n*x)/sqrt( - c**2*x**2 + 1),x)/sqrt(d)
```

3.342 $\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^{1+n}}{bcd(1+n)\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^(1+n)/b/c/d/(1+n)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{arccosh}(cx))^{1+n}}{bc(1+n)\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*S
qrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}dx$$

$$\Big\downarrow \text{ 6307}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(a+b\text{arccosh}(cx))^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*S
qrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 6307
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\,\text{arccosh}(cx))^{1+n}}{bc(1+n)\sqrt{-d(cx-1)(cx+1)}}$	54

input
```
int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/c/(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1+n)/(-d*(c*x-
1)*(c*x+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.62

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\,dx = \frac{\left(\sqrt{-c^2dx^2 + d}\sqrt{c^2x^2 - 1}b\log\left(cx + \sqrt{c^2x^2 - 1}\right) + \sqrt{-c^2dx^2 + d}\sqrt{c^2x^2 - 1}a\right)\cosh\left(n\log\left(b\log\left(cx + \sqrt{c\right.\right.\right.}{}$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
((sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) +
sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*
x^2 - 1)) + a)) + (sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt
(c^2*x^2 - 1)) + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*sinh(n*log(b*lo
g(c*x + sqrt(c^2*x^2 - 1)) + a)))/(b*c*d*n + b*c*d - (b*c^3*d*n + b*c^3*d)
*x^2)
```

Sympy [F]

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\,dx = \int \frac{(a + b\,\text{acosh}\,(cx))^n}{\sqrt{-d\,(cx - 1)\,(cx + 1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arccosh}(cx))^n}{\sqrt{d - c^2dx^2}}\, dx = \frac{\int \frac{(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}}dx}{\sqrt{d}}$$

input
```
int((a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((acosh(c*x)*b + a)**n/sqrt( - c**2*x**2 + 1),x)/sqrt(d)
```

3.343 $\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx = \text{Int}\left(\frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}dx$$

input
```
Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^n}{x\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^3 - d*x), x
)
```

Sympy [N/A]

Not integrable

Time = 6.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acosh}(cx))^n}{x\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**n/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Giac [N/A]

Not integrable

Time = 9.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{(a + b\operatorname{arccosh}(cx))^n}{x\sqrt{d - c^2dx^2}}\,dx = \frac{\int \frac{(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,x}dx}{\sqrt{d}}$$

input
```
int((a+b*acosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((acosh(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x),x)/sqrt(d)
```

3.344 $\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx = \text{Int}\left(\frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx$$

input
```
Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^n}{x^2\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^4 - d*x^2),
 x)
```

Sympy [N/A]

Not integrable

Time = 28.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acosh}(cx))^n}{x^2\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 9.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\operatorname{arccosh}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{\int \frac{(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,x^2}dx}{\sqrt{d}}$$

input
```
int((a+b*acosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((acosh(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)/sqrt(d)
```

3.345 $\int \frac{x^2(a+b\,\mathrm{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{x^2(a+b\,\mathrm{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\mathrm{Int}\left(\frac{x^2(a+b\,\mathrm{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^2(a+b\,\mathrm{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^2(a+b\,\mathrm{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 6375}$$
$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}dx$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^2(a+b\,\text{arccosh}\,(cx))^n}{(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)^n x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^4*d^2*x^4 - 2*
c^2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 63.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^2(a+b\operatorname{acosh}(cx))^n}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 9.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{acosh}(c\,x))^n}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \frac{x^2(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=-\frac{\int \frac{(acosh(cx)b+a)^n x^2}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx}{\sqrt{d}\,d}$$

input
```
int(x^2*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - int(((acosh(c*x)*b + a)**n*x**2)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - s
qrt( - c**2*x**2 + 1)),x))/(sqrt(d)*d)
```

3.346 $\int \frac{x(a+b\,\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6375}$$

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}dx$$

input
```
Int[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x(a+b\,\text{arccosh}\,(cx))^n}{(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^4*d^2*x^4 - 2*c^
2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 60.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\,\text{acosh}\,(cx))^n}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 9.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x\,(a+b\,\text{acosh}(c\,x))^n}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \frac{x(a+b\operatorname{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{\int \frac{(acosh(cx)b+a)^n x}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx}{\sqrt{d}\,d}$$

input
```
int(x*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - int(((acosh(c*x)*b + a)**n*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt
( - c**2*x**2 + 1)),x))/(sqrt(d)*d)
```

3.347 $\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \text{Int}\left(\frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 6325$$

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^n}{(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*
d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 40.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\operatorname{acosh}(cx))^n}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 9.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^n}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int \frac{(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{\int \frac{(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx}{\sqrt{d}\,d}$$

input
```
int((a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - int((acosh(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( -
c**2*x**2 + 1)),x))/(sqrt(d)*d)
```

3.348 $\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}dx$$

input
```
Int[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^n}{x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^5 - 2*c^2*
d^2*x^3 + d^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Giac [N/A]

Not integrable

Time = 9.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^n}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acosh}(c\,x)\right)^n}{x\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\int \frac{(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,c^2x^3-\sqrt{-c^2x^2+1}\,x}dx}{\sqrt{d}\,d}$$

input
```
int((a+b*acosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - int((acosh(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*c**2*x**3 - sqrt( -
c**2*x**2 + 1)*x),x))/(sqrt(d)*d)
```

3.349 $\int \frac{(a+b\text{arccosh}(cx))^n}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2(d-c^2dx^2)^{3/2}}\,dx = \text{Int}\left(\frac{(a+b\text{arccosh}(cx))^n}{x^2(d-c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\text{arccosh}(cx))^n}{x^2(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2 (d-c^2dx^2)^{3/2}} \, dx$$
$$\Big\downarrow \text{6375}$$
$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2 (d-c^2dx^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b \,\text{arccosh}\,(cx))^n}{x^2 \left(-c^2 d \,x^2+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^6 - 2*c^2*
d^2*x^4 + d^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 9.76 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac
")
```

output
```
integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\,\text{acosh}(c\,x)\right)^n}{x^2\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \frac{(a+b\text{arccosh}(cx))^n}{x^2(d-c^2dx^2)^{3/2}}\,dx = -\frac{\int \frac{(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,c^2x^4-\sqrt{-c^2x^2+1}\,x^2}dx}{\sqrt{d}\,d}$$

input
```
int((a+b*acosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - int((acosh(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*c**2*x**4 - sqrt( -
c**2*x**2 + 1)*x**2),x))/(sqrt(d)*d)
```

3.350 $\int \frac{(fx)^m(a+b\,\mathbf{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m\,(a+b\,\text{arccosh}\,(cx))^n}{\sqrt{-c^2x^2+1}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*x^2 - 1),
 x)
```

Sympy [N/A]

Not integrable

Time = 78.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m\,(a+b\,\text{acosh}\,(cx))^n}{\sqrt{-\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 9.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n\,(f\,x)^m}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{1-c^2x^2}}\,dx = f^m\left(\int \frac{x^m(acosh(cx)\,b+a)^n}{\sqrt{-c^2x^2+1}}dx\right)$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

output
```
f**m*int((x**m*(acosh(c*x)*b + a)**n)/sqrt( - c**2*x**2 + 1),x)
```

3.351 $\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))^n \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))^n \, dx = \text{Int}\left(d^2(fx)^m (1-c^2x^2)^2 (a+b\text{arccosh}(cx))^n, x\right)$$

output
```
Defer(Int)(d^2*(f*x)^m*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))^n \, dx = \int (fx)^m (d-c^2dx^2)^2 (a+b\text{arccosh}(cx))^n \, dx$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
```

output
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (fx)^m (a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{6375}$$

$$\int \left(d-c^2dx^2\right)^2 (fx)^m (a+b\text{arccosh}(cx))^n dx$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(-c^2d\,x^2+d\right)^2 (a+b\,\text{arccosh}\,(cx))^n\,dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int (fx)^m \left(d - c^2dx^2\right)^2 (a + b\text{arccosh}(cx))^n \, dx = \int \left(c^2dx^2 - d\right)^2 (fx)^m \left(b \operatorname{arcosh}(cx) + a\right)^n \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="fric
as")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*(f*x)^m*(b*arccosh(c*x) + a)^
n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d - c^2dx^2\right)^2 (a + b\text{arccosh}(cx))^n \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int (fx)^m \left(d - c^2dx^2\right)^2 (a + b\text{arccosh}(cx))^n \, dx = \int \left(c^2dx^2 - d\right)^2 (fx)^m \left(b \operatorname{arcosh}(cx) + a\right)^n \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="maxi
ma")
```

output
```
integrate((c^2*d*x^2 - d)^2*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (fx)^m \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))^n\,dx = \int (a+b\,\text{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^2\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^2*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^2*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int (fx)^m \left(d-c^2dx^2\right)^2 (a+b\text{arccosh}(cx))^n\,dx$$
$$= f^m d^2 \left(\left(\int x^m (acosh(cx)\,b+a)^n\,x^4dx\right)c^4 - 2\left(\int x^m (acosh(cx)\,b+a)^n\,x^2dx\right)c^2 + \int x^m (acosh(cx)\,b+a)^n\,dx\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*acosh(c*x))^n,x)
```

output
```
f**m*d**2*(int(x**m*(acosh(c*x)*b + a)**n*x**4,x)*c**4 - 2*int(x**m*(acosh
(c*x)*b + a)**n*x**2,x)*c**2 + int(x**m*(acosh(c*x)*b + a)**n,x))
```

3.352 $\int (fx)^m (d - c^2dx^2) (a+b\text{arccosh}(cx))^n dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (fx)^m (d - c^2dx^2) (a + b\text{arccosh}(cx))^n dx = \text{Int}\left(d(fx)^m (1 - c^2x^2) (a + b\text{arccosh}(cx))^n, x\right)$$

output
```
Defer(Int)(d*(f*x)^m*(-c^2*x^2+1)*(a+b*arccosh(c*x))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (fx)^m (d-c^2dx^2) (a+b\text{arccosh}(cx))^n dx = \int (fx)^m (d-c^2dx^2) (a+b\text{arccosh}(cx))^n dx$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)(fx)^m(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{6375}$$

$$\int \left(d-c^2dx^2\right)(fx)^m(a+b\text{arccosh}(cx))^ndx$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (fx)^m\left(-c^2d\,x^2+d\right)(a+b\operatorname{arccosh}(cx))^n\,dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\text{arccosh}(cx))^n \, dx = \int -\left(c^2dx^2-d\right)(fx)^m \left(b\,\text{arcosh}\left(cx\right)+a\right)^n \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="fricas
")
```

output
```
integral(-(c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\text{arccosh}(cx))^n \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\text{arccosh}(cx))^n \, dx = \int -\left(c^2dx^2-d\right)(fx)^m \left(b\,\text{arcosh}\left(cx\right)+a\right)^n \, dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="maxima
")
```

output
```
-integrate((c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\text{arccosh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\text{arccosh}(cx))^n\,dx=\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\left(d-c^2\,d\,x^2\right)\,{\left(f\,x\right)}^m \,d x$$

input
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int (fx)^m \left(d-c^2dx^2\right) (a+b\text{arccosh}(cx))^n \, dx$$
$$= f^m d\left(-\left(\int x^m (acosh(cx)\, b+a)^n \, x^2 dx\right) c^2+\int x^m (acosh(cx)\, b+a)^n \, dx\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)*(a+b*acosh(c*x))^n,x)
```

output
```
f**m*d*( - int(x**m*(acosh(c*x)*b + a)**n*x**2,x)*c**2 + int(x**m*(acosh(c
*x)*b + a)**n,x))
```

3.353 $\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx = \text{Int}((fx)^m(a+b\text{arccosh}(cx))^n,x)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx = \int (fx)^m(a+b\text{arccosh}(cx))^n\,dx$$

input
```
Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
```

output
```
Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{6303}$$

$$\int (fx)^m(a+b\text{arccosh}(cx))^ndx$$

input
```
Int[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (fx)^m\,(a+b\,\text{arccosh}\,(cx))^n\,dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx=\int (fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

output
```
integral((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [N/A]

Not integrable

Time = 27.75 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx=\int (fx)^m\,(a+b\,\text{acosh}\,(cx))^n\,\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**n,x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))**n, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx=\int (fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-1)]

Timed out.

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx = \text{Timed out}$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx = \int (a+b\,\text{acosh}(c\,x))^n\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))^n*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^n*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (fx)^m(a+b\text{arccosh}(cx))^n\,dx = f^m\left(\int x^m(acosh(cx)\,b+a)^n\,dx\right)$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^n,x)
```

output

```
f**m*int(x**m*(acosh(c*x)*b + a)**n,x)
```

3.354 $\int \frac{(fx)^m(a+b\,\mathbf{arccosh}(cx))^n}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{d-c^2dx^2}\,dx=\operatorname{Int}\left(\frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{d\,(1-c^2x^2)},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^n/d/(-c^2*x^2+1),x)
```

Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{d-c^2dx^2}\,dx=\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{d-c^2dx^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d-c^2dx^2}dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d-c^2dx^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\,\text{arccosh}\,(cx))^n}{-c^2 d\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d-c^2dx^2}\,dx=\int -\frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{c^2dx^2-d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="fricas
")
```

output
```
integral(-(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Sympy [N/A]

Not integrable

Time = 69.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d-c^2dx^2}\,dx=-\frac{\int \frac{(fx)^m(a+b\,\text{acosh}\,(cx))^n}{c^2x^2-1}\,dx}{d}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d),x)
```

output
```
-Integral((f*x)**m*(a + b*acosh(c*x))**n/(c**2*x**2 - 1), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d-c^2dx^2}\,dx=\int -\frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{c^2dx^2-d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="maxima
")
```

output
```
-integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Giac [N/A]

Not integrable

Time = 9.81 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d-c^2dx^2}\,dx=\int -\frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{c^2dx^2-d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Mupad [N/A]

Not integrable

Time = 3.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d-c^2dx^2}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))^n\,(f\,x)^m}{d-c^2\,d\,x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2),x)
```

output
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{d-c^2dx^2}\,dx = -\frac{f^m\left(\int \frac{x^m(acosh(cx)b+a)^n}{c^2x^2-1}dx\right)}{d}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d),x)
```

output
```
( - f**m*int((x**m*(acosh(c*x)*b + a)**n)/(c**2*x**2 - 1),x))/d
```

3.355 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))^n}{d^2(1-c^2x^2)^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^n/d^2/(-c^2*x^2+1)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2} \, dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b \,\text{arccosh}\left(cx\right)\right)^n}{\left(-c^2d\,x^2+d\right)^2}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx = \int \frac{(fx)^m\,(b\operatorname{arcosh}(cx)+a)^n}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="fric
as")
```

output
```
integral((f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx = \int \frac{(fx)^m\,(b\operatorname{arcosh}(cx)+a)^n}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="maxi
ma")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d)^2, x)
```

Giac [N/A]

Not integrable

Time = 9.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="giac
")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^n\,(f\,x)^m}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^2,x)
```

output
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^2}\,dx = \frac{f^m\left(\int \frac{x^m(acosh(cx)b+a)^n}{c^4x^4-2c^2x^2+1}dx\right)}{d^2}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

output
```
(f**m*int((x**m*(acosh(c*x)*b + a)**n)/(c**4*x**4 - 2*c**2*x**2 + 1),x))/d
**2
```

3.356 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n \, dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n \, dx = \text{Int}\left((fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n, x\right)$$

output
```
Defer(Int)((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n \, dx = \int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n \, dx$$

input
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(fx)^m(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \left(d-c^2dx^2\right)^{3/2}(fx)^m(a+b\text{arccosh}(cx))^ndx$$

input
```
Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (fx)^m\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\,\text{arccosh}\,(cx))^n\,dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int (fx)^m (d - c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^n dx = \int (-c^2dx^2 + d)^{\frac{3}{2}} (fx)^m (b \operatorname{arcosh}(cx) + a)^n dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="
fricas")
```

output
```
integral(-(c^2*d*x^2 - d)*sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a
)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d - c^2dx^2)^{3/2} (a + b\text{arccosh}(cx))^n dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m (d - c^2dx^2)^{3/2} (a+b\text{arccosh}(cx))^n dx = \int (-c^2dx^2 + d)^{\frac{3}{2}} (fx)^m (b \operatorname{arcosh}(cx) + a)^n dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="
maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-1)]

Timed out.

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n\,dx = \text{Timed out}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} (a+b\text{arccosh}(cx))^n\,dx = \int \left(a+b\,\text{acosh}(c\,x)\right)^n \left(d-c^2\,d\,x^2\right)^{3/2} (f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int (fx)^m \left(d - c^2dx^2\right)^{3/2} (a + b\text{arccosh}(cx))^n \, dx = f^m \sqrt{d}\, d \left(-\left(\int x^m (acosh(cx)\, b + a)^n \sqrt{-c^2x^2+1}\, x^2 dx\right) c^2 + \int x^m (acosh(cx)\, b + a)^n \sqrt{-c^2x^2+1} dx\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*acosh(c*x))^n,x)
```

output
```
f**m*sqrt(d)*d*( - int(x**m*(acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x
**2,x)*c**2 + int(x**m*(acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x))
```

3.357 $\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = \text{Int}\Big((fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n, x\Big)$$

output
```
Defer(Int)((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = \int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx$$

input
```
Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(fx)^m(a+b\text{arccosh}(cx))^n\,dx$$

$$\Big\downarrow \; 6375$$

$$\int \sqrt{d-c^2dx^2}(fx)^m(a+b\text{arccosh}(cx))^ndx$$

input
```
Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (fx)^m \sqrt{-c^2d\,x^2+d}\,(a+b\ \text{arccosh}\,(cx))^n\,dx$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

output
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\,dx = \int \sqrt{-c^2dx^2+d}(fx)^m\,(b\operatorname{arcosh}(cx)+a)^n\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="
fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d-c^2dx^2}(a+b\operatorname{arccosh}(cx))^n\,dx = \int \sqrt{-c^2dx^2+d}(fx)^m\,(b\operatorname{arcosh}(cx)+a)^n\,dx$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="
maxima")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Giac [F(-1)]

Timed out.

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = \text{Timed out}$$

input
```
integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = \int (a+b\,\text{acosh}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,(f\,x)^m\,dx$$

input
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
```

output
```
int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (fx)^m \sqrt{d-c^2dx^2}(a+b\text{arccosh}(cx))^n\,dx = f^m\sqrt{d}\left(\int x^m(a\cosh(cx)\,b + a)^n\,\sqrt{-c^2x^2+1}dx\right)$$

input
```
int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*acosh(c*x))^n,x)
```

output
```
f**m*sqrt(d)*int(x**m*(acosh(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x)
```

3.358 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(fx)^m\,(a+b\,\text{arccosh}\,(cx))^n}{\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="
fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 -
 d), x)
```

Sympy [N/A]

Not integrable

Time = 78.97 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^m\,(a+b\,\text{acosh}\,(cx))^n}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 10.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="
giac")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))^n\,(f\,x)^m}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{f^m\left(\int \frac{x^m(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}}dx\right)}{\sqrt{d}}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(f**m*int((x**m*(acosh(c*x)*b + a)**n)/sqrt( - c**2*x**2 + 1),x))/sqrt(d)
```

3.359 $\int \frac{(fx)^m(a+b\mathbf{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(fx)^m (a+b \operatorname{arccosh}(cx))^n}{(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4
- 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="
maxima")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 10.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))^n}{(d - c^2dx^2)^{3/2}}\, dx = \int \frac{(fx)^m\,(b\,\text{arcosh}\,(cx) + a)^n}{(-c^2dx^2 + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="
giac")
```

output
```
integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))^n}{(d - c^2dx^2)^{3/2}}\, dx = \int \frac{(a + b\,\text{acosh}(c\,x))^n\,(f\,x)^m}{(d - c^2\,d\,x^2)^{3/2}}\, dx$$

input
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))^n}{(d - c^2dx^2)^{3/2}}\,dx = -\frac{f^m\left(\int \frac{x^m(acosh(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)}{\sqrt{d}\,d}$$

input
```
int((f*x)^m*(a+b*acosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - f**m*int((x**m*(acosh(c*x)*b + a)**n)/(sqrt( - c**2*x**2 + 1)*c**2*x**
2 - sqrt( - c**2*x**2 + 1)),x))/(sqrt(d)*d)
```

3.360 $\int x^4(d+ex^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int x^4(d+ex^2)(a+b\text{arccosh}(cx))\,dx = -\frac{8b(49c^2d+30e)\sqrt{-1+cx}\sqrt{1+cx}}{3675c^7} - \frac{4b(49c^2d+30e)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^5} - \frac{b(49c^2d+30e)\,x^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c^3} - \frac{bex^6\sqrt{-1+cx}\sqrt{1+cx}}{49c} + \frac{1}{5}dx^5(a+b\text{arccosh}(cx)) + \frac{1}{7}ex^7(a+b\text{arccosh}(cx))$$

output

```
-8/3675*b*(49*c^2*d+30*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^7-4/3675*b*(49*c^2
*d+30*e)*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-1/1225*b*(49*c^2*d+30*e)*x^4*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/49*b*e*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
+1/5*d*x^5*(a+b*arccosh(c*x))+1/7*e*x^7*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int x^4(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx=\frac{1}{35}ax^5(7d+5ex^2)-\frac{b\sqrt{-1+cx}\sqrt{1+cx}(240e+8c^2(49d+15ex^2)+2c^4(98dx^2+45ex^4)+3c^6(49dx^4+25ex^6))}{3675c^7}+\frac{1}{35}bx^5(7d+5ex^2)\operatorname{arccosh}(cx)$$

input
```
Integrate[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(a*x^5*(7*d + 5*e*x^2))/35 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(240*e + 8*c^
2*(49*d + 15*e*x^2) + 2*c^4*(98*d*x^2 + 45*e*x^4) + 3*c^6*(49*d*x^4 + 25*e
*x^6)))/(3675*c^7) + (b*x^5*(7*d + 5*e*x^2)*ArcCosh[c*x])/35
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6371, 960, 111, 27, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6371}$$

$$-\frac{1}{35}bc\int\frac{x^5(5ex^2+7d)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}dx^5(a+b\operatorname{arccosh}(cx))+\frac{1}{7}ex^7(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \text{960}$$

$$-\frac{1}{35}bc\left(\frac{1}{7}\left(\frac{30e}{c^2}+49d\right)\int\frac{x^5}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{5ex^6\sqrt{cx-1}\sqrt{cx+1}}{7c^2}\right)+\frac{1}{5}dx^5(a+b\operatorname{arccosh}(cx))+\frac{1}{7}ex^7(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 111$$

$$-\frac{1}{35}bc\left(\frac{1}{7}\left(\frac{30e}{c^2}+49d\right)\left(\frac{\int\frac{4x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)+\frac{5ex^6\sqrt{cx-1}\sqrt{cx+1}}{7c^2}\right)+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))+\frac{1}{7}ex^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{35}bc\left(\frac{1}{7}\left(\frac{30e}{c^2}+49d\right)\left(\frac{4\int\frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)+\frac{5ex^6\sqrt{cx-1}\sqrt{cx+1}}{7c^2}\right)+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))+\frac{1}{7}ex^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 111$$

$$-\frac{1}{35}bc\left(\frac{1}{7}\left(\frac{30e}{c^2}+49d\right)\left(\frac{4\left(\frac{\int\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)+\frac{5ex^6\sqrt{cx-1}\sqrt{cx+1}}{7c^2}\right)+\frac{1}{5}dx^5(a+b\text{arccosh}(cx))+\frac{1}{7}ex^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{35}bc\left(\frac{1}{7}\left(\frac{30e}{c^2}+49d\right)\left(\frac{4\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)+\frac{5ex^6\sqrt{cx-1}\sqrt{cx+}}{7c^2}\right.\;\frac{1}{5}dx^5(a+b\text{arccosh}(cx))+\frac{1}{7}ex^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 83$$

$$\frac{1}{5}dx^5(a+b\text{arccosh}(cx))+\frac{1}{7}ex^7(a+b\text{arccosh}(cx))-\frac{1}{35}bc\left(\frac{5ex^6\sqrt{cx-1}\sqrt{cx+1}}{7c^2}+\frac{1}{7}\left(\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{4\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)}{5c^2}\right)\left(\frac{30e}{c^2}+49d\right)\right)$$

input
```
Int[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/35*(b*c*((5*e*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(7*c^2) + ((49*d + (30*
e)/c^2)*((x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*c^2) + (4*((2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2)))/(5
*c^2)))/7)) + (d*x^5*(a + b*ArcCosh[c*x]))/5 + (e*x^7*(a + b*ArcCosh[c*x])
)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6371
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (Simp[e*(f*x)^(m + 3)*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x] - Sim
p[b*(c/(f*(m + 1)*(m + 3)))   Int[(f*x)^(m + 1)*((d*(m + 3) + e*(m + 1)*x^2
)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

method	result
parts	$a\left(\frac{1}{7}e\,x^7+\frac{1}{5}d\,x^5\right)+\frac{b\,(\frac{c^5\operatorname{arccosh}(cx)e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)c^5x^5d}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(75x^6c^6e+147x^4c^6d+90x^4c^4e+196x^2c^4 \cdots}{3675c^2}}{c^5}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,(\frac{\operatorname{arccosh}(cx)d\,c^7x^5}{5}+\frac{\operatorname{arccosh}(cx)e\,c^7x^7}{7}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(75x^6c^6e+147x^4c^6d+90x^4c^4e+196x^2c^4d+12 \cdots}{3675}}{c^2}}{c^5}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,(\frac{\operatorname{arccosh}(cx)d\,c^7x^5}{5}+\frac{\operatorname{arccosh}(cx)e\,c^7x^7}{7}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(75x^6c^6e+147x^4c^6d+90x^4c^4e+196x^2c^4d+12 \cdots}{3675}}{c^2}}{c^5}$
orering	$\frac{(975c^8e^2x^{10}+2442c^8de\,x^8+1323c^8d^2x^6+90x^8e^2c^6+354x^6e\,c^6d+196c^6d^2x^4+180c^4e^2x^6+1296c^4de\,x^4+784c^4d^2x^2+ \cdots}{3675x(e\,x^2+d)c^8}$

input
```
int(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e*x^7+1/5*d*x^5)+b/c^5*(1/7*c^5*arccosh(c*x)*e*x^7+1/5*arccosh(c*x)
*c^5*x^5*d-1/3675/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*e*x^6+147*c^6*d*
x^4+90*c^4*e*x^4+196*c^4*d*x^2+120*c^2*e*x^2+392*c^2*d+240*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int x^4(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx = \frac{525\,ac^7ex^7+735\,ac^7dx^5+105\,(5\,bc^7ex^7+7\,bc^7dx^5)\log\left(cx+\sqrt{c^2x^2-1}\right)-(75\,bc^6ex^6+3\,(49\,bc^6d+30\ldots}{3675\,c^7}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*e*x^7 + 735*a*c^7*d*x^5 + 105*(5*b*c^7*e*x^7 + 7*b*c^7*d
*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*e*x^6 + 3*(49*b*c^6*d + 30*
b*c^4*e)*x^4 + 392*b*c^2*d + 4*(49*b*c^4*d + 30*b*c^2*e)*x^2 + 240*b*e)*sq
rt(c^2*x^2 - 1))/c^7
```

Sympy [F]

$$\int x^4(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx = \int x^4(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**4*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\begin{aligned}\int x^4(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx &= \frac{1}{7}aex^7+\frac{1}{5}adx^5\\ &\quad+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd\\ &\quad+\frac{1}{245}\left(35\,x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)be\end{aligned}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 -
1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d
 + 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*
x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)
*c)*b*e
```

Giac [F(-2)]

Exception generated.

$$\int x^4(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx = \int x^4\,(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d + e*x^2),x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int x^4(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx$$
$$= \frac{735acosh(cx)\,b\,c^7d\,x^5+525acosh(cx)\,b\,c^7e\,x^7-147\sqrt{c^2x^2-1}\,b\,c^6d\,x^4-75\sqrt{c^2x^2-1}\,b\,c^6e\,x^6-196\sqrt{c^2}}{}$$

input
```
int(x^4*(e*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(735*acosh(c*x)*b*c**7*d*x**5 + 525*acosh(c*x)*b*c**7*e*x**7 - 147*sqrt(c*
*2*x**2 - 1)*b*c**6*d*x**4 - 75*sqrt(c**2*x**2 - 1)*b*c**6*e*x**6 - 196*sq
rt(c**2*x**2 - 1)*b*c**4*d*x**2 - 90*sqrt(c**2*x**2 - 1)*b*c**4*e*x**4 - 3
92*sqrt(c**2*x**2 - 1)*b*c**2*d - 120*sqrt(c**2*x**2 - 1)*b*c**2*e*x**2 -
240*sqrt(c**2*x**2 - 1)*b*e + 735*a*c**7*d*x**5 + 525*a*c**7*e*x**7)/(3675
*c**7)
```

3.361 $\int x^3(d+ex^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^3(d+ex^2)(a+b\text{arccosh}(cx))\,dx = -\frac{b(9c^2d+5e)\,x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5} - \frac{b(9c^2d+5e)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3} - \frac{bex^5\sqrt{-1+cx}\sqrt{1+cx}}{36c} - \frac{b(9c^2d+5e)\,\text{arccosh}(cx)}{96c^6} + \frac{1}{4}dx^4(a+b\text{arccosh}(cx)) + \frac{1}{6}ex^6(a+b\text{arccosh}(cx))$$

output
```
-1/96*b*(9*c^2*d+5*e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-1/144*b*(9*c^2*d+5
*e)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*b*e*x^5*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c-1/96*b*(9*c^2*d+5*e)*arccosh(c*x)/c^6+1/4*d*x^4*(a+b*arccosh(c*x)
)+1/6*e*x^6*(a+b*arccosh(c*x))
```

Mathematica [A] (warning: unable to verify)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int x^3(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx$$
$$=\frac{24ac^6x^4(3d+2ex^2)-bcx\sqrt{-1+cx}\sqrt{1+cx}(15e+c^2(27d+10ex^2)+2c^4(9dx^2+4ex^4))+24bc^6x^4(3d}{288c^6}$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(24*a*c^6*x^4*(3*d + 2*e*x^2) - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(15*e +
 c^2*(27*d + 10*e*x^2) + 2*c^4*(9*d*x^2 + 4*e*x^4)) + 24*b*c^6*x^4*(3*d +
2*e*x^2)*ArcCosh[c*x] - 6*b*(9*c^2*d + 5*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c
*x)]])/(288*c^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6371, 27, 960, 111, 27, 101, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx$$

$$\Big\downarrow \; 6371$$

$$-\frac{1}{24}bc\int \frac{2x^4(2ex^2+3d)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))+\frac{1}{6}ex^6(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{12}bc\int \frac{x^4(2ex^2+3d)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{4}dx^4(a+b\operatorname{arccosh}(cx))+\frac{1}{6}ex^6(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \; 960$$

$$-\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{5e}{c^2}+9d\right)\int\frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{ex^5\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{1}{4}dx^4(a+b\text{arccosh}(cx))+\frac{1}{6}ex^6(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{111}$$

$$-\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{5e}{c^2}+9d\right)\left(\frac{\int\frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{ex^5\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{1}{4}dx^4(a+b\text{arccosh}(cx))+\frac{1}{6}ex^6(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{5e}{c^2}+9d\right)\left(\frac{3\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{ex^5\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{1}{4}dx^4(a+b\text{arccosh}(cx))+\frac{1}{6}ex^6(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{101}$$

$$-\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{5e}{c^2}+9d\right)\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{ex^5\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{1}{4}dx^4(a+b\text{arccosh}(cx))+\frac{1}{6}ex^6(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}dx^4(a+b\text{arccosh}(cx))+\frac{1}{6}ex^6(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)\left(\frac{5e}{c^2}+9d\right)+\frac{ex^5\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)$$

input

```
Int[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output

```
(d*x^4*(a + b*ArcCosh[c*x]))/4 + (e*x^6*(a + b*ArcCosh[c*x]))/6 - (b*c*((e
*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) + ((9*d + (5*e)/c^2)*((x^3*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(4*c^2) + (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(
2*c^2) + ArcCosh[c*x]/(2*c^3)))/(4*c^2)))/3))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6371
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (Simp[e*(f*x)^(m + 3)*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x] - Sim
p[b*(c/(f*(m + 1)*(m + 3)))   Int[(f*x)^(m + 1)*((d*(m + 3) + e*(m + 1)*x^2
)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.34

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\operatorname{arccosh}(cx)e\,x^6}{6}+\frac{\operatorname{arccosh}(cx)c^4x^4d}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(18c^5d\sqrt{c^2x^2-1}\,x^3+8e\,c^5x^5\sqrt{c^2x^2-1}+\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}x^4c^6d+\frac{1}{6}x^6c^6e\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d\,c^6x^4}{4}+\frac{\operatorname{arccosh}(cx)e\,c^6x^6}{6}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(18c^5d\sqrt{c^2x^2-1}\,x^3+8e\,c^5x^5\sqrt{c^2x^2-1}+27\sqrt{c^2\cdots}\right.}{\cdots}\right.}{c^2\cdots}}{c^4}$
default	$\frac{\frac{a\left(\frac{1}{4}x^4c^6d+\frac{1}{6}x^6c^6e\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d\,c^6x^4}{4}+\frac{\operatorname{arccosh}(cx)e\,c^6x^6}{6}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(18c^5d\sqrt{c^2x^2-1}\,x^3+8e\,c^5x^5\sqrt{c^2x^2-1}+27\sqrt{c^2\cdots}\right.}{\cdots}\right.}{c^2\cdots}}{c^4}$
orering	$\frac{\left(88x^8e^2c^6+234x^6e\,c^6d+126c^6d^2x^4+10c^4e^2x^6+51c^4de\,x^4+27c^4d^2x^2+25c^2e^2x^4-147c^2de\,x^2-108c^2d^2-90e^2x^2-60a\cdots\right.}{288(e\,x^2+d)c^6}$

input
```
int(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arccosh(c*x)*e*x^6+1/4*arccosh(c*x)
*c^4*x^4*d-1/288/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(18*c^5*d*(c^2*x^2-1)^(1/
2)*x^3+8*e*c^5*x^5*(c^2*x^2-1)^(1/2)+27*(c^2*x^2-1)^(1/2)*c^3*d*x+10*e*c^3
*x^3*(c^2*x^2-1)^(1/2)+27*ln(c*x+(c^2*x^2-1)^(1/2))*c^2*d+15*(c^2*x^2-1)^(
1/2)*e*c*x+15*ln(c*x+(c^2*x^2-1)^(1/2))*e)/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int x^3(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx = \frac{48\,ac^6ex^6+72\,ac^6dx^4+3\,(16\,bc^6ex^6+24\,bc^6dx^4-9\,bc^2d-5\,be)\log\left(cx+\sqrt{c^2x^2-1}\right)-(8\,bc^5ex^5+2\,(}{288\,c^6}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/288*(48*a*c^6*e*x^6 + 72*a*c^6*d*x^4 + 3*(16*b*c^6*e*x^6 + 24*b*c^6*d*x^
4 - 9*b*c^2*d - 5*b*e)*log(c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5*e*x^5 + 2*(
9*b*c^5*d + 5*b*c^3*e)*x^3 + 3*(9*b*c^3*d + 5*b*c*e)*x)*sqrt(c^2*x^2 - 1))
/c^6
```

Sympy [F]

$$\int x^3(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx = \int x^3(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.22

$$\int x^3(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx = \frac{1}{6}\,aex^6+\frac{1}{4}\,adx^4$$
$$+\frac{1}{32}\left(8\,x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}x}{c^4}+\frac{3\log\left(2\,c^2x+2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bd$$
$$+\frac{1}{288}\left(48\,x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}x}{c^6}+\frac{15\log\left(2\,c^2x+2\sqrt{\right.}{c^7}\right.\right.$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1
)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1
)*c)/c^5)*c)*b*d + 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c
^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*
c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*e
```

Giac [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)(a+b\text{arccosh}(cx))\,dx = \int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^2+d\right) \,dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d + e*x^2),x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.17

$$\int x^3(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx$$
$$= \frac{72acosh(cx)\,b\,c^6d\,x^4+48acosh(cx)\,b\,c^6e\,x^6-18\sqrt{c^2x^2-1}\,b\,c^5d\,x^3-8\sqrt{c^2x^2-1}\,b\,c^5e\,x^5-27\sqrt{c^2x^2-1}}{}$$

input
```
int(x^3*(e*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(72*acosh(c*x)*b*c**6*d*x**4 + 48*acosh(c*x)*b*c**6*e*x**6 - 18*sqrt(c**2*
x**2 - 1)*b*c**5*d*x**3 - 8*sqrt(c**2*x**2 - 1)*b*c**5*e*x**5 - 27*sqrt(c*
*2*x**2 - 1)*b*c**3*d*x - 10*sqrt(c**2*x**2 - 1)*b*c**3*e*x**3 - 15*sqrt(c
**2*x**2 - 1)*b*c*e*x - 27*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d - 15*lo
g(sqrt(c**2*x**2 - 1) + c*x)*b*e + 72*a*c**6*d*x**4 + 48*a*c**6*e*x**6)/(2
88*c**6)
```

3.362 $\int x^2(d+ex^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int x^2(d+ex^2)(a+b\text{arccosh}(cx))\,dx = -\frac{2b(25c^2d+12e)\sqrt{-1+cx}\sqrt{1+cx}}{225c^5} - \frac{b(25c^2d+12e)x^2\sqrt{-1+cx}\sqrt{1+cx}}{225c^3} - \frac{bex^4\sqrt{-1+cx}\sqrt{1+cx}}{25c} + \frac{1}{3}dx^3(a+b\text{arccosh}(cx)) + \frac{1}{5}ex^5(a+b\text{arccosh}(cx))$$

output
```
-2/225*b*(25*c^2*d+12*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-1/225*b*(25*c^2*d
+12*e)*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/25*b*e*x^4*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c+1/3*d*x^3*(a+b*arccosh(c*x))+1/5*e*x^5*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int x^2(d+ex^2)(a+b\text{arccosh}(cx))\,dx = \frac{1}{225}\left(15ax^3(5d+3ex^2) - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(24e+2c^2(25d+6ex^2)+c^4(25dx^2+9ex^4))}{c^5} + 15bx^3(5d+3ex^2)\,\text{arccosh}(cx)\right)$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(15*a*x^3*(5*d + 3*e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(24*e + 2*c^2*
(25*d + 6*e*x^2) + c^4*(25*d*x^2 + 9*e*x^4)))/c^5 + 15*b*x^3*(5*d + 3*e*x^
2)*ArcCosh[c*x])/225
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6371, 960, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex^2)(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6371}$$

$$-\frac{1}{15}bc\int \frac{x^3(3ex^2+5d)}{\sqrt{cx-1}\sqrt{cx+1}}dx + \frac{1}{3}dx^3(a+b\text{arccosh}(cx)) + \frac{1}{5}ex^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{960}$$

$$-\frac{1}{15}bc\left(\frac{1}{5}\left(\frac{12e}{c^2}+25d\right)\int\frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{3ex^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)+\frac{1}{3}dx^3(a+b\text{arccosh}(cx))+\frac{1}{5}ex^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{111}$$

$$-\frac{1}{15}bc\left(\frac{1}{5}\left(\frac{12e}{c^2}+25d\right)\left(\frac{\int\frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{3ex^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)+\frac{1}{3}dx^3(a+b\text{arccosh}(cx))+\frac{1}{5}ex^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bc\left(\frac{1}{5}\left(\frac{12e}{c^2}+25d\right)\left(\frac{2\int\frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{3ex^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}\right)+\frac{1}{3}dx^3(a+b\text{arccosh}(cx))+\frac{1}{5}ex^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{3}dx^3(a+b\text{arccosh}(cx))+\frac{1}{5}ex^5(a+b\text{arccosh}(cx))-\frac{1}{15}bc\left(\frac{3ex^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{1}{5}\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\left(\frac{12e}{c^2}+25d\right)\right)$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/15*(b*c*((3*e*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*c^2) + ((25*d + (12*
e)/c^2)*((2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])/(3*c^2)))/5)) + (d*x^3*(a + b*ArcCosh[c*x]))/3 + (e*x^5*(a +
b*ArcCosh[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6371
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (Simp[e*(f*x)^(m + 3)*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x] - Sim
p[b*(c/(f*(m + 1)*(m + 3)))   Int[(f*x)^(m + 1)*((d*(m + 3) + e*(m + 1)*x^2
)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\left(\frac{c^3\,\mathrm{arccosh}(cx)e\,x^5}{5}+\frac{\mathrm{arccosh}(cx)c^3x^3d}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9x^4c^4e+25x^2c^4d+12c^2e\,x^2+50c^2d+24}{225c^2}\right.}{c^3}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccosh}(cx)d\,c^5x^3}{3}+\frac{\mathrm{arccosh}(cx)e\,c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9x^4c^4e+25x^2c^4d+12c^2e\,x^2+50c^2d+24e\right)}{225}\right)}{c^2}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccosh}(cx)d\,c^5x^3}{3}+\frac{\mathrm{arccosh}(cx)e\,c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9x^4c^4e+25x^2c^4d+12c^2e\,x^2+50c^2d+24e\right)}{225}\right)}{c^2}}{c^3}$
orering	$\frac{(81x^8e^2c^6+238x^6e\,c^6d+125c^6d^2x^4+12c^4e^2x^6+106c^4de\,x^4+50c^4d^2x^2+48c^2e^2x^4-176c^2de\,x^2-100c^2d^2-96e^2x^2-48}{225x(e\,x^2+d)c^6}$

input
```
int(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+b/c^3*(1/5*c^3*arccosh(c*x)*e*x^5+1/3*arccosh(c*x)
*c^3*x^3*d-1/225/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e*x^4+25*c^4*d*x^2
+12*c^2*e*x^2+50*c^2*d+24*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int x^2\left(d+ex^2\right)\left(a+b\,\mathrm{arccosh}(cx)\right)dx$$
$$=\frac{45\,ac^5ex^5+75\,ac^5dx^3+15\left(3\,bc^5ex^5+5\,bc^5dx^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^4ex^4+50\,bc^2d+(25\,bc^4d\right.}{225\,c^5}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*e*x^5 + 75*a*c^5*d*x^3 + 15*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3
)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^4*e*x^4 + 50*b*c^2*d + (25*b*c^4*d
 + 12*b*c^2*e)*x^2 + 24*b*e)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [F]

$$\int x^2(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx=\int x^2(a+b\,\text{acosh}\,(cx))\,(d+ex^2)\;dx$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int x^2(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{1}{5}\,aex^5+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\,\text{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bd+\frac{1}{75}\left(15\,x^5\,\text{arcosh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)
*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d + 1/75*(15*x^5*arccosh(c*x) - (3*
sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 -
 1)/c^6)*c)*b*e
```

Giac [F(-2)]

Exception generated.

$$\int x^2(d+ex^2)(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)(a+b\text{arccosh}(cx))\,dx = \int x^2\,(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d + e*x^2),x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$\int x^2(d+ex^2)(a+b\text{arccosh}(cx))\,dx = \frac{75acosh(cx)\,b\,c^5d\,x^3+45acosh(cx)\,b\,c^5e\,x^5-25\sqrt{c^2x^2-1}\,b\,c^4d\,x^2-9\sqrt{c^2x^2-1}\,b\,c^4e\,x^4-50\sqrt{c^2x^2-1}}{225c^5}$$

input
```
int(x^2*(e*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(75*acosh(c*x)*b*c**5*d*x**3 + 45*acosh(c*x)*b*c**5*e*x**5 - 25*sqrt(c**2*
x**2 - 1)*b*c**4*d*x**2 - 9*sqrt(c**2*x**2 - 1)*b*c**4*e*x**4 - 50*sqrt(c*
*2*x**2 - 1)*b*c**2*d - 12*sqrt(c**2*x**2 - 1)*b*c**2*e*x**2 - 24*sqrt(c**
2*x**2 - 1)*b*e + 75*a*c**5*d*x**3 + 45*a*c**5*e*x**5)/(225*c**5)
```

3.363 $\int x(d+ex^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 122

$$\int x(d+ex^2)(a+b\text{arccosh}(cx))\,dx = -\frac{b(8c^2d+3e)x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3} - \frac{bex^3\sqrt{-1+cx}\sqrt{1+cx}}{16c} - \frac{b(8c^2d+3e)\text{arccosh}(cx)}{32c^4} + \frac{1}{2}dx^2(a+b\text{arccosh}(cx)) + \frac{1}{4}ex^4(a+b\text{arccosh}(cx))$$

output
```
-1/32*b*(8*c^2*d+3*e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/16*b*e*x^3*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c-1/32*b*(8*c^2*d+3*e)*arccosh(c*x)/c^4+1/2*d*x^2*(
a+b*arccosh(c*x))+1/4*e*x^4*(a+b*arccosh(c*x))
```

Mathematica [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int x(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{cx\left(8ac^3x(2d+ex^2)-b\sqrt{-1+cx}\sqrt{1+cx}(3e+2c^2(4d+ex^2))\right)+8bc^4x^2(2d+ex^2)\,\text{arccosh}(cx)-2b(8}{32c^4}$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(c*x*(8*a*c^3*x*(2*d + e*x^2) - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*e + 2*c^
2*(4*d + e*x^2))) + 8*b*c^4*x^2*(2*d + e*x^2)*ArcCosh[c*x] - 2*b*(8*c^2*d
+ 3*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(32*c^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6371, 27, 960, 101, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)\,(a+b\text{arccosh}(cx))\,dx$$
$$\Big\downarrow \text{ 6371}$$
$$-\frac{1}{8}bc\int\frac{2x^2(ex^2+2d)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{2}dx^2(a+b\text{arccosh}(cx))+\frac{1}{4}ex^4(a+b\text{arccosh}(cx))$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{1}{4}bc\int\frac{x^2(ex^2+2d)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{2}dx^2(a+b\text{arccosh}(cx))+\frac{1}{4}ex^4(a+b\text{arccosh}(cx))$$
$$\Big\downarrow \text{ 960}$$

$$-\frac{1}{4}bc\left(\frac{1}{4}\left(\frac{3e}{c^2}+8d\right)\int\frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{ex^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{1}{2}dx^2(a+\text{barccosh}(cx))+\frac{1}{4}ex^4(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{101}$$

$$-\frac{1}{4}bc\left(\frac{1}{4}\left(\frac{3e}{c^2}+8d\right)\left(\frac{\int\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)+\frac{ex^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{1}{2}dx^2(a+\text{barccosh}(cx))+\frac{1}{4}ex^4(a+\text{barccosh}(cx))$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{2}dx^2(a+\text{barccosh}(cx))+\frac{1}{4}ex^4(a+\text{barccosh}(cx))-\frac{1}{4}bc\left(\frac{1}{4}\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)\left(\frac{3e}{c^2}+8d\right)+\frac{ex^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*x^2*(a + b*ArcCosh[c*x]))/2 + (e*x^4*(a + b*ArcCosh[c*x]))/4 - (b*c*((e
*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^2) + ((8*d + (3*e)/c^2)*((x*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/4))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6371
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (Simp[e*(f*x)^(m + 3)*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x] - Sim
p[b*(c/(f*(m + 1)*(m + 3)))   Int[(f*x)^(m + 1)*((d*(m + 3) + e*(m + 1)*x^2
)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.61

method	result
orering	$\frac{\left(14c^4e^2x^6+50c^4de\,x^4+24c^4d^2x^2+3c^2e^2x^4-31c^2de\,x^2-16c^2d^2-12e^2x^2-6de\right)\left(a+b\,\operatorname{arccosh}(cx)\right)}{32(e\,x^2+d)c^4}-\frac{\left(2c^2e\,x^2+8c^2d\cdots\right.}{\cdots}$
parts	$\frac{a\left(e\,x^2+d\right)^2}{4e}+\frac{b\left(\frac{c^2e\,\operatorname{arccosh}(cx)x^4}{4}+\frac{\operatorname{arccosh}(cx)c^2x^2d}{2}+\frac{c^2\operatorname{arccosh}(cx)d^2}{4e}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8d^2c^4\ln\left(cx+\sqrt{c^2x^2-1}\right)+8c^3de\cdots\right.}{\cdots}\right.}{c^2}$
derivativedivides	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^2c^4}{4e}+\frac{\operatorname{arccosh}(cx)c^4d\,x^2}{2}+\frac{e\,\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8d^2c^4\ln\left(cx+\sqrt{c^2x^2-1}\right)+8c^3d\cdots\right.}{\cdots}\right.}{c^2}}{c^2}$
default	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^2c^4}{4e}+\frac{\operatorname{arccosh}(cx)c^4d\,x^2}{2}+\frac{e\,\operatorname{arccosh}(cx)c^4x^4}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8d^2c^4\ln\left(cx+\sqrt{c^2x^2-1}\right)+8c^3d\cdots\right.}{\cdots}\right.}{c^2}}{c^2}$

input
```
int(x*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/32*(14*c^4*e^2*x^6+50*c^4*d*e*x^4+24*c^4*d^2*x^2+3*c^2*e^2*x^4-31*c^2*d*
e*x^2-16*c^2*d^2-12*e^2*x^2-6*d*e)/(e*x^2+d)/c^4*(a+b*arccosh(c*x))-1/32*(
2*c^2*e*x^2+8*c^2*d+3*e)/c^4/(e*x^2+d)*(c*x-1)*(c*x+1)*((a+b*arccosh(c*x))
*(e*x^2+d)+2*e*x^2*(a+b*arccosh(c*x))+x*(e*x^2+d)*b*c/(c*x-1)^(1/2)/(c*x+1
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int x\left(d+ex^2\right)(a+b\operatorname{arccosh}(cx))\,dx = \frac{8\,ac^4ex^4+16\,ac^4dx^2+\left(8\,bc^4ex^4+16\,bc^4dx^2-8\,bc^2d-3\,be\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2\,bc^3ex^3+\left(8\,bc^3a\right.\right.}{32\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/32*(8*a*c^4*e*x^4 + 16*a*c^4*d*x^2 + (8*b*c^4*e*x^4 + 16*b*c^4*d*x^2 - 8
*b*c^2*d - 3*b*e)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*e*x^3 + (8*b*c^3
*d + 3*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [F]

$$\int x\left(d+ex^2\right)(a+b\operatorname{arccosh}(cx))\,dx = \int x(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
Integral(x*(a + b*acosh(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int x(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx = \frac{1}{4}aex^4 + \frac{1}{2}adx^2 + \frac{1}{4}\left(2x^2\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x}{c^2} + \frac{\log\left(2c^2x + 2\sqrt{c^2x^2-1}c\right)}{c^3}\right)\right)bd + \frac{1}{32}\left(8x^4\operatorname{arcosh}(cx) - \left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2} + \frac{3\sqrt{c^2x^2-1}x}{c^4} + \frac{3\log\left(2c^2x + 2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)
*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d + 1/32*(8*x^4*arcc
osh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*lo
g(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*e
```

Giac [F(-2)]

Exception generated.

$$\int x(d+ex^2)(a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)(a+b\text{arccosh}(cx))\,dx=\int x\,(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d + e*x^2),x)
```

output
```
int(x*(a + b*acosh(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.22

$$\int x\left(d+ex^2\right)(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{16acosh(cx)\,b\,c^4d\,x^2+8acosh(cx)\,b\,c^4e\,x^4-8\sqrt{c^2x^2-1}\,b\,c^3dx-2\sqrt{c^2x^2-1}\,b\,c^3e\,x^3-3\sqrt{c^2x^2-1}\,bcex}{32c^4}$$

input
```
int(x*(e*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(16*acosh(c*x)*b*c**4*d*x**2 + 8*acosh(c*x)*b*c**4*e*x**4 - 8*sqrt(c**2*x*
*2 - 1)*b*c**3*d*x - 2*sqrt(c**2*x**2 - 1)*b*c**3*e*x**3 - 3*sqrt(c**2*x**
2 - 1)*b*c*e*x - 8*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d - 3*log(sqrt(c*
*2*x**2 - 1) + c*x)*b*e + 16*a*c**4*d*x**2 + 8*a*c**4*e*x**4)/(32*c**4)
```

3.364 $\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx = -\frac{b(9c^2d+2e)\sqrt{-1+cx}\sqrt{1+cx}}{9c^3} - \frac{bex^2\sqrt{-1+cx}\sqrt{1+cx}}{9c} + dx(a+b\text{arccosh}(cx)) + \frac{1}{3}ex^3(a+b\text{arccosh}(cx))$$

output
```
-1/9*b*(9*c^2*d+2*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/9*b*e*x^2*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/c+d*x*(a+b*arccosh(c*x))+1/3*e*x^3*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx = \frac{1}{9}\left(3ax(3d+ex^2) - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(2e+c^2(9d+ex^2))}{c^3} + 3bx(3d+ex^2)\text{arccosh}(cx)\right)$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(3*a*x*(3*d + e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*(9*d + e
*x^2)))/c^3 + 3*b*x*(3*d + e*x^2)*ArcCosh[c*x])/9
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6323, 27, 960, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6323}$$

$$-bc\int \frac{x(ex^2+3d)}{3\sqrt{cx-1}\sqrt{cx+1}}dx+dx(a+b\text{arccosh}(cx))+\frac{1}{3}ex^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bc\int \frac{x(ex^2+3d)}{\sqrt{cx-1}\sqrt{cx+1}}dx+dx(a+b\text{arccosh}(cx))+\frac{1}{3}ex^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{960}$$

$$-\frac{1}{3}bc\left(\frac{1}{3}\left(\frac{2e}{c^2}+9d\right)\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{ex^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+dx(a+b\text{arccosh}(cx))+\frac{1}{3}ex^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{83}$$

$$dx(a+b\text{arccosh}(cx))+\frac{1}{3}ex^3(a+b\text{arccosh}(cx))-\frac{1}{3}bc\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{2e}{c^2}+9d\right)}{3c^2}+\frac{ex^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/3*(b*c*(((9*d + (2*e)/c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) + (e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2))) + d*x*(a + b*ArcCosh[c*x]) + (e
*x^3*(a + b*ArcCosh[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6323
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0]
|| ILtQ[p + 1/2, 0])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

method	result
parts	$a\left(\frac{1}{3}e\,x^{3}+dx\right)+\frac{b\left(\frac{c\,\operatorname{arccosh}(cx)e\,x^{3}}{3}+\operatorname{arccosh}(cx)cxd-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^{2}e\,x^{2}+9c^{2}d+2e\right)}{9c^{2}}\right)}{c}$
derivativedivides	$\frac{\frac{a\left(c^{3}dx+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\operatorname{arccosh}(cx)d\,c^{3}x+\frac{\operatorname{arccosh}(cx)e\,c^{3}x^{3}}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^{2}e\,x^{2}+9c^{2}d+2e\right)}{9}\right)}{c^{2}}}{c}$
default	$\frac{\frac{a\left(c^{3}dx+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\operatorname{arccosh}(cx)d\,c^{3}x+\frac{\operatorname{arccosh}(cx)e\,c^{3}x^{3}}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^{2}e\,x^{2}+9c^{2}d+2e\right)}{9}\right)}{c^{2}}}{c}$
orering	$\frac{x\left(5c^{4}e^{2}x^{4}+30c^{4}de\,x^{2}+9c^{4}d^{2}+2c^{2}e^{2}x^{2}-18c^{2}de-4e^{2}\right)\left(a+b\,\operatorname{arccosh}(cx)\right)}{9\left(e\,x^{2}+d\right)c^{4}}-\frac{\left(c^{2}e\,x^{2}+9c^{2}d+2e\right)(cx-1)(cx+1)\left(\frac{(e}{\sqrt{cx}}\right.}{9c^{4}\left(e\,x^{2}\right.}$

input
```
int((a+b*arccosh(c*x))*(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arccosh(c*x)*e*x^3+arccosh(c*x)*c*x*d-1/9/c^2
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*e*x^2+9*c^2*d+2*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)\left(a+b\operatorname{arccosh}(cx)\right)dx$$
$$=\frac{3\,ac^3ex^3+9\,ac^3dx+3\left(bc^3ex^3+3\,bc^3dx\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^2ex^2+9\,bc^2d+2\,be\right)\sqrt{c^2x^2-1}}{9\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
 sqrt(c^2*x^2 - 1)) - (b*c^2*e*x^2 + 9*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))
/c^3
```

Sympy [F]

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx=\int (a+b\operatorname{acosh}(cx))(d+ex^2)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx = \frac{1}{3}aex^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)be + adx+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*s
qrt(c^2*x^2 - 1)/c^4))*b*e + a*d*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1)
)*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx = \int (a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*acosh(c*x))*(d + e*x^2),x)
```

output
```
int((a + b*acosh(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int (d+ex^2)(a+b\text{arccosh}(cx))\,dx = \frac{9acosh(cx)\,b\,c^3dx+3acosh(cx)\,b\,c^3e\,x^3-\sqrt{c^2x^2-1}\,b\,c^2e\,x^2-2\sqrt{c^2x^2-1}\,be-9\sqrt{cx+1}\,\sqrt{cx-1}\,b\,c^2d}{9c^3}$$

input
```
int((e*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(9*acosh(c*x)*b*c**3*d*x + 3*acosh(c*x)*b*c**3*e*x**3 - sqrt(c**2*x**2 - 1
)*b*c**2*e*x**2 - 2*sqrt(c**2*x**2 - 1)*b*e - 9*sqrt(c*x + 1)*sqrt(c*x - 1
)*b*c**2*d + 9*a*c**3*d*x + 3*a*c**3*e*x**3)/(9*c**3)
```

3.365 $\int \frac{(d+ex^2)(a+b\,\text{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 264

$$
\begin{aligned}
\int \frac{(d+ex^2)(a+b\,\text{arccosh}(cx))}{x}\,dx = & -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be\,\text{arccosh}(cx)}{4c^2} \\
& + \frac{1}{2}ex^2(a+b\,\text{arccosh}(cx)) \\
& - \frac{ibd\sqrt{1-c^2x^2}\arcsin(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} \\
& + \frac{bd\sqrt{1-c^2x^2}\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
& + d(a+b\,\text{arccosh}(cx))\log(x) \\
& - \frac{bd\sqrt{1-c^2x^2}\arcsin(cx)\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}} \\
& - \frac{ibd\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}
$$

output
```
-1/4*b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*e*arccosh(c*x)/c^2+1/2*e*x^
2*(a+b*arccosh(c*x))-1/2*I*b*d*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+b*d*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2
+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d*(a+b*arccosh(c*x))*ln(x)-b*d*(
-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*d*
(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x}\,dx = \frac{1}{2}aex^2 - \frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c} + \frac{1}{2}bex^2\text{arccosh}(cx) - \frac{be\,\text{arctanh}\left(\frac{\sqrt{-1+cx}}{\sqrt{1+cx}}\right)}{2c^2} + ad\log(x) + \frac{1}{2}bd\Big(\text{arccosh}(cx)\left(\text{arccosh}(cx) + 2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right) - \text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\Big)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(a*e*x^2)/2 - (b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) + (b*e*x^2*ArcCos
h[c*x])/2 - (b*e*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(2*c^2) + a*d*Log[
x] + (b*d*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) -
PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x}dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int \frac{ex^2+2d\log(x)}{2\sqrt{cx-1}\sqrt{cx+1}}dx+d\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}ex^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}bc\int \frac{ex^2+2d\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}dx+d\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}ex^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}bc\int \left(\frac{ex^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2d\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}\right)dx+d\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}ex^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}ex^2(a+b\text{arccosh}(cx))-\frac{1}{2}bc\left(\frac{e\text{arccosh}(cx)}{2c^3}+\frac{id\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{id\sqrt{1-c^2x^2}\arcsin(cx)^2}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2d\sqrt{1-c^2x^2}\arcsin(ca}{c\sqrt{cx-}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(e*x^2*(a + b*ArcCosh[c*x]))/2 + d*(a + b*ArcCosh[c*x])*Log[x] - (b*c*((e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + (e*ArcCosh[c*x])/(2*c^3) + (I*d*
Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*d*S
qrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (2*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(c*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (I*d*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSi
n[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.48

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln(x)-\frac{db\,\mathrm{arccosh}(cx)^2}{2}+\frac{b\,\mathrm{arccosh}(cx)e\,x^2}{2}-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}-\frac{be\,\mathrm{arccosh}(cx)}{4c^2}+db$ ar
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)-\frac{db\,\mathrm{arccosh}(cx)^2}{2}-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}+\frac{b\,\mathrm{arccosh}(cx)e\,x^2}{2}-\frac{be\,\mathrm{arccosh}(cx)}{4c^2}+db$ a
default	$\frac{ae\,x^2}{2}+ad\ln(cx)-\frac{db\,\mathrm{arccosh}(cx)^2}{2}-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}+\frac{b\,\mathrm{arccosh}(cx)e\,x^2}{2}-\frac{be\,\mathrm{arccosh}(cx)}{4c^2}+db$ a

input
```
int((e*x^2+d)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(x)-1/2*d*b*arccosh(c*x)^2+1/2*b*arccosh(c*x)*e*x^2-1/4*
b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*e*arccosh(c*x)/c^2+d*b*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d*polylog(2,-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\mathrm{arccosh}(cx))}{x}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acosh(c*x))/x,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) + integrate(b*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1)) + b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(e\,x^2+d)}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x}\,dx$$
$$= \frac{2acosh(cx)\,b\,c^2e\,x^2 - \sqrt{c^2x^2-1}\,bcex + 4\left(\int \frac{acosh(cx)}{x}dx\right)b\,c^2d - \log\left(\sqrt{c^2x^2-1}+cx\right)be + 4\log(x)\,a\,c^2d}{4c^2}$$

input
```
int((e*x^2+d)*(a+b*acosh(c*x))/x,x)
```

output
```
(2*acosh(c*x)*b*c**2*e*x**2 - sqrt(c**2*x**2 - 1)*b*c*e*x + 4*int(acosh(c*
x)/x,x)*b*c**2*d - log(sqrt(c**2*x**2 - 1) + c*x)*b*e + 4*log(x)*a*c**2*d
+ 2*a*c**2*e*x**2)/(4*c**2)
```

3.366 $\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c}-\frac{d(a+b\text{arccosh}(cx))}{x}+ex(a+b\text{arccosh}(cx))+bcd\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
-b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-d*(a+b*arccosh(c*x))/x+e*x*(a+b*arccosh
(c*x))+b*c*d*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{ad}{x}+aex-\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c}-\frac{bd\text{arccosh}(cx)}{x}+bex\text{arccosh}(cx)+\frac{bcd\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*d*ArcCosh[c
*x])/x + b*e*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c
^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6371, 960, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{6371}$$

$$bc\int \frac{d-ex^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d(a+b\text{arccosh}(cx))}{x}+ex(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{960}$$

$$bc\left(d\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{e\sqrt{cx-1}\sqrt{cx+1}}{c^2}\right)-\frac{d(a+b\text{arccosh}(cx))}{x}+ex(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{103}$$

$$bc\left(cd\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{e\sqrt{cx-1}\sqrt{cx+1}}{c^2}\right)-\frac{d(a+b\text{arccosh}(cx))}{x}+ex(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{218}$$

$$-\frac{d(a+b\text{arccosh}(cx))}{x}+ex(a+b\text{arccosh}(cx))+bc\left(d\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{e\sqrt{cx-1}\sqrt{cx+1}}{c^2}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcCosh[c*x]))/x) + e*x*(a + b*ArcCosh[c*x]) + b*c*(-((e*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/c^2) + d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6371
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (Simp[e*(f*x)^(m + 3)*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x] - Sim
p[b*(c/(f*(m + 1)*(m + 3)))   Int[(f*x)^(m + 1)*((d*(m + 3) + e*(m + 1)*x^2
)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

method	result
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\operatorname{arccosh}(cx)ex}{c}-\frac{\operatorname{arccosh}(cx)d}{cx}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(d\,c^{2}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)+e\sqrt{c^{2}x^{2}-1}\right)}{c^{2}\sqrt{c^{2}x^{2}-1}}\right)$
derivativedivides	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(\operatorname{arccosh}(cx)ecx-\frac{\operatorname{arccosh}(cx)dc}{x}+\frac{\left(-d\,c^{2}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)-e\sqrt{c^{2}x^{2}-1}\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^{2}x^{2}-1}}\right)}{c^{2}}\right)$
default	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(\operatorname{arccosh}(cx)ecx-\frac{\operatorname{arccosh}(cx)dc}{x}+\frac{\left(-d\,c^{2}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)-e\sqrt{c^{2}x^{2}-1}\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^{2}x^{2}-1}}\right)}{c^{2}}\right)$

input
```
int((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(e*x-d/x)+b*c*(1/c*arccosh(c*x)*e*x-arccosh(c*x)*d/c/x-1/c^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*(d*c^2*arctan(1/(c^2*x^2-1)^(1/2))+e*(c^2*x^2-1)^(1/2))/(
c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex^2)\,(a+b\operatorname{arccosh}(cx))}{x^2}\,dx$$
$$=\frac{2\,bc^2dx\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+acex^2-\sqrt{c^2x^2-1}bex-acd+(bcd-bce)x\log\left(-cx+\sqrt{c^2x^2-1}\right.}{cx}$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

output
```
(2*b*c^2*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + a*c*e*x^2 - sqrt(c^2*x^2 -
 1)*b*e*x - a*c*d + (b*c*d - b*c*e)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c
*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*log(c*x + sqrt(c^2*x^2 - 1)))/(c*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\operatorname{acosh}(cx))\,(d+ex^2)}{x^2}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x^2}\,dx = -\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)bd+aex + \frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)be}{c}-\frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

output
```
-(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d + a*e*x + (c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*e/c - a*d/x
```

Giac [F]

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcosh}(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\operatorname{acosh}(c\,x))\,(e\,x^2+d)}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{-acosh(cx)\,bcd+acosh(cx)\,bce\,x^2-2atan\left(\sqrt{c^2x^2-1}+cx\right)b\,c^2dx-\sqrt{cx+1}\,\sqrt{cx-1}\,bex-acd+ace}{cx}$$

input
```
int((e*x^2+d)*(a+b*acosh(c*x))/x^2,x)
```

output
```
( - acosh(c*x)*b*c*d + acosh(c*x)*b*c*e*x**2 - 2*atan(sqrt(c**2*x**2 - 1)
+ c*x)*b*c**2*d*x - sqrt(c*x + 1)*sqrt(c*x - 1)*b*e*x - a*c*d + a*c*e*x**2
)/(c*x)
```

3.367 $\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 251

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^3} dx = \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d(a+b\text{arccosh}(cx))}{2x^2} - \frac{ibe\sqrt{1-c^2x^2}\arcsin(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be\sqrt{1-c^2x^2}\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + e(a+b\text{arccosh}(cx))\log(x) - \frac{be\sqrt{1-c^2x^2}\arcsin(cx)\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ibe\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/2*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x-1/2*d*(a+b*arccosh(c*x))/x^2-1/2*I
*b*e*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*e*(-c^
2*x^2+1)^(1/2)*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+e*(a+b*arccosh(c*x))*ln(x)-b*e*(-c^2*x^2+1)^(1/2)*arcsin(c
*x)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*e*(-c^2*x^2+1)^(1/2)*polylog
(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^3}\,dx = -\frac{ad}{2x^2}+\frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x}-\frac{bd\text{arccosh}(cx)}{2x^2}+ae\log(x)+\frac{1}{2}be\left(\text{arccosh}(cx)\left(\text{arccosh}(cx)+2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right)-\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*d*ArcCosh
[c*x])/(2*x^2) + a*e*Log[x] + (b*e*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 +
 E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int -\frac{\frac{d}{x^2}-2e\log(x)}{2\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d(a+b\text{arccosh}(cx))}{2x^2}+e\log(x)(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}bc\int \frac{\frac{d}{x^2}-2e\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d(a+b\text{arccosh}(cx))}{2x^2}+e\log(x)(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}bc\int\left(\frac{d}{x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{2e\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}\right)dx-\frac{d(a+b\text{arccosh}(cx))}{2x^2}+e\log(x)(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\text{arccosh}(cx))}{2x^2}+e\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(-\frac{ie\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{ie\sqrt{1-c^2x^2}\arcsin(cx)^2}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{2e\sqrt{1-c^2x^2}\arcsin(cx)\log\left(1-e^{2i\text{ar}}\right.}{c\sqrt{cx-1}\sqrt{cx+1}}\right.$$

input
```
Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcCosh[c*x]))/x^2 + e*(a + b*ArcCosh[c*x])*Log[x] + (b*c*(
(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x - (I*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)
/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log
[1 - E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*e*Sqrt[
1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*e*S
qrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

method	result
derivativedivides	$c^2\left(\frac{ae\ln(cx)}{c^2}-\frac{ad}{2c^2x^2}+\frac{b\left(-\frac{e\,\mathrm{arccosh}(cx)^2}{2}-\frac{d\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\mathrm{arccosh}(cx)\right)}{2x^2}+\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right.\right.\right.}{c^2}\right.$
default	$c^2\left(\frac{ae\ln(cx)}{c^2}-\frac{ad}{2c^2x^2}+\frac{b\left(-\frac{e\,\mathrm{arccosh}(cx)^2}{2}-\frac{d\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\mathrm{arccosh}(cx)\right)}{2x^2}+\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right.\right.\right.}{c^2}\right.$
parts	$ae\ln(x)-\frac{ad}{2x^2}+b\,c^2\left(-\frac{e\,\mathrm{arccosh}(cx)^2}{2c^2}-\frac{d\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\mathrm{arccosh}(cx)\right)}{2c^2x^2}+\frac{e\ \mathrm{arccosh}(cx)\ln(}{}\right.$

input
```
int((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/c^2*e*ln(c*x)-1/2*a*d/c^2/x^2+b/c^2*(-1/2*e*arccosh(c*x)^2-1/2*d*(-
(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x+c^2*x^2+arccosh(c*x))/x^2+ln(1+(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2)*e*arccosh(c*x)+1/2*polylog(2,-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*e))
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\mathrm{arccosh}(cx))}{x^3}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{acosh}(cx))(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acosh(c*x))/x**3,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcosh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + b*e*integrate(log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

Giac [F]

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcosh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)\,(a+b\text{arccosh}(cx))}{x^3}\,dx = \frac{-acosh(cx)\,bd-\sqrt{c^2x^2-1}\,bcdx+2\left(\int \frac{acosh(cx)}{x}dx\right)be\,x^2+2\log(x)\,ae\,x^2-ad-b\,c^2d\,x^2}{2x^2}$$

input
```
int((e*x^2+d)*(a+b*acosh(c*x))/x^3,x)
```

output
```
( - acosh(c*x)*b*d - sqrt(c**2*x**2 - 1)*b*c*d*x + 2*int(acosh(c*x)/x,x)*b
*e*x**2 + 2*log(x)*a*e*x**2 - a*d - b*c**2*d*x**2)/(2*x**2)
```

3.368 $\int \frac{(d+ex^2)(a+b\,\mathbf{arccosh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{(d+ex^2)(a+b\,\mathrm{arccosh}(cx))}{x^4}\,dx = \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d(a+b\,\mathrm{arccosh}(cx))}{3x^3} - \frac{e(a+b\,\mathrm{arccosh}(cx))}{x} + \frac{1}{6}bc\left(c^2d+6e\right)\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-1/3*d*(a+b*arccosh(c*x))/x^3-e*(
a+b*arccosh(c*x))/x+1/6*b*c*(c^2*d+6*e)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2)
)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex^2)(a+b\,\mathrm{arccosh}(cx))}{x^4}\,dx = \frac{-2b(d+3ex^2)\,\mathrm{arccosh}(cx) + \frac{bcdx(-1+c^2x^2)-2a\sqrt{-1+cx}\sqrt{1+cx}(d+3ex^2)+bc(c^2d+6e)x^3\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}}{6x^3}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
(-2*b*(d + 3*e*x^2)*ArcCosh[c*x] + (b*c*d*x*(-1 + c^2*x^2) - 2*a*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(d + 3*e*x^2) + b*c*(c^2*d + 6*e)*x^3*Sqrt[-1 + c^2*x^
2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(6*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6371, 25, 956, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6371}$$

$$-\frac{1}{3}bc\int -\frac{3ex^2+d}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d(a+b\text{arccosh}(cx))}{3x^3}-\frac{e(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}bc\int \frac{3ex^2+d}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d(a+b\text{arccosh}(cx))}{3x^3}-\frac{e(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{3}bc\left(\frac{1}{2}(c^2d+6e)\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{d\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)-\frac{d(a+b\text{arccosh}(cx))}{3x^3}-\frac{e(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{3}bc\left(\frac{1}{2}c(c^2d+6e)\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{d\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)-\frac{d(a+b\text{arccosh}(cx))}{3x^3}-\frac{e(a+b\text{arccosh}(cx))}{x}$$

$$\Big\downarrow \text{218}$$

$$-\frac{d(a+b\operatorname{arccosh}(cx))}{3x^3}-\frac{e(a+b\operatorname{arccosh}(cx))}{x}+\frac{1}{3}bc\left(\frac{1}{2}\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)\left(c^2d+6e\right)+\frac{d\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcCosh[c*x]))/x^3 - (e*(a + b*ArcCosh[c*x]))/x + (b*c*((d*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((c^2*d + 6*e)*ArcTan[Sqrt[-1 + c*
x]*Sqrt[1 + c*x]])/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 6371
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (Simp[e*(f*x)^(m + 3)*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x] - Sim
p[b*(c/(f*(m + 1)*(m + 3)))   Int[(f*x)^(m + 1)*((d*(m + 3) + e*(m + 1)*x^2
)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.50

method	result
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arccosh}(cx)e}{c^3x}-\frac{\operatorname{arccosh}(cx)d}{3c^3x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^4d\,x^2+6\,\mathrm{arct}\cdots\right.}{6c^4\sqrt{c^2x^2-1}\cdots}\right.$
derivativedivides	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)e}{cx}-\frac{\operatorname{arccosh}(cx)d}{3c\,x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^4d\,x^2+6\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right.\right.}{6\sqrt{c^2x^2-1}\,c^2x^2}\cdots\right.}{c^2}\right.$
default	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)e}{cx}-\frac{\operatorname{arccosh}(cx)d}{3c\,x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^4d\,x^2+6\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right.\right.}{6\sqrt{c^2x^2-1}\,c^2x^2}\cdots\right.}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arccosh(c*x)*e/x-1/3*arccosh(c*x)*d/c^3/x
^3-1/6/c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*c^4*d*
x^2+6*arctan(1/(c^2*x^2-1)^(1/2))*e*c^2*x^2-d*c^2*(c^2*x^2-1)^(1/2))/(c^2*
x^2-1)^(1/2)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^4}\,dx$$
$$= \frac{2\,(bc^3d+6\,bce)x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,(bd+3\,be)x^3\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcdx}{6\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*(b*c^3*d + 6*b*c*e)*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*d +
 3*b*e)*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2 - 1)*b*c*d*x - 6*
a*e*x^2 - 2*a*d - 2*(3*b*e*x^2 - (b*d + 3*b*e)*x^3 + b*d)*log(c*x + sqrt(c
^2*x^2 - 1)))/x^3
```

Sympy [F]

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\operatorname{acosh}(cx))(d+ex^2)}{x^4}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^4}\,dx$$
$$= -\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\text{arcosh}\,(cx)}{x^3}\right)bd$$
$$-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\text{arcosh}\,(cx)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x
)/x^3)*b*d - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*e - a*e/x - 1/3*a
*d/x^3
```

Giac [F]

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)(b\,\text{arcosh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(e\,x^2+d)}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)(a+b\text{arccosh}(cx))}{x^4}\,dx$$
$$= \frac{-2acosh(cx)\,bd-6acosh(cx)\,be\,x^2-2atan\left(\sqrt{c^2x^2-1}+cx\right)b\,c^3d\,x^3-12atan\left(\sqrt{c^2x^2-1}+cx\right)bce\,x^3}{6x^3}$$

input
```
int((e*x^2+d)*(a+b*acosh(c*x))/x^4,x)
```

output
```
( - 2*acosh(c*x)*b*d - 6*acosh(c*x)*b*e*x**2 - 2*atan(sqrt(c**2*x**2 - 1)
+ c*x)*b*c**3*d*x**3 - 12*atan(sqrt(c**2*x**2 - 1) + c*x)*b*c*e*x**3 - sqr
t(c**2*x**2 - 1)*b*c*d*x - 2*a*d - 6*a*e*x**2)/(6*x**3)
```

3.369 $\int x^4(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\begin{aligned}\int x^4(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx &= -\frac{b(63c^4d^2+90c^2de+35e^2)\sqrt{-1+cx}\sqrt{1+cx}}{315c^9}\\&\quad-\frac{2b(63c^4d^2+135c^2de+70e^2)(-1+cx)^{3/2}(1+cx)^{3/2}}{945c^9}\\&\quad-\frac{b(21c^4d^2+90c^2de+70e^2)(-1+cx)^{5/2}(1+cx)^{5/2}}{525c^9}\\&\quad-\frac{2be(9c^2d+14e)(-1+cx)^{7/2}(1+cx)^{7/2}}{441c^9}-\frac{be^2(-1+cx)^{9/2}(1+cx)^{9/2}}{81c^9}\\&\quad+\frac{1}{5}d^2x^5(a+b\text{arccosh}(cx))+\frac{2}{7}dex^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^2x^9(a+b\text{arccosh}(cx))\end{aligned}$$

output
```
-1/315*b*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^9-2/
945*b*(63*c^4*d^2+135*c^2*d*e+70*e^2)*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^9-1/52
5*b*(21*c^4*d^2+90*c^2*d*e+70*e^2)*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^9-2/441*b
*e*(9*c^2*d+14*e)*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^9-1/81*b*e^2*(c*x-1)^(9/2)
*(c*x+1)^(9/2)/c^9+1/5*d^2*x^5*(a+b*arccosh(c*x))+2/7*d*e*x^7*(a+b*arccosh
(c*x))+1/9*e^2*x^9*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.74

$$\int x^4 \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$
$$= \frac{315ax^5(63d^2+90dex^2+35e^2x^4)-\frac{b\sqrt{-1+cx}\sqrt{1+cx}(4480e^2+160c^2e(81d+14ex^2)+24c^4(441d^2+270dex^2+70e^2x^4)+4c^6(1323d}{c^9}}{99225}$$

input
```
Integrate[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(315*a*x^5*(63*d^2 + 90*d*e*x^2 + 35*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(4480*e^2 + 160*c^2*e*(81*d + 14*e*x^2) + 24*c^4*(441*d^2 + 270*d*e*
x^2 + 70*e^2*x^4) + 4*c^6*(1323*d^2*x^2 + 1215*d*e*x^4 + 350*e^2*x^6) + c^
8*(3969*d^2*x^4 + 4050*d*e*x^6 + 1225*e^2*x^8)))/c^9 + 315*b*x^5*(63*d^2 +
 90*d*e*x^2 + 35*e^2*x^4)*ArcCosh[c*x])/99225
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6373, 27, 1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6373}$$

$$-bc\int \frac{x^5(35e^2x^4+90dex^2+63d^2)}{315\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}d^2x^5(a+b\text{arccosh}(cx))+\frac{2}{7}dex^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^2x^9(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{315}bc\int \frac{x^5\left(35e^2x^4+90dex^2+63d^2\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}d^2x^5(a+b\text{arccosh}(cx))+\frac{2}{7}dex^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^2x^9(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1905}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^5\left(35e^2x^4+90dex^2+63d^2\right)}{\sqrt{c^2x^2-1}}dx}{315\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}d^2x^5(a+b\text{arccosh}(cx))+\frac{2}{7}dex^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^2x^9(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1578}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^4\left(35e^2x^4+90dex^2+63d^2\right)}{\sqrt{c^2x^2-1}}dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}d^2x^5(a+b\text{arccosh}(cx))+\frac{2}{7}dex^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^2x^9(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1195}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \left(\frac{35e^2\left(c^2x^2-1\right)^{7/2}}{c^8}+\frac{10e\left(9dc^2+14e\right)\left(c^2x^2-1\right)^{5/2}}{c^8}+\frac{3\left(21d^2c^4+90dec^2+70e^2\right)\left(c^2x^2-1\right)^{3/2}}{c^8}+\frac{2\left(63d^2c^4+135dec^2+70\right.}{c^8}\right.}{630\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{5}d^2x^5(a+b\text{arccosh}(cx))+\frac{2}{7}dex^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^2x^9(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}d^2x^5(a+b\text{arccosh}(cx))+\frac{2}{7}dex^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^2x^9(a+b\text{arccosh}(cx))-$$
$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{20e\left(c^2x^2-1\right)^{7/2}\left(9c^2d+14e\right)}{7c^{10}}+\frac{70e^2\left(c^2x^2-1\right)^{9/2}}{9c^{10}}+\frac{6\left(c^2x^2-1\right)^{5/2}\left(21c^4d^2+90c^2de+70e^2\right)}{5c^{10}}+\frac{4\left(c^2x^2-1\right)^{3/2}\left(63c^4d^2+135\right.}{3c^{10}}\right.}{630\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/630*(b*c*Sqrt[-1 + c^2*x^2]*((2*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*Sqrt
[-1 + c^2*x^2])/c^10 + (4*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(-1 + c^2*x^
2)^(3/2))/(3*c^10) + (6*(21*c^4*d^2 + 90*c^2*d*e + 70*e^2)*(-1 + c^2*x^2)^
(5/2))/(5*c^10) + (20*e*(9*c^2*d + 14*e)*(-1 + c^2*x^2)^(7/2))/(7*c^10) +
(70*e^2*(-1 + c^2*x^2)^(9/2))/(9*c^10)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(d^2*x^5*(a + b*ArcCosh[c*x]))/5 + (2*d*e*x^7*(a + b*ArcCosh[c*x]))/7 + (e
^2*x^9*(a + b*ArcCosh[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec), antiderivative size = 214, normalized size of antiderivative = 0.82

method	result
parts	$a\left(\frac{1}{9}e^{2}x^{9}+\frac{2}{7}de\,x^{7}+\frac{1}{5}d^{2}x^{5}\right)+\frac{b\left(\frac{c^{5}\operatorname{arccosh}(cx)e^{2}x^{9}}{9}+\frac{2c^{5}\operatorname{arccosh}(cx)de\,x^{7}}{7}+\frac{\operatorname{arccosh}(cx)c^{5}x^{5}d^{2}}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+}}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d^{2}c^{9}x^{5}+\frac{2}{7}d\,c^{9}e\,x^{7}+\frac{1}{9}e^{2}c^{9}x^{9}\right)}{c^{4}}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^{2}c^{9}x^{5}}{5}+\frac{2\operatorname{arccosh}(cx)d\,c^{9}e\,x^{7}}{7}+\frac{\operatorname{arccosh}(cx)e^{2}c^{9}x^{9}}{9}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1225e}{}\right.}{}}{}$
default	$\frac{\frac{a\left(\frac{1}{5}d^{2}c^{9}x^{5}+\frac{2}{7}d\,c^{9}e\,x^{7}+\frac{1}{9}e^{2}c^{9}x^{9}\right)}{c^{4}}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^{2}c^{9}x^{5}}{5}+\frac{2\operatorname{arccosh}(cx)d\,c^{9}e\,x^{7}}{7}+\frac{\operatorname{arccosh}(cx)e^{2}c^{9}x^{9}}{9}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1225e}{}\right.}{}}{}$
orering	$\frac{(20825c^{10}e^{3}x^{12}+76675c^{10}d\,e^{2}x^{10}+96147c^{10}d^{2}e\,x^{8}+1400c^{8}e^{3}x^{10}+35721c^{10}d^{3}x^{6}+7180c^{8}d\,e^{2}x^{8}+13824c^{8}d^{2}e\,x^{6}+22}{}$

input
```
int(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^2*x^9+2/7*d*e*x^7+1/5*d^2*x^5)+b/c^5*(1/9*c^5*arccosh(c*x)*e^2*x^
9+2/7*c^5*arccosh(c*x)*d*e*x^7+1/5*arccosh(c*x)*c^5*x^5*d^2-1/99225/c^4*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^2*x^8+4050*c^8*d*e*x^6+3969*c^8*d^2*
x^4+1400*c^6*e^2*x^6+4860*c^6*d*e*x^4+5292*c^6*d^2*x^2+1680*c^4*e^2*x^4+64
80*c^4*d*e*x^2+10584*c^4*d^2+2240*c^2*e^2*x^2+12960*c^2*d*e+4480*e^2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec), antiderivative size = 231, normalized size of antiderivative = 0.89

$$\int x^4\left(d+ex^2\right)^2\left(a+b\operatorname{arccosh}(cx)\right)dx$$
$$=\frac{11025\,ac^9e^2x^9+28350\,ac^9dex^7+19845\,ac^9d^2x^5+315\left(35\,bc^9e^2x^9+90\,bc^9dex^7+63\,bc^9d^2x^5\right)\log\left(cx+\right.}{}$$

input
```
integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*e^2*x^9 + 28350*a*c^9*d*e*x^7 + 19845*a*c^9*d^2*x^5 +
 315*(35*b*c^9*e^2*x^9 + 90*b*c^9*d*e*x^7 + 63*b*c^9*d^2*x^5)*log(c*x + sq
rt(c^2*x^2 - 1)) - (1225*b*c^8*e^2*x^8 + 10584*b*c^4*d^2 + 50*(81*b*c^8*d*
e + 28*b*c^6*e^2)*x^6 + 12960*b*c^2*d*e + 3*(1323*b*c^8*d^2 + 1620*b*c^6*d
*e + 560*b*c^4*e^2)*x^4 + 4480*b*e^2 + 4*(1323*b*c^6*d^2 + 1620*b*c^4*d*e
+ 560*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

Sympy [F]

$$\int x^4 (d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx = \int x^4 (a+b\operatorname{acosh}(cx)) \left(d+ex^2\right)^2\,dx$$

input
```
integrate(x**4*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.17

$$\int x^4 (d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx = \frac{1}{9}ae^2x^9 + \frac{2}{7}adex^7 + \frac{1}{5}ad^2x^5 + \frac{1}{75}\left(15x^5\operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2 + \frac{2}{245}\left(35x^7\operatorname{arcosh}(cx) - \left(\frac{5\sqrt{c^2x^2-1}x^6}{c^2} + \frac{6\sqrt{c^2x^2-1}x^4}{c^4} + \frac{8\sqrt{c^2x^2-1}x^2}{c^6} + \frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bd + \frac{1}{2835}\left(315x^9\operatorname{arcosh}(cx) - \left(\frac{35\sqrt{c^2x^2-1}x^8}{c^2} + \frac{40\sqrt{c^2x^2-1}x^6}{c^4} + \frac{48\sqrt{c^2x^2-1}x^4}{c^6} + \frac{64\sqrt{c^2x^2-1}x}{c^8}\right.\right.$$

input
```
integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^2*x^9 + 2/7*a*d*e*x^7 + 1/5*a*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x)
- (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*
x^2 - 1)/c^6)*c)*b*d^2 + 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)
*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*
sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e + 1/2835*(315*x^9*arccosh(c*x) - (35*sqrt(
c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)
*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b
*e^2
```

Giac [F(-2)]

Exception generated.

$$\int x^4(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx = \int x^4\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.29

$$\int x^4(d+ex^2)^2(a+b\operatorname{arccosh}(cx))\,dx$$
$$=\frac{19845acosh(cx)\,b\,c^9d^2x^5+28350acosh(cx)\,b\,c^9de\,x^7+11025acosh(cx)\,b\,c^9e^2x^9-3969\sqrt{c^2x^2-1}\,b\,c^8d^2x}{}$$

input
```
int(x^4*(e*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(19845*acosh(c*x)*b*c**9*d**2*x**5 + 28350*acosh(c*x)*b*c**9*d*e*x**7 + 11
025*acosh(c*x)*b*c**9*e**2*x**9 - 3969*sqrt(c**2*x**2 - 1)*b*c**8*d**2*x**
4 - 4050*sqrt(c**2*x**2 - 1)*b*c**8*d*e*x**6 - 1225*sqrt(c**2*x**2 - 1)*b*
c**8*e**2*x**8 - 5292*sqrt(c**2*x**2 - 1)*b*c**6*d**2*x**2 - 4860*sqrt(c**
2*x**2 - 1)*b*c**6*d*e*x**4 - 1400*sqrt(c**2*x**2 - 1)*b*c**6*e**2*x**6 -
10584*sqrt(c**2*x**2 - 1)*b*c**4*d**2 - 6480*sqrt(c**2*x**2 - 1)*b*c**4*d*
e*x**2 - 1680*sqrt(c**2*x**2 - 1)*b*c**4*e**2*x**4 - 12960*sqrt(c**2*x**2
- 1)*b*c**2*d*e - 2240*sqrt(c**2*x**2 - 1)*b*c**2*e**2*x**2 - 4480*sqrt(c*
*2*x**2 - 1)*b*e**2 + 19845*a*c**9*d**2*x**5 + 28350*a*c**9*d*e*x**7 + 110
25*a*c**9*e**2*x**9)/(99225*c**9)
```

3.370 $\int x^3(d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 301

$$\begin{aligned}\int x^3(d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx &= -\frac{b(288c^4d^2+320c^2de+105e^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{3072c^7}\\ &\quad-\frac{b(288c^4d^2+320c^2de+105e^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{4608c^5}\\ &\quad-\frac{be(64c^2d+21e)\,x^5\sqrt{-1+cx}\sqrt{1+cx}}{1152c^3}-\frac{be^2x^7\sqrt{-1+cx}\sqrt{1+cx}}{64c}\\ &\quad+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))\\ &\quad-\frac{b(288c^4d^2+320c^2de+105e^2)\sqrt{-1+c^2x^2}\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{3072c^8\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^7-1/4608*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x^3*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c^5-1/1152*b*e*(64*c^2*d+21*e)*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/
64*b*e^2*x^7*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/4*d^2*x^4*(a+b*arccosh(c*x))+
1/3*d*e*x^6*(a+b*arccosh(c*x))+1/8*e^2*x^8*(a+b*arccosh(c*x))-1/3072*b*(28
8*c^4*d^2+320*c^2*d*e+105*e^2)*(c^2*x^2-1)^(1/2)*arctanh(c*x/(c^2*x^2-1)^(
1/2))/c^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.71

$$\int x^3 \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$
$$= \frac{384ac^8x^4(6d^2+8dex^2+3e^2x^4) - bcx\sqrt{-1+cx}\sqrt{1+cx}(315e^2+30c^2e(32d+7ex^2)+8c^4(108d^2+80d$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(384*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) - b*c*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(315*e^2 + 30*c^2*e*(32*d + 7*e*x^2) + 8*c^4*(108*d^2 + 80*d*e*x
^2 + 21*e^2*x^4) + 16*c^6*(36*d^2*x^2 + 32*d*e*x^4 + 9*e^2*x^6)) + 384*b*c
^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x] - 6*b*(288*c^4*d^2 + 3
20*c^2*d*e + 105*e^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(9216*c^8)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6373, 27, 1905, 1590, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int \frac{x^4(3e^2x^4+8dex^2+6d^2)}{24\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{24}bc\int \frac{x^4\left(3e^2x^4+8dex^2+6d^2\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1905}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^4\left(3e^2x^4+8dex^2+6d^2\right)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1590}$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int \frac{x^4\left(48c^2d^2+e\left(64dc^2+21e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{8c^2}+\frac{3e^2x^7\sqrt{c^2x^2-1}}{8c^2}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{363}$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)\int \frac{x^4}{\sqrt{c^2x^2-1}}dx}{6c^2}+\frac{ex^5\sqrt{c^2x^2-1}\left(64c^2d+21e\right)}{6c^2}}{8c^2}+\frac{3e^2x^7\sqrt{c^2x^2-1}}{8c^2}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{262}$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)\left(\frac{3\int \frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{6c^2}+\frac{ex^5\sqrt{c^2x^2-1}\left(64c^2d+21e\right)}{6c^2}}{8c^2}+\frac{3e^2x^7\sqrt{c^2x^2-1}}{8c^2}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{262}$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{6c^2}+\frac{ex^5\sqrt{c^2x^2-1}\left(64c^2d+21e\right)}{6c^2}}{8c^2}+\frac{3e^2x^7\sqrt{c^2x^2-1}}{8c^2}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 224}$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)\left(\frac{3\left(\frac{\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{6c^2}+\frac{ex^5\sqrt{c^2x^2-1}\left(64c^2d+21e\right)}{6c^2}}{8c^2}+\frac{3e^2x^7\sqrt{c^2x^2-1}}{8c^2}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}d^2x^4(a+b\text{arccosh}(cx))+\frac{1}{3}dex^6(a+b\text{arccosh}(cx))+\frac{1}{8}e^2x^8(a+b\text{arccosh}(cx))-$$
$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)}{6c^2}+\frac{ex^5\sqrt{c^2x^2-1}\left(64c^2d+21e\right)}{6c^2}}{8c^2}+\frac{3e^2x^7\sqrt{c^2x^2-1}}{8c^2}\right)}{24\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcCosh[c*x]))/4 + (d*e*x^6*(a + b*ArcCosh[c*x]))/3 + (e^2
*x^8*(a + b*ArcCosh[c*x]))/8 - (b*c*Sqrt[-1 + c^2*x^2]*((3*e^2*x^7*Sqrt[-1
 + c^2*x^2])/(8*c^2) + ((e*(64*c^2*d + 21*e)*x^5*Sqrt[-1 + c^2*x^2])/(6*c^
2) + ((288*c^4*d^2 + 5*e*(64*c^2*d + 21*e))*((x^3*Sqrt[-1 + c^2*x^2])/(4*c
^2) + (3*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2
]]/(2*c^3)))/(4*c^2)))/(6*c^2))/(8*c^2)))/(24*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.21

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}d^2x^4\right)+\frac{b\left(\frac{c^4\,\mathrm{arccosh}(cx)e^2x^8}{8}+\frac{c^4\,\mathrm{arccosh}(cx)de\,x^6}{3}+\frac{\mathrm{arccosh}(cx)c^4x^4d^2}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}}{}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8de\,x^6+\frac{1}{8}e^2x^8c^8\right)}{c^4}+\frac{b\left(\frac{\mathrm{arccosh}(cx)d^2c^8x^4}{4}+\frac{\mathrm{arccosh}(cx)d\,c^8e\,x^6}{3}+\frac{\mathrm{arccosh}(cx)e^2c^8x^8}{8}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(576c^7d^2}{}\right.}{}$
default	$\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8de\,x^6+\frac{1}{8}e^2x^8c^8\right)}{c^4}+\frac{b\left(\frac{\mathrm{arccosh}(cx)d^2c^8x^4}{4}+\frac{\mathrm{arccosh}(cx)d\,c^8e\,x^6}{3}+\frac{\mathrm{arccosh}(cx)e^2c^8x^8}{8}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(576c^7d^2}{}\right.}{}$
orering	$\frac{(2160c^8e^3x^{10}+8240c^8d\,e^2x^8+10944c^8d^2e\,x^6+168x^8e^3c^6+4032c^8d^3x^4+968x^6e^2c^6d+2400x^4e\,c^6d^2+294x^6e^3c^4+864}{92}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*d^2*x^4)+b/c^4*(1/8*c^4*arccosh(c*x)*e^2*x^
8+1/3*c^4*arccosh(c*x)*d*e*x^6+1/4*arccosh(c*x)*c^4*x^4*d^2-1/9216/c^4*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*(576*c^7*d^2*(c^2*x^2-1)^(1/2)*x^3+512*c^7*d*e*(c
^2*x^2-1)^(1/2)*x^5+144*e^2*c^7*x^7*(c^2*x^2-1)^(1/2)+864*(c^2*x^2-1)^(1/2
)*c^5*d^2*x+640*e*(c^2*x^2-1)^(1/2)*c^5*d*x^3+168*e^2*c^5*x^5*(c^2*x^2-1)^
(1/2)+864*d^2*c^4*ln(c*x+(c^2*x^2-1)^(1/2))+960*c^3*d*e*x*(c^2*x^2-1)^(1/2
)+210*e^2*(c^2*x^2-1)^(1/2)*c^3*x^3+960*d*c^2*e*ln(c*x+(c^2*x^2-1)^(1/2))+
315*e^2*c*x*(c^2*x^2-1)^(1/2)+315*e^2*ln(c*x+(c^2*x^2-1)^(1/2)))/(c^2*x^2-
1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.75

$$\int x^3\left(d+ex^2\right)^2\left(a+b\,\mathrm{arccosh}(cx)\right)dx$$
$$=\frac{1152\,ac^8e^2x^8+3072\,ac^8dex^6+2304\,ac^8d^2x^4+3\,(384\,bc^8e^2x^8+1024\,bc^8dex^6+768\,bc^8d^2x^4-288\,bc^4d^2}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/9216*(1152*a*c^8*e^2*x^8 + 3072*a*c^8*d*e*x^6 + 2304*a*c^8*d^2*x^4 + 3*(
384*b*c^8*e^2*x^8 + 1024*b*c^8*d*e*x^6 + 768*b*c^8*d^2*x^4 - 288*b*c^4*d^2
 - 320*b*c^2*d*e - 105*b*e^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (144*b*c^7*e^
2*x^7 + 8*(64*b*c^7*d*e + 21*b*c^5*e^2)*x^5 + 2*(288*b*c^7*d^2 + 320*b*c^5
*d*e + 105*b*c^3*e^2)*x^3 + 3*(288*b*c^5*d^2 + 320*b*c^3*d*e + 105*b*c*e^2
)*x)*sqrt(c^2*x^2 - 1))/c^8
```

Sympy [F]

$$\int x^3(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx=\int x^3(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.10

$$\int x^3(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx=\frac{1}{8}ae^2x^8+\frac{1}{3}adex^6+\frac{1}{4}ad^2x^4$$
$$+\frac{1}{32}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bd^2$$
$$+\frac{1}{144}\left(48x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{\right.}}{c^7}\right.\right.$$
$$+\frac{1}{3072}\left(384x^8\operatorname{arcosh}(cx)-\left(\frac{48\sqrt{c^2x^2-1}x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}}{c^8}\right.\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/32*(8*x^4*arccosh(c*x) -
 (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x
+ 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^2 + 1/144*(48*x^6*arccosh(c*x) - (8*s
qrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2
- 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*d*e + 1/307
2*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2
- 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8
+ 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*e^2
```

Giac [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^2(a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^2(a+b\operatorname{arccosh}(cx))\,dx = \int x^3\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.14

$$\int x^3 (d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx$$
$$= \frac{2304acosh(cx)\,b\,c^8d^2x^4+3072acosh(cx)\,b\,c^8de\,x^6+1152acosh(cx)\,b\,c^8e^2x^8-576\sqrt{c^2x^2-1}\,b\,c^7d^2x^3-5}{}$$

input
```
int(x^3*(e*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(2304*acosh(c*x)*b*c**8*d**2*x***4 + 3072*acosh(c*x)*b*c**8*d*e*x**6 + 1152
*acosh(c*x)*b*c**8*e**2*x**8 - 576*sqrt(c**2*x**2 - 1)*b*c**7*d**2*x**3 -
512*sqrt(c**2*x**2 - 1)*b*c**7*d*e*x**5 - 144*sqrt(c**2*x**2 - 1)*b*c**7*e
**2*x**7 - 864*sqrt(c**2*x**2 - 1)*b*c**5*d**2*x - 640*sqrt(c**2*x**2 - 1)
*b*c**5*d*e*x**3 - 168*sqrt(c**2*x**2 - 1)*b*c**5*e**2*x**5 - 960*sqrt(c**
2*x**2 - 1)*b*c**3*d*e*x - 210*sqrt(c**2*x**2 - 1)*b*c**3*e**2*x**3 - 315*
sqrt(c**2*x**2 - 1)*b*c*e**2*x - 864*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**4
*d**2 - 960*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d*e - 315*log(sqrt(c**2*
x**2 - 1) + c*x)*b*e**2 + 2304*a*c**8*d**2*x**4 + 3072*a*c**8*d*e*x**6 + 1
152*a*c**8*e**2*x**8)/(9216*c**8)
```

3.371 $\int x^2(d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\begin{aligned}\int x^2(d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx &= -\frac{b(35c^4d^2+42c^2de+15e^2)\sqrt{-1+cx}\sqrt{1+cx}}{105c^7} \\ &\quad -\frac{b(35c^4d^2+84c^2de+45e^2)(-1+cx)^{3/2}(1+cx)^{3/2}}{315c^7} \\ &\quad -\frac{be(14c^2d+15e)(-1+cx)^{5/2}(1+cx)^{5/2}}{175c^7}-\frac{be^2(-1+cx)^{7/2}(1+cx)^{7/2}}{49c^7} \\ &\quad +\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))+\frac{2}{5}dex^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^2x^7(a+b\text{arccosh}(cx))\end{aligned}$$

output
```
-1/105*b*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^7-1/
315*b*(35*c^4*d^2+84*c^2*d*e+45*e^2)*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^7-1/175
*b*e*(14*c^2*d+15*e)*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^7-1/49*b*e^2*(c*x-1)^(7
/2)*(c*x+1)^(7/2)/c^7+1/3*d^2*x^3*(a+b*arccosh(c*x))+2/5*d*e*x^5*(a+b*arcc
osh(c*x))+1/7*e^2*x^7*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.76

$$\int x^2\left(d+ex^2\right)^2(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{105ax^3(35d^2+42dex^2+15e^2x^4)-\frac{b\sqrt{-1+cx}\sqrt{1+cx}(720e^2+24c^2e(98d+15ex^2)+2c^4(1225d^2+588dex^2+135e^2x^4)+c^6(1225d^2x}{c^7}}{11025}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(105*a*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(720*e^2 + 24*c^2*e*(98*d + 15*e*x^2) + 2*c^4*(1225*d^2 + 588*d*e*x^
2 + 135*e^2*x^4) + c^6*(1225*d^2*x^2 + 882*d*e*x^4 + 225*e^2*x^6)))/c^7 +
105*b*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCosh[c*x])/11025
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6373, 27, 1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^2(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int \frac{x^3\left(15e^2x^4+42dex^2+35d^2\right)}{105\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))+\frac{2}{5}dex^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^2x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{105}bc\int \frac{x^3\left(15e^2x^4+42dex^2+35d^2\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))+\frac{2}{5}dex^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^2x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 1905$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^3(15e^2x^4+42dex^2+35d^2)}{\sqrt{c^2x^2-1}}dx}{105\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))+\frac{2}{5}dex^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^2x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 1578$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^2(15e^2x^4+42dex^2+35d^2)}{\sqrt{c^2x^2-1}}dx^2}{210\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))+\frac{2}{5}dex^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^2x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 1195$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \left(\frac{15e^2(c^2x^2-1)^{5/2}}{c^6}+\frac{3e(14dc^2+15e)(c^2x^2-1)^{3/2}}{c^6}+\frac{(35d^2c^4+84dec^2+45e^2)\sqrt{c^2x^2-1}}{c^6}+\frac{35d^2c^4+42dec^2+15e^2}{c^6\sqrt{c^2x^2-1}}\right)dx^2}{210\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))+\frac{2}{5}dex^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^2x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}d^2x^3(a+b\text{arccosh}(cx))+\frac{2}{5}dex^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^2x^7(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{6e(c^2x^2-1)^{5/2}(14c^2d+15e)}{5c^8}+\frac{30e^2(c^2x^2-1)^{7/2}}{7c^8}+\frac{2(c^2x^2-1)^{3/2}(35c^4d^2+84c^2de+45e^2)}{3c^8}+\frac{2\sqrt{c^2x^2-1}(35c^4d^2+42c^2de+15e^2)}{c^8}\right)}{210\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/210*(b*c*Sqrt[-1 + c^2*x^2]*((2*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*Sqrt
[-1 + c^2*x^2])/c^8 + (2*(35*c^4*d^2 + 84*c^2*d*e + 45*e^2)*(-1 + c^2*x^2)
^(3/2))/(3*c^8) + (6*e*(14*c^2*d + 15*e)*(-1 + c^2*x^2)^(5/2))/(5*c^8) + (
30*e^2*(-1 + c^2*x^2)^(7/2))/(7*c^8)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d
^2*x^3*(a + b*ArcCosh[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCosh[c*x]))/5 + (e^2
*x^7*(a + b*ArcCosh[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}d^2x^3\right)+\frac{b\left(\frac{c^3\,\mathrm{arccosh}(cx)e^2x^7}{7}+\frac{2c^3\,\mathrm{arccosh}(cx)de\,x^5}{5}+\frac{\mathrm{arccosh}(cx)c^3x^3d^2}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+}}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\mathrm{arccosh}(cx)d^2c^7x^3}{3}+\frac{2\,\mathrm{arccosh}(cx)d\,c^7e\,x^5}{5}+\frac{\mathrm{arccosh}(cx)e^2c^7x^7}{7}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(225x^6\right.}{}\right.}{}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\mathrm{arccosh}(cx)d^2c^7x^3}{3}+\frac{2\,\mathrm{arccosh}(cx)d\,c^7e\,x^5}{5}+\frac{\mathrm{arccosh}(cx)e^2c^7x^7}{7}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(225x^6\right.}{}\right.}{}}{c^3}$
orering	$\frac{(2925c^8e^3x^{10}+11727c^8d\,e^2x^8+17199c^8d^2e\,x^6+270x^8e^3c^6+6125c^8d^3x^4+1854x^6e^2c^6d+7938x^4e\,c^6d^2+540x^6e^3c^4+2}{}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*d^2*x^3)+b/c^3*(1/7*c^3*arccosh(c*x)*e^2*x^
7+2/5*c^3*arccosh(c*x)*d*e*x^5+1/3*arccosh(c*x)*c^3*x^3*d^2-1/11025/c^4*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*(225*c^6*e^2*x^6+882*c^6*d*e*x^4+1225*c^6*d^2*x^
2+270*c^4*e^2*x^4+1176*c^4*d*e*x^2+2450*c^4*d^2+360*c^2*e^2*x^2+2352*c^2*d
*e+720*e^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int x^2(d+ex^2)^2\,(a+b\mathrm{arccosh}(cx))\,dx$$
$$=\frac{1575\,ac^7e^2x^7+4410\,ac^7dex^5+3675\,ac^7d^2x^3+105\,(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3)\log\left(cx+\sqrt{c^2}\right.}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/11025*(1575*a*c^7*e^2*x^7 + 4410*a*c^7*d*e*x^5 + 3675*a*c^7*d^2*x^3 + 10
5*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3)*log(c*x + sqrt(
c^2*x^2 - 1)) - (225*b*c^6*e^2*x^6 + 2450*b*c^4*d^2 + 2352*b*c^2*d*e + 18*
(49*b*c^6*d*e + 15*b*c^4*e^2)*x^4 + 720*b*e^2 + (1225*b*c^6*d^2 + 1176*b*c
^4*d*e + 360*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [F]

$$\int x^2(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx = \int x^2(a+b\,\text{acosh}\,(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.15

$$\begin{aligned}\int x^2(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx &= \frac{1}{7}\,ae^2x^7+\frac{2}{5}\,adex^5+\frac{1}{3}\,ad^2x^3\\ &+\frac{1}{9}\left(3\,x^3\,\text{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^2\\ &+\frac{2}{75}\left(15\,x^5\,\text{arcosh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bde\\ &+\frac{1}{245}\left(35\,x^7\,\text{arcosh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2-1}x^6}{c^2}+\frac{6\,\sqrt{c^2x^2-1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2-1}x^2}{c^6}+\frac{16\,\sqrt{c^2x^2-1}}{c^8}\right)c\right)be\end{aligned}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) -
c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2 + 2/75*(15*
x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d*e + 1/245*(35*x^7*arccosh(c*x) - (5*
sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 -
 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^2
```

Giac [F(-2)]

Exception generated.

$$\int x^2(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx=\int x^2\,(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.27

$$\int x^2(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx$$

$$= \frac{3675acosh(cx)\,b\,c^7d^2x^3+4410acosh(cx)\,b\,c^7de\,x^5+1575acosh(cx)\,b\,c^7e^2x^7-1225\sqrt{c^2x^2-1}\,b\,c^6d^2x^2-}{}$$

input

```
int(x^2*(e*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output

```
(3675*acosh(c*x)*b*c**7*d**2*x**3 + 4410*acosh(c*x)*b*c**7*d*e*x**5 + 1575
*acosh(c*x)*b*c**7*e**2*x**7 - 1225*sqrt(c**2*x**2 - 1)*b*c**6*d**2*x**2 -
 882*sqrt(c**2*x**2 - 1)*b*c**6*d*e*x**4 - 225*sqrt(c**2*x**2 - 1)*b*c**6*
e**2*x**6 - 2450*sqrt(c**2*x**2 - 1)*b*c**4*d**2 - 1176*sqrt(c**2*x**2 - 1
)*b*c**4*d*e*x**2 - 270*sqrt(c**2*x**2 - 1)*b*c**4*e**2*x**4 - 2352*sqrt(c
**2*x**2 - 1)*b*c**2*d*e - 360*sqrt(c**2*x**2 - 1)*b*c**2*e**2*x**2 - 720*
sqrt(c**2*x**2 - 1)*b*e**2 + 3675*a*c**7*d**2*x**3 + 4410*a*c**7*d*e*x**5
+ 1575*a*c**7*e**2*x**7)/(11025*c**7)
```

3.372 $\int x(d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 233

$$\begin{aligned}\int x(d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx &= -\frac{b(24c^4d^2+18c^2de+5e^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5} \\ &\quad - \frac{be(18c^2d+5e)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3} \\ &\quad - \frac{be^2x^5\sqrt{-1+cx}\sqrt{1+cx}}{36c} + \frac{(d+ex^2)^3\,(a+b\text{arccosh}(cx))}{6e} \\ &\quad - \frac{b(2c^2d+e)\,(8c^4d^2+8c^2de+5e^2)\,\sqrt{-1+c^2x^2}\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{96c^6e\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/96*b*(24*c^4*d^2+18*c^2*d*e+5*e^2)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-1/
144*b*e*(18*c^2*d+5*e)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*b*e^2*x^5*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/6*(e*x^2+d)^3*(a+b*arccosh(c*x))/e-1/96*b*
(2*c^2*d+e)*(8*c^4*d^2+8*c^2*d*e+5*e^2)*(c^2*x^2-1)^(1/2)*arctanh(c*x/(c^2
*x^2-1)^(1/2))/c^6/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.79

$$\int x(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx$$
$$=\frac{cx\left(48ac^5x(3d^2+3dex^2+e^2x^4)-b\sqrt{-1+cx}\sqrt{1+cx}(15e^2+2c^2e(27d+5ex^2)+4c^4(18d^2+9dex^2+2\right.}{288c}$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(c*x*(48*a*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - b*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(15*e^2 + 2*c^2*e*(27*d + 5*e*x^2) + 4*c^4*(18*d^2 + 9*d*e*x^2 + 2*e
^2*x^4))) + 48*b*c^6*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCosh[c*x] - 6*b*
(24*c^4*d^2 + 18*c^2*d*e + 5*e^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(28
8*c^6)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6372, 648, 318, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6372}$$

$$\frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{6e}-\frac{bc\int\frac{(ex^2+d)^3}{\sqrt{cx-1}\sqrt{cx+1}}dx}{6e}$$

$$\Big\downarrow \text{648}$$

$$\frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{6e}-\frac{bc\sqrt{c^2x^2-1}\int\frac{(ex^2+d)^3}{\sqrt{c^2x^2-1}}dx}{6e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 318$$

$$\frac{(d+ex^2)^3(a+b\operatorname{arccosh}(cx))}{6e}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int\frac{(ex^2+d)\left(5e(2dc^2+e)x^2+d(6dc^2+e)\right)}{\sqrt{c^2x^2-1}}dx}{6c^2}+\frac{ex\sqrt{c^2x^2-1}(d+ex^2)^2}{6c^2}\right)}{6e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 403$$

$$\frac{(d+ex^2)^3(a+b\operatorname{arccosh}(cx))}{6e}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\int\frac{e(44d^2c^4+44dec^2+15e^2)x^2+d(24d^2c^4+14dec^2+5e^2)}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{5ex\sqrt{c^2x^2-1}(2c^2d+e)(d+ex^2)}{4c^2}}{6c^2}+\frac{ex\sqrt{c^2x^2-1}(d+ex^2)^2}{6c^2}\right)}{6e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 299$$

$$\frac{(d+ex^2)^3(a+b\operatorname{arccosh}(cx))}{6e}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{3(2c^2d+e)(8c^4d^2+8c^2de+5e^2)\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}(44c^4d^2+44c^2de+15e^2)}{2c^2}}{4c^2}+\frac{5ex\sqrt{c^2x^2-1}(2c^2d+e)(d+ex^2)}{4c^2}}{6c^2}+\cdots\right.}{6e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 224$$

$$\frac{(d+ex^2)^3(a+b\operatorname{arccosh}(cx))}{6e}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{3(2c^2d+e)(8c^4d^2+8c^2de+5e^2)\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}(44c^4d^2+44c^2de+15e^2)}{2c^2}}{4c^2}+\frac{5ex\sqrt{c^2x^2-1}(2c^2d+e)(d+ex^2)}{4c^2}}{6c^2}+\cdots\right.}{6e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{6e}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{3\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(2c^2d+e\right)\left(8c^4d^2+8c^2de+5e^2\right)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}\left(44c^4d^2+44c^2de+15e^2\right)}{2c^2}}{4c^2}+\frac{5ex\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(d+ex^2\right)}{4c^2}}{6c^2}+\frac{e}{}\right.}{6e\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/(6*e) - (b*c*Sqrt[-1 + c^2*x^2]*((e*x
*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^2)/(6*c^2) + ((5*e*(2*c^2*d + e)*x*Sqrt[-1
 + c^2*x^2]*(d + e*x^2))/(4*c^2) + ((e*(44*c^4*d^2 + 44*c^2*d*e + 15*e^2)*
x*Sqrt[-1 + c^2*x^2])/(2*c^2) + (3*(2*c^2*d + e)*(8*c^4*d^2 + 8*c^2*d*e +
5*e^2)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(2*c^3))/(4*c^2))/(6*c^2)))/(6*e
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

rule 6372
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.30

method	result
orering	$\frac{(88x^8e^3c^6+380x^6e^2c^6d+684x^4e\,c^6d^2+10x^6e^3c^4+216c^6d^3x^2+92x^4e^2c^4d-414x^2e\,c^4d^2+25x^4e^3c^2-144c^4d^3-319x^2e}{288(e\,x^2+d)c^6}$
parts	$\frac{a(e\,x^2+d)^3}{6e}+\frac{b\left(\frac{c^2e^2\operatorname{arccosh}(cx)x^6}{6}+\frac{c^2e\operatorname{arccosh}(cx)x^4d}{2}+\frac{\operatorname{arccosh}(cx)c^2x^2d^2}{2}+\frac{c^2\operatorname{arccosh}(cx)d^3}{6e}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(48c^6d^3}{}\right.}{}$
derivativedivides	$\frac{\frac{a(c^2e\,x^2+c^2d)^3}{6c^4e}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^6d^3}{6e}+\frac{\operatorname{arccosh}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arccosh}(cx)c^6d\,x^4}{2}+\frac{e^2\operatorname{arccosh}(cx)c^6x^6}{6}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(48c^6d^3}{}\right.}{}}{}$
default	$\frac{\frac{a(c^2e\,x^2+c^2d)^3}{6c^4e}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^6d^3}{6e}+\frac{\operatorname{arccosh}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arccosh}(cx)c^6d\,x^4}{2}+\frac{e^2\operatorname{arccosh}(cx)c^6x^6}{6}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(48c^6d^3}{}\right.}{}}{}$

input
```
int(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/288*(88*c^6*e^3*x^8+380*c^6*d*e^2*x^6+684*c^6*d^2*e*x^4+10*c^4*e^3*x^6+2
16*c^6*d^3*x^2+92*c^4*d*e^2*x^4-414*c^4*d^2*e*x^2+25*c^2*e^3*x^4-144*c^4*d
^3-319*c^2*d*e^2*x^2-108*c^2*d^2*e-90*e^3*x^2-30*d*e^2)/(e*x^2+d)/c^6*(a+b
*arccosh(c*x))-1/288*(8*c^4*e^2*x^4+36*c^4*d*e*x^2+72*c^4*d^2+10*c^2*e^2*x
^2+54*c^2*d*e+15*e^2)/c^6/(e*x^2+d)^2*(c*x-1)*(c*x+1)*((e*x^2+d)^2*(a+b*ar
ccosh(c*x))+4*x^2*(e*x^2+d)*(a+b*arccosh(c*x))*e+x*(e*x^2+d)^2*b*c/(c*x-1)
^(1/2)/(c*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

$$\int x\left(d+ex^2\right)^2\left(a+b\operatorname{arccosh}(cx)\right)dx$$
$$=\frac{48\,ac^6e^2x^6+144\,ac^6dex^4+144\,ac^6d^2x^2+3\,(16\,bc^6e^2x^6+48\,bc^6dex^4+48\,bc^6d^2x^2-24\,bc^4d^2-18\,bc^2de}{}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/288*(48*a*c^6*e^2*x^6 + 144*a*c^6*d*e*x^4 + 144*a*c^6*d^2*x^2 + 3*(16*b*
c^6*e^2*x^6 + 48*b*c^6*d*e*x^4 + 48*b*c^6*d^2*x^2 - 24*b*c^4*d^2 - 18*b*c^
2*d*e - 5*b*e^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5*e^2*x^5 + 2*(18*b
*c^5*d*e + 5*b*c^3*e^2)*x^3 + 3*(24*b*c^5*d^2 + 18*b*c^3*d*e + 5*b*c*e^2)*
x)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [F]

$$\int x(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx=\int x(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
Integral(x*(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.17

$$\begin{aligned}\int x(d+ex^2)^2(a+b\text{arccosh}(cx))\,dx &= \frac{1}{6}ae^2x^6+\frac{1}{2}adex^4+\frac{1}{2}ad^2x^2\\
&+\frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}c\right)}{c^3}\right)\right)bd^2\\
&+\frac{1}{16}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bde\\
&+\frac{1}{288}\left(48x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{\right.}}{c^7}\right.\right.\end{aligned}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arccosh(c*x) -
c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*
d^2 + 1/16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2
*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d*e + 1
/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2
 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x
^2 - 1)*c)/c^7)*c)*b*e^2
```

Giac [F(-2)]

Exception generated.

$$\int x(d+ex^2)^2 (a+b\operatorname{arccosh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2 (a+b\operatorname{arccosh}(cx))\, dx = \int x\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^2 dx$$

input
```
int(x*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.19

$$\int x(d+ex^2)^2(a+b\operatorname{arccosh}(cx))\,dx$$
$$= \frac{144acosh(cx)\,b\,c^6d^2x^2 + 144acosh(cx)\,b\,c^6de\,x^4 + 48acosh(cx)\,b\,c^6e^2x^6 - 72\sqrt{c^2x^2-1}\,b\,c^5d^2x - 36\sqrt{c^2x}$$

input
```
int(x*(e*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(144*acosh(c*x)*b*c**6*d**2*x**2 + 144*acosh(c*x)*b*c**6*d*e*x**4 + 48*aco
sh(c*x)*b*c**6*e**2*x**6 - 72*sqrt(c**2*x**2 - 1)*b*c**5*d**2*x - 36*sqrt(
c**2*x**2 - 1)*b*c**5*d*e*x**3 - 8*sqrt(c**2*x**2 - 1)*b*c**5*e**2*x**5 -
54*sqrt(c**2*x**2 - 1)*b*c**3*d*e*x - 10*sqrt(c**2*x**2 - 1)*b*c**3*e**2*x
**3 - 15*sqrt(c**2*x**2 - 1)*b*c*e**2*x - 72*log(sqrt(c**2*x**2 - 1) + c*x
)*b*c**4*d**2 - 54*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d*e - 15*log(sqrt
(c**2*x**2 - 1) + c*x)*b*e**2 + 144*a*c**6*d**2*x**2 + 144*a*c**6*d*e*x**4
 + 48*a*c**6*e**2*x**6)/(288*c**6)
```

3.373 $\int (d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\begin{aligned}\int (d+ex^2)^2 (a+b\text{arccosh}(cx))\,dx &= -\frac{b(15c^4d^2+10c^2de+3e^2)\sqrt{-1+cx}\sqrt{1+cx}}{15c^5} \\ &\quad -\frac{2be(5c^2d+3e)(-1+cx)^{3/2}(1+cx)^{3/2}}{45c^5}-\frac{be^2(-1+cx)^{5/2}(1+cx)^{5/2}}{25c^5} \\ &\quad +d^2x(a+b\text{arccosh}(cx))+\frac{2}{3}dex^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arccosh}(cx))\end{aligned}$$

output
```
-1/15*b*(15*c^4*d^2+10*c^2*d*e+3*e^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-2/45
*b*e*(5*c^2*d+3*e)*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^5-1/25*b*e^2*(c*x-1)^(5/2
)*(c*x+1)^(5/2)/c^5+d^2*x*(a+b*arccosh(c*x))+2/3*d*e*x^3*(a+b*arccosh(c*x)
)+1/5*e^2*x^5*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx = \frac{1}{225}\left(15ax\left(15d^2+10dex^2+3e^2x^4\right) - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5} + 15bx\left(15d^2+10dex^2+3e^2x^4\right)\text{arccosh}(cx)\right)$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x
^4)))/c^5 + 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x])/225
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6323, 27, 1905, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6323}$$

$$-bc\int \frac{x\left(3e^2x^4+10dex^2+15d^2\right)}{15\sqrt{cx-1}\sqrt{cx+1}}dx + d^2x(a+b\text{arccosh}(cx)) + \frac{2}{3}dex^3(a+b\text{arccosh}(cx)) + \frac{1}{5}e^2x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{15}bc\int \frac{x(3e^2x^4+10dex^2+15d^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx+d^2x(a+b\text{arccosh}(cx))+\frac{2}{3}dex^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \textbf{1905}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x(3e^2x^4+10dex^2+15d^2)}{\sqrt{c^2x^2-1}}dx}{15\sqrt{cx-1}\sqrt{cx+1}}+d^2x(a+b\text{arccosh}(cx))+\frac{2}{3}dex^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \textbf{1576}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{c^2x^2-1}}dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+d^2x(a+b\text{arccosh}(cx))+\frac{2}{3}dex^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \textbf{1140}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \left(\frac{3(c^2x^2-1)^{3/2}e^2}{c^4}+\frac{2(5dc^2+3e)\sqrt{c^2x^2-1}e}{c^4}+\frac{15d^2c^4+10dec^2+3e^2}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{30\sqrt{cx-1}\sqrt{cx+1}}+d^2x(a+b\text{arccosh}(cx))+\frac{2}{3}dex^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \textbf{2009}$$

$$d^2x(a+b\text{arccosh}(cx))+\frac{2}{3}dex^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arccosh}(cx))-\frac{bc\sqrt{c^2x^2-1}\left(\frac{4e(c^2x^2-1)^{3/2}(5c^2d+3e)}{3c^6}+\frac{6e^2(c^2x^2-1)^{5/2}}{5c^6}+\frac{2\sqrt{c^2x^2-1}(15c^4d^2+10c^2de+3e^2)}{c^6}\right)}{30\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/30*(b*c*Sqrt[-1 + c^2*x^2]*((2*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[-
1 + c^2*x^2])/c^6 + (4*e*(5*c^2*d + 3*e)*(-1 + c^2*x^2)^(3/2))/(3*c^6) + (
6*e^2*(-1 + c^2*x^2)^(5/2))/(5*c^6)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2
*x*(a + b*ArcCosh[c*x]) + (2*d*e*x^3*(a + b*ArcCosh[c*x]))/3 + (e^2*x^5*(a
 + b*ArcCosh[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6323
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0]
|| ILtQ[p + 1/2, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\left(\frac{c\,\operatorname{arccosh}(cx)e^2x^5}{5}+\frac{2c\,\operatorname{arccosh}(cx)de\,x^3}{3}+\operatorname{arccosh}(cx)cx\,d^2-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4\right.}{}\right.}{c}$
derivativedivides	$\frac{\frac{a\left(c^5d^2x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arccosh}(cx)d^2c^5x+\frac{2\,\operatorname{arccosh}(cx)d\,c^5e\,x^3}{3}+\frac{\operatorname{arccosh}(cx)e^2c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4e^2x^4+5\right.}{}\right.}{c^4}}{c}$
default	$\frac{\frac{a\left(c^5d^2x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arccosh}(cx)d^2c^5x+\frac{2\,\operatorname{arccosh}(cx)d\,c^5e\,x^3}{3}+\frac{\operatorname{arccosh}(cx)e^2c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4e^2x^4+5\right.}{}\right.}{c^4}}{c}$
orering	$\frac{x\left(81c^6e^3x^6+395c^6d\,e^2x^4+1275c^6d^2e\,x^2+12c^4e^3x^4+225c^6d^3+200c^4d\,e^2x^2-900c^4d^2e+48c^2e^3x^2-400c^2d\,e^2-96e^3\right)(}{225(e\,x^2+d)c^6}$

input
```
int((e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*c*arccosh(c*x)*e^2*x^5+2/3*c*ar
ccosh(c*x)*d*e*x^3+arccosh(c*x)*c*x*d^2-1/225/c^4*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*(9*c^4*e^2*x^4+50*c^4*d*e*x^2+225*c^4*d^2+12*c^2*e^2*x^2+100*c^2*d*e+2
4*e^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int \left(d+ex^2\right)^2\left(a+b\operatorname{arccosh}(cx)\right)dx$$
$$=\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-}{225\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^
5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1)
) - (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 + 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*
c^4*d*e + 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [F]

$$\int \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx=\int (a+b\operatorname{acosh}(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11

$$\begin{aligned}&\int \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx\\&=\frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{2}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bde\\&\quad+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)be^2\\&\quad+ad^2x+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd^2}{c}\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2
- 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d*e + 1/75*(15*x^5*arccosh(c*x)
 - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 - 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))
*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)^2 (a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^2 (a+b\operatorname{arccosh}(cx))\,dx = \int (a+b\operatorname{acosh}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

output
```
int((a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.26

$$\int (d+ex^2)^2 (a+b\operatorname{arccosh}(cx))\,dx = \frac{225 acosh(cx)\,b\,c^5 d^2 x + 150 acosh(cx)\,b\,c^5 de\,x^3 + 45 acosh(cx)\,b\,c^5 e^2 x^5 - 50\sqrt{c^2x^2-1}\,b\,c^4 de\,x^2 - 9\sqrt{c^2x^2}}{}$$

input
```
int((e*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(225*acosh(c*x)*b*c**5*d**2*x + 150*acosh(c*x)*b*c**5*d*e*x**3 + 45*acosh(
c*x)*b*c**5*e**2*x**5 - 50*sqrt(c**2*x**2 - 1)*b*c**4*d*e*x**2 - 9*sqrt(c*
*2*x**2 - 1)*b*c**4*e**2*x**4 - 100*sqrt(c**2*x**2 - 1)*b*c**2*d*e - 12*sq
rt(c**2*x**2 - 1)*b*c**2*e**2*x**2 - 24*sqrt(c**2*x**2 - 1)*b*e**2 - 225*s
qrt(c*x + 1)*sqrt(c*x - 1)*b*c**4*d**2 + 225*a*c**5*d**2*x + 150*a*c**5*d*
e*x**3 + 45*a*c**5*e**2*x**5)/(225*c**5)
```

3.374 $\int \frac{(d+ex^2)^2(a+b\text{arccosh}(cx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 342

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\text{arccosh}(cx))}{x}\,dx = &-\frac{be(16c^2d+3e)\,x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3} \\ &-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c} \\ &-\frac{be(16c^2d+3e)\,\text{arccosh}(cx)}{32c^4} \\ &+dex^2(a+b\text{arccosh}(cx)) \\ &+\frac{1}{4}e^2x^4(a+b\text{arccosh}(cx)) \\ &-\frac{ibd^2\sqrt{1-c^2x^2}\arcsin(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} \\ &+\frac{bd^2\sqrt{1-c^2x^2}\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \\ &+d^2(a+b\text{arccosh}(cx))\log(x) \\ &-\frac{bd^2\sqrt{1-c^2x^2}\arcsin(cx)\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}} \\ &-\frac{ibd^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/32*b*e*(16*c^2*d+3*e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/16*b*e^2*x^3*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/32*b*e*(16*c^2*d+3*e)*arccosh(c*x)/c^4+d*e
*x^2*(a+b*arccosh(c*x))+1/4*e^2*x^4*(a+b*arccosh(c*x))-1/2*I*b*d^2*(-c^2*x
^2+1)^(1/2)*arcsin(c*x)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d^2*(-c^2*x^2+1)^(
1/2)*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+d^2*(a+b*arccosh(c*x))*ln(x)-b*d^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln
(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*d^2*(-c^2*x^2+1)^(1/2)*polylog(2,(
I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x} dx$$
$$= adex^2 + \frac{1}{4}ae^2x^4 + bdex^2\text{arccosh}(cx) + \frac{1}{4}be^2x^4\text{arccosh}(cx) - \frac{bde\left(cx\sqrt{-1+cx}\sqrt{1+cx} + 2\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{2c^2} - \frac{be^2\left(cx\sqrt{\frac{-1+cx}{1+cx}}(3+3cx+2c^2x^2+2c^3x^3) + 6\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{32c^4} + \frac{1}{2}bd^2\text{arccosh}(cx)\left(\text{arccosh}(cx) + 2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right) + ad^2\log(x) - \frac{1}{2}bd^2\,\text{PolyLog}\left(2, -e^{-2\text{arccosh}(cx)}\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
a*d*e*x^2 + (a*e^2*x^4)/4 + b*d*e*x^2*ArcCosh[c*x] + (b*e^2*x^4*ArcCosh[c*
x])/4 - (b*d*e*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*
x)/(1 + c*x)]]))/(2*c^2) - (b*e^2*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(3 + 3*c
*x + 2*c^2*x^2 + 2*c^3*x^3) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(32*
c^4) + (b*d^2*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])])
)/2 + a*d^2*Log[x] - (b*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x} \, dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int \frac{e^2x^4+4dex^2+4d^2\log(x)}{4\sqrt{cx-1}\sqrt{cx+1}}dx+d^2\log(x)(a+b\text{arccosh}(cx))+dex^2(a+b\text{arccosh}(cx))+\frac{1}{4}e^2x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}bc\int \frac{e^2x^4+4dex^2+4d^2\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}dx+d^2\log(x)(a+b\text{arccosh}(cx))+dex^2(a+b\text{arccosh}(cx))+\frac{1}{4}e^2x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{4}bc\int \left(\frac{e^2x^4}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{4dex^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{4d^2\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}\right)dx+d^2\log(x)(a+b\text{arccosh}(cx))+dex^2(a+b\text{arccosh}(cx))+\frac{1}{4}e^2x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2\log(x)(a+b\text{arccosh}(cx))+dex^2(a+b\text{arccosh}(cx))+\frac{1}{4}e^2x^4(a+b\text{arccosh}(cx))-\frac{1}{4}bc\left(\frac{3e^2\text{arccosh}(cx)}{8c^5}+\frac{2de\text{arccosh}(cx)}{c^3}+\frac{2id^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{2id^2\sqrt{1-c^2x^2}\arcsin(cx)^2}{c\sqrt{cx-1}\sqrt{cx+1}}\right.$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
d*e*x^2*(a + b*ArcCosh[c*x]) + (e^2*x^4*(a + b*ArcCosh[c*x]))/4 + d^2*(a +
 b*ArcCosh[c*x])*Log[x] - (b*c*((2*d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2
 + (3*e^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (e^2*x^3*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/(4*c^2) + (2*d*e*ArcCosh[c*x])/c^3 + (3*e^2*ArcCosh[c*x])
/(8*c^5) + ((2*I)*d^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (4*d^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcS
in[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*d^2*Sqrt[1 - c^2*x^2]*Arc
Sin[c*x]*Log[x])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*d^2*Sqrt[1 - c^
2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.65

method	result
parts	$a\left(\frac{e^2x^4}{4}+de\,x^2+d^2\ln\left(x\right)\right)-\frac{b\,e^2x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}-\frac{3b\sqrt{cx+1}\,\sqrt{cx-1}\,e^2x}{32c^3}+b\,\operatorname{arccosh}\left(cx\right)de$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\,d^2\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2}+b\,\operatorname{arccosh}\left(cx\right)de\,x^2-$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\,d^2\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2}+b\,\operatorname{arccosh}\left(cx\right)de\,x^2-$

input
```
int((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))-1/16*b*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c-3/32*b/c^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*e^2*x+b*arccosh(c*x)*d*e*x^2+1
/4*b*arccosh(c*x)*e^2*x^4-1/2*b/c^2*d*e*arccosh(c*x)-1/2*b/c*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*d*e*x-1/2*d^2*b*arccosh(c*x)^2+1/2*b*d^2*polylog(2,-(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))^2)+d^2*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2)-3/32*b/c^4*e^2*arccosh(c*x)
```

Fricas [F]

$$\int\frac{\left(d+ex^2\right)^2\left(a+b\text{arccosh}(cx)\right)}{x}\,dx=\int\frac{\left(ex^2+d\right)^2\left(b\operatorname{arcosh}\left(cx\right)+a\right)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccosh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x} dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)^2}{x} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x} dx = \int \frac{(ex^2+d)^2 (b\,\text{arcosh}\,(cx)+a)}{x} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate(b*e^2*x^3*log(c*x + s
qrt(c*x + 1)*sqrt(c*x - 1)) + 2*b*d*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x} dx = \int \frac{(ex^2+d)^2 (b\,\text{arcosh}\,(cx)+a)}{x} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x} \, dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)^2}{x} \, dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x} \, dx = \frac{32acosh(cx)\,b\,c^4de\,x^2+8acosh(cx)\,b\,c^4e^2x^4-16\sqrt{c^2x^2-1}\,b\,c^3dex-2\sqrt{c^2x^2-1}\,b\,c^3e^2x^3-3\sqrt{c^2x^2-1}}{}$$

input
```
int((e*x^2+d)^2*(a+b*acosh(c*x))/x,x)
```

output
```
(32*acosh(c*x)*b*c**4*d*e*x**2 + 8*acosh(c*x)*b*c**4*e**2*x**4 - 16*sqrt(c
**2*x**2 - 1)*b*c**3*d*e*x - 2*sqrt(c**2*x**2 - 1)*b*c**3*e**2*x**3 - 3*sq
rt(c**2*x**2 - 1)*b*c*e**2*x + 32*int(acosh(c*x)/x,x)*b*c**4*d**2 - 16*log
(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d*e - 3*log(sqrt(c**2*x**2 - 1) + c*x)*
b*e**2 + 32*log(x)*a*c**4*d**2 + 32*a*c**4*d*e*x**2 + 8*a*c**4*e**2*x**4)/
(32*c**4)
```

3.375 $\int \frac{(d+ex^2)^2(a+b\,\mathbf{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{(d+ex^2)^2\,(a+b\text{arccosh}(cx))}{x^2}\,dx = -\frac{2be(9c^2d+e)\sqrt{-1+cx}\sqrt{1+cx}}{9c^3} - \frac{be^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{9c} - \frac{d^2(a+b\text{arccosh}(cx))}{x} + 2dex(a+b\text{arccosh}(cx)) + \frac{1}{3}e^2x^3(a+b\text{arccosh}(cx)) + bcd^2\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output

```
-2/9*b*e*(9*c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/9*b*e^2*x^2*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/c-d^2*(a+b*arccosh(c*x))/x+2*d*e*x*(a+b*arccosh(c*x))
+1/3*e^2*x^3*(a+b*arccosh(c*x))+b*c*d^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2)
)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)^2 (a+b\operatorname{arccosh}(cx))}{x^2} dx = \frac{1}{3}\left(-\frac{3ad^2}{x}+6adex+ae^2x^3 - \frac{be\sqrt{-1+cx}\sqrt{1+cx}(2e+c^2(18d+ex^2))}{3c^3} + \frac{b(-3d^2+6dex^2+e^2x^4)\operatorname{arccosh}(cx)}{x} - 3bcd^2\arctan\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
((-3*a*d^2)/x + 6*a*d*e*x + a*e^2*x^3 - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(2*e + c^2*(18*d + e*x^2)))/(3*c^3) + (b*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*Ar
cCosh[c*x])/x - 3*b*c*d^2*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])/3
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6373, 27, 1905, 1578, 1192, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\operatorname{arccosh}(cx))}{x^2} dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int -\frac{-e^2x^4-6dex^2+3d^2}{3x\sqrt{cx-1}\sqrt{cx+1}}dx - \frac{d^2(a+b\operatorname{arccosh}(cx))}{x} + 2dex(a+b\operatorname{arccosh}(cx)) + \frac{1}{3}e^2x^3(a+b\operatorname{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-e^2x^4-6dex^2+3d^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^2(a+b\text{arccosh}(cx))}{x}+2dex(a+b\text{arccosh}(cx))+\frac{1}{3}e^2x^3(a+b\text{arccosh}(cx))$$

$\Big\downarrow$ 1905

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-e^2x^4-6dex^2+3d^2}{x\sqrt{c^2x^2-1}}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+b\text{arccosh}(cx))}{x}+2dex(a+b\text{arccosh}(cx))+\frac{1}{3}e^2x^3(a+b\text{arccosh}(cx))$$

$\Big\downarrow$ 1578

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\sqrt{c^2x^2-1}}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+b\text{arccosh}(cx))}{x}+2dex(a+b\text{arccosh}(cx))+\frac{1}{3}e^2x^3(a+b\text{arccosh}(cx))$$

$\Big\downarrow$ 1192

$$\frac{b\sqrt{c^2x^2-1}\int \frac{-e^2x^8-2e(3dc^2+e)x^4+3c^4d^2-e^2-6c^2de}{x^4+1}d\sqrt{c^2x^2-1}}{3c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+b\text{arccosh}(cx))}{x}+2dex(a+b\text{arccosh}(cx))+\frac{1}{3}e^2x^3(a+b\text{arccosh}(cx))$$

$\Big\downarrow$ 1467

$$\frac{b\sqrt{c^2x^2-1}\int \left(\frac{3d^2c^4}{x^4+1}-e^2x^4-e\left(6dc^2+e\right)\right)d\sqrt{c^2x^2-1}}{3c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+b\text{arccosh}(cx))}{x}+2dex(a+b\text{arccosh}(cx))+\frac{1}{3}e^2x^3(a+b\text{arccosh}(cx))$$

$\Big\downarrow$ 2009

$$-\frac{d^2(a+b\text{arccosh}(cx))}{x}+2dex(a+b\text{arccosh}(cx))+\frac{1}{3}e^2x^3(a+b\text{arccosh}(cx))+\frac{b\sqrt{c^2x^2-1}\left(3c^4d^2\arctan\left(\sqrt{c^2x^2-1}\right)-e\sqrt{c^2x^2-1}(6c^2d+e)-\frac{1}{3}e^2x^6\right)}{3c^3\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcCosh[c*x]))/x) + 2*d*e*x*(a + b*ArcCosh[c*x]) + (e^2*x^3*
(a + b*ArcCosh[c*x]))/3 + (b*Sqrt[-1 + c^2*x^2]*(-1/3*(e^2*x^6) - e*(6*c^2
*d + e)*Sqrt[-1 + c^2*x^2] + 3*c^4*d^2*ArcTan[Sqrt[-1 + c^2*x^2]]))/(3*c^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.26

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+bc\,(\frac{\operatorname{arccosh}(cx)e^2x^3}{3c}+\frac{2\operatorname{arccosh}(cx)xde}{c}-\frac{\operatorname{arccosh}(cx)d^2}{cx}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(}{}$
derivativedivides	$c\,(\frac{a\left(2d\,c^3ex+\frac{e^2c^3x^3}{3}-\frac{d^2c^3}{x}\right)}{c^4}+\frac{b\,(2\operatorname{arccosh}(cx)d\,c^3ex+\frac{\operatorname{arccosh}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arccosh}(cx)d^2c^3}{x}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9d^2c}{}}{c^4}$
default	$c\,(\frac{a\left(2d\,c^3ex+\frac{e^2c^3x^3}{3}-\frac{d^2c^3}{x}\right)}{c^4}+\frac{b\,(2\operatorname{arccosh}(cx)d\,c^3ex+\frac{\operatorname{arccosh}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arccosh}(cx)d^2c^3}{x}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9d^2c}{}}{c^4}$

input
```
int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3/c*arccosh(c*x)*e^2*x^3+2/c*arccosh(
c*x)*x*d*e-arccosh(c*x)*d^2/c/x-1/9/c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*d^2
*c^4*arctan(1/(c^2*x^2-1)^(1/2))+18*d*c^2*e*(c^2*x^2-1)^(1/2)+e^2*c^2*x^2*
(c^2*x^2-1)^(1/2)+2*e^2*(c^2*x^2-1)^(1/2))/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex^2)^2 (a+b\operatorname{arccosh}(cx))}{x^2} dx$$
$$= \frac{3\,ac^3e^2x^4 + 18\,bc^4d^2x\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + 18\,ac^3dex^2 - 9\,ac^3d^2 + 3\,(3\,bc^3d^2 - 6\,bc^3de - bc^3e^2)x}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*e^2*x^4 + 18*b*c^4*d^2*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 1
8*a*c^3*d*e*x^2 - 9*a*c^3*d^2 + 3*(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*
x*log(-c*x + sqrt(c^2*x^2 - 1)) + 3*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b
*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*log(c*x + sqrt(c^2*x
^2 - 1)) - (b*c^2*e^2*x^3 + 2*(9*b*c^2*d*e + b*e^2)*x)*sqrt(c^2*x^2 - 1))/
(c^3*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\operatorname{arccosh}(cx))}{x^2} dx = \int \frac{(a+b\,\operatorname{acosh}(cx))\,(d+ex^2)^2}{x^2} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^2} dx = \frac{1}{3} ae^2x^3 - \left(c \arcsin\left(\frac{1}{c|x|}\right) + \frac{\text{arcosh}(cx)}{x}\right)bd^2 + \frac{1}{9}\left(3x^3 \text{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)be^2 + 2\,adex + \frac{2\left(cx\,\text{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bde}{c} - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d^2 + 1/9*(3*x
^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))
*b*e^2 + 2*a*d*e*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d*e/c - a*
d^2/x
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^2} dx = \int \frac{(ex^2+d)^2(b\,\text{arcosh}(cx)+a)}{x^2} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)^2}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{-9acosh(cx)\,b\,c^3d^2+18acosh(cx)\,b\,c^3de\,x^2+3acosh(cx)\,b\,c^3e^2x^4-18atan\left(\sqrt{c^2x^2-1}+cx\right)b\,c^4d^2x-}{9c}$$

input
```
int((e*x^2+d)^2*(a+b*acosh(c*x))/x^2,x)
```

output
```
( - 9*acosh(c*x)*b*c**3*d**2 + 18*acosh(c*x)*b*c**3*d*e*x**2 + 3*acosh(c*x
)*b*c**3*e**2*x**4 - 18*atan(sqrt(c**2*x**2 - 1) + c*x)*b*c**4*d**2*x - sq
rt(c**2*x**2 - 1)*b*c**2*e**2*x**3 - 2*sqrt(c**2*x**2 - 1)*b*e**2*x - 18*s
qrt(c*x + 1)*sqrt(c*x - 1)*b*c**2*d*e*x - 9*a*c**3*d**2 + 18*a*c**3*d*e*x*
*2 + 3*a*c**3*e**2*x**4)/(9*c**3*x)
```

3.376 $\int \frac{(d+ex^2)^2(a+b\,\text{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 321

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\,\text{arccosh}(cx))}{x^3}\,dx &= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c}\\&\quad-\frac{be^2\text{arccosh}(cx)}{4c^2}-\frac{d^2(a+b\,\text{arccosh}(cx))}{2x^2}\\&\quad+\frac{1}{2}e^2x^2(a+b\,\text{arccosh}(cx))\\&\quad-\frac{ibde\sqrt{1-c^2x^2}\arcsin(cx)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\\&\quad+\frac{2bde\sqrt{1-c^2x^2}\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\\&\quad+2de(a+b\,\text{arccosh}(cx))\log(x)\\&\quad-\frac{2bde\sqrt{1-c^2x^2}\arcsin(cx)\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}}\\&\quad-\frac{ibde\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
1/2*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x-1/4*b*e^2*x*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c-1/4*b*e^2*arccosh(c*x)/c^2-1/2*d^2*(a+b*arccosh(c*x))/x^2+1/2*e^
2*x^2*(a+b*arccosh(c*x))-I*b*d*e*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+2*b*d*e*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(1-(I*c*x+(-c
^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*d*e*(a+b*arccosh(c*x))*l
n(x)-2*b*d*e*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-I*b*d*e*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^3} dx$$
$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2} + 2ae^2x^2 + \frac{2bd^2\left(cx\sqrt{-1+cx}\sqrt{1+cx} - \text{arccosh}(cx)\right)}{x^2} + \frac{be^2\left(-cx\sqrt{-1+cx}\sqrt{1+cx} + 2c^2x^2\text{arccosh}(cx) - 2\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{c^2} + 8ade\log(x) + 4bde\left(\text{arccosh}(cx)\left(\text{arccosh}(cx) + 2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right) - \text{PolyLog}\left(2, -e^{-2\text{arccosh}(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
((-2*a*d^2)/x^2 + 2*a*e^2*x^2 + (2*b*d^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
 - ArcCosh[c*x]))/x^2 + (b*e^2*(-(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 2*c^
2*x^2*ArcCosh[c*x] - 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^2 + 8*a*d*e
*Log[x] + 4*b*d*e*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*
x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/4
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^3}dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int -\frac{\frac{d^2}{x^2}-4e\log(x)d-e^2x^2}{2\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^2(a+b\text{arccosh}(cx))}{2x^2}+2de\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}e^2x^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}bc\int \frac{\frac{d^2}{x^2}-4e\log(x)d-e^2x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^2(a+b\text{arccosh}(cx))}{2x^2}+2de\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}e^2x^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}bc\int \left(\frac{d^2}{x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{4e\log(x)d}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{e^2x^2}{\sqrt{cx-1}\sqrt{cx+1}}\right)dx-\frac{d^2(a+b\text{arccosh}(cx))}{2x^2}+2de\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}e^2x^2(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\text{arccosh}(cx))}{2x^2}+2de\log(x)(a+b\text{arccosh}(cx))+\frac{1}{2}e^2x^2(a+b\text{arccosh}(cx))+\frac{1}{2}bc\left(-\frac{e^2\text{arccosh}(cx)}{2c^3}-\frac{2ide\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ide\sqrt{1-c^2x^2}\arcsin(cx)^2}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{4de\sqrt{1-c^2x^2}\ldots}{c\ldots}\right.$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + b*ArcCosh[c*x]))/x^2 + (e^2*x^2*(a + b*ArcCosh[c*x]))/2 + 2
*d*e*(a + b*ArcCosh[c*x])*Log[x] + (b*c*((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)/x - (e^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) - (e^2*ArcCosh[c*x])/(2
*c^3) - ((2*I)*d*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(c*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) + (4*d*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[
c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*d*e*Sqrt[1 - c^2*x^2]*ArcSin
[c*x]*Log[x])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*d*e*Sqrt[1 - c^2*x
^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/
2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.61

method	result
parts	$a\left(\frac{e^{2}x^{2}}{2}+2de\ln\left(x\right)-\frac{d^{2}}{2x^{2}}\right)-b\,\mathrm{arccosh}\left(cx\right)^{2}de+\frac{b\,e^{2}\,\mathrm{arccosh}(cx)x^{2}}{2}-\frac{b\,e^{2}x\sqrt{cx-1}\,\sqrt{cx+1}}{4c}-$
derivativedivides	$c^{2}\left(\frac{a\,x^{2}e^{2}}{2c^{2}}+\frac{2ade\ln(cx)}{c^{2}}-\frac{a\,d^{2}}{2c^{2}x^{2}}-\frac{bde\,\mathrm{arccosh}(cx)^{2}}{c^{2}}-\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,e^{2}x}{4c^{3}}+\frac{b\,\mathrm{arccosh}(cx)x^{2}e^{2}}{2c^{2}}-\right.$
default	$c^{2}\left(\frac{a\,x^{2}e^{2}}{2c^{2}}+\frac{2ade\ln(cx)}{c^{2}}-\frac{a\,d^{2}}{2c^{2}x^{2}}-\frac{bde\,\mathrm{arccosh}(cx)^{2}}{c^{2}}-\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,e^{2}x}{4c^{3}}+\frac{b\,\mathrm{arccosh}(cx)x^{2}e^{2}}{2c^{2}}-\right.$

input
```
int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2+2*d*e*ln(x)-1/2*d^2/x^2)-b*arccosh(c*x)^2*d*e+1/2*b*e^2*arc
cosh(c*x)*x^2-1/4*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*e^2*arccosh(
c*x)/c^2+1/2*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x-1/2*d^2*b*c^2-1/2*d^2*b
/x^2*arccosh(c*x)+2*b*e*d*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)+b*e*d*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)^2\,(a+b\mathrm{arccosh}(cx))}{x^3}\,dx=\int \frac{(ex^2+d)^2(b\,\mathrm{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccosh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^3} dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)^2}{x^3} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b\,\text{arcosh}\,(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 + 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + 2*a
*d*e*log(x) - 1/2*a*d^2/x^2 + integrate(b*e^2*x*log(c*x + sqrt(c*x + 1)*sq
rt(c*x - 1)) + 2*b*d*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b\,\text{arcosh}\,(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2(a+b\text{arccosh}(cx))}{x^3}\,dx=\int \frac{(a+b\,\text{acosh}(c\,x))\,(e\,x^2+d)^2}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2(a+b\text{arccosh}(cx))}{x^3}\,dx$$
$$=\frac{-2acosh(cx)\,b\,c^2d^2+2acosh(cx)\,b\,c^2e^2x^4-2\sqrt{c^2x^2-1}\,b\,c^3d^2x-\sqrt{c^2x^2-1}\,bc\,e^2x^3+8\left(\int \frac{acosh(cx)}{x}dx\right)b}{4c^2x^2}$$

input
```
int((e*x^2+d)^2*(a+b*acosh(c*x))/x^3,x)
```

output
```
( - 2*acosh(c*x)*b*c**2*d**2 + 2*acosh(c*x)*b*c**2*e**2*x**4 - 2*sqrt(c**2
*x**2 - 1)*b*c**3*d**2*x - sqrt(c**2*x**2 - 1)*b*c*e**2*x**3 + 8*int(acosh
(c*x)/x,x)*b*c**2*d*e*x**2 - log(sqrt(c**2*x**2 - 1) + c*x)*b*e**2*x**2 +
8*log(x)*a*c**2*d*e*x**2 - 2*a*c**2*d**2 + 2*a*c**2*e**2*x**4 - 2*b*c**4*d
**2*x**2)/(4*c**2*x**2)
```

3.377 $\int \frac{(d+ex^2)^2(a+b\,\mathrm{arccosh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{(d+ex^2)^2(a+b\,\mathrm{arccosh}(cx))}{x^4}\,dx = -\frac{be^2\sqrt{-1+cx}\sqrt{1+cx}}{c}+\frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d^2(a+b\,\mathrm{arccosh}(cx))}{3x^3}-\frac{2de(a+b\,\mathrm{arccosh}(cx))}{x} + e^2x(a+b\,\mathrm{arccosh}(cx)) + \frac{1}{6}bcd(c^2d+12e)\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
-b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/6*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/x^2-1/3*d^2*(a+b*arccosh(c*x))/x^3-2*d*e*(a+b*arccosh(c*x))/x+e^2*x*(a+
b*arccosh(c*x))+1/6*b*c*d*(c^2*d+12*e)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2(a+b\text{arccosh}(cx))}{x^4}\,dx = -\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x+b\left(-\frac{e^2}{c}+\frac{cd^2}{6x^2}\right)\sqrt{-1+cx}\sqrt{1+cx}-\frac{b(d^2+6dex^2-3e^2x^4)\,\text{arccosh}(cx)}{3x^3}-\frac{1}{6}bcd(c^2d+12e)\arctan\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(a*d^2)/x^3 - (2*a*d*e)/x + a*e^2*x + b*(-(e^2/c) + (c*d^2)/(6*x^2))*
Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCosh[c*
x])/(3*x^3) - (b*c*d*(c^2*d + 12*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)])/6
```

Rubi [A] (warning: unable to verify)

Time = 0.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6373, 27, 1905, 1578, 1192, 1471, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2(a+b\text{arccosh}(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^2(a+b\text{arccosh}(cx))}{3x^3}-\frac{2de(a+b\text{arccosh}(cx))}{x}+e^2x(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-3e^2x^4+6dex^2+d^2}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^2(a+\text{barccosh}(cx))}{3x^3}-\frac{2de(a+\text{barccosh}(cx))}{x}+e^2x(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 1905

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-3e^2x^4+6dex^2+d^2}{x^3\sqrt{c^2x^2-1}}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+\text{barccosh}(cx))}{3x^3}-\frac{2de(a+\text{barccosh}(cx))}{x}+e^2x(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 1578

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\sqrt{c^2x^2-1}}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+\text{barccosh}(cx))}{3x^3}-\frac{2de(a+\text{barccosh}(cx))}{x}+e^2x(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 1192

$$\frac{b\sqrt{c^2x^2-1}\int \frac{-3e^2x^8+6(c^2d-e)ex^4+c^4d^2-3e^2+6c^2de}{(x^4+1)^2}d\sqrt{c^2x^2-1}}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+\text{barccosh}(cx))}{3x^3}-\frac{2de(a+\text{barccosh}(cx))}{x}+e^2x(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 1471

$$\frac{b\sqrt{c^2x^2-1}\left(\frac{c^4d^2\sqrt{c^2x^2-1}}{2(x^4+1)}-\frac{1}{2}\int -\frac{d^2c^4+12dec^2-6e^2x^4-6e^2}{x^4+1}d\sqrt{c^2x^2-1}\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+\text{barccosh}(cx))}{3x^3}-\frac{2de(a+\text{barccosh}(cx))}{x}+e^2x(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 25

$$\frac{b\sqrt{c^2x^2-1}\left(\frac{1}{2}\int \frac{d^2c^4+12dec^2-6e^2x^4-6e^2}{x^4+1}d\sqrt{c^2x^2-1}+\frac{c^4d^2\sqrt{c^2x^2-1}}{2(x^4+1)}\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+\text{barccosh}(cx))}{3x^3}-\frac{2de(a+\text{barccosh}(cx))}{x}+e^2x(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 299

$$\frac{b\sqrt{c^2x^2-1}\left(\frac{1}{2}\left(c^2d(c^2d+12e)\int \frac{1}{x^4+1}d\sqrt{c^2x^2-1}-6e^2\sqrt{c^2x^2-1}\right)+\frac{c^4d^2\sqrt{c^2x^2-1}}{2(x^4+1)}\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2(a+\text{barccosh}(cx))}{3x^3}-\frac{2de(a+\text{barccosh}(cx))}{x}+e^2x(a+\text{barccosh}(cx))$$

$\Big\downarrow$ 216

$$-\frac{d^2(a+b\operatorname{arccosh}(cx))}{3x^3}-\frac{2de(a+b\operatorname{arccosh}(cx))}{x}+e^2x(a+b\operatorname{arccosh}(cx))+\frac{b\sqrt{c^2x^2-1}\left(\frac{1}{2}\left(c^2d\arctan\left(\sqrt{c^2x^2-1}\right)(c^2d+12e)-6e^2\sqrt{c^2x^2-1}\right)+\frac{c^4d^2\sqrt{c^2x^2-1}}{2(x^4+1)}\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcCosh[c*x]))/x^3 - (2*d*e*(a + b*ArcCosh[c*x]))/x + e^2
*x*(a + b*ArcCosh[c*x]) + (b*Sqrt[-1 + c^2*x^2]*((c^4*d^2*Sqrt[-1 + c^2*x^
2])/(2*(1 + x^4)) + (-6*e^2*Sqrt[-1 + c^2*x^2] + c^2*d*(c^2*d + 12*e)*ArcT
an[Sqrt[-1 + c^2*x^2]])/2))/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.36

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,c^3\left(\frac{\operatorname{arccosh}(cx)x\,e^2}{c^3}-\frac{2\operatorname{arccosh}(cx)de}{c^3x}-\frac{\operatorname{arccosh}(cx)d^2}{3c^3x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(\mathrm{a}\ldots}{}\right.$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{2dce}{x}-\frac{d^2c}{3x^3}\right)}{c^4}+\frac{b\left(\operatorname{arccosh}(cx)e^2cx-\frac{2\operatorname{arccosh}(cx)dce}{x}-\frac{\operatorname{arccosh}(cx)d^2c}{3x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\ldots\right.\right.}{}\right.}{c^4}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{2dce}{x}-\frac{d^2c}{3x^3}\right)}{c^4}+\frac{b\left(\operatorname{arccosh}(cx)e^2cx-\frac{2\operatorname{arccosh}(cx)dce}{x}-\frac{\operatorname{arccosh}(cx)d^2c}{3x^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\ldots\right.\right.}{}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*c^3*(1/c^3*arccosh(c*x)*x*e^2-2/c^3*arccos
h(c*x)*d*e/x-1/3*arccosh(c*x)*d^2/c^3/x^3-1/6/c^6*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*(arctan(1/(c^2*x^2-1)^(1/2))*c^6*d^2*x^2+12*arctan(1/(c^2*x^2-1)^(1/2)
)*c^4*d*e*x^2-d^2*c^4*(c^2*x^2-1)^(1/2)+6*e^2*c^2*x^2*(c^2*x^2-1)^(1/2))/(
c^2*x^2-1)^(1/2)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.54

$$\int\frac{(d+ex^2)^2\,(a+b\operatorname{arccosh}(cx))}{x^4}\,dx$$
$$=\frac{6\,ace^2x^4-12\,acdex^2+2\,(bc^4d^2+12\,bc^2de)x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,(bcd^2+6\,bcde-3\,bce^2)x^3\ldots}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a*c*e^2*x^4 - 12*a*c*d*e*x^2 + 2*(b*c^4*d^2 + 12*b*c^2*d*e)*x^3*arc
tan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*lo
g(-c*x + sqrt(c^2*x^2 - 1)) - 2*a*c*d^2 + 2*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2
 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log(c*x + sqrt(c^2*x^2
 - 1)) + (b*c^2*d^2*x - 6*b*e^2*x^3)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^4} \, dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)^2}{x^4} \, dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^4} \, dx &= -\frac{1}{6}\left(\left(c^2 \arcsin\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2-1}}{x^2}\right)c + \frac{2\,\text{arcosh}\,(cx)}{x^3}\right)bd^2 \\ &\quad - 2\left(c \arcsin\left(\frac{1}{c|x|}\right) + \frac{\text{arcosh}\,(cx)}{x}\right)bde + ae^2x \\ &\quad + \frac{\left(cx\,\text{arcosh}\,(cx) - \sqrt{c^2x^2-1}\right)be^2}{c} - \frac{2\,ade}{x} - \frac{ad^2}{3\,x^3}\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x
)/x^3)*b*d^2 - 2*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d*e + a*e^2*x
 + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/
x^3
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcosh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)^2}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)^2 (a+b\text{arccosh}(cx))}{x^4}\,dx = \frac{-2acosh(cx)\,bc\,d^2-12acosh(cx)\,bcde\,x^2+6acosh(cx)\,bc\,e^2x^4-2atan\left(\sqrt{c^2x^2-1}+cx\right)b\,c^4d^2x^3-24a\cdots}{}$$

input
```
int((e*x^2+d)^2*(a+b*acosh(c*x))/x^4,x)
```

output
```
( - 2*acosh(c*x)*b*c*d**2 - 12*acosh(c*x)*b*c*d*e*x**2 + 6*acosh(c*x)*b*c*
e**2*x**4 - 2*atan(sqrt(c**2*x**2 - 1) + c*x)*b*c**4*d**2*x**3 - 24*atan(s
qrt(c**2*x**2 - 1) + c*x)*b*c**2*d*e*x**3 - sqrt(c**2*x**2 - 1)*b*c**2*d**
2*x - 6*sqrt(c*x + 1)*sqrt(c*x - 1)*b*e**2*x**3 - 2*a*c*d**2 - 12*a*c*d*e*
x**2 + 6*a*c*e**2*x**4)/(6*c*x**3)
```

3.378 $\int x^4(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 365

$$\begin{aligned}
&\int x^4(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx \\
&= -\frac{b(231c^6d^3+495c^4d^2e+385c^2de^2+105e^3)\sqrt{-1+cx}\sqrt{1+cx}}{1155c^{11}} \\
&\quad -\frac{b(462c^6d^3+1485c^4d^2e+1540c^2de^2+525e^3)(-1+cx)^{3/2}(1+cx)^{3/2}}{3465c^{11}} \\
&\quad -\frac{b(77c^6d^3+495c^4d^2e+770c^2de^2+350e^3)(-1+cx)^{5/2}(1+cx)^{5/2}}{1925c^{11}} \\
&\quad -\frac{be(99c^4d^2+308c^2de+210e^2)(-1+cx)^{7/2}(1+cx)^{7/2}}{1617c^{11}} \\
&\quad -\frac{be^2(11c^2d+15e)(-1+cx)^{9/2}(1+cx)^{9/2}}{297c^{11}}-\frac{be^3(-1+cx)^{11/2}(1+cx)^{11/2}}{121c^{11}} \\
&\quad +\frac{1}{5}d^3x^5(a+b\text{arccosh}(cx))+\frac{3}{7}d^2ex^7(a+b\text{arccosh}(cx))+\frac{1}{3}de^2x^9(a+b\text{arccosh}(cx))+\frac{1}{11}e^3x^{11}(a+b\text{arccosh}
\end{aligned}$$

output
```
-1/1155*b*(231*c^6*d^3+495*c^4*d^2*e+385*c^2*d*e^2+105*e^3)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^11-1/3465*b*(462*c^6*d^3+1485*c^4*d^2*e+1540*c^2*d*e^2+525
*e^3)*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^11-1/1925*b*(77*c^6*d^3+495*c^4*d^2*e+
770*c^2*d*e^2+350*e^3)*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^11-1/1617*b*e*(99*c^4
*d^2+308*c^2*d*e+210*e^2)*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^11-1/297*b*e^2*(11
*c^2*d+15*e)*(c*x-1)^(9/2)*(c*x+1)^(9/2)/c^11-1/121*b*e^3*(c*x-1)^(11/2)*(
c*x+1)^(11/2)/c^11+1/5*d^3*x^5*(a+b*arccosh(c*x))+3/7*d^2*e*x^7*(a+b*arcco
sh(c*x))+1/3*d*e^2*x^9*(a+b*arccosh(c*x))+1/11*e^3*x^11*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76

$$\int x^4(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$$

$$= \frac{3465ax^5(231d^3+495d^2ex^2+385de^2x^4+105e^3x^6) - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(134400e^3+4480c^2e^2(121d+15ex^2)+80c^4e(9801d^2}{}}{}$$

input
```
Integrate[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
(3465*a*x^5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6) - (b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(134400*e^3 + 4480*c^2*e^2*(121*d + 15*e*x^2)
+ 80*c^4*e*(9801*d^2 + 3388*d*e*x^2 + 630*e^2*x^4) + 24*c^6*(17787*d^3 + 1
6335*d^2*e*x^2 + 8470*d*e^2*x^4 + 1750*e^3*x^6) + c^10*x^4*(160083*d^3 + 2
45025*d^2*e*x^2 + 148225*d*e^2*x^4 + 33075*e^3*x^6) + 2*c^8*(106722*d^3*x^
2 + 147015*d^2*e*x^4 + 84700*d*e^2*x^6 + 18375*e^3*x^8)))/c^11 + 3465*b*x^
5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6)*ArcCosh[c*x])/40
02075
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6373, 27, 2113, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx)) \, dx$$

$$\Big\downarrow \text{6373}$$

$$-bc \int \frac{x^5 \left(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3\right)}{1155\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{5}d^3x^5(a+b\text{arccosh}(cx))+\frac{3}{7}d^2ex^7(a+b\text{arccosh}(cx))+\frac{1}{3}de^2x^9(a+b\text{arccosh}(cx))+\frac{1}{11}e^3x^{11}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x^5\left(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{1155}+\frac{1}{5}d^3x^5(a+b\text{arccosh}(cx))+\frac{3}{7}d^2ex^7(a+b\text{arccosh}(cx))+\frac{1}{3}de^2x^9(a+b\text{arccosh}(cx))+\frac{1}{11}e^3x^{11}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2113}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^5\left(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3\right)}{\sqrt{c^2x^2-1}}dx}{1155\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}d^3x^5(a+b\text{arccosh}(cx))+\frac{3}{7}d^2ex^7(a+b\text{arccosh}(cx))+\frac{1}{3}de^2x^9(a+b\text{arccosh}(cx))+\frac{1}{11}e^3x^{11}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^4\left(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3\right)}{\sqrt{c^2x^2-1}}dx^2}{2310\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}d^3x^5(a+b\text{arccosh}(cx))+\frac{3}{7}d^2ex^7(a+b\text{arccosh}(cx))+\frac{1}{3}de^2x^9(a+b\text{arccosh}(cx))+\frac{1}{11}e^3x^{11}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2123}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int\left(\frac{105e^3\left(c^2x^2-1\right)^{9/2}}{c^{10}}+\frac{35e^2\left(11dc^2+15e\right)\left(c^2x^2-1\right)^{7/2}}{c^{10}}+\frac{5e\left(99d^2c^4+308dec^2+210e^2\right)\left(c^2x^2-1\right)^{5/2}}{c^{10}}+\frac{3\left(77d^3c^6+495d\right.}{}}{2310\sqrt{cx}}$$

$$\frac{1}{5}d^3x^5(a+b\text{arccosh}(cx))+\frac{3}{7}d^2ex^7(a+b\text{arccosh}(cx))+\frac{1}{3}de^2x^9(a+b\text{arccosh}(cx))+\frac{1}{11}e^3x^{11}(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}d^3x^5(a+b\text{arccosh}(cx))+\frac{3}{7}d^2ex^7(a+b\text{arccosh}(cx))+\frac{1}{3}de^2x^9(a+b\text{arccosh}(cx))+\frac{1}{11}e^3x^{11}(a+b\text{arccosh}(cx))-$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{70e^2\left(c^2x^2-1\right)^{9/2}\left(11c^2d+15e\right)}{9c^{12}}+\frac{210e^3\left(c^2x^2-1\right)^{11/2}}{11c^{12}}+\frac{10e\left(c^2x^2-1\right)^{7/2}\left(99c^4d^2+308c^2de+210e^2\right)}{7c^{12}}+\frac{6\left(c^2x^2-1\right)^{5/2}\left(77c\right.}{}}{2310\sqrt{c}}$$

input
```
Int[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/2310*(b*c*Sqrt[-1 + c^2*x^2]*((2*(231*c^6*d^3 + 495*c^4*d^2*e + 385*c^2
*d*e^2 + 105*e^3)*Sqrt[-1 + c^2*x^2])/c^12 + (2*(462*c^6*d^3 + 1485*c^4*d^
2*e + 1540*c^2*d*e^2 + 525*e^3)*(-1 + c^2*x^2)^(3/2))/(3*c^12) + (6*(77*c^
6*d^3 + 495*c^4*d^2*e + 770*c^2*d*e^2 + 350*e^3)*(-1 + c^2*x^2)^(5/2))/(5*
c^12) + (10*e*(99*c^4*d^2 + 308*c^2*d*e + 210*e^2)*(-1 + c^2*x^2)^(7/2))/(
7*c^12) + (70*e^2*(11*c^2*d + 15*e)*(-1 + c^2*x^2)^(9/2))/(9*c^12) + (210*
e^3*(-1 + c^2*x^2)^(11/2))/(11*c^12)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d
^3*x^5*(a + b*ArcCosh[c*x]))/5 + (3*d^2*e*x^7*(a + b*ArcCosh[c*x]))/7 + (d
*e^2*x^9*(a + b*ArcCosh[c*x]))/3 + (e^3*x^11*(a + b*ArcCosh[c*x]))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.87

method	result
parts	$a\left(\frac{1}{11}e^3x^{11}+\frac{1}{3}d\,e^2x^9+\frac{3}{7}d^2e\,x^7+\frac{1}{5}d^3x^5\right)+\frac{b\left(\frac{c^5\operatorname{arccosh}(cx)e^3x^{11}}{11}+\frac{c^5\operatorname{arccosh}(cx)d\,e^2x^9}{3}+\frac{3c^5\operatorname{arccosh}(cx}{7}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{5}c^{11}d^3x^5+\frac{3}{7}c^{11}d^2e\,x^7+\frac{1}{3}c^{11}d\,e^2x^9+\frac{1}{11}e^3c^{11}x^{11}\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^{11}d^3x^5}{5}+\frac{3\operatorname{arccosh}(cx)c^{11}d^2e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)c^{11}d\,e^2x}{3}\right.}{}$
default	$\frac{a\left(\frac{1}{5}c^{11}d^3x^5+\frac{3}{7}c^{11}d^2e\,x^7+\frac{1}{3}c^{11}d\,e^2x^9+\frac{1}{11}e^3c^{11}x^{11}\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^{11}d^3x^5}{5}+\frac{3\operatorname{arccosh}(cx)c^{11}d^2e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)c^{11}d\,e^2x}{3}\right.}{}$
orering	$(694575c^{12}e^4x^{14}+3312400c^{12}d\,e^3x^{12}+6092350c^{12}d^2e^2x^{10}+36750c^{10}e^4x^{12}+5096520c^{12}d^3e\,x^8+226450c^{10}d\,e^3x^{10}+$

input
```
int(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/11*e^3*x^11+1/3*d*e^2*x^9+3/7*d^2*e*x^7+1/5*d^3*x^5)+b/c^5*(1/11*c^5*
arccosh(c*x)*e^3*x^11+1/3*c^5*arccosh(c*x)*d*e^2*x^9+3/7*c^5*arccosh(c*x)*
d^2*e*x^7+1/5*arccosh(c*x)*c^5*x^5*d^3-1/4002075/c^6*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*(33075*c^10*e^3*x^10+148225*c^10*d*e^2*x^8+245025*c^10*d^2*e*x^6+36
750*c^8*e^3*x^8+160083*c^10*d^3*x^4+169400*c^8*d*e^2*x^6+294030*c^8*d^2*e*
x^4+42000*c^6*e^3*x^6+213444*c^8*d^3*x^2+203280*c^6*d*e^2*x^4+392040*c^6*d
^2*e*x^2+50400*c^4*e^3*x^4+426888*c^6*d^3+271040*c^4*d*e^2*x^2+784080*c^4*
d^2*e+67200*c^2*e^3*x^2+542080*c^2*d*e^2+134400*e^3))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.92

$$\int x^4(d+ex^2)^3\,(a+b\operatorname{arccosh}(cx))\,dx$$

$$=\frac{363825\,ac^{11}e^3x^{11}+1334025\,ac^{11}de^2x^9+1715175\,ac^{11}d^2ex^7+800415\,ac^{11}d^3x^5+3465\,(105\,bc^{11}e^3x^{11}+3}{}$$

input
```
integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/4002075*(363825*a*c^11*e^3*x^11 + 1334025*a*c^11*d*e^2*x^9 + 1715175*a*c
^11*d^2*e*x^7 + 800415*a*c^11*d^3*x^5 + 3465*(105*b*c^11*e^3*x^11 + 385*b*
c^11*d*e^2*x^9 + 495*b*c^11*d^2*e*x^7 + 231*b*c^11*d^3*x^5)*log(c*x + sqrt
(c^2*x^2 - 1)) - (33075*b*c^10*e^3*x^10 + 426888*b*c^6*d^3 + 1225*(121*b*c
^10*d*e^2 + 30*b*c^8*e^3)*x^8 + 784080*b*c^4*d^2*e + 25*(9801*b*c^10*d^2*e
 + 6776*b*c^8*d*e^2 + 1680*b*c^6*e^3)*x^6 + 542080*b*c^2*d*e^2 + 3*(53361*
b*c^10*d^3 + 98010*b*c^8*d^2*e + 67760*b*c^6*d*e^2 + 16800*b*c^4*e^3)*x^4
+ 134400*b*e^3 + 4*(53361*b*c^8*d^3 + 98010*b*c^6*d^2*e + 67760*b*c^4*d*e^
2 + 16800*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^11
```

Sympy [F(-1)]

Timed out.

$$\int x^4 (d + ex^2)^3 (a + b\operatorname{arccosh}(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.24

$$\int x^4 (d + ex^2)^3 (a + b\operatorname{arccosh}(cx))\,dx = \frac{1}{11}ae^3x^{11} + \frac{1}{3}ade^2x^9 + \frac{3}{7}ad^2ex^7 + \frac{1}{5}ad^3x^5$$
$$+ \frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^3$$
$$+ \frac{3}{245}\left(35\,x^7\operatorname{arcosh}(cx) - \left(\frac{5\sqrt{c^2x^2-1}x^6}{c^2} + \frac{6\sqrt{c^2x^2-1}x^4}{c^4} + \frac{8\sqrt{c^2x^2-1}x^2}{c^6} + \frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bd$$
$$+ \frac{1}{945}\left(315\,x^9\operatorname{arcosh}(cx) - \left(\frac{35\sqrt{c^2x^2-1}x^8}{c^2} + \frac{40\sqrt{c^2x^2-1}x^6}{c^4} + \frac{48\sqrt{c^2x^2-1}x^4}{c^6} + \frac{64\sqrt{c^2x^2-1}x^2}{c^8}\right.\right.$$
$$+ \frac{1}{7623}\left(693\,x^{11}\operatorname{arcosh}(cx) - \left(\frac{63\sqrt{c^2x^2-1}x^{10}}{c^2} + \frac{70\sqrt{c^2x^2-1}x^8}{c^4} + \frac{80\sqrt{c^2x^2-1}x^6}{c^6} + \frac{96\sqrt{c^2x^2-1}}{c^8}\right.\right.$$

input
```
integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/11*a*e^3*x^11 + 1/3*a*d*e^2*x^9 + 3/7*a*d^2*e*x^7 + 1/5*a*d^3*x^5 + 1/75
*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)
*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3 + 3/245*(35*x^7*arccosh(c*x)
- (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*
x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d^2*e + 1/945*(315*x^9*a
rccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4
 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(
c^2*x^2 - 1)/c^10)*c)*b*d*e^2 + 1/7623*(693*x^11*arccosh(c*x) - (63*sqrt(c
^2*x^2 - 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)*x^8/c^4 + 80*sqrt(c^2*x^2 - 1)
*x^6/c^6 + 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 128*sqrt(c^2*x^2 - 1)*x^2/c^10 +
 256*sqrt(c^2*x^2 - 1)/c^12)*c)*b*e^3
```

Giac [F(-2)]

Exception generated.

$$\int x^4(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx = \int x^4\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.40

$$\int x^4 (d + ex^2)^3 (a + b\text{arccosh}(cx))\, dx = \frac{-134400\sqrt{c^2x^2-1}\, b\, e^3 - 36750\sqrt{c^2x^2-1}\, b\, c^8e^3x^8 - 42000\sqrt{c^2x^2-1}\, b\, c^6e^3x^6 - 784080\sqrt{c^2x^2-1}\, b\, c^4 d}{}$$

input
```
int(x^4*(e*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(800415*acosh(c*x)*b*c**11*d**3*x**5 + 1715175*acosh(c*x)*b*c**11*d**2*e*x
**7 + 1334025*acosh(c*x)*b*c**11*d*e**2*x**9 + 363825*acosh(c*x)*b*c**11*e
**3*x**11 - 160083*sqrt(c**2*x**2 - 1)*b*c**10*d**3*x**4 - 245025*sqrt(c**
2*x**2 - 1)*b*c**10*d**2*e*x**6 - 148225*sqrt(c**2*x**2 - 1)*b*c**10*d*e**
2*x**8 - 33075*sqrt(c**2*x**2 - 1)*b*c**10*e**3*x**10 - 213444*sqrt(c**2*x
**2 - 1)*b*c**8*d**3*x**2 - 294030*sqrt(c**2*x**2 - 1)*b*c**8*d**2*e*x**4
- 169400*sqrt(c**2*x**2 - 1)*b*c**8*d*e**2*x**6 - 36750*sqrt(c**2*x**2 - 1
)*b*c**8*e**3*x**8 - 426888*sqrt(c**2*x**2 - 1)*b*c**6*d**3 - 392040*sqrt(
c**2*x**2 - 1)*b*c**6*d**2*e*x**2 - 203280*sqrt(c**2*x**2 - 1)*b*c**6*d*e*
*2*x**4 - 42000*sqrt(c**2*x**2 - 1)*b*c**6*e**3*x**6 - 784080*sqrt(c**2*x*
*2 - 1)*b*c**4*d**2*e - 271040*sqrt(c**2*x**2 - 1)*b*c**4*d*e**2*x**2 - 50
400*sqrt(c**2*x**2 - 1)*b*c**4*e**3*x**4 - 542080*sqrt(c**2*x**2 - 1)*b*c*
*2*d*e**2 - 67200*sqrt(c**2*x**2 - 1)*b*c**2*e**3*x**2 - 134400*sqrt(c**2*
x**2 - 1)*b*e**3 + 800415*a*c**11*d**3*x**5 + 1715175*a*c**11*d**2*e*x**7
+ 1334025*a*c**11*d*e**2*x**9 + 363825*a*c**11*e**3*x**11)/(4002075*c**11)
```

3.379 $\int x^3(d+ex^2)^3\,(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 399

$$\begin{aligned}\int x^3\left(d+ex^2\right)^3(a+b\text{arccosh}(cx))\,dx &= -\frac{b(480c^6d^3+800c^4d^2e+525c^2de^2+126e^3)\,x\sqrt{-1+cx}\sqrt{1+cx}}{5120c^9}\\ &\quad-\frac{b(480c^6d^3+800c^4d^2e+525c^2de^2+126e^3)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{7680c^7}\\ &\quad-\frac{be(800c^4d^2+525c^2de+126e^2)\,x^5\sqrt{-1+cx}\sqrt{1+cx}}{9600c^5}\\ &\quad-\frac{3be^2(25c^2d+6e)\,x^7\sqrt{-1+cx}\sqrt{1+cx}}{1600c^3}-\frac{be^3x^9\sqrt{-1+cx}\sqrt{1+cx}}{100c}\\ &\quad-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}+\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}\\ &\quad+\frac{b(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5)\sqrt{-1+c^2x^2}\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{5120c^{10}e^2\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output
```
-1/5120*b*(480*c^6*d^3+800*c^4*d^2*e+525*c^2*d*e^2+126*e^3)*x*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c^9-1/7680*b*(480*c^6*d^3+800*c^4*d^2*e+525*c^2*d*e^2+126*
e^3)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^7-1/9600*b*e*(800*c^4*d^2+525*c^2*d
*e+126*e^2)*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-3/1600*b*e^2*(25*c^2*d+6*e
)*x^7*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/100*b*e^3*x^9*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c-1/8*d*(e*x^2+d)^4*(a+b*arccosh(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*a
rccosh(c*x))/e^2+1/5120*b*(128*c^10*d^5-480*c^6*d^3*e^2-800*c^4*d^2*e^3-52
5*c^2*d*e^4-126*e^5)*(c^2*x^2-1)^(1/2)*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^10
/e^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.74

$$\int x^3 (d + ex^2)^3 (a + b\text{arccosh}(cx)) \, dx$$

$$= \frac{1920ax^4(10d^3 + 20d^2ex^2 + 15de^2x^4 + 4e^3x^6) - \frac{bx\sqrt{-1+cx}\sqrt{1+cx}(1890e^3+315c^2e^2(25d+4ex^2)+6c^4e(2000d^2+875dex^2+1}{}}{}$$

input
```
Integrate[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
(1920*a*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) - (b*x*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(1890*e^3 + 315*c^2*e^2*(25*d + 4*e*x^2) + 6*c^4*
e*(2000*d^2 + 875*d*e*x^2 + 168*e^2*x^4) + 8*c^6*(900*d^3 + 1000*d^2*e*x^2
 + 525*d*e^2*x^4 + 108*e^3*x^6) + 16*c^8*(300*d^3*x^2 + 400*d^2*e*x^4 + 22
5*d*e^2*x^6 + 48*e^3*x^8)))/c^9 + 1920*b*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d
*e^2*x^4 + 4*e^3*x^6)*ArcCosh[c*x] - (30*b*(480*c^6*d^3 + 800*c^4*d^2*e +
525*c^2*d*e^2 + 126*e^3)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/c^10)/76800
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6373, 27, 2041, 403, 27, 403, 403, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d+ex^2)^3 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int -\frac{(d-4ex^2)\,(ex^2+d)^4}{40e^2\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{(d+ex^2)^5 (a+b\text{arccosh}(cx))}{10e^2}-\frac{d(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(d-4ex^2)(ex^2+d)^4}{\sqrt{cx-1}\sqrt{cx+1}}dx}{40e^2}+\frac{(d+ex^2)^5 (a+b\text{arccosh}(cx))}{10e^2}-\frac{d(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e^2}$$

$$\Big\downarrow \text{2041}$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{(d-4ex^2)(ex^2+d)^4}{\sqrt{c^2x^2-1}}dx}{40e^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{(d+ex^2)^5 (a+b\text{arccosh}(cx))}{10e^2}-\frac{d(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int \frac{2(ex^2+d)^3\left(d(5c^2d-2e)-e(11dc^2+18e)x^2\right)}{\sqrt{c^2x^2-1}}dx}{10c^2}-\frac{2ex\sqrt{c^2x^2-1}(d+ex^2)^4}{5c^2}\right)}{40e^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{(d+ex^2)^5 (a+b\text{arccosh}(cx))}{10e^2}-\frac{d(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int \frac{\left(ex^2+d\right)^3\left(d\left(5c^2d-2e\right)-e\left(11dc^2+18e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{5c^2}-\frac{2ex\sqrt{c^2x^2-1}\left(d+ex^2\right)^4}{5c^2}\right)}{40e^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}$$

$\big\downarrow$ 403

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\int \frac{\left(ex^2+d\right)^2\left(d\left(40d^2c^4-27dec^2-18e^2\right)-e\left(26d^2c^4+201dec^2+126e^2\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{8c^2}-\frac{ex\sqrt{c^2x^2-1}\left(11c^2d+18e\right)\left(d+ex^2\right)^3}{8c^2}}{5c^2}-\frac{2ex\sqrt{c^2x^2-1}(d+\cdots}{5c^2}\right)}{40e^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}$$

$\big\downarrow$ 403

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\int \frac{\left(ex^2+d\right)\left(e\left(136d^3c^6-1096d^2ec^4-1617de^2c^2-630e^3\right)x^2+d\left(240d^3c^6-188d^2ec^4-309de^2c^2-126e^3\right)\right)}{\sqrt{c^2x^2-1}}dx}{6c^2}-\frac{ex\sqrt{c^2x^2-1}\left(26c^4d^2+201c^2d\cdots\right)}{6c^2}}{8c^2}}{5c^2}\right)}{40e^2\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}$$

$\big\downarrow$ 403

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\frac{\int \frac{e\left(1232d^4c^8-2536d^3ec^6-7758d^2e^2c^4-6615de^3c^2-1890e^4\right)x^2+d\left(960d^4c^8-616d^3ec^6-2332d^2e^2c^4-2121de^3c^2-630e^4\right)}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{ex\sqrt{c^2x^2-\cdots}}{}}{6c^2}}{8c^2}}{}\right)}{40e^2\cdots}$$

$$\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}$$

$\big\downarrow$ 299

$$bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\frac{15\left(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5\right)\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-\right.}{2c^2}}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}$$

$$\Big\downarrow \; 224$$

$$bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\frac{15\left(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5\right)\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6\right.}{2c^2}}{4c^2}}{6c^2}}{8c}\right.$$

$$\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(d+ex^2\right)^5(a+b\text{arccosh}(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\text{arccosh}(cx))}{8e^2}+$$

$$bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\frac{15\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5\right)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-66\right.}{2c^2}}{4c^2}}{6c^2}}{8c}\right.$$

input

```
Int[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/8*(d*(d + e*x^2)^4*(a + b*ArcCosh[c*x]))/e^2 + ((d + e*x^2)^5*(a + b*Ar
cCosh[c*x]))/(10*e^2) + (b*c*Sqrt[-1 + c^2*x^2]*((-2*e*x*Sqrt[-1 + c^2*x^2
]*(d + e*x^2)^4)/(5*c^2) + (-1/8*(e*(11*c^2*d + 18*e)*x*Sqrt[-1 + c^2*x^2]
*(d + e*x^2)^3)/c^2 + (-1/6*(e*(26*c^4*d^2 + 201*c^2*d*e + 126*e^2)*x*Sqrt
[-1 + c^2*x^2]*(d + e*x^2)^2)/c^2 + ((e*(136*c^6*d^3 - 1096*c^4*d^2*e - 16
17*c^2*d*e^2 - 630*e^3)*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2))/(4*c^2) + ((e*(1
232*c^8*d^4 - 2536*c^6*d^3*e - 7758*c^4*d^2*e^2 - 6615*c^2*d*e^3 - 1890*e^
4)*x*Sqrt[-1 + c^2*x^2])/(2*c^2) + (15*(128*c^10*d^5 - 480*c^6*d^3*e^2 - 8
00*c^4*d^2*e^3 - 525*c^2*d*e^4 - 126*e^5)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]
])/(2*c^3))/(4*c^2))/(6*c^2))/(8*c^2))/(5*c^2)))/(40*e^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2041
```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
Simp[(e1 + f1*x^(n/2))^FracPart[r]*((e2 + f2*x^(n/2))^FracPart[r]/(e1*e2 +
f1*f2*x^n)^FracPart[r])   Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^
n)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n
2, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.35

method	result
parts	$a\left(\frac{1}{10}e^3x^{10}+\frac{3}{8}d\,e^2x^8+\frac{1}{2}d^2e\,x^6+\frac{1}{4}d^3x^4\right)+\frac{b\left(\frac{c^4\operatorname{arccosh}(cx)e^3x^{10}}{10}+\frac{3c^4\operatorname{arccosh}(cx)d\,e^2x^8}{8}+\frac{c^4\operatorname{arccosh}(cx}{2}\right.}{}$
orering	$\frac{(4864c^{10}e^4x^{12}+23728c^{10}d\,e^3x^{10}+45200c^{10}d^2e^2x^8+288c^8e^4x^{10}+40000c^{10}d^3e\,x^6+1896c^8d\,e^3x^8+11200c^{10}d^4x^4+54}{}$
derivativedivides	$\frac{a\left(\frac{1}{4}c^{10}d^3x^4+\frac{1}{2}c^{10}d^2e\,x^6+\frac{3}{8}c^{10}d\,e^2x^8+\frac{1}{10}e^3x^{10}c^{10}\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^{10}d^3x^4}{4}+\frac{\operatorname{arccosh}(cx)c^{10}d^2e\,x^6}{2}+\frac{3\operatorname{arccosh}(cx)c^{10}d\,e^2x}{8}\right.}{}$
default	$\frac{a\left(\frac{1}{4}c^{10}d^3x^4+\frac{1}{2}c^{10}d^2e\,x^6+\frac{3}{8}c^{10}d\,e^2x^8+\frac{1}{10}e^3x^{10}c^{10}\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^{10}d^3x^4}{4}+\frac{\operatorname{arccosh}(cx)c^{10}d^2e\,x^6}{2}+\frac{3\operatorname{arccosh}(cx)c^{10}d\,e^2x}{8}\right.}{}$

input
```
int(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/10*e^3*x^10+3/8*d*e^2*x^8+1/2*d^2*e*x^6+1/4*d^3*x^4)+b/c^4*(1/10*c^4*
arccosh(c*x)*e^3*x^10+3/8*c^4*arccosh(c*x)*d*e^2*x^8+1/2*c^4*arccosh(c*x)*
d^2*e*x^6+1/4*arccosh(c*x)*c^4*x^4*d^3-1/76800/c^6*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*(4800*c^9*d^3*(c^2*x^2-1)^(1/2)*x^3+6400*c^9*d^2*e*(c^2*x^2-1)^(1/2)*
x^5+3600*c^9*d*e^2*(c^2*x^2-1)^(1/2)*x^7+768*e^3*c^9*x^9*(c^2*x^2-1)^(1/2)
+7200*(c^2*x^2-1)^(1/2)*c^7*d^3*x+8000*e*(c^2*x^2-1)^(1/2)*c^7*d^2*x^3+420
0*e^2*(c^2*x^2-1)^(1/2)*c^7*d*x^5+864*e^3*c^7*x^7*(c^2*x^2-1)^(1/2)+7200*c
^6*d^3*ln(c*x+(c^2*x^2-1)^(1/2))+12000*c^5*d^2*e*x*(c^2*x^2-1)^(1/2)+5250*
c^5*d*e^2*(c^2*x^2-1)^(1/2)*x^3+1008*e^3*(c^2*x^2-1)^(1/2)*c^5*x^5+12000*c
^4*d^2*e*ln(c*x+(c^2*x^2-1)^(1/2))+7875*c^3*d*e^2*x*(c^2*x^2-1)^(1/2)+1260
*e^3*c^3*x^3*(c^2*x^2-1)^(1/2)+7875*c^2*d*e^2*ln(c*x+(c^2*x^2-1)^(1/2))+18
90*e^3*c*x*(c^2*x^2-1)^(1/2)+1890*e^3*ln(c*x+(c^2*x^2-1)^(1/2)))/(c^2*x^2-
1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.83

$$\int x^3(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$$
$$= \frac{7680\,ac^{10}e^3x^{10}+28800\,ac^{10}de^2x^8+38400\,ac^{10}d^2ex^6+19200\,ac^{10}d^3x^4+15\,(512\,bc^{10}e^3x^{10}+1920\,bc^{10}de}{}$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/76800*(7680*a*c^10*e^3*x^10 + 28800*a*c^10*d*e^2*x^8 + 38400*a*c^10*d^2*
e*x^6 + 19200*a*c^10*d^3*x^4 + 15*(512*b*c^10*e^3*x^10 + 1920*b*c^10*d*e^2
*x^8 + 2560*b*c^10*d^2*e*x^6 + 1280*b*c^10*d^3*x^4 - 480*b*c^6*d^3 - 800*b
*c^4*d^2*e - 525*b*c^2*d*e^2 - 126*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (
768*b*c^9*e^3*x^9 + 144*(25*b*c^9*d*e^2 + 6*b*c^7*e^3)*x^7 + 8*(800*b*c^9*
d^2*e + 525*b*c^7*d*e^2 + 126*b*c^5*e^3)*x^5 + 10*(480*b*c^9*d^3 + 800*b*c
^7*d^2*e + 525*b*c^5*d*e^2 + 126*b*c^3*e^3)*x^3 + 15*(480*b*c^7*d^3 + 800*
b*c^5*d^2*e + 525*b*c^3*d*e^2 + 126*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^10
```

Sympy [F]

$$\int x^3(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx = \int x^3(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)^3\,dx$$

input
```
integrate(x**3*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))*(d + e*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.22

$$\int x^3(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx = \frac{1}{10}ae^3x^{10} + \frac{3}{8}ade^2x^8 + \frac{1}{2}ad^2ex^6 + \frac{1}{4}ad^3x^4 + \frac{1}{32}\left(8x^4\operatorname{arcosh}(cx) - \left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2} + \frac{3\sqrt{c^2x^2-1}x}{c^4} + \frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bd^3 + \frac{1}{96}\left(48x^6\operatorname{arcosh}(cx) - \left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2} + \frac{10\sqrt{c^2x^2-1}x^3}{c^4} + \frac{15\sqrt{c^2x^2-1}x}{c^6} + \frac{15\log\left(2c^2x+2\sqrt{c\right.}}{c^7}\right.\right. + \frac{1}{1024}\left(384x^8\operatorname{arcosh}(cx) - \left(\frac{48\sqrt{c^2x^2-1}x^7}{c^2} + \frac{56\sqrt{c^2x^2-1}x^5}{c^4} + \frac{70\sqrt{c^2x^2-1}x^3}{c^6} + \frac{105\sqrt{c^2x^2-1.}}{c^8}\right.\right. + \frac{1}{12800}\left(1280x^{10}\operatorname{arcosh}(cx) - \left(\frac{128\sqrt{c^2x^2-1}x^9}{c^2} + \frac{144\sqrt{c^2x^2-1}x^7}{c^4} + \frac{168\sqrt{c^2x^2-1}x^5}{c^6} + \frac{210\sqrt{c^2}}{c}\right.\right.$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/32
*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*
x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^3 + 1/96*(48*x^
6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c
^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/
c^7)*c)*b*d^2*e + 1/1024*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7
/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*s
qrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*
b*d*e^2 + 1/12800*(1280*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2
 + 144*sqrt(c^2*x^2 - 1)*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqr
t(c^2*x^2 - 1)*x^3/c^8 + 315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*c)/c^11)*c)*b*e^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^3(a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^3(a+b\operatorname{arccosh}(cx))\,dx = \int x^3\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.31

$$\int x^3(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$$
$$= \frac{38400acosh(cx)\,b\,c^{10}d^2e\,x^6+28800acosh(cx)\,b\,c^{10}d\,e^2x^8-6400\sqrt{c^2x^2-1}\,b\,c^9d^2e\,x^5-3600\sqrt{c^2x^2-1}\,b\,c}{}$$

input
```
int(x^3*(e*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(19200*acosh(c*x)*b*c**10*d**3*x**4 + 38400*acosh(c*x)*b*c**10*d**2*e*x**6
 + 28800*acosh(c*x)*b*c**10*d*e**2*x**8 + 7680*acosh(c*x)*b*c**10*e**3*x**
10 - 4800*sqrt(c**2*x**2 - 1)*b*c**9*d**3*x**3 - 6400*sqrt(c**2*x**2 - 1)*
b*c**9*d**2*e*x**5 - 3600*sqrt(c**2*x**2 - 1)*b*c**9*d*e**2*x**7 - 768*sqr
t(c**2*x**2 - 1)*b*c**9*e**3*x**9 - 7200*sqrt(c**2*x**2 - 1)*b*c**7*d**3*x
 - 8000*sqrt(c**2*x**2 - 1)*b*c**7*d**2*e*x**3 - 4200*sqrt(c**2*x**2 - 1)*
b*c**7*d*e**2*x**5 - 864*sqrt(c**2*x**2 - 1)*b*c**7*e**3*x**7 - 12000*sqrt
(c**2*x**2 - 1)*b*c**5*d**2*e*x - 5250*sqrt(c**2*x**2 - 1)*b*c**5*d*e**2*x
**3 - 1008*sqrt(c**2*x**2 - 1)*b*c**5*e**3*x**5 - 7875*sqrt(c**2*x**2 - 1)
*b*c**3*d*e**2*x - 1260*sqrt(c**2*x**2 - 1)*b*c**3*e**3*x**3 - 1890*sqrt(c
**2*x**2 - 1)*b*c*e**3*x - 7200*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**6*d**3
 - 12000*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**4*d**2*e - 7875*log(sqrt(c**2
*x**2 - 1) + c*x)*b*c**2*d*e**2 - 1890*log(sqrt(c**2*x**2 - 1) + c*x)*b*e*
*3 + 19200*a*c**10*d**3*x**4 + 38400*a*c**10*d**2*e*x**6 + 28800*a*c**10*d
*e**2*x**8 + 7680*a*c**10*e**3*x**10)/(76800*c**10)
```

3.380 $\int x^2(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 307

$$\begin{aligned}\int x^2(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx &= -\frac{b(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3)\sqrt{-1+cx}\sqrt{1+cx}}{315c^9} \\ &- \frac{b(105c^6d^3+378c^4d^2e+405c^2de^2+140e^3)(-1+cx)^{3/2}(1+cx)^{3/2}}{945c^9} \\ &- \frac{be(63c^4d^2+135c^2de+70e^2)(-1+cx)^{5/2}(1+cx)^{5/2}}{525c^9} \\ &- \frac{be^2(27c^2d+28e)(-1+cx)^{7/2}(1+cx)^{7/2}}{441c^9} - \frac{be^3(-1+cx)^{9/2}(1+cx)^{9/2}}{81c^9} \\ &+ \frac{1}{3}d^3x^3(a+b\text{arccosh}(cx)) + \frac{3}{5}d^2ex^5(a+b\text{arccosh}(cx)) + \frac{3}{7}de^2x^7(a+b\text{arccosh}(cx)) + \frac{1}{9}e^3x^9(a+b\text{arccosh}(c\end{aligned}$$

output

```
-1/315*b*(105*c^6*d^3+189*c^4*d^2*e+135*c^2*d*e^2+35*e^3)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/c^9-1/945*b*(105*c^6*d^3+378*c^4*d^2*e+405*c^2*d*e^2+140*e^3)*
(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^9-1/525*b*e*(63*c^4*d^2+135*c^2*d*e+70*e^2)*
(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^9-1/441*b*e^2*(27*c^2*d+28*e)*(c*x-1)^(7/2)*
(c*x+1)^(7/2)/c^9-1/81*b*e^3*(c*x-1)^(9/2)*(c*x+1)^(9/2)/c^9+1/3*d^3*x^3*(
a+b*arccosh(c*x))+3/5*d^2*e*x^5*(a+b*arccosh(c*x))+3/7*d*e^2*x^7*(a+b*arcc
osh(c*x))+1/9*e^3*x^9*(a+b*arccosh(c*x))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.77

$$\int x^2 \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx)) \, dx$$
$$= \frac{315ax^3(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6) - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(4480e^3+80c^2e^2(243d+28ex^2)+24c^4e(1323d^2+405de}{}}{}$$

input
```
Integrate[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
(315*a*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3*x^6) - (b*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(4480*e^3 + 80*c^2*e^2*(243*d + 28*e*x^2) + 24*c
^4*e*(1323*d^2 + 405*d*e*x^2 + 70*e^2*x^4) + 2*c^6*(11025*d^3 + 7938*d^2*e
*x^2 + 3645*d*e^2*x^4 + 700*e^3*x^6) + c^8*(11025*d^3*x^2 + 11907*d^2*e*x^
4 + 6075*d*e^2*x^6 + 1225*e^3*x^8)))/c^9 + 315*b*x^3*(105*d^3 + 189*d^2*e*
x^2 + 135*d*e^2*x^4 + 35*e^3*x^6)*ArcCosh[c*x])/99225
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6373, 27, 2113, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx)) \, dx$$

$$\Big\downarrow \ 6373$$

$$-bc\int \frac{x^3\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{315\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{3}d^3x^3(a+b\text{arccosh}(cx))+\frac{3}{5}d^2ex^5(a+b\text{arccosh}(cx))+\frac{3}{7}de^2x^7(a+b\text{arccosh}(cx))+\frac{1}{9}e^3x^9(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{315}bc\int \frac{x^3(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3)}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{1}{3}d^3x^3(a+b\operatorname{arccosh}(cx))+\frac{3}{5}d^2ex^5(a+b\operatorname{arccosh}(cx))+\frac{3}{7}de^2x^7(a+b\operatorname{arccosh}(cx))+\frac{1}{9}e^3x^9(a+b\operatorname{arccosh}(cx))$$

$\big\downarrow$ 2113

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^3(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3)}{\sqrt{c^2x^2-1}}dx}{315\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}d^3x^3(a+b\operatorname{arccosh}(cx))+\frac{3}{5}d^2ex^5(a+b\operatorname{arccosh}(cx))+\frac{3}{7}de^2x^7(a+b\operatorname{arccosh}(cx))+\frac{1}{9}e^3x^9(a+b\operatorname{arccosh}(cx))$$

$\big\downarrow$ 2331

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^2(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3)}{\sqrt{c^2x^2-1}}dx^2}{630\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}d^3x^3(a+b\operatorname{arccosh}(cx))+\frac{3}{5}d^2ex^5(a+b\operatorname{arccosh}(cx))+\frac{3}{7}de^2x^7(a+b\operatorname{arccosh}(cx))+\frac{1}{9}e^3x^9(a+b\operatorname{arccosh}(cx))$$

$\big\downarrow$ 2123

$$-\frac{bc\sqrt{c^2x^2-1}\int \left(\frac{35e^3(c^2x^2-1)^{7/2}}{c^8}+\frac{5e^2(27dc^2+28e)(c^2x^2-1)^{5/2}}{c^8}+\frac{3e(63d^2c^4+135dec^2+70e^2)(c^2x^2-1)^{3/2}}{c^8}+\frac{(105d^3c^6+378d^2ec}{}\right.}{630\sqrt{cx-1}\sqrt{cx+1}}$$
$$\frac{1}{3}d^3x^3(a+b\operatorname{arccosh}(cx))+\frac{3}{5}d^2ex^5(a+b\operatorname{arccosh}(cx))+\frac{3}{7}de^2x^7(a+b\operatorname{arccosh}(cx))+\frac{1}{9}e^3x^9(a+b\operatorname{arccosh}(cx))$$

$\big\downarrow$ 2009

$$\frac{1}{3}d^3x^3(a+b\operatorname{arccosh}(cx))+\frac{3}{5}d^2ex^5(a+b\operatorname{arccosh}(cx))+\frac{3}{7}de^2x^7(a+b\operatorname{arccosh}(cx))+\frac{1}{9}e^3x^9(a+b\operatorname{arccosh}(cx))-$$
$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{10e^2(c^2x^2-1)^{7/2}(27c^2d+28e)}{7c^{10}}+\frac{70e^3(c^2x^2-1)^{9/2}}{9c^{10}}+\frac{6e(c^2x^2-1)^{5/2}(63c^4d^2+135c^2de+70e^2)}{5c^{10}}+\frac{2(c^2x^2-1)^{3/2}(105c^6d^3}{}\right.}{630\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/630*(b*c*Sqrt[-1 + c^2*x^2]*((2*(105*c^6*d^3 + 189*c^4*d^2*e + 135*c^2*
d*e^2 + 35*e^3)*Sqrt[-1 + c^2*x^2])/c^10 + (2*(105*c^6*d^3 + 378*c^4*d^2*e
 + 405*c^2*d*e^2 + 140*e^3)*(-1 + c^2*x^2)^(3/2))/(3*c^10) + (6*e*(63*c^4*
d^2 + 135*c^2*d*e + 70*e^2)*(-1 + c^2*x^2)^(5/2))/(5*c^10) + (10*e^2*(27*c
^2*d + 28*e)*(-1 + c^2*x^2)^(7/2))/(7*c^10) + (70*e^3*(-1 + c^2*x^2)^(9/2)
)/(9*c^10)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]
))/3 + (3*d^2*e*x^5*(a + b*ArcCosh[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcCosh[
c*x]))/7 + (e^3*x^9*(a + b*ArcCosh[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.89

method	result
parts	$a\left(\frac{1}{9}e^3x^9+\frac{3}{7}d\,e^2x^7+\frac{3}{5}d^2e\,x^5+\frac{1}{3}d^3x^3\right)+\frac{b\left(\frac{c^3\operatorname{arccosh}(cx)e^3x^9}{9}+\frac{3c^3\operatorname{arccosh}(cx)d\,e^2x^7}{7}+\frac{3c^3\operatorname{arccosh}(cx)d^2}{5}\right.}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}c^9d^3x^3+\frac{3}{5}c^9d^2e\,x^5+\frac{3}{7}c^9d\,e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^9d^3x^3}{3}+\frac{3\operatorname{arccosh}(cx)c^9d^2e\,x^5}{5}+\frac{3\operatorname{arccosh}(cx)c^9d\,e^2x^7}{7}+\operatorname{arcc}\right.}{}}{}$
default	$\frac{\frac{a\left(\frac{1}{3}c^9d^3x^3+\frac{3}{5}c^9d^2e\,x^5+\frac{3}{7}c^9d\,e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^9d^3x^3}{3}+\frac{3\operatorname{arccosh}(cx)c^9d^2e\,x^5}{5}+\frac{3\operatorname{arccosh}(cx)c^9d\,e^2x^7}{7}+\operatorname{arcc}\right.}{}}{}$
orering	$\frac{(20825c^{10}e^4x^{12}+104600c^{10}d\,e^3x^{10}+209466c^{10}d^2e^2x^8+1400c^8e^4x^{10}+204624c^{10}d^3e\,x^6+10070c^8d\,e^3x^8+55125c^{10}d^4}{}$

input
```
int(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^3*x^9+3/7*d*e^2*x^7+3/5*d^2*e*x^5+1/3*d^3*x^3)+b/c^3*(1/9*c^3*arc
cosh(c*x)*e^3*x^9+3/7*c^3*arccosh(c*x)*d*e^2*x^7+3/5*c^3*arccosh(c*x)*d^2*
e*x^5+1/3*arccosh(c*x)*c^3*x^3*d^3-1/99225/c^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*(1225*c^8*e^3*x^8+6075*c^8*d*e^2*x^6+11907*c^8*d^2*e*x^4+1400*c^6*e^3*x^6
+11025*c^8*d^3*x^2+7290*c^6*d*e^2*x^4+15876*c^6*d^2*e*x^2+1680*c^4*e^3*x^4
+22050*c^6*d^3+9720*c^4*d*e^2*x^2+31752*c^4*d^2*e+2240*c^2*e^3*x^2+19440*c
^2*d*e^2+4480*e^3))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.94

$$\int x^2 (d + ex^2)^3 (a + b\text{arccosh}(cx))\,dx$$
$$= \frac{11025\,ac^9e^3x^9 + 42525\,ac^9de^2x^7 + 59535\,ac^9d^2ex^5 + 33075\,ac^9d^3x^3 + 315\,(35\,bc^9e^3x^9 + 135\,bc^9de^2x^7 + 1}{}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*e^3*x^9 + 42525*a*c^9*d*e^2*x^7 + 59535*a*c^9*d^2*e*x
^5 + 33075*a*c^9*d^3*x^3 + 315*(35*b*c^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 1
89*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (12
25*b*c^8*e^3*x^8 + 22050*b*c^6*d^3 + 31752*b*c^4*d^2*e + 25*(243*b*c^8*d*e
^2 + 56*b*c^6*e^3)*x^6 + 19440*b*c^2*d*e^2 + 3*(3969*b*c^8*d^2*e + 2430*b*
c^6*d*e^2 + 560*b*c^4*e^3)*x^4 + 4480*b*e^3 + (11025*b*c^8*d^3 + 15876*b*c
^6*d^2*e + 9720*b*c^4*d*e^2 + 2240*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

Sympy [F]

$$\int x^2 (d + ex^2)^3 (a + b\text{arccosh}(cx))\,dx = \int x^2(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)^3\,dx$$

input
```
integrate(x**2*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))*(d + e*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.22

$$\int x^2(d+ex^2)^3(a+b\operatorname{arccosh}(cx))\,dx = \frac{1}{9}ae^3x^9 + \frac{3}{7}ade^2x^7 + \frac{3}{5}ad^2ex^5 + \frac{1}{3}ad^3x^3 + \frac{1}{9}\left(3x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3 + \frac{1}{25}\left(15x^5\operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2e + \frac{3}{245}\left(35x^7\operatorname{arcosh}(cx) - \left(\frac{5\sqrt{c^2x^2-1}x^6}{c^2} + \frac{6\sqrt{c^2x^2-1}x^4}{c^4} + \frac{8\sqrt{c^2x^2-1}x^2}{c^6} + \frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bd + \frac{1}{2835}\left(315x^9\operatorname{arcosh}(cx) - \left(\frac{35\sqrt{c^2x^2-1}x^8}{c^2} + \frac{40\sqrt{c^2x^2-1}x^6}{c^4} + \frac{48\sqrt{c^2x^2-1}x^4}{c^6} + \frac{64\sqrt{c^2x^2-1}x}{c^8}\right.\right.$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/9*(3
*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4
))*b*d^3 + 1/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sq
rt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^2*e + 3/245*(35*
x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/
c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e^2 +
 1/2835*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^
2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x
^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^3
```

Giac [F(-2)]

Exception generated.

$$\int x^2(d+ex^2)^3(a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx = \int x^2\,(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.37

$$\int x^2(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$$

$$= \frac{33075acosh(cx)\,b\,c^9d^3x^3+59535acosh(cx)\,b\,c^9d^2e\,x^5+42525acosh(cx)\,b\,c^9d\,e^2x^7+11025acosh(cx)\,b\,c^9}{}$$

input
```
int(x^2*(e*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(33075*acosh(c*x)*b*c**9*d**3*x**3 + 59535*acosh(c*x)*b*c**9*d**2*e*x**5 +
 42525*acosh(c*x)*b*c**9*d*e**2*x**7 + 11025*acosh(c*x)*b*c**9*e**3*x**9 -
 11025*sqrt(c**2*x**2 - 1)*b*c**8*d**3*x**2 - 11907*sqrt(c**2*x**2 - 1)*b*
c**8*d**2*e*x**4 - 6075*sqrt(c**2*x**2 - 1)*b*c**8*d*e**2*x**6 - 1225*sqrt
(c**2*x**2 - 1)*b*c**8*e**3*x**8 - 22050*sqrt(c**2*x**2 - 1)*b*c**6*d**3 -
 15876*sqrt(c**2*x**2 - 1)*b*c**6*d**2*e*x**2 - 7290*sqrt(c**2*x**2 - 1)*b
*c**6*d*e**2*x**4 - 1400*sqrt(c**2*x**2 - 1)*b*c**6*e**3*x**6 - 31752*sqrt
(c**2*x**2 - 1)*b*c**4*d**2*e - 9720*sqrt(c**2*x**2 - 1)*b*c**4*d*e**2*x**
2 - 1680*sqrt(c**2*x**2 - 1)*b*c**4*e**3*x**4 - 19440*sqrt(c**2*x**2 - 1)*
b*c**2*d*e**2 - 2240*sqrt(c**2*x**2 - 1)*b*c**2*e**3*x**2 - 4480*sqrt(c**2
*x**2 - 1)*b*e**3 + 33075*a*c**9*d**3*x**3 + 59535*a*c**9*d**2*e*x**5 + 42
525*a*c**9*d*e**2*x**7 + 11025*a*c**9*e**3*x**9)/(99225*c**9)
```

3.381 $\int x(d+ex^2)^3\,(a+b\,\mathrm{arccosh}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 311

$$\begin{aligned}\int x\left(d+ex^2\right)^3(a+b\,\mathrm{arccosh}(cx))\,dx &= -\frac{b(256c^6d^3+288c^4d^2e+160c^2de^2+35e^3)\,x\sqrt{-1+cx}\sqrt{1+cx}}{1024c^7}\\ &\quad-\frac{be(288c^4d^2+160c^2de+35e^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{1536c^5}\\ &\quad-\frac{be^2(32c^2d+7e)\,x^5\sqrt{-1+cx}\sqrt{1+cx}}{384c^3}\\ &\quad-\frac{be^3x^7\sqrt{-1+cx}\sqrt{1+cx}}{64c}+\frac{\left(d+ex^2\right)^4(a+b\,\mathrm{arccosh}(cx))}{8e}\\ &\quad-\frac{b(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4)\sqrt{-1+c^2x^2}\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1024c^8e\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

output

```
-1/1024*b*(256*c^6*d^3+288*c^4*d^2*e+160*c^2*d*e^2+35*e^3)*x*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^7-1/1536*b*e*(288*c^4*d^2+160*c^2*d*e+35*e^2)*x^3*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/c^5-1/384*b*e^2*(32*c^2*d+7*e)*x^5*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c^3-1/64*b*e^3*x^7*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/8*(e*x^2+d)^4
*(a+b*arccosh(c*x))/e-1/1024*b*(128*c^8*d^4+256*c^6*d^3*e+288*c^4*d^2*e^2+
160*c^2*d*e^3+35*e^4)*(c^2*x^2-1)^(1/2)*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^8
/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.82

$$\int x(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$$
$$= \frac{cx(384ac^7x(4d^3+6d^2ex^2+4de^2x^4+e^3x^6)-b\sqrt{-1+cx}\sqrt{1+cx}(105e^3+10c^2e^2(48d+7ex^2)+8c^4e(1}{}$$

input
```
Integrate[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
(c*x*(384*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6) - b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(105*e^3 + 10*c^2*e^2*(48*d + 7*e*x^2) + 8*c^4*e*(1
08*d^2 + 40*d*e*x^2 + 7*e^2*x^4) + 16*c^6*(48*d^3 + 36*d^2*e*x^2 + 16*d*e^
2*x^4 + 3*e^3*x^6))) + 384*b*c^8*x^2*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 +
e^3*x^6)*ArcCosh[c*x] - 6*b*(256*c^6*d^3 + 288*c^4*d^2*e + 160*c^2*d*e^2 +
 35*e^3)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(3072*c^8)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6372, 648, 318, 403, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)^3(a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{6372}$$

$$\frac{(d+ex^2)^4(a+b\text{arccosh}(cx))}{8e}-\frac{bc\int \frac{(ex^2+d)^4}{\sqrt{cx-1}\sqrt{cx+1}}dx}{8e}$$

$$\Big\downarrow \text{648}$$

$$\frac{(d+ex^2)^4(a+b\text{arccosh}(cx))}{8e}-\frac{bc\sqrt{c^2x^2-1}\int \frac{(ex^2+d)^4}{\sqrt{c^2x^2-1}}dx}{8e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 318$$

$$\frac{(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e} - \frac{bc\sqrt{c^2x^2-1}\left(\frac{\int \frac{(ex^2+d)^2\left(7e\left(2dc^2+e\right)x^2+d\left(8dc^2+e\right)\right)}{\sqrt{c^2x^2-1}}dx}{8c^2} + \frac{ex\sqrt{c^2x^2-1}(d+ex^2)^3}{8c^2}\right)}{8e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 403$$

$$\frac{(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e} - \frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\int \frac{(ex^2+d)\left(e\left(104d^2c^4+104dec^2+35e^2\right)x^2+d\left(48d^2c^4+20dec^2+7e^2\right)\right)}{\sqrt{c^2x^2-1}}dx}{6c^2} + \frac{7ex\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(d+ex^2\right)^2}{6c^2}}{8c^2} + \frac{ex\sqrt{c^2x^2-1}(d+ex^2)^{3}}{8c^2}\right.}{8e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 403$$

$$\frac{(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e} - \frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\int \frac{5e\left(2dc^2+e\right)\left(40d^2c^4+40dec^2+21e^2\right)x^2+d\left(192d^3c^6+184d^2ec^4+132de^2c^2+35e^3\right)}{\sqrt{c^2x^2-1}}dx}{4c^2} + \frac{ex\sqrt{c^2x^2-1}\left(104c^4d^2+104c^2de+35e^2\right)\left(d+ex^2\right)}{4c^2}}{6c^2}}{8c^2}\right.}{8e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 299$$

$$\frac{(d+ex^2)^4 (a+b\text{arccosh}(cx))}{8e} - \frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\frac{3\left(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4\right)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2} + \frac{5ex\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)}{2c^2}}{4c^2} + \frac{ex\sqrt{c^2x^2-1}(104}{}}{6c^2}}{8c^2}\right.}{8e\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow 224$$

$$\frac{(d+ex^2)^4(a+b\text{arccosh}(cx))}{8e}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\frac{3\left(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4\right)\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{5ex\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)}{2c^2}}{4c^2}+\frac{ex\sqrt{c^2\cdots}}{\cdots}}{6c^2}\cdots}{8c^2}\cdots\right)}{8e\sqrt{cx-1}\sqrt{cx+1}}$$

$\downarrow$ 219

$$\frac{(d+ex^2)^4(a+b\text{arccosh}(cx))}{8e}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\frac{\frac{3\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4\right)}{2c^3}+\frac{5ex\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)}{2c^2}}{4c^2}+\frac{ex\sqrt{c^2\cdots}}{\cdots}}{6c^2}\cdots}{8c^2}\cdots\right)}{8e\sqrt{cx-1}\sqrt{cx+1}}$$

input

```
Int[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output

```
((d + e*x^2)^4*(a + b*ArcCosh[c*x]))/(8*e) - (b*c*Sqrt[-1 + c^2*x^2]*((e*x
*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^3)/(8*c^2) + ((7*e*(2*c^2*d + e)*x*Sqrt[-1
 + c^2*x^2]*(d + e*x^2)^2)/(6*c^2) + ((e*(104*c^4*d^2 + 104*c^2*d*e + 35*e
^2)*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2))/(4*c^2) + ((5*e*(2*c^2*d + e)*(40*c^
4*d^2 + 40*c^2*d*e + 21*e^2)*x*Sqrt[-1 + c^2*x^2])/(2*c^2) + (3*(128*c^8*d
^4 + 256*c^6*d^3*e + 288*c^4*d^2*e^2 + 160*c^2*d*e^3 + 35*e^4)*ArcTanh[(c*
x)/Sqrt[-1 + c^2*x^2]])/(2*c^3))/(4*c^2))/(6*c^2))/(8*c^2)))/(8*e*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

rule 6372
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.38

method	result
orering	$\frac{(720c^8e^4x^{10}+3760c^8d\,e^3x^8+8128c^8d^2e^2x^6+56c^6e^4x^8+9792c^8d^3e\,x^4+456c^6d\,e^3x^6+2304c^8d^4x^2+2080c^6d^2e^2x^4+98}{}$
parts	$\frac{a(e\,x^2+d)^4}{8e}+\frac{b\left(\frac{c^2e^3\operatorname{arccosh}(cx)x^8}{8}+\frac{c^2e^2\operatorname{arccosh}(cx)x^6d}{2}+\frac{3c^2e\operatorname{arccosh}(cx)x^4d^2}{4}+\frac{\operatorname{arccosh}(cx)c^2x^2d^3}{2}+\frac{c^2\operatorname{arccosh}(cx)d^4}{8e}\right.}{}$
derivativedivides	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^4}{8c^6e}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^4c^8}{8e}+\frac{\operatorname{arccosh}(cx)c^8d^3x^2}{2}+\frac{3e\operatorname{arccosh}(cx)c^8d^2x^4}{4}+\frac{e^2\operatorname{arccosh}(cx)c^8d\,x^6}{2}+\frac{e^3\operatorname{arccosh}(cx)c^8x^8}{8}\right.}{}}{}$
default	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^4}{8c^6e}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^4c^8}{8e}+\frac{\operatorname{arccosh}(cx)c^8d^3x^2}{2}+\frac{3e\operatorname{arccosh}(cx)c^8d^2x^4}{4}+\frac{e^2\operatorname{arccosh}(cx)c^8d\,x^6}{2}+\frac{e^3\operatorname{arccosh}(cx)c^8x^8}{8}\right.}{}}{}$

input
```
int(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3072*(720*c^8*e^4*x^10+3760*c^8*d*e^3*x^8+8128*c^8*d^2*e^2*x^6+56*c^6*e^
4*x^8+9792*c^8*d^3*e*x^4+456*c^6*d*e^3*x^6+2304*c^8*d^4*x^2+2080*c^6*d^2*e
^2*x^4+98*c^4*e^4*x^6-5856*c^6*d^3*e*x^2+1134*c^4*d*e^3*x^4-1536*c^6*d^4-6
752*c^4*d^2*e^2*x^2+245*c^2*e^4*x^4-1728*c^4*d^3*e-3805*c^2*d*e^3*x^2-960*
c^2*d^2*e^2-840*e^4*x^2-210*d*e^3)/(e*x^2+d)/c^8*(a+b*arccosh(c*x))-1/3072
*(48*c^6*e^3*x^6+256*c^6*d*e^2*x^4+576*c^6*d^2*e*x^2+56*c^4*e^3*x^4+768*c^
6*d^3+320*c^4*d*e^2*x^2+864*c^4*d^2*e+70*c^2*e^3*x^2+480*c^2*d*e^2+105*e^3
)/c^8/(e*x^2+d)^3*(c*x-1)*(c*x+1)*((e*x^2+d)^3*(a+b*arccosh(c*x))+6*x^2*(e
*x^2+d)^2*(a+b*arccosh(c*x))*e+x*(e*x^2+d)^3*b*c/(c*x-1)^(1/2)/(c*x+1)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.92

$$\int x(d+ex^2)^3\,(a+b\operatorname{arccosh}(cx))\,dx$$
$$=\frac{384\,ac^8e^3x^8+1536\,ac^8de^2x^6+2304\,ac^8d^2ex^4+1536\,ac^8d^3x^2+3\,(128\,bc^8e^3x^8+512\,bc^8de^2x^6+768\,bc^8d}{}$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/3072*(384*a*c^8*e^3*x^8 + 1536*a*c^8*d*e^2*x^6 + 2304*a*c^8*d^2*e*x^4 +
1536*a*c^8*d^3*x^2 + 3*(128*b*c^8*e^3*x^8 + 512*b*c^8*d*e^2*x^6 + 768*b*c^
8*d^2*e*x^4 + 512*b*c^8*d^3*x^2 - 256*b*c^6*d^3 - 288*b*c^4*d^2*e - 160*b*
c^2*d*e^2 - 35*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (48*b*c^7*e^3*x^7 + 8
*(32*b*c^7*d*e^2 + 7*b*c^5*e^3)*x^5 + 2*(288*b*c^7*d^2*e + 160*b*c^5*d*e^2
 + 35*b*c^3*e^3)*x^3 + 3*(256*b*c^7*d^3 + 288*b*c^5*d^2*e + 160*b*c^3*d*e^
2 + 35*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^8
```

Sympy [F]

$$\int x\left(d+ex^2\right)^3\left(a+b\text{arccosh}(cx)\right)dx=\int x(a+b\,\text{acosh}\,(cx))\left(d+ex^2\right)^3\,dx$$

input
```
integrate(x*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Integral(x*(a + b*acosh(c*x))*(d + e*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.32

$$\int x\left(d+ex^2\right)^3\left(a+b\text{arccosh}(cx)\right)dx=\frac{1}{8}\,ae^3x^8+\frac{1}{2}\,ade^2x^6+\frac{3}{4}\,ad^2ex^4+\frac{1}{2}\,ad^3x^2$$
$$+\frac{1}{4}\left(2\,x^2\,\text{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c^3}\right)\right)bd^3$$
$$+\frac{3}{32}\left(8\,x^4\,\text{arcosh}\,(cx)-\left(\frac{2\,\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c^5}\right)c\right)bd^2e$$
$$+\frac{1}{96}\left(48\,x^6\,\text{arcosh}\,(cx)-\left(\frac{8\,\sqrt{c^2x^2-1}x^5}{c^2}+\frac{10\,\sqrt{c^2x^2-1}x^3}{c^4}+\frac{15\,\sqrt{c^2x^2-1}x}{c^6}+\frac{15\,\log\left(2\,c^2x+2\,\sqrt{c\right.}{c^7}\right.\right.$$
$$+\frac{1}{3072}\left(384\,x^8\,\text{arcosh}\,(cx)-\left(\frac{48\,\sqrt{c^2x^2-1}x^7}{c^2}+\frac{56\,\sqrt{c^2x^2-1}x^5}{c^4}+\frac{70\,\sqrt{c^2x^2-1}x^3}{c^6}+\frac{105\,\sqrt{c^2x^2-1}}{c^8}\right.\right.$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/4*(2
*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*
x^2 - 1)*c)/c^3))*b*d^3 + 3/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*
x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*
c)/c^5)*c)*b*d^2*e + 1/96*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/
c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2
*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*d*e^2 + 1/3072*(384*x^8*arccosh(
c*x) - (48*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*s
qrt(c^2*x^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x +
 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*e^3
```

Giac [F(-2)]

Exception generated.

$$\int x(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))\,dx = \int x\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)^3 (a+b\text{arccosh}(cx))\,dx$$
$$= \frac{1536acosh(cx)\,b\,c^8d^3x^2+2304acosh(cx)\,b\,c^8d^2e\,x^4+1536acosh(cx)\,b\,c^8d\,e^2x^6+384acosh(cx)\,b\,c^8e^3x^8-}{}$$

input
```
int(x*(e*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(1536*acosh(c*x)*b*c**8*d**3*x**2 + 2304*acosh(c*x)*b*c**8*d**2*e*x**4 + 1
536*acosh(c*x)*b*c**8*d*e**2*x**6 + 384*acosh(c*x)*b*c**8*e**3*x**8 - 768*
sqrt(c**2*x**2 - 1)*b*c**7*d**3*x - 576*sqrt(c**2*x**2 - 1)*b*c**7*d**2*e*
x**3 - 256*sqrt(c**2*x**2 - 1)*b*c**7*d*e**2*x**5 - 48*sqrt(c**2*x**2 - 1)
*b*c**7*e**3*x**7 - 864*sqrt(c**2*x**2 - 1)*b*c**5*d**2*e*x - 320*sqrt(c**
2*x**2 - 1)*b*c**5*d*e**2*x**3 - 56*sqrt(c**2*x**2 - 1)*b*c**5*e**3*x**5 -
 480*sqrt(c**2*x**2 - 1)*b*c**3*d*e**2*x - 70*sqrt(c**2*x**2 - 1)*b*c**3*e
**3*x**3 - 105*sqrt(c**2*x**2 - 1)*b*c*e**3*x - 768*log(sqrt(c**2*x**2 - 1
) + c*x)*b*c**6*d**3 - 864*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**4*d**2*e -
480*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d*e**2 - 105*log(sqrt(c**2*x**2
- 1) + c*x)*b*e**3 + 1536*a*c**8*d**3*x**2 + 2304*a*c**8*d**2*e*x**4 + 153
6*a*c**8*d*e**2*x**6 + 384*a*c**8*e**3*x**8)/(3072*c**8)
```

3.382 $\int (d+ex^2)^3 (a+b\text{arccosh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 241

$$\begin{aligned}\int (d+ex^2)^3 (a+b\text{arccosh}(cx))\,dx &= -\frac{b(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3)\sqrt{-1+cx}\sqrt{1+cx}}{35c^7} \\ &\quad -\frac{be(35c^4d^2+42c^2de+15e^2)(-1+cx)^{3/2}(1+cx)^{3/2}}{105c^7} \\ &\quad -\frac{3be^2(7c^2d+5e)(-1+cx)^{5/2}(1+cx)^{5/2}}{175c^7}-\frac{be^3(-1+cx)^{7/2}(1+cx)^{7/2}}{49c^7} \\ &\quad +d^3x(a+b\text{arccosh}(cx))+d^2ex^3(a+b\text{arccosh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arccosh}(cx))\end{aligned}$$

output
```
-1/35*b*(35*c^6*d^3+35*c^4*d^2*e+21*c^2*d*e^2+5*e^3)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^7-1/105*b*e*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(c*x-1)^(3/2)*(c*x+1)^
(3/2)/c^7-3/175*b*e^2*(7*c^2*d+5*e)*(c*x-1)^(5/2)*(c*x+1)^(5/2)/c^7-1/49*b
*e^3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^7+d^3*x*(a+b*arccosh(c*x))+d^2*e*x^3*(a
+b*arccosh(c*x))+3/5*d*e^2*x^5*(a+b*arccosh(c*x))+1/7*e^3*x^7*(a+b*arccosh
(c*x))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx = a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)$$
$$-\frac{b\sqrt{-1+cx}\sqrt{1+cx}(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1}{3675c^7}$$
$$+\frac{1}{35}bx\left(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6\right)\text{arccosh}(cx)$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 +
 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^
4 + 75*e^3*x^6)))/(3675*c^7) + (b*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4
+ 5*e^3*x^6)*ArcCosh[c*x])/35
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6323, 27, 2113, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \text{ 6323}$$

$$-bc\int \frac{x\left(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3\right)}{35\sqrt{cx-1}\sqrt{cx+1}}dx+d^3x(a+b\text{arccosh}(cx))+d^2ex^3(a+b\text{arccosh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{35}bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{\sqrt{cx-1}\sqrt{cx+1}}dx+d^3x(a+b\text{arccosh}(cx))+d^2ex^3(a+b\text{arccosh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2113}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{\sqrt{c^2x^2-1}}dx}{35\sqrt{cx-1}\sqrt{cx+1}}+d^3x(a+b\text{arccosh}(cx))+d^2ex^3(a+b\text{arccosh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{5e^3x^6+21de^2x^4+35d^2ex^2+35d^3}{\sqrt{c^2x^2-1}}dx^2}{70\sqrt{cx-1}\sqrt{cx+1}}+d^3x(a+b\text{arccosh}(cx))+d^2ex^3(a+b\text{arccosh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2389}$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \left(\frac{5(c^2x^2-1)^{5/2}e^3}{c^6}+\frac{3(7dc^2+5e)(c^2x^2-1)^{3/2}e^2}{c^6}+\frac{(35d^2c^4+42dec^2+15e^2)\sqrt{c^2x^2-1}e}{c^6}+\frac{35d^3c^6+35d^2ec^4+21de^2c^2+}{c^6\sqrt{c^2x^2-1}}\right.}{70\sqrt{cx-1}\sqrt{cx+1}}$$
$$d^3x(a+b\text{arccosh}(cx))+d^2ex^3(a+b\text{arccosh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^3x(a+b\text{arccosh}(cx))+d^2ex^3(a+b\text{arccosh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arccosh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arccosh}(cx))-$$
$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{6e^2(c^2x^2-1)^{5/2}(7c^2d+5e)}{5c^8}+\frac{10e^3(c^2x^2-1)^{7/2}}{7c^8}+\frac{2e(c^2x^2-1)^{3/2}(35c^4d^2+42c^2de+15e^2)}{3c^8}+\frac{2\sqrt{c^2x^2-1}(35c^6d^3+35c^4d^2e}{c^8}\right.}{70\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
-1/70*(b*c*Sqrt[-1 + c^2*x^2]*((2*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^
2 + 5*e^3)*Sqrt[-1 + c^2*x^2])/c^8 + (2*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^
2)*(-1 + c^2*x^2)^(3/2))/(3*c^8) + (6*e^2*(7*c^2*d + 5*e)*(-1 + c^2*x^2)^(
5/2))/(5*c^8) + (10*e^3*(-1 + c^2*x^2)^(7/2))/(7*c^8)))/(Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + d^3*x*(a + b*ArcCosh[c*x]) + d^2*e*x^3*(a + b*ArcCosh[c*x])
 + (3*d*e^2*x^5*(a + b*ArcCosh[c*x]))/5 + (e^3*x^7*(a + b*ArcCosh[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6323
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCosh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0]
|| ILtQ[p + 1/2, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.88

method	result
parts	$a\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b\left(\frac{c\,\operatorname{arccosh}(cx)e^3x^7}{7}+\frac{3c\,\operatorname{arccosh}(cx)d\,e^2x^5}{5}+c\,\operatorname{arccosh}(cx)d^2e\,x^3+\right.}{}$
derivativedivides	$\frac{\frac{a\left(c^7d^3x+c^7d^2e\,x^3+\frac{3}{5}c^7d\,e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arccosh}(cx)c^7d^3x+\operatorname{arccosh}(cx)c^7d^2e\,x^3+\frac{3\operatorname{arccosh}(cx)c^7d\,e^2x^5}{5}+\frac{\operatorname{arccosh}(cx)e^3}{7}\right.}{}}{}$
default	$\frac{\frac{a\left(c^7d^3x+c^7d^2e\,x^3+\frac{3}{5}c^7d\,e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arccosh}(cx)c^7d^3x+\operatorname{arccosh}(cx)c^7d^2e\,x^3+\frac{3\operatorname{arccosh}(cx)c^7d\,e^2x^5}{5}+\frac{\operatorname{arccosh}(cx)e^3}{7}\right.}{}}{}$
orering	$\frac{x\left(325e^4x^8c^8+1792c^8d\,e^3x^6+4410c^8d^2e^2x^4+30c^6e^4x^6+9800c^8d^3e\,x^2+294c^6d\,e^3x^4+1225c^8d^4+2450c^6d^2e^2x^2+60c^4\right.}{1225(e\,x^2+d)c^8}$

input
```
int((e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^3*x^7+3/5*d*e^2*x^5+d^2*e*x^3+d^3*x)+b/c*(1/7*c*arccosh(c*x)*e^3*
x^7+3/5*c*arccosh(c*x)*d*e^2*x^5+c*arccosh(c*x)*d^2*e*x^3+arccosh(c*x)*c*x
*d^3-1/3675/c^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*e^3*x^6+441*c^6*d*e^2*
x^4+1225*c^6*d^2*e*x^2+90*c^4*e^3*x^4+3675*c^6*d^3+588*c^4*d*e^2*x^2+2450*
c^4*d^2*e+120*c^2*e^3*x^2+1176*c^2*d*e^2+240*e^3))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$
$$=\frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\left(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex\right.}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 +
3675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^
2*e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*e^3*x^6
 + 3675*b*c^6*d^3 + 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^
2 + 10*b*c^4*e^3)*x^4 + 240*b*e^3 + (1225*b*c^6*d^2*e + 588*b*c^4*d*e^2 +
120*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [F]

$$\int \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx=\int \left(a+b\operatorname{acosh}\left(cx\right)\right)\left(d+ex^2\right)^3\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx &= \frac{1}{7}\,ae^3x^7+\frac{3}{5}\,ade^2x^5+ad^2ex^3\\ &+\frac{1}{3}\left(3\,x^3\operatorname{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^2e\\ &+\frac{1}{25}\left(15\,x^5\operatorname{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bde^2\\ &+\frac{1}{245}\left(35\,x^7\operatorname{arcosh}\left(cx\right)-\left(\frac{5\,\sqrt{c^2x^2-1}x^6}{c^2}+\frac{6\,\sqrt{c^2x^2-1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2-1}x^2}{c^6}+\frac{16\,\sqrt{c^2x^2-1}}{c^8}\right)c\right)be\\ &+ad^3x+\frac{\left(cx\operatorname{arcosh}\left(cx\right)-\sqrt{c^2x^2-1}\right)bd^3}{c}\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arccosh(c*x) -
c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2*e + 1/25*(1
5*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^
2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arccosh(c*x) -
 (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x
^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arcc
osh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \int (a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

output
```
int((a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.37

$$\int (d + ex^2)^3 (a + b\text{arccosh}(cx)) \, dx$$
$$= \frac{3675acosh(cx)\, b\, c^7 d^3 x + 3675acosh(cx)\, b\, c^7 d^2 e\, x^3 + 2205acosh(cx)\, b\, c^7 d\, e^2 x^5 + 525acosh(cx)\, b\, c^7 e^3 x^7 -}{}$$

input
```
int((e*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(3675*acosh(c*x)*b*c**7*d**3*x + 3675*acosh(c*x)*b*c**7*d**2*e*x**3 + 2205
*acosh(c*x)*b*c**7*d*e**2*x**5 + 525*acosh(c*x)*b*c**7*e**3*x**7 - 1225*sq
rt(c**2*x**2 - 1)*b*c**6*d**2*e*x**2 - 441*sqrt(c**2*x**2 - 1)*b*c**6*d*e*
*2*x**4 - 75*sqrt(c**2*x**2 - 1)*b*c**6*e**3*x**6 - 2450*sqrt(c**2*x**2 -
1)*b*c**4*d**2*e - 588*sqrt(c**2*x**2 - 1)*b*c**4*d*e**2*x**2 - 90*sqrt(c*
*2*x**2 - 1)*b*c**4*e**3*x**4 - 1176*sqrt(c**2*x**2 - 1)*b*c**2*d*e**2 - 1
20*sqrt(c**2*x**2 - 1)*b*c**2*e**3*x**2 - 240*sqrt(c**2*x**2 - 1)*b*e**3 -
 3675*sqrt(c*x + 1)*sqrt(c*x - 1)*b*c**6*d**3 + 3675*a*c**7*d**3*x + 3675*
a*c**7*d**2*e*x**3 + 2205*a*c**7*d*e**2*x**5 + 525*a*c**7*e**3*x**7)/(3675
*c**7)
```

3.383 $\int \frac{(d+ex^2)^3(a+b\operatorname{arccosh}(cx))}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 509

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x} \, dx = -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3} - \frac{5be^3x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5} - \frac{3bde^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c} - \frac{5be^3x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3} - \frac{be^3x^5\sqrt{-1+cx}\sqrt{1+cx}}{36c} - \frac{3bd^2e\text{arccosh}(cx)}{4c^2} - \frac{9bde^2\text{arccosh}(cx)}{32c^4} - \frac{5be^3\text{arccosh}(cx)}{96c^6} + \frac{3}{2}d^2ex^2(a+b\text{arccosh}(cx)) + \frac{3}{4}de^2x^4(a+b\text{arccosh}(cx)) + \frac{1}{6}e^3x^6(a+b\text{arccosh}(cx)) - \frac{ibd^3\sqrt{1-c^2x^2}\arcsin(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bd^3\sqrt{1-c^2x^2}\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + d^3(a+b\text{arccosh}(cx))\log(x) - \frac{bd^3\sqrt{1-c^2x^2}\arcsin(cx)\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ibd^3\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-3/4*b*d^2*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-9/32*b*d*e^2*x*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^3-5/96*b*e^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-3/16*b*d*e^
2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-5/144*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/c^3-1/36*b*e^3*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/4*b*d^2*e*arccosh
(c*x)/c^2-9/32*b*d*e^2*arccosh(c*x)/c^4-5/96*b*e^3*arccosh(c*x)/c^6+3/2*d^
2*e*x^2*(a+b*arccosh(c*x))+3/4*d*e^2*x^4*(a+b*arccosh(c*x))+1/6*e^3*x^6*(a
+b*arccosh(c*x))-1/2*I*b*d^3*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+b*d^3*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2
+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d^3*(a+b*arccosh(c*x))*ln(x)-b*d
^3*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*
b*d^3*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x} dx = \frac{3}{2}ad^2ex^2 + \frac{3}{4}ade^2x^4 + \frac{1}{6}ae^3x^6 - \frac{3bd^2e\left(cx\sqrt{-1+cx}\sqrt{1+cx} - 2c^2x^2\text{arccosh}(cx) + 2\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{4c^2} - \frac{3bde^2\left(cx\sqrt{\frac{-1+cx}{1+cx}}(3+3cx+2c^2x^2+2c^3x^3) - 8c^4x^4\text{arccosh}(cx) + 6\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{32c^4} - \frac{be^3\left(cx\sqrt{\frac{-1+cx}{1+cx}}(15+15cx+10c^2x^2+10c^3x^3+8c^4x^4+8c^5x^5) - 48c^6x^6\text{arccosh}(cx) + 30\text{arctanh}\left(\sqrt{\right.\right.}{288c^6} + ad^3\log(x) + \frac{1}{2}bd^3\left(\text{arccosh}(cx)\left(\text{arccosh}(cx) + 2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right) - \text{PolyLog}\left(2, -e^{-2\text{arccosh}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 + (a*e^3*x^6)/6 - (3*b*d^2*e*(c*x*Sq
rt[-1 + c*x]*Sqrt[1 + c*x] - 2*c^2*x^2*ArcCosh[c*x] + 2*ArcTanh[Sqrt[(-1 +
 c*x)/(1 + c*x)]]))/(4*c^2) - (3*b*d*e^2*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(
3 + 3*c*x + 2*c^2*x^2 + 2*c^3*x^3) - 8*c^4*x^4*ArcCosh[c*x] + 6*ArcTanh[Sq
rt[(-1 + c*x)/(1 + c*x)]]))/(32*c^4) - (b*e^3*(c*x*Sqrt[(-1 + c*x)/(1 + c*
x)]*(15 + 15*c*x + 10*c^2*x^2 + 10*c^3*x^3 + 8*c^4*x^4 + 8*c^5*x^5) - 48*c
^6*x^6*ArcCosh[c*x] + 30*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(288*c^6) +
 a*d^3*Log[x] + (b*d^3*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCo
sh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x} \, dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int \frac{2e^3x^6+9de^2x^4+18d^2ex^2+12d^3\log(x)}{12\sqrt{cx-1}\sqrt{cx+1}}dx+d^3\log(x)(a+b\text{arccosh}(cx))+\frac{3}{2}d^2ex^2(a+b\text{arccosh}(cx))+\frac{3}{4}de^2x^4(a+b\text{arccosh}(cx))+\frac{1}{6}e^3x^6(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bc\int \frac{2e^3x^6+9de^2x^4+18d^2ex^2+12d^3\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}dx+d^3\log(x)(a+b\text{arccosh}(cx))+\frac{3}{2}d^2ex^2(a+b\text{arccosh}(cx))+\frac{3}{4}de^2x^4(a+b\text{arccosh}(cx))+\frac{1}{6}e^3x^6(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{12}bc\int\left(\frac{2e^3x^6}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{9de^2x^4}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{18d^2ex^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{12d^3\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}\right)dx+ d^3\log(x)(a+b\text{arccosh}(cx))+\frac{3}{2}d^2ex^2(a+b\text{arccosh}(cx))+\frac{3}{4}de^2x^4(a+b\text{arccosh}(cx))+\frac{1}{6}e^3x^6(a+b\text{arccosh}(cx))$$

$\big\downarrow$ 2009

$$d^3\log(x)(a+b\text{arccosh}(cx))+\frac{3}{2}d^2ex^2(a+b\text{arccosh}(cx))+\frac{3}{4}de^2x^4(a+b\text{arccosh}(cx))+\frac{1}{6}e^3x^6(a+b\text{arccosh}(cx))-\frac{1}{12}bc\left(\frac{5e^3\text{arccosh}(cx)}{8c^7}+\frac{27de^2\text{arccosh}(cx)}{8c^5}+\frac{9d^2e\text{arccosh}(cx)}{c^3}+\frac{6id^3\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{6id\right.$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

output
```
(3*d^2*e*x^2*(a + b*ArcCosh[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/
4 + (e^3*x^6*(a + b*ArcCosh[c*x]))/6 + d^3*(a + b*ArcCosh[c*x])*Log[x] - (
b*c*((9*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + (27*d*e^2*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(8*c^4) + (5*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^
6) + (9*d*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^2) + (5*e^3*x^3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(12*c^4) + (e^3*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
/(3*c^2) + (9*d^2*e*ArcCosh[c*x])/c^3 + (27*d*e^2*ArcCosh[c*x])/(8*c^5) +
(5*e^3*ArcCosh[c*x])/(8*c^7) + ((6*I)*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)
/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (12*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*
Log[1 - E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (12*d^3
*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
((6*I)*d^3*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.68

method	result
parts	$a\left(\frac{e^3x^6}{6}+\frac{3d\,e^2x^4}{4}+\frac{3d^2e\,x^2}{2}+d^3\ln(x)\right)+\frac{b\,d^3\operatorname{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2}+d^3b\operatorname{arccosh}($
derivativedivides	$\frac{a\left(\frac{3c^6d^2e\,x^2}{2}+\frac{3c^6d\,e^2x^4}{4}+\frac{c^6e^3x^6}{6}+c^6d^3\ln(cx)\right)}{c^6}-\frac{3b\,d^2e\operatorname{arccosh}(cx)}{4c^2}-\frac{3bd\,e^2x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}+\frac{3b\operatorname{arccosh}(cx}{2}$
default	$\frac{a\left(\frac{3c^6d^2e\,x^2}{2}+\frac{3c^6d\,e^2x^4}{4}+\frac{c^6e^3x^6}{6}+c^6d^3\ln(cx)\right)}{c^6}-\frac{3b\,d^2e\operatorname{arccosh}(cx)}{4c^2}-\frac{3bd\,e^2x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}+\frac{3b\operatorname{arccosh}(cx}{2}$

input
```
int((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e^3*x^6+3/4*d*e^2*x^4+3/2*d^2*e*x^2+d^3*ln(x))+1/2*b*d^3*polylog(2,
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+d^3*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2)-3/4*b*d^2*e*arccosh(c*x)/c^2+1/6*b*arccosh(c*x)*e
^3*x^6-9/32*b*d*e^2*arccosh(c*x)/c^4-1/36*b*e^3*x^5*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/c-5/144*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-5/96*b*e^3*x*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^5+3/4*b*arccosh(c*x)*d*e^2*x^4+3/2*b*arccosh(c*x)
*d^2*e*x^2-3/16*b*d*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-9/32*b*d*e^2*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3/4*b*d^2*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
-5/96*b*e^3*arccosh(c*x)/c^6-1/2*d^3*b*arccosh(c*x)^2
```

Fricas [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x} dx = \int \frac{(ex^2+d)^3 (b\,\text{arcosh}\,(cx)+a)}{x} dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x} dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)^3}{x} dx$$

input
```
integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x} dx = \int \frac{(ex^2+d)^3 (b\,\text{arcosh}\,(cx)+a)}{x} dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

output
```
1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) + integra
te(b*e^3*x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*e^2*x^3*log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d^2*e*x*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1)) + b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(ex^2+d)^3(b\,\text{arcosh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(e\,x^2+d)^3}{x}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x}\,dx$$
$$= \frac{432acosh(cx)\,b\,c^6d^2e\,x^2+216acosh(cx)\,b\,c^6d\,e^2x^4+48acosh(cx)\,b\,c^6e^3x^6-216\sqrt{c^2x^2-1}\,b\,c^5d^2ex-54}{}$$

input
```
int((e*x^2+d)^3*(a+b*acosh(c*x))/x,x)
```

output
```
(432*acosh(c*x)*b*c**6*d**2*e*x**2 + 216*acosh(c*x)*b*c**6*d*e**2*x**4 + 4
8*acosh(c*x)*b*c**6*e**3*x**6 - 216*sqrt(c**2*x**2 - 1)*b*c**5*d**2*e*x -
54*sqrt(c**2*x**2 - 1)*b*c**5*d*e**2*x**3 - 8*sqrt(c**2*x**2 - 1)*b*c**5*e
**3*x**5 - 81*sqrt(c**2*x**2 - 1)*b*c**3*d*e**2*x - 10*sqrt(c**2*x**2 - 1)
*b*c**3*e**3*x**3 - 15*sqrt(c**2*x**2 - 1)*b*c*e**3*x + 288*int(acosh(c*x)
/x,x)*b*c**6*d**3 - 216*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**4*d**2*e - 81*
log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d*e**2 - 15*log(sqrt(c**2*x**2 - 1)
+ c*x)*b*e**3 + 288*log(x)*a*c**6*d**3 + 432*a*c**6*d**2*e*x**2 + 216*a*c*
*6*d*e**2*x**4 + 48*a*c**6*e**3*x**6)/(288*c**6)
```

3.384 $\int \frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{x^2}\,dx = & -\frac{be(225c^4d^2+50c^2de+8e^2)\sqrt{-1+cx}\sqrt{1+cx}}{75c^5} \\ & -\frac{be^2(25c^2d+4e)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{75c^3} \\ & -\frac{be^3x^4\sqrt{-1+cx}\sqrt{1+cx}}{25c} \\ & -\frac{d^3(a+b\text{arccosh}(cx))}{x}+3d^2ex(a+b\text{arccosh}(cx)) \\ & +de^2x^3(a+b\text{arccosh}(cx)) \\ & +\frac{1}{5}e^3x^5(a+b\text{arccosh}(cx)) \\ & +bcd^3\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\end{aligned}$$

output

```
-1/75*b*e*(225*c^4*d^2+50*c^2*d*e+8*e^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-1
/75*b*e^2*(25*c^2*d+4*e)*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/25*b*e^3*x^
4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-d^3*(a+b*arccosh(c*x))/x+3*d^2*e*x*(a+b*ar
ccosh(c*x))+d*e^2*x^3*(a+b*arccosh(c*x))+1/5*e^3*x^5*(a+b*arccosh(c*x))+b*
c*d^3*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^2} dx$$
$$= -\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{1}{5}ae^3x^5 - \frac{be\sqrt{-1+cx}\sqrt{1+cx}(8e^2+2c^2e(25d+2ex^2)+c^4(225d^2+25dex^2+3e^2x^4))}{75c^5} + \frac{b(-5d^3+15d^2ex^2+5de^2x^4+e^3x^6)\operatorname{arccosh}(cx)}{5x} - bcd^3\arctan\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((a*d^3)/x) + 3*a*d^2*e*x + a*d*e^2*x^3 + (a*e^3*x^5)/5 - (b*e*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(8*e^2 + 2*c^2*e*(25*d + 2*e*x^2) + c^4*(225*d^2 + 25*d
*e*x^2 + 3*e^2*x^4)))/(75*c^5) + (b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^2*x^4 +
 e^3*x^6)*ArcCosh[c*x])/(5*x) - b*c*d^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])]
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6373, 27, 2113, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^2} dx$$
$$\Big\downarrow \text{ 6373}$$

$$-bc\int -\frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{5x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^3(a+b\text{arccosh}(cx))}{x}+3d^2ex(a+b\text{arccosh}(cx))+de^2x^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^3x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}bc\int \frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^3(a+b\text{arccosh}(cx))}{x}+3d^2ex(a+b\text{arccosh}(cx))+de^2x^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^3x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 2113$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x\sqrt{c^2x^2-1}}dx}{5\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{x}+3d^2ex(a+b\text{arccosh}(cx))+de^2x^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^3x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 2331$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x^2\sqrt{c^2x^2-1}}dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{x}+3d^2ex(a+b\text{arccosh}(cx))+de^2x^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^3x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 2123$$

$$\frac{bc\sqrt{c^2x^2-1}\int \left(\frac{5d^3}{x^2\sqrt{c^2x^2-1}}-\frac{e^3(c^2x^2-1)^{3/2}}{c^4}-\frac{e^2(5dc^2+2e)\sqrt{c^2x^2-1}}{c^4}-\frac{e(15d^2c^4+5dec^2+e^2)}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{10\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{x}+3d^2ex(a+b\text{arccosh}(cx))+de^2x^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^3x^5(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \ 2009$$

$$-\frac{d^3(a+b\text{arccosh}(cx))}{x}+3d^2ex(a+b\text{arccosh}(cx))+de^2x^3(a+b\text{arccosh}(cx))+\frac{1}{5}e^3x^5(a+b\text{arccosh}(cx))+\frac{bc\sqrt{c^2x^2-1}\left(10d^3\arctan\left(\sqrt{c^2x^2-1}\right)-\frac{2e^2(c^2x^2-1)^{3/2}(5c^2d+2e)}{3c^6}-\frac{2e^3(c^2x^2-1)^{5/2}}{5c^6}-\frac{2e\sqrt{c^2x^2-1}(15c^4d^2+5c^2de+e^2)}{c^6}\right)}{10\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

output
```
-((d^3*(a + b*ArcCosh[c*x]))/x) + 3*d^2*e*x*(a + b*ArcCosh[c*x]) + d*e^2*x
^3*(a + b*ArcCosh[c*x]) + (e^3*x^5*(a + b*ArcCosh[c*x]))/5 + (b*c*Sqrt[-1
+ c^2*x^2]*((-2*e*(15*c^4*d^2 + 5*c^2*d*e + e^2)*Sqrt[-1 + c^2*x^2])/c^6 -
 (2*e^2*(5*c^2*d + 2*e)*(-1 + c^2*x^2)^(3/2))/(3*c^6) - (2*e^3*(-1 + c^2*x
^2)^(5/2))/(5*c^6) + 10*d^3*ArcTan[Sqrt[-1 + c^2*x^2]]))/(10*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.28

method	result
parts	$a\left(\frac{e^3x^5}{5}+d\,e^2x^3+3d^2ex-\frac{d^3}{x}\right)+bc\left(\frac{\operatorname{arccosh}(cx)e^3x^5}{5c}+\frac{\operatorname{arccosh}(cx)d\,e^2x^3}{c}+\frac{3\operatorname{arccosh}(cx)d^2ex}{c}-\right.$
derivativedivides	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\operatorname{arccosh}(cx)c^5d^2ex+\operatorname{arccosh}(cx)c^5d\,e^2x^3+\frac{\operatorname{arccosh}(cx)e^3c^5x^5}{5}-\right.}{}\right.$
default	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\operatorname{arccosh}(cx)c^5d^2ex+\operatorname{arccosh}(cx)c^5d\,e^2x^3+\frac{\operatorname{arccosh}(cx)e^3c^5x^5}{5}-\right.}{}\right.$

input
```
int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^3*x^5+d*e^2*x^3+3*d^2*e*x-d^3/x)+b*c*(1/5/c*arccosh(c*x)*e^3*x^5+
1/c*arccosh(c*x)*d*e^2*x^3+3/c*arccosh(c*x)*d^2*e*x-arccosh(c*x)*d^3/c/x-1
/75/c^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*d^3*arctan(1/(c^2*x^2-1)^(1/2)
)+225*c^4*d^2*e*(c^2*x^2-1)^(1/2)+25*c^4*d*e^2*x^2*(c^2*x^2-1)^(1/2)+3*e^3
*c^4*x^4*(c^2*x^2-1)^(1/2)+50*c^2*d*e^2*(c^2*x^2-1)^(1/2)+4*e^3*c^2*x^2*(c
^2*x^2-1)^(1/2)+8*e^3*(c^2*x^2-1)^(1/2))/(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^2}\,dx$$
$$= \frac{15\,ac^5e^3x^6 + 75\,ac^5de^2x^4 + 150\,bc^6d^3x\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + 225\,ac^5d^2ex^2 - 75\,ac^5d^3 + 15\,(5\,bc^5d}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/75*(15*a*c^5*e^3*x^6 + 75*a*c^5*d*e^2*x^4 + 150*b*c^6*d^3*x*arctan(-c*x
+ sqrt(c^2*x^2 - 1)) + 225*a*c^5*d^2*e*x^2 - 75*a*c^5*d^3 + 15*(5*b*c^5*d^
3 - 15*b*c^5*d^2*e - 5*b*c^5*d*e^2 - b*c^5*e^3)*x*log(-c*x + sqrt(c^2*x^2
- 1)) + 15*(b*c^5*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c^5*d^2*e*x^2 - 5*b*c
^5*d^3 + (5*b*c^5*d^3 - 15*b*c^5*d^2*e - 5*b*c^5*d*e^2 - b*c^5*e^3)*x)*log
(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^4*e^3*x^5 + (25*b*c^4*d*e^2 + 4*b*c^2*e
^3)*x^3 + (225*b*c^4*d^2*e + 50*b*c^2*d*e^2 + 8*b*e^3)*x)*sqrt(c^2*x^2 - 1
))/(c^5*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)^3}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^2}\,dx$$
$$= \frac{1}{5}ae^3x^5 + ade^2x^3 - \left(c\arcsin\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)bd^3$$
$$+ \frac{1}{3}\left(3x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bde^2$$
$$+ \frac{1}{75}\left(15x^5\operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)be^3$$
$$+ 3ad^2ex + \frac{3\left(cx\operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bd^2e}{c} - \frac{ad^3}{x}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/5*a*e^3*x^5 + a*d*e^2*x^3 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*
d^3 + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*
x^2 - 1)/c^4))*b*d*e^2 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*
x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*e^3
+ 3*a*d^2*e*x + 3*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^2*e/c - a*d^3
/x
```

Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^3(b\operatorname{arcosh}(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(e\,x^2+d)^3}{x^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^2,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{x^2}\,dx = \frac{-75acosh(cx)\,b\,c^5d^3+225acosh(cx)\,b\,c^5d^2e\,x^2+75acosh(cx)\,b\,c^5d\,e^2x^4+15acosh(cx)\,b\,c^5e^3x^6-150ata}{}$$

input
```
int((e*x^2+d)^3*(a+b*acosh(c*x))/x^2,x)
```

output
```
( - 75*acosh(c*x)*b*c**5*d**3 + 225*acosh(c*x)*b*c**5*d**2*e*x**2 + 75*aco
sh(c*x)*b*c**5*d*e**2*x**4 + 15*acosh(c*x)*b*c**5*e**3*x**6 - 150*atan(sqr
t(c**2*x**2 - 1) + c*x)*b*c**6*d**3*x - 25*sqrt(c**2*x**2 - 1)*b*c**4*d*e*
*2*x**3 - 3*sqrt(c**2*x**2 - 1)*b*c**4*e**3*x**5 - 50*sqrt(c**2*x**2 - 1)*
b*c**2*d*e**2*x - 4*sqrt(c**2*x**2 - 1)*b*c**2*e**3*x**3 - 8*sqrt(c**2*x**
2 - 1)*b*e**3*x - 225*sqrt(c*x + 1)*sqrt(c*x - 1)*b*c**4*d**2*e*x - 75*a*c
**5*d**3 + 225*a*c**5*d**2*e*x**2 + 75*a*c**5*d*e**2*x**4 + 15*a*c**5*e**3
*x**6)/(75*c**5*x)
```

3.385 $\int \frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 402

$$\begin{aligned}
\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3}\,dx &= \frac{bcd^3\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{3be^2(8c^2d+e)\,x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3} \\
&\quad - \frac{be^3x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c} - \frac{3be^2(8c^2d+e)\,\text{arccosh}(cx)}{32c^4} - \frac{d^3(a+b\text{arccosh}(cx))}{2x^2} \\
&\quad + \frac{3}{2}de^2x^2(a+b\text{arccosh}(cx)) + \frac{1}{4}e^3x^4(a+b\text{arccosh}(cx)) - \frac{3ibd^2e\sqrt{1-c^2x^2}\arcsin(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} \\
&\quad + \frac{3bd^2e\sqrt{1-c^2x^2}\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + 3d^2e(a+b\text{arccosh}(cx))\log(x) \\
&\quad - \frac{3bd^2e\sqrt{1-c^2x^2}\arcsin(cx)\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3ibd^2e\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

output
```
1/2*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x-3/32*b*e^2*(8*c^2*d+e)*x*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/c^3-1/16*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/32
*b*e^2*(8*c^2*d+e)*arccosh(c*x)/c^4-1/2*d^3*(a+b*arccosh(c*x))/x^2+3/2*d*e
^2*x^2*(a+b*arccosh(c*x))+1/4*e^3*x^4*(a+b*arccosh(c*x))-3/2*I*b*d^2*e*(-c
^2*x^2+1)^(1/2)*arcsin(c*x)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*b*d^2*e*(-c^2*
x^2+1)^(1/2)*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+3*d^2*e*(a+b*arccosh(c*x))*ln(x)-3*b*d^2*e*(-c^2*x^2+1)^(1/2
)*arcsin(c*x)*ln(x)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*I*b*d^2*e*(-c^2*x^2+1)
^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3}\,dx = \frac{1}{4}\left(-\frac{2ad^3}{x^2}+6ade^2x^2+ae^3x^4+\frac{2bd^3\left(cx\sqrt{-1+cx}\sqrt{1+cx}-\text{arccosh}(cx)\right)}{x^2} + 6bde^2x^2\text{arccosh}(cx)+be^3x^4\text{arccosh}(cx) - \frac{3bde^2\left(cx\sqrt{-1+cx}\sqrt{1+cx}+2\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{c^2} - \frac{be^3\left(cx\sqrt{\frac{-1+cx}{1+cx}}(3+3cx+2c^2x^2+2c^3x^3)+6\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{8c^4} + 6bd^2e\text{arccosh}(cx)\left(\text{arccosh}(cx)+2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right)+12ad^2e\log(x) - 6bd^2e\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
((-2*a*d^3)/x^2 + 6*a*d*e^2*x^2 + a*e^3*x^4 + (2*b*d^3*(c*x*Sqrt[-1 + c*x]
*Sqrt[1 + c*x] - ArcCosh[c*x]))/x^2 + 6*b*d*e^2*x^2*ArcCosh[c*x] + b*e^3*x
^4*ArcCosh[c*x] - (3*b*d*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh
[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^2 - (b*e^3*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)
]*(3 + 3*c*x + 2*c^2*x^2 + 2*c^3*x^3) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x
)]]))/(8*c^4) + 6*b*d^2*e*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*Arc
Cosh[c*x])]) + 12*a*d^2*e*Log[x] - 6*b*d^2*e*PolyLog[2, -E^(-2*ArcCosh[c*x
])])/4
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3} \, dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int -\frac{-e^3x^6-6de^2x^4-12d^2e\log(x)x^2+2d^3}{4x^2\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^3(a+b\text{arccosh}(cx))}{2x^2}+3d^2e\log(x)(a+b\text{arccosh}(cx))+\frac{3}{2}de^2x^2(a+b\text{arccosh}(cx))+\frac{1}{4}e^3x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}bc\int \frac{-e^3x^6-6de^2x^4-12d^2e\log(x)x^2+2d^3}{x^2\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^3(a+b\text{arccosh}(cx))}{2x^2}+3d^2e\log(x)(a+b\text{arccosh}(cx))+\frac{3}{2}de^2x^2(a+b\text{arccosh}(cx))+\frac{1}{4}e^3x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{4}bc\int \left(\frac{-e^3x^6-6de^2x^4+2d^3}{x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{12d^2e\log(x)}{\sqrt{cx-1}\sqrt{cx+1}}\right)dx-\frac{d^3(a+b\text{arccosh}(cx))}{2x^2}+3d^2e\log(x)(a+b\text{arccosh}(cx))+\frac{3}{2}de^2x^2(a+b\text{arccosh}(cx))+\frac{1}{4}e^3x^4(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\operatorname{arccosh}(cx))}{2x^2}+3d^2e\log(x)(a+b\operatorname{arccosh}(cx))+\frac{3}{2}de^2x^2(a+b\operatorname{arccosh}(cx))+\frac{1}{4}e^3x^4(a+b\operatorname{arccosh}(cx))+\frac{1}{4}bc\left(-\frac{6id^2e\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{6id^2e\sqrt{1-c^2x^2}\arcsin(cx)^2}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{12d^2e\sqrt{1-c^2x^2}\arcsin(cx)\log}{c\sqrt{cx-1}\sqrt{cx+1}}\right.$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

output
```
-1/2*(d^3*(a + b*ArcCosh[c*x]))/x^2 + (3*d*e^2*x^2*(a + b*ArcCosh[c*x]))/2
 + (e^3*x^4*(a + b*ArcCosh[c*x]))/4 + 3*d^2*e*(a + b*ArcCosh[c*x])*Log[x]
+ (b*c*((-2*d^3*(1 - c^2*x^2))/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*e^2*(
8*c^2*d + e)*x*(1 - c^2*x^2))/(8*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (e^3*
x^3*(1 - c^2*x^2))/(4*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((6*I)*d^2*e*Sqr
t[1 - c^2*x^2]*ArcSin[c*x]^2)/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*e^2*(8
*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(8*c^5*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (12*d^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log
[1 - E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (12*d^2*e*
Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
(6*I)*d^2*e*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(c*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.73

method	result
parts	$a\left(\frac{e^3x^4}{4}+\frac{3d\,e^2x^2}{2}+3d^2e\ln\left(x\right)-\frac{d^3}{2x^2}\right)-\frac{d^3b\,c^2}{2}-\frac{b\,e^3x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}-\frac{3b\,e^3\sqrt{cx+1}\,\sqrt{cx-1}\,x}{32c^3}+$
derivativedivides	$c^2\left(\frac{a\left(\frac{3c^4d\,e^2x^2}{2}+\frac{c^4e^3x^4}{4}+3c^4d^2e\ln(cx)-\frac{c^4d^3}{2x^2}\right)}{c^6}+\frac{d^3b\sqrt{cx+1}\,\sqrt{cx-1}}{2cx}+\frac{3b\operatorname{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2c^2}\right.$
default	$c^2\left(\frac{a\left(\frac{3c^4d\,e^2x^2}{2}+\frac{c^4e^3x^4}{4}+3c^4d^2e\ln(cx)-\frac{c^4d^3}{2x^2}\right)}{c^6}+\frac{d^3b\sqrt{cx+1}\,\sqrt{cx-1}}{2cx}+\frac{3b\operatorname{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2c^2}\right.$

input
```
int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^3*x^4+3/2*d*e^2*x^2+3*d^2*e*ln(x)-1/2*d^3/x^2)-1/2*d^3*b*c^2-1/16
*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/32*b/c^3*e^3*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x+3/2*b*e^2*arccosh(c*x)*x^2*d+1/2*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/x-3/2*b*d^2*e*arccosh(c*x)^2+3/2*b*e*d^2*polylog(2,-(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)+1/4*b*e^3*arccosh(c*x)*x^4-3/4*b/c*e^2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*x*d-3/4*b/c^2*e^2*arccosh(c*x)*d-1/2*d^3*b/x^2*arccosh(c*x)
+3*b*e*d^2*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/32*b/c
^4*e^3*arccosh(c*x)
```

Fricas [F]

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\text{arccosh}(cx)\right)}{x^3}\,dx=\int \frac{\left(ex^2+d\right)^3\left(b\,\text{arcosh}\left(cx\right)+a\right)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3} \, dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)^3}{x^3} \, dx$$

input
```
integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3} \, dx = \int \frac{(ex^2+d)^3(b\,\text{arcosh}\,(cx)+a)}{x^3} \, dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 + 1/2*b*d^3*(sqrt(c^2*x^2 - 1)*c/x - arcco
sh(c*x)/x^2) + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 + integrate(b*e^3*x^3*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*e^2*x*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)) + 3*b*d^2*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^3(b\,\text{arcosh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,\left(e\,x^2+d\right)^3}{x^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^3,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^3}\,dx = \frac{-16acosh(cx)\,b\,c^4d^3+48acosh(cx)\,b\,c^4d\,e^2x^4+8acosh(cx)\,b\,c^4e^3x^6-16\sqrt{c^2x^2-1}\,b\,c^5d^3x-24\sqrt{c^2x^2-}}{}$$

input
```
int((e*x^2+d)^3*(a+b*acosh(c*x))/x^3,x)
```

output
```
( - 16*acosh(c*x)*b*c**4*d**3 + 48*acosh(c*x)*b*c**4*d*e**2*x**4 + 8*acosh
(c*x)*b*c**4*e**3*x**6 - 16*sqrt(c**2*x**2 - 1)*b*c**5*d**3*x - 24*sqrt(c*
*2*x**2 - 1)*b*c**3*d*e**2*x**3 - 2*sqrt(c**2*x**2 - 1)*b*c**3*e**3*x**5 -
 3*sqrt(c**2*x**2 - 1)*b*c*e**3*x**3 + 96*int(acosh(c*x)/x,x)*b*c**4*d**2*
e*x**2 - 24*log(sqrt(c**2*x**2 - 1) + c*x)*b*c**2*d*e**2*x**2 - 3*log(sqrt
(c**2*x**2 - 1) + c*x)*b*e**3*x**2 + 96*log(x)*a*c**4*d**2*e*x**2 - 16*a*c
**4*d**3 + 48*a*c**4*d*e**2*x**4 + 8*a*c**4*e**3*x**6 - 16*b*c**6*d**3*x**
2)/(32*c**4*x**2)
```

3.386 $\int \frac{(d+ex^2)^3(a+b\text{arccosh}(cx))}{x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 208

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^4} dx = -\frac{be^2(27c^2d+2e)\sqrt{-1+cx}\sqrt{1+cx}}{9c^3} + \frac{bcd^3\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{be^3x^2\sqrt{-1+cx}\sqrt{1+cx}}{9c} - \frac{d^3(a+b\text{arccosh}(cx))}{3x^3} - \frac{3d^2e(a+b\text{arccosh}(cx))}{x} + 3de^2x(a+b\text{arccosh}(cx)) + \frac{1}{3}e^3x^3(a+b\text{arccosh}(cx)) + \frac{1}{6}bcd^2(c^2d+18e)\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
-1/9*b*e^2*(27*c^2*d+2*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3+1/6*b*c*d^3*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/x^2-1/9*b*e^3*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/
3*d^3*(a+b*arccosh(c*x))/x^3-3*d^2*e*(a+b*arccosh(c*x))/x+3*d*e^2*x*(a+b*a
rccosh(c*x))+1/3*e^3*x^3*(a+b*arccosh(c*x))+1/6*b*c*d^2*(c^2*d+18*e)*arcta
n((c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^4} \, dx = \frac{1}{6}\left(-\frac{2ad^3}{x^3}-\frac{18ad^2e}{x}+18ade^2x+2ae^3x^3 - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(-3c^4d^3+4e^3x^2+2c^2e^2x^2(27d+ex^2))}{3c^3x^2} + \frac{2b(-d^3-9d^2ex^2+9de^2x^4+e^3x^6)\operatorname{arccosh}(cx)}{x^3} - bcd^2(c^2d+18e)\arctan\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
((-2*a*d^3)/x^3 - (18*a*d^2*e)/x + 18*a*d*e^2*x + 2*a*e^3*x^3 - (b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(-3*c^4*d^3 + 4*e^3*x^2 + 2*c^2*e^2*x^2*(27*d + e*x^
2)))/(3*c^3*x^2) + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcCo
sh[c*x])/x^3 - b*c*d^2*(c^2*d + 18*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*
x])])/6
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6373, 27, 2113, 2331, 2124, 27, 1192, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^4} \, dx$$

$$\Big\downarrow \; 6373$$

$$-bc\int -\frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{3x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2113}$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^3\sqrt{c^2x^2-1}}dx}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^4\sqrt{c^2x^2-1}}dx^2}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2124}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\int \frac{-2e^3x^4-18de^2x^2+d^2(dc^2+18e)}{2x^2\sqrt{c^2x^2-1}}dx^2+\frac{d^3\sqrt{c^2x^2-1}}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{1}{2}\int \frac{-2e^3x^4-18de^2x^2+d^2(dc^2+18e)}{x^2\sqrt{c^2x^2-1}}dx^2+\frac{d^3\sqrt{c^2x^2-1}}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1192}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int \frac{-2e^3x^8-2e^2\left(9dc^2+2e\right)x^4+c^6d^3-2e^3-18c^2de^2+18c^4d^2e}{x^4+1}d\sqrt{c^2x^2-1}}{c^4}+\frac{d^3\sqrt{c^2x^2-1}}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{1467}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int\left(-2e^3x^4-2e^2(9dc^2+e)+\frac{d^3c^6+18d^2ec^4}{x^4+1}\right)d\sqrt{c^2x^2-1}}{c^4}+\frac{d^3\sqrt{c^2x^2-1}}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\text{arccosh}(cx))}{3x^3}-\frac{3d^2e(a+b\text{arccosh}(cx))}{x}+3de^2x(a+b\text{arccosh}(cx))+\frac{1}{3}e^3x^3(a+b\text{arccosh}(cx))+\frac{bc\sqrt{c^2x^2-1}\left(\frac{c^4d^2\arctan\left(\sqrt{c^2x^2-1}\right)(c^2d+18e)-2e^2\sqrt{c^2x^2-1}(9c^2d+e)-\frac{2}{3}e^3x^6}{c^4}+\frac{d^3\sqrt{c^2x^2-1}}{x^2}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

output
```
-1/3*(d^3*(a + b*ArcCosh[c*x]))/x^3 - (3*d^2*e*(a + b*ArcCosh[c*x]))/x + 3
*d*e^2*x*(a + b*ArcCosh[c*x]) + (e^3*x^3*(a + b*ArcCosh[c*x]))/3 + (b*c*Sq
rt[-1 + c^2*x^2]*((d^3*Sqrt[-1 + c^2*x^2])/x^2 + ((-2*e^3*x^6)/3 - 2*e^2*(
9*c^2*d + e)*Sqrt[-1 + c^2*x^2] + c^4*d^2*(c^2*d + 18*e)*ArcTan[Sqrt[-1 +
c^2*x^2]])/c^4))/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.30

method	result
parts	$a\left(\frac{e^3x^3}{3}+3d\,e^2x-\frac{3d^2e}{x}-\frac{d^3}{3x^3}\right)+b\,c^3\left(\frac{\operatorname{arccosh}(cx)e^3x^3}{3c^3}+\frac{3\operatorname{arccosh}(cx)xd\,e^2}{c^3}-\frac{3\operatorname{arccosh}(cx)d^2e}{c^3x}\right.$
derivativedivides	$c^3\left(\frac{a\left(3c^3d\,e^2x+\frac{e^3c^3x^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\operatorname{arccosh}(cx)c^3d\,e^2x+\frac{\operatorname{arccosh}(cx)e^3c^3x^3}{3}-\frac{3\operatorname{arccosh}(cx)c^3d^2e}{x}-\right.}{}\right.$
default	$c^3\left(\frac{a\left(3c^3d\,e^2x+\frac{e^3c^3x^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\operatorname{arccosh}(cx)c^3d\,e^2x+\frac{\operatorname{arccosh}(cx)e^3c^3x^3}{3}-\frac{3\operatorname{arccosh}(cx)c^3d^2e}{x}-\right.}{}\right.$

input
```
int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^3*x^3+3*d*e^2*x-3*d^2*e/x-1/3*d^3/x^3)+b*c^3*(1/3/c^3*arccosh(c*x
)*e^3*x^3+3/c^3*arccosh(c*x)*x*d*e^2-3/c^3*arccosh(c*x)*d^2*e/x-1/3*arccos
h(c*x)*d^3/c^3/x^3-1/18/c^8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*arctan(1/(c^2*x
^2-1)^(1/2))*c^8*d^3*x^2+54*arctan(1/(c^2*x^2-1)^(1/2))*c^6*d^2*e*x^2-3*c^
6*d^3*(c^2*x^2-1)^(1/2)+54*c^4*d*e^2*x^2*(c^2*x^2-1)^(1/2)+2*e^3*c^4*x^4*(
c^2*x^2-1)^(1/2)+4*e^3*c^2*x^2*(c^2*x^2-1)^(1/2))/(c^2*x^2-1)^(1/2)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.55

$$\int\frac{(d+ex^2)^3\,(a+b\operatorname{arccosh}(cx))}{x^4}\,dx$$
$$=\frac{6\,ac^3e^3x^6+54\,ac^3de^2x^4-54\,ac^3d^2ex^2-6\,ac^3d^3+6\,(bc^6d^3+18\,bc^4d^2e)x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/18*(6*a*c^3*e^3*x^6 + 54*a*c^3*d*e^2*x^4 - 54*a*c^3*d^2*e*x^2 - 6*a*c^3*
d^3 + 6*(b*c^6*d^3 + 18*b*c^4*d^2*e)*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1))
+ 6*(b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c^3*e^3)*x^3*log(-c*x +
 sqrt(c^2*x^2 - 1)) + 6*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^3*d^2*e
*x^2 - b*c^3*d^3 + (b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c^3*e^3)
*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^2*e^3*x^5 - 3*b*c^4*d^3*x + 2*
(27*b*c^2*d*e^2 + 2*b*e^3)*x^3)*sqrt(c^2*x^2 - 1))/(c^3*x^3)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{acosh}\,(cx))\,(d+ex^2)^3}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

output
```
Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.95

$$\begin{aligned}
&\int \frac{(d+ex^2)^3 (a+b\text{arccosh}(cx))}{x^4}\,dx \\
&= \frac{1}{3}ae^3x^3 - \frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2-1}}{x^2}\right)c + \frac{2\,\text{arcosh}\,(cx)}{x^3}\right)bd^3 \\
&\quad - 3\left(c\arcsin\left(\frac{1}{c|x|}\right) + \frac{\text{arcosh}\,(cx)}{x}\right)bd^2e \\
&\quad + \frac{1}{9}\left(3\,x^3\,\text{arcosh}\,(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)be^3 \\
&\quad + 3\,ade^2x + \frac{3\left(cx\,\text{arcosh}\,(cx) - \sqrt{c^2x^2-1}\right)bde^2}{c} - \frac{3\,ad^2e}{x} - \frac{ad^3}{3\,x^3}
\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/3*a*e^3*x^3 - 1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c
+ 2*arccosh(c*x)/x^3)*b*d^3 - 3*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*
b*d^2*e + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(
c^2*x^2 - 1)/c^4))*b*e^3 + 3*a*d*e^2*x + 3*(c*x*arccosh(c*x) - sqrt(c^2*x^
2 - 1))*b*d*e^2/c - 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^4}\,dx=\int \frac{(ex^2+d)^3(b\operatorname{arcosh}(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^4}\,dx=\int \frac{(a+b\operatorname{acosh}(c\,x))\,(e\,x^2+d)^3}{x^4}\,dx$$

input
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^4,x)
```

output
```
int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)^3 (a+b\operatorname{arccosh}(cx))}{x^4}\,dx$$
$$= \frac{-6acosh(cx)\,b\,c^3d^3 - 54acosh(cx)\,b\,c^3d^2e\,x^2 + 54acosh(cx)\,b\,c^3d\,e^2x^4 + 6acosh(cx)\,b\,c^3e^3x^6 - 6atan(\sqrt{c}}{}$$

input
```
int((e*x^2+d)^3*(a+b*acosh(c*x))/x^4,x)
```

output
```
( - 6*acosh(c*x)*b*c**3*d**3 - 54*acosh(c*x)*b*c**3*d**2*e*x**2 + 54*acosh
(c*x)*b*c**3*d*e**2*x**4 + 6*acosh(c*x)*b*c**3*e**3*x**6 - 6*atan(sqrt(c**
2*x**2 - 1) + c*x)*b*c**6*d**3*x**3 - 108*atan(sqrt(c**2*x**2 - 1) + c*x)*
b*c**4*d**2*e*x**3 - 3*sqrt(c**2*x**2 - 1)*b*c**4*d**3*x - 2*sqrt(c**2*x**
2 - 1)*b*c**2*e**3*x**5 - 4*sqrt(c**2*x**2 - 1)*b*e**3*x**3 - 54*sqrt(c*x
+ 1)*sqrt(c*x - 1)*b*c**2*d*e**2*x**3 - 6*a*c**3*d**3 - 54*a*c**3*d**2*e*x
**2 + 54*a*c**3*d*e**2*x**4 + 6*a*c**3*e**3*x**6)/(18*c**3*x**3)
```

3.387 $\int \frac{x^4(a+b\,\mathbf{arccosh}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 627

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx = -\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\sqrt{1+cx}}{ce^2} - \frac{2b\sqrt{-1+cx}\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9ce} - \frac{bdx\operatorname{arccosh}(cx)}{e^2}+\frac{x^3(a+b\operatorname{arccosh}(cx))}{3e} + \frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{5/2}} - \frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{5/2}} + \frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{5/2}} - \frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{5/2}} - \frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{5/2}} + \frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{5/2}} - \frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{5/2}} + \frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{5/2}}$$

output
```
-a*d*x/e^2+b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2-2/9*b*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/c^3/e-1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e-b*d*x*arccosh(c*x
)/e^2+1/3*x^3*(a+b*arccosh(c*x))/e+1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1-
e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))
/e^(5/2)-1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*(-d)^(3/2)*(a
+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1
/2)+(-c^2*d-e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1+e^(1
/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(
5/2)-1/2*b*(-d)^(3/2)*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*b*(-d)^(3/2)*polylog(2,e^(1/
2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5
/2)-1/2*b*(-d)^(3/2)*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/
(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*b*(-d)^(3/2)*polylog(2,e^(1/2
)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/
2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.84

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx = -\frac{adx}{e^2}+\frac{ax^3}{3e}+\frac{ad^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}} + \frac{b\left(\frac{4d\sqrt{e}\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)-cx\operatorname{arccosh}(cx)\right)}{c}-\frac{4e^{3/2}\left(\sqrt{-1+cx}\sqrt{1+cx}(2+c^2x^2)-3c^3x^3\operatorname{arccosh}(cx)\right)}{9c^3}-id^{3/2}\left(\operatorname{arccosh}(cx)\right.\right.}{}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
-((a*d*x)/e^2) + (a*x^3)/(3*e) + (a*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e
^(5/2) + (b*((4*d*Sqrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcC
osh[c*x]))/c - (4*e^(3/2)*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c^2*x^2) - 3*
c^3*x^3*ArcCosh[c*x]))/(9*c^3) - I*d^(3/2)*(ArcCosh[c*x]*(-ArcCosh[c*x] +
2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] + Log
[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) + 2*PolyL
og[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + 2*Pol
yLog[2, ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])]) + I*
d^(3/2)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x
])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (I*Sqrt[e]*E^ArcCosh[c*x])/
(c*Sqrt[d] + Sqrt[c^2*d + e])])) + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[d] - Sqrt[c^2*d + e])] + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/
(c*Sqrt[d] + Sqrt[c^2*d + e])])))/(4*e^(5/2))
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{d+ex^2}dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{d^2(a+b\operatorname{arccosh}(cx))}{e^2\left(d+ex^2\right)}-\frac{d(a+b\operatorname{arccosh}(cx))}{e^2}+\frac{x^2(a+b\operatorname{arccosh}(cx))}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}-\frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^{5/2}}+\frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{5/2}}-\frac{(-d)^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^{5/2}}+\frac{x^3(a+b\operatorname{arccosh}(cx))}{3e}-\frac{adx}{e^2}-\frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{5/2}}+\frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{5/2}}-\frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^{5/2}}+\frac{b(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^{5/2}}-\frac{bdx\operatorname{arccosh}(cx)}{e^2}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}}{9c^3e}+\frac{bd\sqrt{cx-1}\sqrt{cx+1}}{ce^2}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{9ce}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
-((a*d*x)/e^2) + (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) - (2*b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])/(9*c^3*e) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9
*c*e) - (b*d*x*ArcCosh[c*x])/e^2 + (x^3*(a + b*ArcCosh[c*x]))/(3*e) + ((-d
)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
- Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/
2)) + ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh
[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(2*e^(5/2)) - (b*(-d)^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - (b*
(-d)^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) - e]))])/(2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.09 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.58

method	result
parts	$\frac{a x^3}{3e} - \frac{a d x}{e^2} + \frac{a d^2 \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{e^2 \sqrt{d e}} + \frac{b c d^2 \left(\sum_{\_R1=\text{RootOf}\left(e\_Z^4+\left(4c^2 d+2e\right)\_Z^2+e\right)} \frac{\_R1\left(\text{arccosh}(cx)\ln\left(\_R1 \cdots\right.\right.}{\cdots}\right.}{2e^2}$
derivativedivides	$\frac{-\frac{a c^5 d x}{e^2} + \frac{a c^5 x^3}{3e} + \frac{a c^5 d^2 \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{e^2 \sqrt{d e}} - \frac{b c^5 \text{arccosh}(cx) d x}{e^2} + \frac{b c^4 \sqrt{cx+1}\,\sqrt{cx-1}\, d}{e^2} + \frac{b c^5 \text{arccosh}(cx) x^3}{3e} - \frac{2 b c^2 \sqrt{cx-1}\,\sqrt{cx+1}}{9e} + \cdots}{\cdots}$
default	$\frac{-\frac{a c^5 d x}{e^2} + \frac{a c^5 x^3}{3e} + \frac{a c^5 d^2 \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{e^2 \sqrt{d e}} - \frac{b c^5 \text{arccosh}(cx) d x}{e^2} + \frac{b c^4 \sqrt{cx+1}\,\sqrt{cx-1}\, d}{e^2} + \frac{b c^5 \text{arccosh}(cx) x^3}{3e} - \frac{2 b c^2 \sqrt{cx-1}\,\sqrt{cx+1}}{9e} + \cdots}{\cdots}$

input
```
int(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/e*x^3-a*d*x/e^2+a*d^2/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*
c/e^2*d^2*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),
_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/2*b*c/e^2*d^2*sum(1/_R1/(_R1^2*
e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+d
ilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*
d+2*e)*_Z^2+e))+1/3*b/e*arccosh(c*x)*x^3-2/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3/e-1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e-b*d*x*arccosh(c*x)/e^2+b*
d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{ex^2+d}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^4*arccosh(c*x) + a*x^4)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{x^4(a+b\,\text{acosh}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{d + ex^2} \, dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)x^4}{ex^2 + d} \, dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{d + ex^2} \, dx = \int \frac{x^4\,(a + b\,\text{acosh}(c\,x))}{e\,x^2 + d} \, dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{d+ex^2}\,dx$$
$$=\frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+3\left(\int\frac{acosh(cx)x^4}{e\,x^2+d}dx\right)b\,e^3-3adex+a\,e^2x^3}{3e^3}$$

input
```
int(x^4*(a+b*acosh(c*x))/(e*x^2+d),x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + 3*int((acosh(c*x)*x
**4)/(d + e*x**2),x)*b*e**3 - 3*a*d*e*x + a*e**2*x**3)/(3*e**3)
```

3.388 $\int \frac{x^3(a+b\,\mathbf{arccosh}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 521

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx = -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce}-\frac{b\operatorname{arccosh}(cx)}{4c^2e}+\frac{x^2(a+b\operatorname{arccosh}(cx))}{2e}+\frac{d(a+b\operatorname{arccosh}(cx))^2}{2be^2}-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2}-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2}-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2}-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2}$$

output
```
-1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e-1/4*b*arccosh(c*x)/c^2/e+1/2*x^2*
(a+b*arccosh(c*x))/e+1/2*d*(a+b*arccosh(c*x))^2/b/e^2-1/2*d*(a+b*arccosh(c
*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-
e)^(1/2)))/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2-1/2*d*(a+b*arccosh(c*x))
*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(
1/2)))/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*b*d*polylog(2,-e^(1/2)*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2-1/2*
b*d*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^
2*d-e)^(1/2)))/e^2-1/2*b*d*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*b*d*polylog(2,e^(1/2)*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.98

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d+ex^2}\,dx =$$

$$-\frac{-2ac^2ex^2+bcex\sqrt{-1+cx}\sqrt{1+cx}-2bc^2ex^2\text{arccosh}(cx)-2bc^2d\text{arccosh}(cx)^2+2be\text{arctanh}\left(\sqrt{\frac{-1+c}{1+c}}\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
-1/4*(-2*a*c^2*e*x^2 + b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*e*x^
2*ArcCosh[c*x] - 2*b*c^2*d*ArcCosh[c*x]^2 + 2*b*e*ArcTanh[Sqrt[(-1 + c*x)/
(1 + c*x)]] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E
^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c
*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] +
2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) - e])] + 2*a*c^2*d*Log[d + e*x^2] + 2*b*c^2*d*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2
*d*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
)] + 2*b*c^2*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^
2*d) - e])])/(c^2*e^2)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{x(a+b\text{arccosh}(cx))}{e}-\frac{dx(a+b\text{arccosh}(cx))}{e\,(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^2}-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}-\frac{d(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^2}+\frac{d(a+b\operatorname{arccosh}(cx))^2}{2be^2}+\frac{x^2(a+b\operatorname{arccosh}(cx))}{2e}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{b\operatorname{arccosh}(cx)}{4c^2e}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{4ce}
$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
-1/4*(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e) - (b*ArcCosh[c*x])/(4*c^2*e)
 + (x^2*(a + b*ArcCosh[c*x]))/(2*e) + (d*(a + b*ArcCosh[c*x])^2)/(2*b*e^2)
 - (d*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E
^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (d*(a + b*Arc
Cosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(2*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) - (b*d*PolyLog[
2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) -
(b*d*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
]))])/(2*e^2) - (b*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) - e])])/(2*e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.45 (sec) , antiderivative size = 2111, normalized size of antiderivative = 4.05

method	result	size
derivativedivides	Expression too large to display	2111
default	Expression too large to display	2111
parts	Expression too large to display	2118

input
```
int(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
1/c^4*(1/2*a*c^4/e*x^2-1/2*a*c^4*d/e^2*ln(c^2*e*x^2+c^2*d)+b*c^2*(-(-2*(c^
2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*
d*c^2/e^3/(c^2*d+e)*arccosh(c*x)^2+1/2*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2
*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*d^2*c^4/e^4/(c^2*d+e)*polylo
g(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1
/2)-e))-(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*
d+e))^(1/2)*e)*d^2*c^4/e^4/(c^2*d+e)*arccosh(c*x)^2-(2*c^2*d-2*(c^2*d*(c^2
*d+e))^(1/2)+e)/e^4*c^4*d^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2
*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-1/2*(2*c^2*d-2*(c^2*d*(c
^2*d+e))^(1/2)+e)/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d
-2*(c^2*d*(c^2*d+e))^(1/2)-e))*c^2*d*arccosh(c*x)+1/2*(-2*(c^2*d*(c^2*d+e)
)^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*d*c^2/e^3/(c^
2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*
(c^2*d+e))^(1/2)-e))-1/2*(c^2*d*(c^2*d+e))^(1/2)*d*c^2/e^2/(c^2*d+e)*arcco
sh(c*x)^2+1/4*(c^2*d*(c^2*d+e))^(1/2)*d*c^2/e^2/(c^2*d+e)*polylog(2,e*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+(-
2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2
)*e)*d*c^2/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^
2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)+(-2*(c^2*d*(c^2*d+e))^(1/2)
*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*d^2*c^4/e^4/(c^2*...
```

Fricas [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{d + ex^2} \, dx = \int \frac{(b \operatorname{arcosh}(cx) + a)x^3}{ex^2 + d} \, dx$$

input

```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output

```
integral((b*x^3*arccosh(c*x) + a*x^3)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{x^3(a+b\operatorname{acosh}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + b*integrate(x^3*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/(e*x^2 + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{x^3\,(a+b\,\mathrm{acosh}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\frac{2\left(\int \frac{acosh(cx)x^3}{e\,x^2+d}dx\right)b\,e^2-\log(e\,x^2+d)\,ad+ae\,x^2}{2e^2}$$

input
```
int(x^3*(a+b*acosh(c*x))/(e*x^2+d),x)
```

output
```
(2*int((acosh(c*x)*x**3)/(d + e*x**2),x)*b*e**2 - log(d + e*x**2)*a*d + a*
e*x**2)/(2*e**2)
```

3.389 $\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 544

$$
\begin{aligned}
\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx &= \frac{ax}{e}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce}+\frac{bx\text{arccosh}(cx)}{e} \\
&+\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{3/2}} \\
&+\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{3/2}} \\
&-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{3/2}} \\
&+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{3/2}} \\
&-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{3/2}} \\
&+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^{3/2}}
\end{aligned}
$$

output
```
a*x/e-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e+b*x*arccosh(c*x)/e+1/2*(-d)^(1/2)*
(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^
(1/2)-(-c^2*d-e)^(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*arccosh(c*x))*ln(1+e^
(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e
^(3/2)+1/2*(-d)^(1/2)*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b
*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2
)+(-c^2*d-e)^(1/2)))/e^(3/2)-1/2*b*(-d)^(1/2)*polylog(2,-e^(1/2)*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(3/2)+1/2*b*(-
d)^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)
-(-c^2*d-e)^(1/2)))/e^(3/2)-1/2*b*(-d)^(1/2)*polylog(2,-e^(1/2)*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(3/2)+1/2*b*(-d
)^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+
(-c^2*d-e)^(1/2)))/e^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.84

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx$$
$$= \frac{4ac\sqrt{e}x - 4ac\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + ib\left(4i\sqrt{e}\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx) - cx\operatorname{arccosh}(cx)\right) - c\sqrt{d}\left(\operatorname{arccosh}(cx)\left(\operatorname{ar}\right.\right.\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
(4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*((4*I)*
Sqrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcCosh[c*x]) - c*Sqrt
[d]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[d] - Sqrt[c^2*d + e])] + Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
d] + Sqrt[c^2*d + e])])) - 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sq
rt[d]) + Sqrt[c^2*d + e])] - 2*PolyLog[2, ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[d] + Sqrt[c^2*d + e])]) + c*Sqrt[d]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*
(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Lo
g[1 - (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) - 2*Poly
Log[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] - 2*PolyL
og[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])))/(4*c*e^
(3/2))
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{a+b\text{arccosh}(cx)}{e}-\frac{d(a+b\text{arccosh}(cx))}{e\left(d+ex^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^{3/2}}+\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^{3/2}}+\frac{ax}{e}-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^{3/2}}+\frac{bx\text{arccosh}(cx)}{e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{ce}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
(a*x)/e - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e) + (b*x*ArcCosh[c*x])/e +
(Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/
2)) + (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x
])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2
*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
- Sqrt[-(c^2*d) - e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (b*Sqrt[-d]
*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]
)/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.50 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.52

method	result
parts	$\frac{ax}{e}-\frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+\frac{bx\,\operatorname{arccosh}(cx)}{e}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{ce}-\frac{bcd\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{e}-\frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}-\frac{b\,c^2\sqrt{cx-1}\,\sqrt{cx+1}}{e}+\frac{b\,c^3\operatorname{arccosh}(cx)x}{e}-\frac{b\,c^4d\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1}{}\right.}{}}{}$
default	$\frac{\frac{a\,c^3x}{e}-\frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}-\frac{b\,c^2\sqrt{cx-1}\,\sqrt{cx+1}}{e}+\frac{b\,c^3\operatorname{arccosh}(cx)x}{e}-\frac{b\,c^4d\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1}{}\right.}{}}{}$

input
```
int(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a*x/e-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*x*arccosh(c*x)/e-b*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c/e-1/2*b*c*d/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccos
h(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/2*
b*c*d/e*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),
_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccosh(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{x^2(a+b\,\text{acosh}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = \int \frac{x^2\,(a+b\,\text{acosh}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a+\left(\int \frac{acosh(cx)x^2}{e\,x^2+d}dx\right)b\,e^2+aex}{e^2}$$

input
```
int(x^2*(a+b*acosh(c*x))/(e*x^2+d),x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int((acosh(c*x)*x**2
)/(d + e*x**2),x)*b*e**2 + a*e*x)/e**2
```

3.390 $\int \frac{x(a+b\,\text{arccosh}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 449

$$\begin{aligned}\int \frac{x(a+b\,\text{arccosh}(cx))}{d+ex^2}\,dx &= -\frac{(a+b\,\text{arccosh}(cx))^2}{2be}\\
&+\frac{(a+b\,\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}\\
&+\frac{(a+b\,\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}\\
&+\frac{(a+b\,\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}\\
&+\frac{(a+b\,\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}\end{aligned}$$

output
```
-1/2*(a+b*arccosh(c*x))^2/b/e+1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*(a+b*arcc
osh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c
^2*d-e)^(1/2)))/e+1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+1/2*(a+b*arccosh(c*x))*ln
(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2
)))/e+1/2*b*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(
1/2)-(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,-e^(1/2)*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+1/2*b*pol
ylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^
(1/2)))/e
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = -\frac{b\text{arccosh}(cx)^2}{2e}+\frac{b\text{arccosh}(cx)\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\text{arccosh}(cx)\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\text{arccosh}(cx)\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\text{arccosh}(cx)\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}+\frac{a\log(d+ex^2)}{2e}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
-1/2*(b*ArcCosh[c*x]^2)/e + (b*ArcCosh[c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x
])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCosh[c*x]*Log[1 + (Sq
rt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCo
sh[c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
])/(2*e) + (b*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*e) + (a*Log[d + e*x^2])/(2*e) + (b*PolyLog[2, -((
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e
) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
 e]))])/(2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(2*e)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{a+b\text{arccosh}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\text{arccosh}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e}-\frac{(a+b\operatorname{arccosh}(cx))^2}{2be}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
-1/2*(a + b*ArcCosh[c*x])^2/(b*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCo
sh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]
)])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e) + (b*PolyLog[2
, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) +
(b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])
/(2*e) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e]))])/(2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e}-\frac{b\,\mathrm{arccosh}(cx)^2}{2e}+\frac{b\,\mathrm{arccosh}(cx)\ln\left(\frac{-2c^2d-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2+2\sqrt{c^4d^2+c^2de}-e}{-2c^2d+2\sqrt{c^4d^2+c^2de}-e}\right)}{2e}+\frac{b\,\mathrm{arccosh}(}{}$
derivativedivides	$\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{\mathrm{arccosh}(cx)^2}{2e}+\frac{\mathrm{arccosh}(cx)\ln\left(\frac{-2c^2d-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2+2\sqrt{c^4d^2+c^2de}-e}{-2c^2d+2\sqrt{c^4d^2+c^2de}-e}\right)}{2e}+\frac{\mathrm{arccosh}(cx)}{}\right.}{}$
default	$\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{\mathrm{arccosh}(cx)^2}{2e}+\frac{\mathrm{arccosh}(cx)\ln\left(\frac{-2c^2d-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2+2\sqrt{c^4d^2+c^2de}-e}{-2c^2d+2\sqrt{c^4d^2+c^2de}-e}\right)}{2e}+\frac{\mathrm{arccosh}(cx)}{}\right.}{}$

input
```
int(x*(a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e*ln(e*x^2+d)-1/2*b*arccosh(c*x)^2/e+1/2*b/e*arccosh(c*x)*ln((-2*c^2
*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*(c^4*d^2+c^2*d*e)^(1/2)-e)/(-2*
c^2*d+2*(c^4*d^2+c^2*d*e)^(1/2)-e))+1/2*b/e*arccosh(c*x)*ln((2*c^2*d+e*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*(c^4*d^2+c^2*d*e)^(1/2)+e)/(2*c^2*d+2*(
c^4*d^2+c^2*d*e)^(1/2)+e))+1/4*b/e*dilog((-2*c^2*d-e*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))^2+2*(c^4*d^2+c^2*d*e)^(1/2)-e)/(-2*c^2*d+2*(c^4*d^2+c^2*d*e)^
(1/2)-e))+1/4*b/e*dilog((2*c^2*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*(
c^4*d^2+c^2*d*e)^(1/2)+e)/(2*c^2*d+2*(c^4*d^2+c^2*d*e)^(1/2)+e))
```

Fricas [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arccosh(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{x(a+b\,\text{acosh}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*acosh(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2
*a*log(e*x^2 + d)/e
```

Giac [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = \int \frac{x\,(a+b\operatorname{acosh}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{d+ex^2}\,dx = \frac{2\left(\int \frac{acosh(cx)x}{e\,x^2+d}dx\right)be+\log(e\,x^2+d)\,a}{2e}$$

input
```
int(x*(a+b*acosh(c*x))/(e*x^2+d),x)
```

output
```
(2*int((acosh(c*x)*x)/(d + e*x**2),x)*b*e + log(d + e*x**2)*a)/(2*e)
```

3.391 $\int \frac{a+b\text{arccosh}(cx)}{d+ex^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 501

$$\begin{aligned}\int \frac{a+b\text{arccosh}(cx)}{d+ex^2}\,dx = {} & \frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & +\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & +\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & +\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccosh(c*x))*ln(
1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*
(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^
(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-e^(1/2)*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e
^(1/2)+1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(
1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-e^(1/2)*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^
(1/2)+1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1
/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.79

$$\int \frac{a+b\text{arccosh}(cx)}{d+ex^2}\,dx$$
$$= \frac{-\left((a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)\right)+(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)+(a+b}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2),x]
```

output
```
(-((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])]) + (a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])
/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + (a + b*ArcCosh[c*x])*Log[1 - (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] - (a + b*ArcCosh[c
*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] +
 b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]
- b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e
])] - b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
- e]))] + b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d
) - e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6324, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{d+ex^2}dx$$

$$\Big\downarrow \text{ 6324}$$

$$\int \left(\frac{\sqrt{-d}(a+b\text{arccosh}(cx))}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\text{arccosh}(cx))}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[
e]) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1
+ (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]
*Sqrt[e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) - e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[
2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*Sqrt
[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6324
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.46

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{bc\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(\operatorname{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{cx-1}\sqrt{cx+1}}{\_R1}\right)+\mathrm{di}\ldots\right.}{\_R1^2e+2c^2d+e}\ldots\right.}{2}\ldots$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(\operatorname{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{cx-1}\sqrt{cx+1}}{\_R1}\right)+\mathrm{c}\ldots\right.}{\_R1^2e+2c^2d+e}\ldots\right.}{2}\ldots\right.}{\ldots}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(\operatorname{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{cx-1}\sqrt{cx+1}}{\_R1}\right)+\mathrm{c}\ldots\right.}{\_R1^2e+2c^2d+e}\ldots\right.}{2}\ldots\right.}{\ldots}$

input
```
int((a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c*sum(_R1/(_R1^2*e+2*c^2*d+e)*
(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e
))-1/2*b*c*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)
^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1
)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{d+ex^2}\,dx=\int \frac{b\operatorname{arcosh}(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\mathrm{acosh}\,(cx)}{d+ex^2}\,dx$$

input
```
integrate((a+b*acosh(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*acosh(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arccosh}(cx)}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{d + ex^2} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{ex^2 + d} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{d + ex^2} \, dx = \int \frac{a + b \operatorname{acosh}(c x)}{e x^2 + d} \, dx$$

input
```
int((a + b*acosh(c*x))/(d + e*x^2),x)
```

output
```
int((a + b*acosh(c*x))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{d + ex^2} \, dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) a + \left(\int \frac{acosh(cx)}{e\,x^2+d} dx\right) bde}{de}$$

input
```
int((a+b*acosh(c*x))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int(acosh(c*x)/(d + e*x
**2),x)*b*d*e)/(d*e)
```

3.392 $\int \frac{a+b\text{arccosh}(cx)}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 472

$$\int \frac{a+b\text{arccosh}(cx)}{x(d+ex^2)}\,dx = -\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d} - \frac{(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d} + \frac{(a+b\text{arccosh}(cx))\log\left(1+e^{2\text{arccosh}(cx)}\right)}{d} - \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d} + \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(cx)}\right)}{2d}$$

output
```
-1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*
(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-1/2*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-1/2*(a+b*ar
ccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(
-c^2*d-e)^(1/2)))/d-1/2*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d+(a+b*arccosh(c*x))*ln(1
+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-1/2*b*polylog(2,-e^(1/2)*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-1/2*b*polylog(
2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/d-1/2*b*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1
/2)+(-c^2*d-e)^(1/2)))/d-1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d+1/2*b*polylog(2,-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.87

$$\int \frac{a + b\text{arccosh}(cx)}{x\,(d + ex^2)}\,dx =$$
$$-\frac{-2b\text{arccosh}(cx)^2 - 2b\text{arccosh}(cx)\log\left(1 + e^{-2\text{arccosh}(cx)}\right) + b\text{arccosh}(cx)\log\left(1 + \frac{i\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{d}-\sqrt{c^2d+e}}\right) + b\text{arc}}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

output
```
-1/2*(-2*b*ArcCosh[c*x]^2 - 2*b*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])]
+ b*ArcCosh[c*x]*Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*
d + e])] + b*ArcCosh[c*x]*Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d])
 + Sqrt[c^2*d + e])] + b*ArcCosh[c*x]*Log[1 - (I*Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[d] + Sqrt[c^2*d + e])] + b*ArcCosh[c*x]*Log[1 + (I*Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])] - 2*a*Log[x] + a*Log[d + e*x^2] + b
*PolyLog[2, -E^(-2*ArcCosh[c*x])] + b*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[d] - Sqrt[c^2*d + e])] + b*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])
/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + b*PolyLog[2, ((-I)*Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])] + b*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])/d
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x(d+ex^2)}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{a+b\text{arccosh}(cx)}{dx}-\frac{ex(a+b\text{arccosh}(cx))}{d(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d}-\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d}-\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d}+\frac{(a+b\text{arccosh}(cx))^2}{bd}+\frac{\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))}{d}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)}{2d}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

output
```
(a + b*ArcCosh[c*x])^2/(b*d) + ((a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh
[c*x])])/d - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*Arc
Cosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] + Sqrt[-(c^2*d) - e])])/(2*d) - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])]
)/(2*d) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^
2*d) - e]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
 Sqrt[-(c^2*d) - e])])/(2*d) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.81

method	result
parts	$-\frac{a\ln(e\,x^2+d)}{2d}+\frac{a\ln(x)}{d}+b\left(-\frac{e\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2+1\right)\left(\mathrm{arccosh}(cx)\ln\left(\frac{\_R1}{}\right.\right.}{}\right)}{4d}\right.$
derivativedivides	$\frac{a\ln(cx)}{d}-\frac{a\ln(c^2e\,x^2+c^2d)}{2d}-\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2+1\right)\left(\mathrm{arccosh}(cx)\ln\left(\frac{\_R1-c}{}\right.\right.}{}\right)}{4d}$
default	$\frac{a\ln(cx)}{d}-\frac{a\ln(c^2e\,x^2+c^2d)}{2d}-\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2+1\right)\left(\mathrm{arccosh}(cx)\ln\left(\frac{\_R1-c}{}\right.\right.}{}\right)}{4d}$

input
```
int((a+b*arccosh(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d*ln(e*x^2+d)+a/d*ln(x)+b*(-1/4*e/d*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+
e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-
c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^
2+e))+1/d*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/d*arcco
sh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/d*dilog(1+I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))+1/d*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-
1/4/d*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(
1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{b\operatorname{arcosh}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\,(d+ex^2)}\,dx=\int \frac{a+b\operatorname{acosh}(cx)}{x\,(d+ex^2)}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*acosh(c*x))/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\,(d+ex^2)}\,dx=\int \frac{b\operatorname{arcosh}(cx)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))/(e*x^3 + d*x), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\,(d+ex^2)}\,dx=\int \frac{b\operatorname{arcosh}(cx)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d+ex^2\right)}\,dx=\frac{2\left(\int \frac{acosh(cx)}{e\,x^3+dx}dx\right)bd-\log\left(e\,x^2+d\right)a+2\log\left(x\right)a}{2d}$$

input
```
int((a+b*acosh(c*x))/x/(e*x^2+d),x)
```

output
```
(2*int(acosh(c*x)/(d*x + e*x**3),x)*b*d - log(d + e*x**2)*a + 2*log(x)*a)/
(2*d)
```

3.393 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{x^2(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 543

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2(d+ex^2)}\,dx = -\frac{a+b\operatorname{arccosh}(cx)}{dx}+\frac{bc\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d}$$
$$+\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$
$$-\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$
$$+\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$
$$-\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$
$$-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$
$$+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$
$$-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$
$$+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{3/2}}$$

output

```
-(a+b*arccosh(c*x))/d/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d+1/2*e^(1
/2)*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arccosh(c*x))*ln(
1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/(-d)^(3/2)+1/2*e^(1/2)*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)-1/2*e^(1/2)*
(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^
(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)-1/2*b*e^(1/2)*polylog(2,-e^(1/2)*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)+1
/2*b*e^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(
1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)-1/2*b*e^(1/2)*polylog(2,-e^(1/2)*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)+1/
2*b*e^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1
/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.01

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d+ex^2)}\,dx = \frac{1}{2}\left(-\frac{2(a+b\text{arccosh}(cx))}{dx} + \frac{2bc\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}} + \frac{d\sqrt{e}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{\sqrt{e}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}} + \frac{\sqrt{e}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}} + \frac{d\sqrt{e}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}} + \frac{bd\sqrt{e}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{bd\sqrt{e}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}}\right)$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
((-2*(a + b*ArcCosh[c*x]))/(d*x) + (2*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-
1 + c^2*x^2]])/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d*Sqrt[e]*(a + b*ArcCos
h[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])
])/(-d)^(5/2) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(3/2) + (Sqrt[e]*(a + b*A
rcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
- e])])/(-d)^(3/2) + (d*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (b*Sqrt[e]*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(-d)
^(3/2) + (b*d*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) +
 Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (b*d*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(-d)^(5/2) + (b*Sqrt[e]*
PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-
d)^(3/2))/2
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2(d+ex^2)}dx$$

$$\Big\downarrow \text{ 6374}$$

$$\int \left(\frac{a+b\text{arccosh}(cx)}{dx^2}-\frac{e(a+b\text{arccosh}(cx))}{d(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2(-d)^{3/2}}-\frac{a+b\operatorname{arccosh}(cx)}{dx}-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
-((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
/d + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh[c*x
])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2
*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*A
rcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
- e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/
(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/
2)) - (b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.63 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.61

method	result
parts	$-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{a}{dx}+bc\left(-\frac{\operatorname{arccosh}(cx)}{dcx}+\frac{2\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}-\frac{e\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+(4c\right.}\right.}{}\right.$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\operatorname{arccosh}(cx)}{cxd}+\frac{2b\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{be\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+(4\right.}\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\operatorname{arccosh}(cx)}{cxd}+\frac{2b\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{be\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+(4\right.}\right.}{}\right.$

input
```
int((a+b*arccosh(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/d/x+b*c*(-arccosh(c*x)/d/c/x+
2/d*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8/d^2*e/c^2*sum((4*_R1^2*c^2
*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),
_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/8/d^2*e/c^2*sum((_R1^2*e+4*c^2*
d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootO
f(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\text{acosh}\left(cx\right)}{x^2\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d+ex^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \frac{b\operatorname{arcosh}\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \frac{a+b\operatorname{acosh}(c\,x)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d+ex^2\right)}\,dx = \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ax+\left(\int\frac{acosh(cx)}{e\,x^4+d\,x^2}dx\right)b\,d^2x-ad}{d^2x}$$

input
```
int((a+b*acosh(c*x))/x^2/(e*x^2+d),x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*x + int(acosh(c*x)/(d*
x**2 + e*x**4),x)*b*d**2*x - a*d)/(d**2*x)
```

3.394 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{x^3\left(d+ex^2\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 531

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^3(d+ex^2)}\,dx = \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\operatorname{arccosh}(cx)}{2dx^2} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1+e^{2\operatorname{arccosh}(cx)}\right)}{d^2} + \frac{be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{be\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)}{2d^2}$$

output
```
1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x-1/2*(a+b*arccosh(c*x))/d/x^2+1/2*e
*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)
^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*e*(a+
b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/
2)+(-c^2*d-e)^(1/2)))/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-e*(a+b*arccos
h(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2+1/2*b*e*polylog(2,-e
^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/
d^2+1/2*b*e*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1
/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,e^(1/
2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-
1/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.98

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d+ex^2)}\,dx$$
$$= -\frac{a}{2dx^2} - \frac{ae\log(x)}{d^2} + \frac{ae\log(d+ex^2)}{2d^2} + b\Bigg(\frac{cx\sqrt{-1+cx}\sqrt{1+cx}-\text{arccosh}(cx)}{2dx^2}$$
$$-\frac{e\left(\text{arccosh}(cx)\left(\text{arccosh}(cx)+2\log\left(1+e^{-2\text{arccosh}(cx)}\right)\right)-\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)\right)}{2d^2}$$
$$+\frac{e\left(\text{arccosh}(cx)\left(-\text{arccosh}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\,\text{PolyLog}}{4d^2}$$
$$+\frac{e\left(\text{arccosh}(cx)\left(-\text{arccosh}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\,\text{PolyLog}}{4d^2}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
-1/2*a/(d*x^2) - (a*e*Log[x])/d^2 + (a*e*Log[d + e*x^2])/(2*d^2) + b*((c*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - ArcCosh[c*x])/(2*d*x^2) - (e*(ArcCosh[c*x]
*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCo
sh[c*x])]))/(2*d^2) + (e*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e
]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E
^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -
((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]))/(4*d^2) +
 (e*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-
I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*
c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x
])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCo
sh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/(4*d^2))
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{e^2x(a+b\text{arccosh}(cx))}{d^2\left(d+ex^2\right)}-\frac{e(a+b\text{arccosh}(cx))}{d^2x}+\frac{a+b\text{arccosh}(cx)}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{e(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^2}+\frac{e(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}+\frac{e(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d^2}-\frac{e(a+b\text{arccosh}(cx))^2}{bd^2}-\frac{e\log\left(e^{-2\text{arccosh}(cx)}+1\right)(a+b\text{arccosh}(cx))}{d^2}-\frac{a+b\text{arccosh}(cx)}{2dx^2}+\frac{be\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}(cx)}\right)}{2d^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2
) - (e*(a + b*ArcCosh[c*x])^2)/(b*d^2) - (e*(a + b*ArcCosh[c*x])*Log[1 + E
^(-2*ArcCosh[c*x])])/d^2 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[
c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])
/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) + (b*e*
PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d^2) + (b*e*PolyLog[2, -((Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) + (b*e*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (
b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]
))])/(2*d^2) + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) - e])])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.88

method	result
parts	$a\left(\frac{e\ln(e\,x^2+d)}{2d^2}-\frac{1}{2d\,x^2}-\frac{e\ln(x)}{d^2}\right)+b\,c^2\left(-\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^2x^2d}+\frac{e^2\left(\_R1=\operatorname{RootOf}(\right.}{}\right.$
derivativedivides	$c^2\left(-\frac{a}{2d\,c^2x^2}-\frac{ae\ln(cx)}{c^2d^2}+\frac{ae\ln(c^2e\,x^2+c^2d)}{2c^2d^2}+b\,c^2\left(-\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^4x^2d}+\frac{e^2\left(-\right.}{}\right.\right.$
default	$c^2\left(-\frac{a}{2d\,c^2x^2}-\frac{ae\ln(cx)}{c^2d^2}+\frac{ae\ln(c^2e\,x^2+c^2d)}{2c^2d^2}+b\,c^2\left(-\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,cx+c^2x^2+\operatorname{arccosh}(cx)}{2c^4x^2d}+\frac{e^2\left(-\right.}{}\right.\right.$

input
```
int((a+b*arccosh(c*x))/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e/d^2*ln(e*x^2+d)-1/2/d/x^2-e/d^2*ln(x))+b*c^2*(-1/2*(-(c*x-1)^(1/2
)*(c*x+1)^(1/2)*c*x+c^2*x^2+arccosh(c*x))/c^2/x^2/d+1/4*e^2/d^2/c^2*sum((_
R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf
(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-e/d^2/c^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2)))-e/d^2/c^2*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))-e/d^2/c^2*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-e/d^2/
c^2*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/4*e/d^2/c^2*sum((_R1^2*
e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=R
ootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d + ex^2)} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(ex^2 + d)x^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d + ex^2)} \, dx = \int \frac{a + b \operatorname{acosh}(cx)}{x^3 (d + ex^2)} \, dx$$

input
```
integrate((a+b*acosh(c*x))/x**3/(e*x**2+d),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**3*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d + ex^2)} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(ex^2 + d)x^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + b*integrate(lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^5 + d*x^3), x)
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d + ex^2)} \, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(ex^2 + d)x^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^3 (d + ex^2)} \, dx = \int \frac{a + b \operatorname{acosh}(c\,x)}{x^3\,(e\,x^2 + d)} \, dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^3\,(d+ex^2)}\,dx$$
$$= \frac{2\left(\int \frac{acosh(cx)}{e\,x^5+d\,x^3}dx\right)b\,d^2x^2+\log(e\,x^2+d)\,ae\,x^2-2\log(x)\,ae\,x^2-ad}{2d^2x^2}$$

input
```
int((a+b*acosh(c*x))/x^3/(e*x^2+d),x)
```

output
```
(2*int(acosh(c*x)/(d*x**3 + e*x**5),x)*b*d**2*x**2 + log(d + e*x**2)*a*e*x
**2 - 2*log(x)*a*e*x**2 - a*d)/(2*d**2*x**2)
```

3.395 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{x^4(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 624

$$
\begin{aligned}
\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\left(d+ex^2\right)}\,dx &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\operatorname{arccosh}(cx)}{3dx^3} \\
&+\frac{e(a+b\operatorname{arccosh}(cx))}{d^2x}+\frac{bc^3\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d} \\
&-\frac{bce\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2} \\
&+\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}} \\
&-\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}} \\
&+\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}} \\
&-\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}} \\
&-\frac{be^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}} \\
&+\frac{be^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}} \\
&-\frac{be^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}} \\
&+\frac{be^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2(-d)^{5/2}}
\end{aligned}
$$

output

```
1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2-1/3*(a+b*arccosh(c*x))/d/x^3+e*(
a+b*arccosh(c*x))/d^2/x+1/6*b*c^3*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d-b*
c*e*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+1/2*e^(3/2)*(a+b*arccosh(c*x))
*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(
1/2)))/(-d)^(5/2)-1/2*e^(3/2)*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)+1/2*e^(3
/2)*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(
-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*e^(3/2)*(a+b*arccosh(c*x))*ln(
1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)
))/(-d)^(5/2)-1/2*b*e^(3/2)*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)+1/2*b*e^(3/2)*polylog(2
,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2))
)/(-d)^(5/2)-1/2*b*e^(3/2)*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)+1/2*b*e^(3/2)*polylog(2,
e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))
/(-d)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.05

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d+ex^2)}\,dx = \frac{1}{6}\left(-\frac{2a}{dx^3}+\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{dx^2}-\frac{2b\text{arccosh}(cx)}{dx^3}+\frac{6e(a+b\text{arccosh}(cx))}{d^2x}+\frac{bc^3\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{d\sqrt{-1+cx}\sqrt{1+cx}}-\frac{6bce\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3e^{3/2}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3e^{3/2}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3e^{3/2}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}-\frac{3e^{3/2}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}-\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}-\frac{3be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}\right)$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```

output
```
((-2*a)/(d*x^3) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(d*x^2) - (2*b*ArcCos
h[c*x])/(d*x^3) + (6*e*(a + b*ArcCosh[c*x]))/(d^2*x) + (b*c^3*Sqrt[-1 + c^
2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (6*b
*c*e*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(d^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (3*e^(3/2)*(a + b*A
rcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*
d) - e])])/(-d)^(5/2) + (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E
^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) - (3*e^(3/2)
*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(-d)^(5/2) + (3*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(-d)^(5/2) - (3*b*e^(3/2)*PolyLog
[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(
5/2) - (3*b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sq
rt[-(c^2*d) - e]))])/(-d)^(5/2) + (3*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2))/6
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^4(d+ex^2)}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{e^2(a+b\text{arccosh}(cx))}{d^2(d+ex^2)}-\frac{e(a+b\text{arccosh}(cx))}{d^2x^2}+\frac{a+b\text{arccosh}(cx)}{dx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}-\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2(-d)^{5/2}}+\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{5/2}}-\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2(-d)^{5/2}}+\frac{e(a+b\operatorname{arccosh}(cx))}{d^2x}-\frac{a+b\operatorname{arccosh}(cx)}{3dx^3}-\frac{be^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}-\frac{be^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}+\frac{bc^3\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{6d}-\frac{bce\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6dx^2}$$

input

```
Int[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```

output

```
(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x
^3) + (e*(a + b*ArcCosh[c*x]))/(d^2*x) + (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(6*d) - (b*c*e*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (e^
(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1
 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5
/2)) + (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c
*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/
(2*(-d)^(5/2)) - (b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e]))])/(2*(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (b
*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
 - e]))])/(2*(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/
(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.45 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.68

method	result
parts	$a\left(\frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d^2\sqrt{de}}-\frac{1}{3d\,x^3}+\frac{e}{d^2x}\right)+\frac{b\left(8\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)c^7d^2x^3+4\sqrt{cx+1}\,\sqrt{cx-1}\,c^5d^2x-48\,\mathrm{arc}\right.}{}$
derivativedivides	$c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ae}{c^3d^2x}+\frac{a\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{c^3d^2\sqrt{de}}+\frac{b\left(8\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)c^7d^2x^3+4\sqrt{cx+1}\,\sqrt{cx-1}\,c^5d^2x\right.}{}\right.$
default	$c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ae}{c^3d^2x}+\frac{a\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{c^3d^2\sqrt{de}}+\frac{b\left(8\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)c^7d^2x^3+4\sqrt{cx+1}\,\sqrt{cx-1}\,c^5d^2x\right.}{}\right.$

input
```
int((a+b*arccosh(c*x))/x^4/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a*(e^2/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/3/d/x^3+e/d^2/x)+1/24*b/c
^4*(8*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^7*d^2*x^3+4*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*c^5*d^2*x-48*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^5*d*e
*x^3-8*c^4*d^2*arccosh(c*x)+24*arccosh(c*x)*c^4*d*e*x^2+3*sum((4*_R1^2*c^2
*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),
_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e^2*c^3*x^3-3*sum((_R1^2*e+4*c^2*
d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootO
f(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e^2*c^3*x^3)/d^3/x^3
```

Fricas [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\,(d+ex^2)}\,dx=\int \frac{b\operatorname{arcosh}\,(cx)+a}{(ex^2+d)x^4}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e*x^6 + d*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^4\,(d+ex^2)}\,dx=\int \frac{a+b\operatorname{acosh}\,(cx)}{x^4\,(d+ex^2)}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x**4/(e*x**2+d),x)
```

output
```
Integral((a + b*acosh(c*x))/(x**4*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d + ex^2)} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d + ex^2)} dx = \int \frac{b \,\text{arcosh}\,(cx) + a}{(ex^2 + d)x^4} dx$$

input
```
integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d + ex^2)} dx = \int \frac{a + b \,\text{acosh}(c\, x)}{x^4 \, (e\, x^2 + d)} dx$$

input
```
int((a + b*acosh(c*x))/(x^4*(d + e*x^2)),x)
```

output
```
int((a + b*acosh(c*x))/(x^4*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^4 (d + ex^2)} \, dx$$
$$= \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) ae\,x^3 + 3\left(\int \frac{acosh(cx)}{e\,x^6+d\,x^4}dx\right) b\,d^3x^3 - a\,d^2 + 3ade\,x^2}{3d^3x^3}$$

input
```
int((a+b*acosh(c*x))/x^4/(e*x^2+d),x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**3 + 3*int(acosh(c*
x)/(d*x**4 + e*x**6),x)*b*d**3*x**3 - a*d**2 + 3*a*d*e*x**2)/(3*d**3*x**3)
```

3.396 $\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 562

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^2}\,dx = \frac{d(a+b\operatorname{arccosh}(cx))}{2e^2(d+ex^2)} - \frac{(a+b\operatorname{arccosh}(cx))^2}{2be^2} - \frac{bc\sqrt{d}\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^2}$$

output
```
1/2*d*(a+b*arccosh(c*x))/e^2/(e*x^2+d)-1/2*(a+b*arccosh(c*x))^2/b/e^2-1/2*
b*c*d^(1/2)*(c^2*x^2-1)^(1/2)*arctanh((c^2*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1
)^(1/2))/e^2/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*(a+b*arccosh(
c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d
-e)^(1/2)))/e^2+1/2*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*(a+b*arccosh(c*x))*ln
(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2
)))/e^2+1/2*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2+1/2*b*polylog(2,-e^(1/2)*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*b*polyl
og(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1
/2)))/e^2+1/2*b*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2+1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.23

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + 2a\log(d+ex^2) + b\left(-2\text{arccosh}(cx)^2 + 2\text{arccosh}(cx)\left(\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right) + \log\left(1-\frac{\sqrt{e}e}{ic\sqrt{d}}\right.\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
((2*a*d)/(d + e*x^2) + 2*a*Log[d + e*x^2] + b*(-2*ArcCosh[c*x]^2 + 2*ArcCo
sh[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d)
- e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])]) + 2*ArcCosh[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqr
t[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-
(c^2*d) - e])]) - I*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*
Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^
2*d) - e]) - I*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(
2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) -
 e]) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d)
- e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^
2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[
-(c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sq
rt[-(c^2*d) - e])]))/(4*e^2)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{(d + ex^2)^2} \, dx$$

$$\Big\downarrow \text{ 6374}$$

$$\int \left(\frac{x(a + b\text{arccosh}(cx))}{e\,(d + ex^2)} - \frac{dx(a + b\text{arccosh}(cx))}{e\,(d + ex^2)^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^2}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^2}+\frac{d(a+b\operatorname{arccosh}(cx))}{2e^2(d+ex^2)}-\frac{(a+b\operatorname{arccosh}(cx))^2}{2be^2}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bc\sqrt{d}\sqrt{c^2x^2-1}\operatorname{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2e^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}$$

input

```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output

```
(d*(a + b*ArcCosh[c*x]))/(2*e^2*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b
*e^2) - (b*c*Sqrt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[
d]*Sqrt[-1 + c^2*x^2])])/(2*e^2*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
 Sqrt[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCos
h[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
])/(2*e^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] + Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^
2) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(2*e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 2132, normalized size of antiderivative = 3.79

method	result	size
derivativedivides	Expression too large to display	2132
default	Expression too large to display	2132
parts	Expression too large to display	2144

input
```
int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/2*a*c^4/e^2*ln(c^2*e*x^2+c^2*d)+1/2*a*c^6*d/e^2/(c^2*e*x^2+c^2*d)
+b*c^4*(1/2*arccosh(c*x)*d*c^2/e^2/(c^2*e*x^2+c^2*d)-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)/e^4*d*c^2*arccosh(c*x)^2-1/2*(c^2*d*(c^2*d+e))^(1/2)/e^2/
(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/2*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/
e^4*d*c^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2
*d*(c^2*d+e))^(1/2)-e))-(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*
d*e-(c^2*d*(c^2*d+e))^(1/2)*e)/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)+(2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^4*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*c^2*d*arccosh(c*x)+1/4*(2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-1/2*(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)/e^3*arccosh(c*x)^2-1/4*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+
2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)/d/c^2/e^2/(c^2*d+e)*ln(1-e*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e)
)*arccosh(c*x)-(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*
d*(c^2*d+e))^(1/2)*e)*d*c^2/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-1/4*(c^2*d
*(c^2*d+e))^(1/2)/d/c^2/e/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1...
```

Fricas [F]

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arccosh(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{(d + ex^2)^2}\,dx = \int \frac{x^3(a + b\,\text{acosh}\,(cx))}{(d + ex^2)^2}\,dx$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)x^3}{(ex^2 + d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{(d + ex^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^3\,(a+b\,\text{acosh}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \frac{2\left(\int \frac{acosh(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^2+2\left(\int \frac{acosh(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)b\,e^3x^2+\log(e\,x^2+d)\,ad+\log(e\,x^2+d)\,ae\,x^2-ae\,x}{2e^2\,(e\,x^2+d)}$$

input
```
int(x^3*(a+b*acosh(c*x))/(e*x^2+d)^2,x)
```

output
```
(2*int((acosh(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**2 + 2*i
nt((acosh(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*e**3*x**2 + log(
d + e*x**2)*a*d + log(d + e*x**2)*a*e*x**2 - a*e*x**2)/(2*e**2*(d + e*x**2
))
```

3.397 $\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = -\frac{a+b\text{arccosh}(cx)}{2e\,(d+ex^2)}+\frac{bc\sqrt{-1+c^2x^2}\text{arctanh}\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2\sqrt{d}e\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/2*(a+b*arccosh(c*x))/e/(e*x^2+d)+1/2*b*c*(c^2*x^2-1)^(1/2)*arctanh((c^2
*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(1/2)/e/(c^2*d+e)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = -\frac{\frac{a}{d+ex^2}+\frac{b\text{arccosh}(cx)}{d+ex^2}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}\arctan\left(\frac{\sqrt{-c^2d-e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}\sqrt{-c^2d-e}\sqrt{-1+c^2x^2}}}{2e}$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a/(d + e*x^2) + (b*ArcCosh[c*x])/(d + e*x^2) - (b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/
(Sqrt[d]*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c^2*x^2]))/e
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6372, 648, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{6372}$$

$$\frac{bc\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}(ex^2+d)}dx}{2e}-\frac{a+b\text{arccosh}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{648}$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{1}{\sqrt{c^2x^2-1}(ex^2+d)}dx}{2e\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{291}$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{1}{d-\frac{\left(dc^2+e\right)x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2e\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{bc\sqrt{c^2x^2-1}\text{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2\sqrt{d}e\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}-\frac{a+b\text{arccosh}(cx)}{2e\left(d+ex^2\right)}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcCosh[c*x])/(e*(d + e*x^2)) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTan
h[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*Sqrt[d]*e*Sqrt[c^2
*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

rule 6372
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(93) = 186$.

Time = 2.26 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.53

method	result
parts	$-\frac{a}{2e(e\,x^2+d)}+\frac{b\left(-\frac{c^4\,\mathrm{arccosh}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{c^4\sqrt{cx-1}\,\sqrt{cx+1}\left(\ln\left(\frac{2\sqrt{-\frac{c^2d+e}{e}}\,\sqrt{c^2x^2-1}\,e+2\sqrt{-c^2de}\,cx-2e}{ecx-\sqrt{-c^2de}}\right)c^2d-\ln\left(-\frac{2\left(-\sqrt{\ldots}\right.}{\ldots}\right.\right.}{\ldots}\right.}{\ldots}$
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arccosh}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\ln\left(\frac{2\sqrt{-\frac{c^2d+e}{e}}\,\sqrt{c^2x^2-1}\,e+2\sqrt{-c^2de}\,cx-2e}{ecx-\sqrt{-c^2de}}\right)c^2d-\ln\left(\frac{2\sqrt{-\ldots}}{\ldots}\right.\right.}{\ldots}\right.}{\ldots}$
default	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arccosh}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\ln\left(\frac{2\sqrt{-\frac{c^2d+e}{e}}\,\sqrt{c^2x^2-1}\,e+2\sqrt{-c^2de}\,cx-2e}{ecx-\sqrt{-c^2de}}\right)c^2d-\ln\left(\frac{2\sqrt{-\ldots}}{\ldots}\right.\right.}{\ldots}\right.}{\ldots}$

input
```
int(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arccosh(c*x)-1/4*c^
4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2
)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^2*d-ln(-2*(-(-(c^2
*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*
e)^(1/2)))*c^2*d+ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)
^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*e-ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^
2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*e)/(c^2
*x^2-1)^(1/2)/((-c^2*d*e)^(1/2)+e)/(-(-c^2*d*e)^(1/2)+e)/(-c^2*d*e)^(1/2)/
(-(c^2*d+e)/e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(93) = 186$.

Time = 0.12 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.75

$$\int \frac{x(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^2}\,dx$$

$$= \left[-\frac{2\,ac^2d^2 - 2\,(bc^2de + be^2)x^2\log\left(cx + \sqrt{c^2x^2 - 1}\right) + 2\,ade - (bcex^2 + bcd)\sqrt{c^2d^2 + de}\log\left(-\frac{2\,c^2d^2 - (4\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{ac^2d^2 - (bc^2de + be^2)x^2\log\left(cx + \sqrt{c^2x^2 - 1}\right) + ade - (bcex^2 + bcd)\sqrt{-c^2d^2 - de}\arctan\left(\frac{\sqrt{-c^2d^2 - de}\sqrt{\cdots}}{\cdots}\right.}{2\,(c^2d^3e + d^2e^2 + (c^2d^2e^2\cdots}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 - 2*(b*c^2*d*e + b*e^2)*x^2*log(c*x + sqrt(c^2*x^2 - 1)
) + 2*a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(c^2*d^2 + d*e)*log(-(2*c^2*d^2 - (4
*c^4*d^2 + 4*c^2*d*e + e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)*((2*c^3*d +
c*e)*x^2 - c*d) - 2*sqrt(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*c^2*d + e)*x
 + 2*(c^3*d^2 + c*d*e)*x))/(e*x^2 + d)) - 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*
e + b*e^2)*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2
*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 - (b*c^2*d*e + b*e^2)*x^2*log(c*x
+ sqrt(c^2*x^2 - 1)) + a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(-c^2*d^2 - d*e)*ar
ctan((sqrt(-c^2*d^2 - d*e)*sqrt(c^2*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c
*e*x^2 + c*d))/(c^2*d^2 + d*e)) - (b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)
*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 +
 d*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^2}\, dx = \int \frac{x(a + b\operatorname{acosh}(cx))}{(d + ex^2)^2}\, dx$$

input
```
integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^2}\, dx = \int \frac{(b\operatorname{arcosh}(cx) + a)x}{(ex^2 + d)^2}\, dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x\,(a+b\,\text{acosh}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \frac{2\left(\int \frac{acosh(cx)x}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^2+2\left(\int \frac{acosh(cx)x}{e^2x^4+2de\,x^2+d^2}dx\right)bde\,x^2+a\,x^2}{2d\,(e\,x^2+d)}$$

input
```
int(x*(a+b*acosh(c*x))/(e*x^2+d)^2,x)
```

output
```
(2*int((acosh(c*x)*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d**2 + 2*int((a
cosh(c*x)*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e*x**2 + a*x**2)/(2*d*
(d + e*x**2))
```

3.398 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 581

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx = \frac{a+b\operatorname{arccosh}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1+e^{2\operatorname{arccosh}(cx)}\right)}{d^2} - \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{b\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)}{2d^2}$$

output
```
1/2*(a+b*arccosh(c*x))/d/(e*x^2+d)-1/2*b*c*(c^2*x^2-1)^(1/2)*arctanh((c^2*
d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*(a+b*arccosh(c*x))*
ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1
/2)))/d^2-1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-1/2*(a+b*arccosh(c*x))*ln(1+e^(
1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^
2+(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-1/2*b*p
olylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-
e)^(1/2)))/d^2-1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(
c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,-e^(1/2)*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,
e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))
/d^2+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.30

$$\int \frac{a+b\text{arccosh}(cx)}{x(d+ex^2)^2}\,dx = \frac{a}{2d^2+2dex^2}+\frac{a\log(x)}{d^2}-\frac{a\log(d+ex^2)}{2d^2} - \frac{b\left(-\frac{\sqrt{d}\text{arccosh}(cx)}{\sqrt{d}-i\sqrt{e}x}-\frac{\sqrt{d}\text{arccosh}(cx)}{\sqrt{d}+i\sqrt{e}x}-4\text{arccosh}(cx)^2-4\text{arccosh}(cx)\log\left(1+e^{-2\text{arccosh}(cx)}\right)+2\text{arccosh}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
a/(2*d^2 + 2*d*e*x^2) + (a*Log[x])/d^2 - (a*Log[d + e*x^2])/(2*d^2) - (b*(
-((Sqrt[d]*ArcCosh[c*x])/(Sqrt[d] - I*Sqrt[e]*x)) - (Sqrt[d]*ArcCosh[c*x])
/(Sqrt[d] + I*Sqrt[e]*x) - 4*ArcCosh[c*x]^2 - 4*ArcCosh[c*x]*Log[1 + E^(-2
*ArcCosh[c*x])] + 2*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqr
t[d] - Sqrt[-(c^2*d) - e])] + 2*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*
x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*ArcCosh[c*x]*Log[1 - (Sqrt[
e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*ArcCosh[c*x]*Lo
g[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + (I*c*
Sqrt[d]*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/S
qrt[-(c^2*d) - e] - (I*c*Sqrt[d]*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sq
rt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*S
qrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + 2*PolyLog[2, -E^(-2*ArcCosh[c*
x])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d)
- e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^
2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[
-(c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sq
rt[-(c^2*d) - e])]))/(4*d^2)
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \; 6374$$

$$\int \left(-\frac{ex(a+b\operatorname{arccosh}(cx))}{d^2\left(d+ex^2\right)}+\frac{a+b\operatorname{arccosh}(cx)}{d^2x}-\frac{ex(a+b\operatorname{arccosh}(cx))}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^2}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d^2}+\frac{(a+b\operatorname{arccosh}(cx))^2}{bd^2}+\frac{\log\left(e^{-2\operatorname{arccosh}(cx)}+1\right)(a+b\operatorname{arccosh}(cx))}{d^2}+\frac{a+b\operatorname{arccosh}(cx)}{2d(d+ex^2)}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}(cx)}\right)}{2d^2}-\frac{bc\sqrt{c^2x^2-1}\operatorname{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2d^{3/2}\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
(a + b*ArcCosh[c*x])/(2*d*(d + e*x^2)) + (a + b*ArcCosh[c*x])^2/(b*d^2) -
(b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2
*x^2])])/(2*d^(3/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((a +
b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^2 - ((a + b*ArcCosh[c*x])*
Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^
2) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh
[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(2*d^2) - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d^2) - (b*PolyLog[2, -
((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) - (
b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
(2*d^2) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^
2*d) - e]))])/(2*d^2) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 505, normalized size of antiderivative = 0.87

method	result
parts	$\frac{a}{2d(e\,x^2+d)}-\frac{a\ln(e\,x^2+d)}{2d^2}+\frac{a\ln(x)}{d^2}+b\left(\frac{c^2\,\mathrm{arccosh}(cx)}{2d(c^2e\,x^2+c^2d)}+\frac{\sqrt{c^2d(c^2d+e)}\,\mathrm{arctanh}\left(\frac{4c^2d+2e(cx+\sqrt{cx-1}\,\sqrt{cx}}{4\sqrt{c^4d^2+c^2de}}\right.}{2d^2(c^2d+e)}\right.$
derivativedivides	$\frac{a\ln(cx)}{d^2}+\frac{a\,c^2}{2d(c^2e\,x^2+c^2d)}-\frac{a\ln(c^2e\,x^2+c^2d)}{2d^2}+\frac{b\,c^2\,\mathrm{arccosh}(cx)}{2d(c^2e\,x^2+c^2d)}+\frac{b\sqrt{c^2d(c^2d+e)}\,\mathrm{arctanh}\left(\frac{4c^2d+2e(cx+\sqrt{cx}}{4\sqrt{c^4d^2}}\right.}{2d^2(c^2d+e)}$
default	$\frac{a\ln(cx)}{d^2}+\frac{a\,c^2}{2d(c^2e\,x^2+c^2d)}-\frac{a\ln(c^2e\,x^2+c^2d)}{2d^2}+\frac{b\,c^2\,\mathrm{arccosh}(cx)}{2d(c^2e\,x^2+c^2d)}+\frac{b\sqrt{c^2d(c^2d+e)}\,\mathrm{arctanh}\left(\frac{4c^2d+2e(cx+\sqrt{cx}}{4\sqrt{c^4d^2}}\right.}{2d^2(c^2d+e)}$

input
```
int((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/d/(e*x^2+d)-1/2*a/d^2*ln(e*x^2+d)+a/d^2*ln(x)+b*(1/2*c^2*arccosh(c*x
)/d/(c^2*e*x^2+c^2*d)+1/2*(c^2*d*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arctanh(1/
4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^
(1/2))-1/4/d^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*l
n((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/d^2*arccos
h(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/d^2*arccosh(c*x)*ln(1-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/d^2*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))+1/d^2*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-1/4/d^2*e*
sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=
RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\operatorname{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\operatorname{acosh}\left(cx\right)}{x\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((a+b*acosh(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acosh(c*x))/(x*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrat
e(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^2}\,dx$$
$$= \frac{2\left(\int \frac{acosh(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right) b\,d^3 + 2\left(\int \frac{acosh(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right) b\,d^2e\,x^2 - \log(e\,x^2+d)\,ad - \log(e\,x^2+d)\,ae\,x^2 + 2}{2d^2\left(e\,x^2+d\right)}$$

input
```
int((a+b*acosh(c*x))/x/(e*x^2+d)^2,x)
```

output
```
(2*int(acosh(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**3 + 2*int(acos
h(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**2*e*x**2 - log(d + e*x**2
)*a*d - log(d + e*x**2)*a*e*x**2 + 2*log(x)*a*d + 2*log(x)*a*e*x**2 - a*e*
x**2)/(2*d**2*(d + e*x**2))
```

3.399 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{x^3(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 616

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^3(d+ex^2)^2}\,dx = \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\operatorname{arccosh}(cx)}{2d^2x^2} - \frac{e(a+b\operatorname{arccosh}(cx))}{2d^2(d+ex^2)} + \frac{bce\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{d^3} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{d^3} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{d^3} + \frac{e(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{d^3} - \frac{2e(a+b\operatorname{arccosh}(cx))\log\left(1+e^{2\operatorname{arccosh}(cx)}\right)}{d^3} + \frac{be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{d^3} + \frac{be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{d^3} + \frac{be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{d^3} + \frac{be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{d^3} - \frac{be\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)}{d^3}$$

output
```
1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x-1/2*(a+b*arccosh(c*x))/d^2/x^2-1
/2*e*(a+b*arccosh(c*x))/d^2/(e*x^2+d)+1/2*b*c*e*(c^2*x^2-1)^(1/2)*arctanh(
(c^2*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)+e*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3+e*(a+b*arccosh(c*x))
*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(
1/2)))/d^3+e*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+e*(a+b*arccosh(c*x))*ln(1+e^(1/2
)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-2
*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3+b*e*po
lylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e
)^(1/2)))/d^3+b*e*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3+b*e*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+b*e*polylog(2,e^(1/2)
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-b*
e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.29

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{2ad}{x^2}-\frac{2ade}{d+ex^2}-8ae\log(x)+4ae\log(d+ex^2)+b\left(\frac{2d\left(cx\sqrt{-1+cx}\sqrt{1+cx}-\text{arccosh}(cx)\right)}{x^2}-4e\text{arccosh}(cx)^2-4\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
```

output
```
((-2*a*d)/x^2 - (2*a*d*e)/(d + e*x^2) - 8*a*e*Log[x] + 4*a*e*Log[d + e*x^2
] + b*((2*d*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - ArcCosh[c*x]))/x^2 - 4*e*A
rcCosh[c*x]^2 - 4*e*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c
*x])]) + 4*e*ArcCosh[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d
] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] +
 Sqrt[-(c^2*d) - e])]) + 4*e*ArcCosh[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x]
)/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(
I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]) + I*Sqrt[d]*e*(ArcCosh[c*x]/((-I)*Sqrt
[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d
) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*S
qrt[e]*x))])/Sqrt[-(c^2*d) - e]) + I*Sqrt[d]*e*(-(ArcCosh[c*x]/(I*Sqrt[d]
+ Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) -
e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[
e]*x))])/Sqrt[-(c^2*d) - e]) + 4*e*PolyLog[2, -E^(-2*ArcCosh[c*x])] + 4*e*
PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] +
4*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + 4*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2
*d) - e]))] + 4*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[
-(c^2*d) - e])]))/(4*d^3)
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \text{ 6374}$$

$$\int \left(\frac{2e^2x(a+b\text{arccosh}(cx))}{d^3\left(d+ex^2\right)}-\frac{2e(a+b\text{arccosh}(cx))}{d^3x}+\frac{e^2x(a+b\text{arccosh}(cx))}{d^2\left(d+ex^2\right)^2}+\frac{a+b\text{arccosh}(cx)}{d^2x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$
\begin{aligned}
&\frac{e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\\
&\frac{e(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{d^3}+\\
&\frac{e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{d^3}+\\
&\frac{e(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{d^3}-\frac{2e(a+b\operatorname{arccosh}(cx))^2}{bd^3}-\\
&\frac{2e\log\left(e^{-2\operatorname{arccosh}(cx)}+1\right)(a+b\operatorname{arccosh}(cx))}{d^3}-\frac{e(a+b\operatorname{arccosh}(cx))}{2d^2(d+ex^2)}-\frac{a+b\operatorname{arccosh}(cx)}{2d^2x^2}+\\
&\frac{be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{d^3}+\frac{be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{d^3}+\\
&\frac{be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{d^3}+\frac{be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{d^3}+\\
&\frac{be\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}(cx)}\right)}{d^3}+\frac{bce\sqrt{c^2x^2-1}\operatorname{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2d^{5/2}\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2d^2x}
\end{aligned}
$$

input

```
Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
```

output

```
(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^2*x) - (a + b*ArcCosh[c*x])/(2*d^2
*x^2) - (e*(a + b*ArcCosh[c*x]))/(2*d^2*(d + e*x^2)) - (2*e*(a + b*ArcCosh
[c*x])^2)/(b*d^3) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/
(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (2*e*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^
3 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
 Sqrt[-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*
x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d
^3 + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e])])/d^3 + (b*e*PolyLog[2, -E^(-2*ArcCosh[c*x])])/d^3 +
 (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) -
e]))])/d^3 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/d^3 + (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e]))])/d^3 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.03

method	result
parts	$-\frac{ae}{2d^2(e\,x^2+d)}+\frac{ae\ln\left(e\,x^2+d\right)}{d^3}-\frac{a}{2d^2x^2}-\frac{2ae\ln\left(x\right)}{d^3}+b\,c^2\left(-\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,c^3dx-\sqrt{cx-1}\,\sqrt{cx+1}\,c^3e\,x}{2c^2x^2}\right.$
derivativedivides	$c^2\left(-\frac{a}{2d^2c^2x^2}-\frac{2ae\ln\left(cx\right)}{c^2d^3}-\frac{ae}{2d^2(c^2e\,x^2+c^2d)}+\frac{ae\ln\left(c^2e\,x^2+c^2d\right)}{c^2d^3}+b\,c^4\left(-\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,c^3dx-\sqrt{c}}{}\right.\right.$
default	$c^2\left(-\frac{a}{2d^2c^2x^2}-\frac{2ae\ln\left(cx\right)}{c^2d^3}-\frac{ae}{2d^2(c^2e\,x^2+c^2d)}+\frac{ae\ln\left(c^2e\,x^2+c^2d\right)}{c^2d^3}+b\,c^4\left(-\frac{-\sqrt{cx-1}\,\sqrt{cx+1}\,c^3dx-\sqrt{c}}{}\right.\right.$

input
```
int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*e/d^2/(e*x^2+d)+a*e/d^3*ln(e*x^2+d)-1/2*a/d^2/x^2-2*a/d^3*e*ln(x)+b
*c^2*(-1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*d*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2)*c^3*e*x^3+x^2*c^4*d+x^4*c^4*e+arccosh(c*x)*c^2*d+2*arccosh(c*x)*c^2*e*x
^2)/c^2/x^2/d^2/(c^2*e*x^2+c^2*d)-1/2*(c^2*d*(c^2*d+e))^(1/2)/d^3/c^2/(c^2
*d+e)*e*arctanh(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/
(c^4*d^2+c^2*d*e)^(1/2))+1/2*e/d^3/c^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*
c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog
((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*
e)*_Z^2+e))-2*e/d^3/c^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))-2*e/d^3/c^2*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-2*
e/d^3/c^2*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-2*e/d^3/c^2*dilog(1
-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/2*e^2/d^3/c^2*sum((_R1^2+1)/(_R1^2
*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+
dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2
*d+2*e)*_Z^2+e)))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d+ex^2)^2} \, dx = \int \frac{b \text{arcosh}(cx) + a}{(ex^2+d)^2 x^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e
*log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^7
 + 2*d*e*x^5 + d^2*x^3), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d+ex^2)^2} \, dx = \int \frac{b \text{arcosh}(cx) + a}{(ex^2+d)^2 x^3} \, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d+ex^2)^2} \, dx = \int \frac{a+b \operatorname{acosh}(c x)}{x^3 (e x^2+d)^2} \, dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^2),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$
$$= \frac{2\left(\int \frac{acosh(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)b\,d^4x^2+2\left(\int \frac{acosh(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)b\,d^3e\,x^4+2\log(e\,x^2+d)\,ade\,x^2+2\log(e\,x^2+a}{2d^3x^2\left(e\,x^2+d\right)}$$

input
```
int((a+b*acosh(c*x))/x^3/(e*x^2+d)^2,x)
```

output
```
(2*int(acosh(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**4*x**2 + 2*
int(acosh(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**3*e*x**4 + 2*l
og(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 - 4*log(x)*a*d*e
*x**2 - 4*log(x)*a*e**2*x**4 - a*d**2 + 2*a*e**2*x**4)/(2*d**3*x**2*(d + e
*x**2))
```

3.400 $\int \frac{x^4(a+b\,\mathbf{arccosh}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 839

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\text{arccosh}(cx)}{e^2} - \frac{d(a+b\text{arccosh}(cx))}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d(a+b\text{arccosh}(cx))}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bcd\text{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{5/2}} - \frac{bcd\text{arctanh}\left(\frac{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{5/2}} + \frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4e^{5/2}} + \frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4e^{5/2}} - \frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4e^{5/2}} + \frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4e^{5/2}} - \frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4e^{5/2}} + \frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4e^{5/2}}$$

output
```
a*x/e^2-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2+b*x*arccosh(c*x)/e^2-1/4*d*(a+
b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*d*(a+b*arccosh(c*x))/e^
(5/2)/((-d)^(1/2)+e^(1/2)*x)+1/2*b*c*d*arctanh((c*(-d)^(1/2)-e^(1/2))^(1/2
)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2))/(c*(-d)^(1/2)-
e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(5/2)-1/2*b*c*d*arctanh((c*(
-d)^(1/2)+e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1
)^(1/2))/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(5/2)
+3/4*(-d)^(1/2)*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)-3/4*(-d)^(1/2)*(a+b*arcco
sh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^
2*d-e)^(1/2)))/e^(5/2)+3/4*(-d)^(1/2)*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)-3/4
*(-d)^(1/2)*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)-3/4*b*(-d)^(1/2)*polylog(2,-e
^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/
e^(5/2)+3/4*b*(-d)^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)-3/4*b*(-d)^(1/2)*polylog(2,-e^
(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e
^(5/2)+3/4*b*(-d)^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 777, normalized size of antiderivative = 0.93

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{8a\sqrt{e}x + \frac{4ad\sqrt{e}x}{d+ex^2} - 12a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(\frac{8\sqrt{e}\left(-\sqrt{\frac{-1+cx}{1+cx}}(1+cx)+cx\text{arccosh}(cx)\right)}{c} + 2d\left(\frac{\text{arccosh}(cx)}{-i\sqrt{d}+\sqrt{e}x} + \frac{c\log}{}\right.\right.}{}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output

```
(8*a*Sqrt[e]*x + (4*a*d*Sqrt[e]*x)/(d + e*x^2) - 12*a*Sqrt[d]*ArcTan[(Sqrt
[e]*x)/Sqrt[d]] + b*((8*Sqrt[e]*(-(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
 c*x*ArcCosh[c*x]))/c + 2*d*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*
Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^
2*d) - e]) + 2*d*(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(-Sqr
t[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3
*I)*Sqrt[d]*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x]
)/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[
c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^A
rcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[d]*(ArcCos
h[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d
] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] +
 Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[
d] - Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqr
t[d] + Sqrt[-(c^2*d) - e])])))/(8*e^(5/2))
```

Rubi [A] (verified)

Time = 2.61 (sec) , antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{d^2(a+b\text{arccosh}(cx))}{e^2(d+ex^2)^2}-\frac{2d(a+b\text{arccosh}(cx))}{e^2(d+ex^2)}+\frac{a+b\text{arccosh}(cx)}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\text{arccosh}(cx)b}{e^2}+\frac{cd\text{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}-\frac{cd\text{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}-\frac{3\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)b}{4e^{5/2}}+\frac{3\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)b}{4e^{5/2}}-\frac{3\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)b}{4e^{5/2}}+\frac{3\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)b}{4e^{5/2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}b}{ce^2}+\frac{ax}{e^2}-\frac{d(a+b\text{arccosh}(cx))}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d(a+b\text{arccosh}(cx))}{4e^{5/2}\left(\sqrt{e}x+\sqrt{-d}\right)}+\frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(\frac{e^{\text{arccosh}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)}{4e^{5/2}}+\frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\text{arccosh}(cx))\log\left(\frac{e^{\text{arccosh}(cx)}\sqrt{e}}{\sqrt{-d}c+\sqrt{-dc^2-e}}+1\right)}{4e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a*x)/e^2 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) + (b*x*ArcCosh[c*x])/
e^2 - (d*(a + b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a
+ b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*d*ArcTanh[(Sq
rt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-
1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/
2)) - (b*c*d*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sq
rt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*S
qrt[-d] + Sqrt[e]]*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sq
rt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3
*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*L
og[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(
5/2)) - (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/
(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2,
 -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*e^(5/2
)) + (3*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*
PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4
*e^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.95 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.07

method	result
parts	$a\left(\frac{x}{e^{2}}-\frac{d\left(-\frac{x}{2\left(e\,x^{2}+d\right)}+\frac{3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}\right)}{e^{2}}\right)+\frac{b\left(\frac{c^{4}(-1+\operatorname{arccosh}(cx))(cx+\sqrt{cx-1}\,\sqrt{cx+1})}{2e^{2}}+\frac{(-\sqrt{cx-1}\,\sqrt{cx+1}+cx)c}{2e^{2}}\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))))+b/c^5*(1/2*c^4*(-1+arccosh(c*x))/e^2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*c^4*(1+arccosh(c*x))/e^2+1/2*c^7*
arccosh(c*x)/e^2*d*x/(c^2*e*x^2+c^2*d)+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(
1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^
2*d*(c^2*d+e))^(1/2)*e)*c^6*d*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(
(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)-1/2*((2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
+e)*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2))*c^6*d/e^5+1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e
)*e)^(1/2)*(2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e+(c^2*d*(c^
2*d+e))^(1/2)*e)*c^6*d*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^
2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)-1/2*(-(2*c^2*d-2*
(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*
arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))
^(1/2)-e)*e)^(1/2))*c^6*d/e^5+3/4*d/e^2*c^6*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*
(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e
))-3/4*d/e^2*c^6*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*
x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/...
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccosh(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\,(a+b\,\text{acosh}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int\frac{acosh(cx)x^4}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^3+2\left(\int\frac{acosh(cx)x^4}{e^2x^4+2de\,x^2+}\right.}{2e^3\,(e\,x^2+d)}$$

input
```
int(x^4*(a+b*acosh(c*x))/(e*x^2+d)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d - 3*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((acosh(c*x)*x**4)/(d**2 +
 2*d*e*x**2 + e**2*x**4),x)*b*d*e**3 + 2*int((acosh(c*x)*x**4)/(d**2 + 2*d
*e*x**2 + e**2*x**4),x)*b*e**4*x**2 + 3*a*d*e*x + 2*a*e**2*x**3)/(2*e**3*(
d + e*x**2))
```

3.401 $\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 792

$$
\begin{aligned}
\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^2}\,dx &= \frac{a+b\operatorname{arccosh}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{a+b\operatorname{arccosh}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} \\
&\quad-\frac{b\operatorname{carctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} \\
&\quad+\frac{b\operatorname{carctanh}\left(\frac{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} \\
&\quad+\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad+\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4\sqrt{-d}e^{3/2}}
\end{aligned}
$$

output
```
1/4*(a+b*arccosh(c*x))/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)-1/4*(a+b*arccosh(c*x
))/e^(3/2)/((-d)^(1/2)+e^(1/2)*x)-1/2*b*c*arctanh((c*(-d)^(1/2)-e^(1/2))^(
1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2))/(c*(-d)^(1/
2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(3/2)+1/2*b*c*arctanh((c*
(-d)^(1/2)+e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-
1)^(1/2))/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(3/2
)+1/4*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c
*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*(a+b*arccosh(c*x))*l
n(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/
2)))/(-d)^(1/2)/e^(3/2)+1/4*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/
4*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d
)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*b*polylog(2,-e^(1/2)*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)
/e^(3/2)+1/4*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)
^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*b*polylog(2,-e^(1/2)*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/
e^(3/2)+1/4*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^
(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 719, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \frac{-\frac{4a\sqrt{e}x}{d+ex^2}+\frac{4a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+b\left(-\frac{2\text{arccosh}(cx)}{i\sqrt{d}+\sqrt{e}x}-2\left(\frac{\text{arccosh}(cx)}{-i\sqrt{d}+\sqrt{e}x}+\frac{c\log\left(\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}x-i\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)-\cdots}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
+ b*((-2*ArcCosh[c*x])/(I*Sqrt[d] + Sqrt[e]*x) - 2*(ArcCosh[c*x]/((-I)*Sqr
t[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*
d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*
Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (2*c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d
]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) -
 e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (I*(ArcCosh[c*x]*(-Arc
Cosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*
d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
- e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqr
t[-(c^2*d) - e]))]))/Sqrt[d] + (I*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 +
 (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 -
 (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) - 2*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] -
2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])
)/Sqrt[d]))/(8*e^(3/2))
```

Rubi [A] (verified)

Time = 3.57 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{a+b\text{arccosh}(cx)}{e(d+ex^2)}-\frac{d(a+b\text{arccosh}(cx))}{e(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{4\sqrt{-d}e^{3/2}}+\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{4\sqrt{-d}e^{3/2}}+\frac{a+b\text{arccosh}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{a+b\text{arccosh}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{bc\,\text{arctanh}\left(\frac{\sqrt{cx+1}\sqrt{c\sqrt{-d}-\sqrt{e}}}{\sqrt{cx-1}\sqrt{c\sqrt{-d}+\sqrt{e}}}\right)}{2e^{3/2}\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}}+\frac{bc\,\text{arctanh}\left(\frac{\sqrt{cx+1}\sqrt{c\sqrt{-d}+\sqrt{e}}}{\sqrt{cx-1}\sqrt{c\sqrt{-d}-\sqrt{e}}}\right)}{2e^{3/2}\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a + b*ArcCosh[c*x])/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcCosh[c
*x])/(4*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] -
Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(2*S
qrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) + (b*c*ArcTa
nh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*
Sqrt[-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]
*e^(3/2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt
[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*
Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sq
rt[-d]*e^(3/2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[
c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] - Sqrt[-(c^2*d) - e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) -
 (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]
))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.75 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{a c^5 x}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{a c^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b c^4\left(-\frac{cx \operatorname{arccosh}(cx)}{2\left(c^2 e x^2+c^2 d\right)e}-\frac{\sqrt{\left(2c^2 d+2\sqrt{c^2 d\left(c^2 d+e\right)}+e\right)e}\left(-2\sqrt{c^2 d\left(c^2 d+e\right)}\, c^2 d+2c^4\right.}{}\right.}{}$
default	$\frac{-\frac{a c^5 x}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{a c^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b c^4\left(-\frac{cx \operatorname{arccosh}(cx)}{2\left(c^2 e x^2+c^2 d\right)e}-\frac{\sqrt{\left(2c^2 d+2\sqrt{c^2 d\left(c^2 d+e\right)}+e\right)e}\left(-2\sqrt{c^2 d\left(c^2 d+e\right)}\, c^2 d+2c^4\right.}{}\right.}{}$
parts	$-\frac{ax}{2e(e x^2+d)}+\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5 \operatorname{arccosh}(cx) x}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{\sqrt{\left(2c^2 d+2\sqrt{c^2 d\left(c^2 d+e\right)}+e\right)e}\left(-2\sqrt{c^2 d\left(c^2 d+e\right)}\, c^2 d+2c^4 d^2\right.}{2e}\right.}{}$

input
```
int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-1/2*a*c^5/e*x/(c^2*e*x^2+c^2*d)+1/2*a*c^3/e/(d*e)^(1/2)*arctan(e*x
/(d*e)^(1/2))+b*c^4*(-1/2*c*x/(c^2*e*x^2+c^2*d)*arccosh(c*x)/e-1/2*((2*c^2
*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d
+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*arctan(e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2
*d+e)+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*
d*(c^2*d+e))^(1/2)+e)*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d
+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4-1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d
+e))^(1/2)+e)*e)^(1/2)*(2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*
e+(c^2*d*(c^2*d+e))^(1/2)*e)*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(
(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)+1/2*(-(2*c^
2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)-e)*e)^(1/2))/e^4+1/4/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c
*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/4/e*s
um(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootO
f(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccosh(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d + ex^2)^2}\,dx = \int \frac{x^2(a + b\operatorname{acosh}(cx))}{(d + ex^2)^2}\,dx$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a + b\text{arccosh}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\operatorname{arcosh}(cx) + a)x^2}{(ex^2 + d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\,\mathrm{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^2\,(a+b\,\mathrm{acosh}(c\,x))}{{(e\,x^2+d)}^2}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\,\mathrm{arccosh}(cx))}{(d+ex^2)^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int \frac{acosh(cx)x^2}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^2e^2+2\left(\int \frac{acosh(cx)x^2}{e^2x^4+2de\,x^2+d^2}dx\right)\cdots}{2d\,e^2\,(e\,x^2+d)}$$

input
```
int(x^2*(a+b*acosh(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((acosh(c*x)*x**2)/(d**2 + 2*d*e*
x**2 + e**2*x**4),x)*b*d**2*e**2 + 2*int((acosh(c*x)*x**2)/(d**2 + 2*d*e*x
**2 + e**2*x**4),x)*b*d*e**3*x**2 - a*d*e*x)/(2*d*e**2*(d + e*x**2))
```

3.402 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 804

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d+ex^2)^2}\,dx = -\frac{a+b\operatorname{arccosh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\operatorname{arccosh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}$$
$$+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}$$
$$-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}$$
$$-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

output
```
-1/4*(a+b*arccosh(c*x))/d/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*(a+b*arccosh(
c*x))/d/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)+1/2*b*c*arctanh((c*(-d)^(1/2)-e^(1/
2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2))/d/(c*(
-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(1/2)-1/2*b*c*arct
anh((c*(-d)^(1/2)+e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2
)/(c*x-1)^(1/2))/d/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/
2)/e^(1/2)-1/4*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccos
h(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2
*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e
^(1/2)+1/4*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-e^
(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(
-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-e^(
1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-
d)^(3/2)/e^(1/2)-1/4*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.30 (sec) , antiderivative size = 734, normalized size of antiderivative = 0.91

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d+ex^2)^2}\,dx = \frac{1}{2}\left(\frac{ax}{d^2+dex^2}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}} + \frac{b\left(2\sqrt{d}\left(\frac{\operatorname{arccosh}(cx)}{-i\sqrt{d}+\sqrt{e}x}+\frac{c\log\left(\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}x-i\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)-2\sqrt{d}\left(-\frac{\operatorname{arccosh}(cx)}{i\sqrt{d}+\sqrt{e}x}-\frac{c\log\left(\frac{2e\left(-\sqrt{e}-ic^2\right.}{c}\right.}{}\right.\right.}{}\right.$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

output
```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*(2*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I
*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e])
- 2*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[
e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(
c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + I*(A
rcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt
[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*
Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(
I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) - I*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*
(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] +
 Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) +
2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])]
+ 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])
])))/(4*d^(3/2)*Sqrt[e]))/2
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 804, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6324, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{ 6324}$$

$$\int \left(-\frac{e(a+b\text{arccosh}(cx))}{2d(-de-e^2x^2)}-\frac{e(a+b\text{arccosh}(cx))}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\text{arccosh}(cx))}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$
-\frac{\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{\sqrt{-d}c+\sqrt{-dc^2-e}}+1\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\operatorname{arccosh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\operatorname{arccosh}(cx)}{4d\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{e}}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}
$$

input
```
Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

output
```
-1/4*(a + b*ArcCosh[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcC
osh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*ArcTanh[(Sqrt[c*Sqrt
[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])
])/(2*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) - (
b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] -
Sqrt[e]]*Sqrt[-1 + c*x])])/(2*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d]
 + Sqrt[e]]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*A
rcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
- e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e]) +
 ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b
*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(
4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e
])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6324
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.81 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.03

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\operatorname{arccosh}(cx)x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{c^2\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(\operatorname{arccosh}(c\right.}{}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arccosh}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(\operatorname{arccosh}(cx\right.}{}}{}\right.}{}$
default	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arccosh}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(\operatorname{arccosh}(cx\right.}{}}{}\right.}{}$

input
```
int((a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arccosh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4/d*c^2*sum(_R1/(_R1^2*e+2*c^2*d+e
)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c
*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d
+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*c^2*arctan
(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e
)*e)^(1/2))/d/(c^2*d+e)/e^3-1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^
(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*c^2/d/e^3+1/
2*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*(c^2*d*(c^2*d+e))^(1
/2)*c^2*d+2*c^4*d^2+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*c^2*arctanh(e*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^
(1/2))/d/(c^2*d+e)/e^3-1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2
)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*c^2/d/e^3-1/4/
d*c^2*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R
1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^2}\, dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{(ex^2 + d)^2}\, dx$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^2}\,dx = \int \frac{a + b\,\text{acosh}\,(cx)}{(d + ex^2)^2}\,dx$$

input
```
integrate((a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^2}\,dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{(ex^2 + d)^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}(c\,x)}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(d + e*x^2)^2,x)
```

output
```
int((a + b*acosh(c*x))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d+ex^2)^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int \frac{acosh(cx)}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^3e+2\left(\int \frac{acosh(cx)}{e^2x^4+2de\,x^2+d^2}dx\right.}{2d^2e\,(e\,x^2+d)}$$

input
```
int((a+b*acosh(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int(acosh(c*x)/(d**2 + 2*d*e*x**2 +
e**2*x**4),x)*b*d**3*e + 2*int(acosh(c*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),
x)*b*d**2*e**2*x**2 + a*d*e*x)/(2*d**2*e*(d + e*x**2))
```

3.403 $\int \frac{a+b\,\mathbf{arccosh}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 846

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2(d+ex^2)^2}\,dx = -\frac{a+b\operatorname{arccosh}(cx)}{d^2x}+\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2} - \frac{bc\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}} + \frac{bc\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{-1+cx}}\right)}{2d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}} - \frac{3\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}} - \frac{3\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}} + \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}} + \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4(-d)^{5/2}}$$

output
```
-(a+b*arccosh(c*x))/d^2/x+1/4*e^(1/2)*(a+b*arccosh(c*x))/d^2/((-d)^(1/2)-e
^(1/2)*x)-1/4*e^(1/2)*(a+b*arccosh(c*x))/d^2/((-d)^(1/2)+e^(1/2)*x)+b*c*ar
ctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2-1/2*b*c*e^(1/2)*arctanh((c*(-d)^(1/2
)-e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2))
/d^2/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)+1/2*b*c*e^(
1/2)*arctanh((c*(-d)^(1/2)+e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)-e^(1
/2))^(1/2)/(c*x-1)^(1/2))/d^2/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e
^(1/2))^(1/2)-3/4*e^(1/2)*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)+3/4*e^(1/2)*
(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^
(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)-3/4*e^(1/2)*(a+b*arccosh(c*x))*ln(1-e^
(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(
-d)^(5/2)+3/4*e^(1/2)*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)+3/4*b*e^(1/2)*po
lylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e
)^(1/2)))/(-d)^(5/2)-3/4*b*e^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)+3/4*b*e^(1/2)*pol
ylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)
^(1/2)))/(-d)^(5/2)-3/4*b*e^(1/2)*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 821, normalized size of antiderivative = 0.97

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d + ex^2)^2} \, dx$$

$$= \frac{-\frac{8a\sqrt{d}}{x} - \frac{4a\sqrt{d}ex}{d+ex^2} - 12a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(8\sqrt{d}\left(-\frac{\text{arccosh}(cx)}{x} + \frac{c\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\right) - 2\sqrt{d}\sqrt{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(S
qrt[e]*x)/Sqrt[d]] + b*(8*Sqrt[d]*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^
2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - 2*Sqrt[d]
*Sqrt[e]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e]
 + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*
Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + 2*Sqrt
[d]*Sqrt[e]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[
e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(
c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3*I
)*Sqrt[e]*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x
])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/
(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*
x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^Arc
Cosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[e]*(ArcCosh[
c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d]
+ Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + S
qrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d]
 - Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[
d] + Sqrt[-(c^2*d) - e])])))/(8*d^(5/2))
```

Rubi [A] (verified)

Time = 2.94 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(-\frac{e(a+b\text{arccosh}(cx))}{d^2\left(d+ex^2\right)}+\frac{a+b\text{arccosh}(cx)}{d^2x^2}-\frac{e(a+b\text{arccosh}(cx))}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{5/2}}-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{\sqrt{-d}c+\sqrt{-dc^2-e}}+1\right)(a+b\operatorname{arccosh}(cx))}{4(-d)^{5/2}}-\frac{a+b\operatorname{arccosh}(cx)}{d^2x}+\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\sqrt{e}(a+b\operatorname{arccosh}(cx))}{4d^2\left(\sqrt{e}x+\sqrt{-d}\right)}+\frac{bc\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d^2}-\frac{bc\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)}{2d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}}+\frac{bc\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)}{2d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
-((a + b*ArcCosh[c*x])/(d^2*x)) + (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(S
qrt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sqrt[-d] +
Sqrt[e]*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 - (b*c*Sqrt[e
]*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sq
rt[e]]*Sqrt[-1 + c*x])]])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d]
 + Sqrt[e]]) + (b*c*Sqrt[e]*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c
*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(2*d^2*Sqrt[c*Sqrt[-d]
- Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcCosh[c*x])*L
og[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d
)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*A
rcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
- e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*
Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
- e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*(-d
)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.89 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.06

method	result
parts	$a\left(-\frac{e\left(\frac{x}{2e\,x^{2}+2d}+\frac{3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}\right)}{d^{2}}-\frac{1}{d^{2}x}\right)+bc\left(-\frac{\operatorname{arccosh}(cx)\left(3c^{2}e\,x^{2}+2c^{2}d\right)}{2cx\,d^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{\sqrt{\left(2c^{2}d+2\sqrt{c^{2}d(c^{2}d-}\right.}}{}\right.$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/d^2/
x)+b*c*(-1/2/c/x*arccosh(c*x)*(3*c^2*e*x^2+2*c^2*d)/d^2/(c^2*e*x^2+c^2*d)+
1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2
*d+e))^(1/2)+e)*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^
2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2/e^2+1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1
/2))/d^2/e^2+3/16/d^3*e/c^2*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e
)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c
*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))-3/16/d^3*e/c^2*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*
(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e
))+2/d^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c
^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2/(c^2*d+e)/e^2-1/2*
(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*(c^2*d*(c^2*d+e))^(1/2
)*c^2*d+2*c^4*d^2+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*arctanh(e*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/...
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d + ex^2)^2}\, dx = \int \frac{b \operatorname{arcosh}(cx) + a}{(ex^2 + d)^2 x^2}\, dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d + ex^2)^2}\, dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(cx)}{x^2 (d + ex^2)^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\operatorname{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\operatorname{acosh}(c\,x)}{x^2\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*acosh(c*x))/(x^2*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$$
$$=\frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)adx-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^3+2\left(\int\frac{acosh(cx)}{e^2x^6+2de\,x^4+d^2x^2}dx\right)b\,d^4x+2\left(\int\frac{acosh}{e^2x^6+2de}\right.}{2d^3x\left(e\,x^2+d\right)}$$

input
```
int((a+b*acosh(c*x))/x^2/(e*x^2+d)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*x - 3*sqrt(e)*sqrt
(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**3 + 2*int(acosh(c*x)/(d**2*x**2 +
 2*d*e*x**4 + e**2*x**6),x)*b*d**4*x + 2*int(acosh(c*x)/(d**2*x**2 + 2*d*e
*x**4 + e**2*x**6),x)*b*d**3*e*x**3 - 2*a*d**2 - 3*a*d*e*x**2)/(2*d**3*x*(
d + e*x**2))
```

3.404 $\int \frac{x^5(a+b\,\mathrm{arccosh}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 727

$$\int \frac{x^5(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcdx\sqrt{-1+cx}\sqrt{1+cx}}{8e^2(c^2d+e)(d+ex^2)} - \frac{d^2(a+b\operatorname{arccosh}(cx))}{4e^3(d+ex^2)^2} + \frac{d(a+b\operatorname{arccosh}(cx))}{e^3(d+ex^2)} - \frac{(a+b\operatorname{arccosh}(cx))^2}{2be^3} - \frac{bc\sqrt{d}\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^3\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc\sqrt{d}(2c^2d+e)\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{8e^3(c^2d+e)^{3/2}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^3} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^3} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^3} + \frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e^3}$$

output
```
-1/8*b*c*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/e^2/(c^2*d+e)/(e*x^2+d)-1/4*d^2*(
a+b*arccosh(c*x))/e^3/(e*x^2+d)^2+d*(a+b*arccosh(c*x))/e^3/(e*x^2+d)-1/2*(
a+b*arccosh(c*x))^2/b/e^3-b*c*d^(1/2)*(c^2*x^2-1)^(1/2)*arctanh((c^2*d+e)^
(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+1/8*b*c*d^(1/2)*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*arctanh((c^2*d+e)^(
1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*d+e)^(3/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b*arccosh(c*x))*ln(1+e^(1/
2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+
1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(
-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^3+1/2*b*po
lylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e
)^(1/2)))/e^3+1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c
*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*b*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^3+1/2*b*polylog(2,e
^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/
e^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.96 (sec) , antiderivative size = 1097, normalized size of antiderivative = 1.51

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-4*a*d^2)/(d + e*x^2)^2 + (16*a*d)/(d + e*x^2) + 8*a*Log[d + e*x^2] + b*
(-((c*d*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] +
 Sqrt[e]*x))) - (c*d*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I
*Sqrt[d] + Sqrt[e]*x)) + (7*Sqrt[d]*ArcCosh[c*x])/(Sqrt[d] - I*Sqrt[e]*x)
- (d*ArcCosh[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 + (7*Sqrt[d]*ArcCosh[c*x])/(S
qrt[d] + I*Sqrt[e]*x) + (d*ArcCosh[c*x])/(I*Sqrt[d] + Sqrt[e]*x)^2 - 8*Arc
Cosh[c*x]^2 + 8*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d]
 - Sqrt[-(c^2*d) - e])] + 8*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/
((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 8*ArcCosh[c*x]*Log[1 - (Sqrt[e]*E
^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 8*ArcCosh[c*x]*Log[1
+ (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] - ((7*I)*c*
Sqrt[d]*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/S
qrt[-(c^2*d) - e] + ((7*I)*c*Sqrt[d]*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x
+ Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*
(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - (c^3*d^(3/2)*Log[(e*Sqrt[c
^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*S
qrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(c^2*d + e)^(3/2) + (c^3*
d^(3/2)*Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d
+ e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))])/(...
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{d^2x(a+b\text{arccosh}(cx))}{e^2(d+ex^2)^3}-\frac{2dx(a+b\text{arccosh}(cx))}{e^2(d+ex^2)^2}+\frac{x(a+b\text{arccosh}(cx))}{e^2(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^3}+\frac{(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^3}+\frac{(a+b\text{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^3}-\frac{d^2(a+b\text{arccosh}(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\text{arccosh}(cx))}{e^3(d+ex^2)}-\frac{(a+b\text{arccosh}(cx))^2}{2be^3}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{bc\sqrt{d}\sqrt{c^2x^2-1}(2c^2d+e)\,\text{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8e^3\sqrt{cx-1}\sqrt{cx+1}(c^2d+e)^{3/2}}-\frac{bc\sqrt{d}\sqrt{c^2x^2-1}\text{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^3\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}+\frac{bcdx(1-c^2x^2)}{8e^2\sqrt{cx-1}\sqrt{cx+1}(c^2d+e)(d+ex^2)}$$

input
```
Int[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
(b*c*d*x*(1 - c^2*x^2))/(8*e^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
 + e*x^2)) - (d^2*(a + b*ArcCosh[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*
ArcCosh[c*x]))/(e^3*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*e^3) - (b*c
*Sqrt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 +
 c^2*x^2])])/(e^3*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqr
t[d]*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])])/(8*e^3*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
 Sqrt[-(c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + ((a + b*ArcCos
h[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
])/(2*e^3) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + (b*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^
3) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(2*e^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.53 (sec) , antiderivative size = 3551, normalized size of antiderivative = 4.88

method	result	size
derivativedivides	Expression too large to display	3551
default	Expression too large to display	3551
parts	Expression too large to display	3556

input
```
int(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^6*(a*c^6*(-1/4*d^2*c^4/e^3/(c^2*e*x^2+c^2*d)^2+1/2/e^3*ln(c^2*e*x^2+c^
2*d)+1/e^3*d*c^2/(c^2*e*x^2+c^2*d))+b*c^6*(3/4*(2*c^2*d-2*(c^2*d*(c^2*d+e)
)^(1/2)+e)/e^4/(c^2*d+e)*c^2*d*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+3/4*(c^2*d*(c^2*d+e))^(1/2)/e
^3/(c^2*d+e)^2*c^2*d*arctanh(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-1/8*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d
+e)^2/c^2/d*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c
^2*d*(c^2*d+e))^(1/2)-e))-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^5/(c^2*d
+e)*c^4*d^2*arccosh(c*x)^2-3/2*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^4/(
c^2*d+e)*c^2*d*arccosh(c*x)^2-1/4*(c^2*d*(c^2*d+e))^(1/2)/e^3/(c^2*d+e)^2*
c^2*d*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)-e))+1/8*d*c^2*(6*c^4*d^2*arccosh(c*x)+8*arccosh(c*x)*c^4*d
*e*x^2-c^3*d*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c^3*e^2*x^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)+c^4*d^2+2*c^4*d*e*x^2+c^4*e^2*x^4+6*c^2*d*e*arccosh(c*x)+8*arcc
osh(c*x)*e^2*c^2*x^2)/e^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)-(-2*(c^2*d*(c^2*d+
e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*c^2*d/e^4/(
c^4*d^2+2*c^2*d*e+e^2)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2
*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-1/8*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d
+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)/c^2/d/e^2/(c^4*d^2+2*c^2*d
*e+e^2)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^...
```

Fricas [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arccosh(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\operatorname{arcosh}(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + b*integrate(x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x
^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5\,(a+b\,\mathrm{acosh}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^5*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^5(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{4\left(\int \frac{acosh(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e^3+8\left(\int \frac{acosh(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)bd\,e^4x^2+4\left(\int \frac{acosh(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}d\right.}{4e^3\left(e^2x^4+2de\right.}$$

input
```
int(x^5*(a+b*acosh(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((acosh(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6
),x)*b*d**2*e**3 + 8*int((acosh(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**
2*x**4 + e**3*x**6),x)*b*d*e**4*x**2 + 4*int((acosh(c*x)*x**5)/(d**3 + 3*d
**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*e**5*x**4 + 2*log(d + e*x**2)
*a*d**2 + 4*log(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 + a
*d**2 - 2*a*e**2*x**4)/(4*e**3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.405 $\int \frac{x^3(a+b\text{arccosh}(cx))}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d+ex^2)^3} dx = \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}}{8e(c^2d+e)(d+ex^2)} + \frac{x^4(a+b\text{arccosh}(cx))}{4d(d+ex^2)^2} - \frac{b\sqrt{-1+c^2x^2}\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{4de^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc(2c^2d+3e)\sqrt{-1+c^2x^2}\text{arctanh}\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{8\sqrt{d}e^2(c^2d+e)^{3/2}\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
1/8*b*c*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)+1/4*x^4*(a+b*a
rccosh(c*x))/d/(e*x^2+d)^2-1/4*b*(c^2*x^2-1)^(1/2)*arctanh(c*x/(c^2*x^2-1)
^(1/2))/d/e^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*(2*c^2*d+3*e)*(c^2*x^2-1
)^(1/2)*arctanh((c^2*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(1/2)/e^2/(
c^2*d+e)^(3/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.83

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^3}\,dx = \frac{\frac{\frac{bcex\sqrt{-1+cx}\sqrt{1+cx}(d+ex^2)}{c^2d+e}-2a(d+2ex^2)}{(d+ex^2)^2}-\frac{2b(d+2ex^2)\operatorname{arccosh}(cx)}{(d+ex^2)^2}-\frac{bc(2c^2d+3e)\sqrt{-1+cx}\sqrt{1+cx}\arctan\left(\frac{\sqrt{-c^2d-e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}(-c^2d-e)^{3/2}\sqrt{-1+c^2x^2}}}{8e^2}$$

input
```
Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
(((b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(c^2*d + e) - 2*a*(d
+ 2*e*x^2))/(d + e*x^2)^2 - (2*b*(d + 2*e*x^2)*ArcCosh[c*x])/(d + e*x^2)^2
 - (b*c*(2*c^2*d + 3*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d)
 - e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*(-(c^2*d) - e)^(3/2)*Sqrt
[-1 + c^2*x^2]))/(8*e^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6373, 27, 2038, 372, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{6373}$$

$$\frac{x^4(a+b\operatorname{arccosh}(cx))}{4d(d+ex^2)^2}-bc\int \frac{x^4}{4d\sqrt{cx-1}\sqrt{cx+1}(ex^2+d)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(a+b\operatorname{arccosh}(cx))}{4d(d+ex^2)^2}-\frac{bc\int \frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \ 2038$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\int \frac{x^4}{\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 372$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int -\frac{d-2\left(dc^2+e\right)x^2}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx}{2e(c^2d+e)}-\frac{dx\sqrt{c^2x^2-1}}{2e(c^2d+e)(d+ex^2)}\right)}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 25$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(-\frac{\int \frac{d-2\left(dc^2+e\right)x^2}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx}{2e(c^2d+e)}-\frac{dx\sqrt{c^2x^2-1}}{2e(c^2d+e)(d+ex^2)}\right)}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 398$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(-\frac{\frac{d\left(2c^2d+3e\right)\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx}{e}-\frac{2\left(c^2d+e\right)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{e}}{2e(c^2d+e)}-\frac{dx\sqrt{c^2x^2-1}}{2e(c^2d+e)(d+ex^2)}\right)}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 224$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(-\frac{\frac{d\left(2c^2d+3e\right)\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx}{e}-\frac{2\left(c^2d+e\right)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{e}}{2e(c^2d+e)}-\frac{dx\sqrt{c^2x^2-1}}{2e(c^2d+e)(d+ex^2)}\right)}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \ 219$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(-\frac{\frac{d\left(2c^2d+3e\right)\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx}{e}-\frac{2\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(c^2d+e\right)}{ce}}{2e(c^2d+e)}-\frac{dx\sqrt{c^2x^2-1}}{2e(c^2d+e)(d+ex^2)}\right)}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{291}$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(-\frac{\frac{d\left(2c^2d+3e\right)\int\frac{1}{d-\frac{\left(dc^2+e\right)x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{e}-\frac{2\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(c^2d+e\right)}{ce}}{2e(c^2d+e)}-\frac{dx\sqrt{c^2x^2-1}}{2e(c^2d+e)(d+ex^2)}\right)}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

$$\Big\downarrow \text{221}$$

$$\frac{x^4(a+b\text{arccosh}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(-\frac{\frac{\sqrt{d}\left(2c^2d+3e\right)\text{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2d+e}}-\frac{2\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(c^2d+e\right)}{ce}}{2e(c^2d+e)}-\frac{dx\sqrt{c^2x^2-1}}{2e(c^2d+e)(d+ex^2)}\right)}{4d\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcCosh[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*Sqrt[-1 + c^2*x^2]*(
-1/2*(d*x*Sqrt[-1 + c^2*x^2])/(e*(c^2*d + e)*(d + e*x^2)) - ((-2*(c^2*d +
e)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(c*e) + (Sqrt[d]*(2*c^2*d + 3*e)*Arc
Tanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(e*Sqrt[c^2*d + e]
))/(2*e*(c^2*d + e))))/(4*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(195) = 390$.

Time = 0.22 (sec) , antiderivative size = 1169, normalized size of antiderivative = 5.06

method	result	size
parts	Expression too large to display	1169
derivativedivides	Expression too large to display	1199
default	Expression too large to display	1199

input
```
int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(1/4*c^8*arccosh(c*x)/e^
2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^6*arccosh(c*x)/e^2/(c^2*e*x^2+c^2*d)-1/16*c^
6*e*(2*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x
-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^6*d^2*e*x^2+2*ln(2*((-(c^2*d+e)/e)^(1/2)*(
c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^6*d
^3-2*ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x
+e)/(e*c*x+(-c^2*d*e)^(1/2)))*c^6*x^2*d^2*e-2*ln(-2*(-(-(c^2*d+e)/e)^(1/2)
*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*c^6
*d^3+5*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x
-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^4*d*e^2*x^2+5*ln(2*((-(c^2*d+e)/e)^(1/2)*(
c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^4*d
^2*e-5*ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c
*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*c^4*x^2*d*e^2-5*ln(-2*(-(-(c^2*d+e)/e)^(1/
2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*c
^4*d^2*e-2*c^3*d*e*(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*(-c^2*d*e)^(1/2)
*x+3*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e
)/(e*c*x-(-c^2*d*e)^(1/2)))*e^3*c^2*x^2+3*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*
x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^2*d*e^2
-3*ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e
)/(e*c*x+(-c^2*d*e)^(1/2)))*c^2*x^2*e^3-3*ln(-2*(-(-(c^2*d+e)/e)^(1/2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(195) = 390.

Time = 0.20 (sec) , antiderivative size = 1217, normalized size of antiderivative = 5.27

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{(d + ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(2*a - b)*c^4*d^4 + 2*(4*a - b)*c^2*d^3*e - 4*(b*c^4*d^2*e^2 + 2
*b*c^2*d*e^3 + b*e^4)*x^4*log(c*x + sqrt(c^2*x^2 - 1)) + 4*a*d^2*e^2 - 2*(
b*c^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 4*((2*a - b)*c^4*d^3*e + (4*a - b)*c^2*
d^2*e^2 + 2*a*d*e^3)*x^2 - (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3
*b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*l
og(-(2*c^2*d^2 - (4*c^4*d^2 + 4*c^2*d*e + e^2)*x^2 + d*e - 2*sqrt(c^2*d^2
+ d*e)*((2*c^3*d + c*e)*x^2 - c*d) - 2*sqrt(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d
*e)*(2*c^2*d + e)*x + 2*(c^3*d^2 + c*d*e)*x))/(e*x^2 + d)) - 4*(b*c^4*d^4
+ 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4
+ 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-c*x + sqrt(c^2*x^2
 - 1)) - 2*sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3
*e + b*c*d^2*e^2)*x))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^
4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)
*x^2), -1/8*((2*a - b)*c^4*d^4 + (4*a - b)*c^2*d^3*e - 2*(b*c^4*d^2*e^2 +
2*b*c^2*d*e^3 + b*e^4)*x^4*log(c*x + sqrt(c^2*x^2 - 1)) + 2*a*d^2*e^2 - (b
*c^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 2*((2*a - b)*c^4*d^3*e + (4*a - b)*c^2*d
^2*e^2 + 2*a*d*e^3)*x^2 - (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*
b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*a
rctan((sqrt(-c^2*d^2 - d*e)*sqrt(c^2*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(
c*e*x^2 + c*d))/(c^2*d^2 + d*e)) - 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{(d + ex^2)^3} dx = \int \frac{(b \operatorname{arcosh}(cx) + a)x^3}{(ex^2 + d)^3} dx$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/8*b*((c^4*d + 2*c^2*e)*log(e*x^2 + d)/(c^4*d^2*e^2 + 2*c^2*d*e^3 + e^4)
 + (c^4*d^3 + c^2*d^2*e + (c^4*d^2*e + c^2*d*e^2)*x^2 + 2*(c^4*d^3 + 2*c^2
*d^2*e + d*e^2 + 2*(c^4*d^2*e + 2*c^2*d*e^2 + e^3)*x^2)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) - (c^4*d^3 + 2*c^2*d^2*e + (c^4*d*e^2 + 2*c^2*e^3)*x^
4 + 2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x + 1) - (c^4*d^3 + 2*c^2*d^2*e
 + (c^4*d*e^2 + 2*c^2*e^3)*x^4 + 2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x
- 1))/(c^4*d^4*e^2 + 2*c^2*d^3*e^3 + d^2*e^4 + (c^4*d^2*e^4 + 2*c^2*d*e^5
+ e^6)*x^4 + 2*(c^4*d^3*e^3 + 2*c^2*d^2*e^4 + d*e^5)*x^2) + 8*integrate(1/
4*(2*c*e*x^2 + c*d)/(c^3*e^4*x^7 - c*d^2*e^2*x + (2*c^3*d*e^3 - c*e^4)*x^5
 + (c^3*d^2*e^2 - 2*c*d*e^3)*x^3 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4
- d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x
 - 1))), x)) - 1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arccosh}(cx))}{(d + ex^2)^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^3\,(a+b\,\text{acosh}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{4\left(\int \frac{acosh(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^3+8\left(\int \frac{acosh(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e\,x^2+4\left(\int \frac{acosh(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right.}{4d\,(e^2x^4+2de\,x^2+d^2)}$$

input
```
int(x^3*(a+b*acosh(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((acosh(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6
),x)*b*d**3 + 8*int((acosh(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**
4 + e**3*x**6),x)*b*d**2*e*x**2 + 4*int((acosh(c*x)*x**3)/(d**3 + 3*d**2*e
*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d*e**2*x**4 + a*x**4)/(4*d*(d**2 +
 2*d*e*x**2 + e**2*x**4))
```

3.406 $\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^3} dx = -\frac{bcx\sqrt{-1+cx}\sqrt{1+cx}}{8d(c^2d+e)(d+ex^2)} - \frac{a+b\text{arccosh}(cx)}{4e(d+ex^2)^2} + \frac{bc(2c^2d+e)\sqrt{-1+c^2x^2}\text{arctanh}\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{8d^{3/2}e(c^2d+e)^{3/2}\sqrt{-1+cx}\sqrt{1+cx}}$$

output
```
-1/8*b*c*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)-1/4*(a+b*arcc
osh(c*x))/e/(e*x^2+d)^2+1/8*b*c*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*arctanh((c^2
*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*d+e)^(3/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^3} dx = \frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx\sqrt{-1+cx}\sqrt{1+cx}(d+ex^2)}{d(c^2d+e)}}{(d+ex^2)^2} - \frac{2b\text{arccosh}(cx)}{e(d+ex^2)^2} - \frac{bc(2c^2d+e)\sqrt{-1+cx}\sqrt{1+cx}\arctan\left(\frac{\sqrt{-c^2d-ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{d^{3/2}(-c^2d-e)^{3/2}e\sqrt{-1+c^2x^2}}\right)$$

input
```
Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
(-(((2*a)/e + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(d*(c^2*d +
 e)))/(d + e*x^2)^2) - (2*b*ArcCosh[c*x])/(e*(d + e*x^2)^2) - (b*c*(2*c^2*
d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])])/(d^(3/2)*(-(c^2*d) - e)^(3/2)*e*Sqrt[-1 + c^2*x^2])
)/8
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6372, 648, 296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arccosh}(cx))}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{ 6372}$$

$$\frac{bc\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}(ex^2+d)^2}dx}{4e}-\frac{a+b\text{arccosh}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 648}$$

$$\frac{bc\sqrt{c^2x^2-1}\int \frac{1}{\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{4e\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 296}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{(2c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx}{2d(c^2d+e)}-\frac{ex\sqrt{c^2x^2-1}}{2d(c^2d+e)(d+ex^2)}\right)}{4e\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{(2c^2d+e)\int \frac{1}{d-\frac{(dc^2+e)x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2d(c^2d+e)}-\frac{ex\sqrt{c^2x^2-1}}{2d(c^2d+e)(d+ex^2)}\right)}{4e\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{4e\,(d+ex^2)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{(2c^2d+e)\text{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2d^{3/2}(c^2d+e)^{3/2}}-\frac{ex\sqrt{c^2x^2-1}}{2d(c^2d+e)(d+ex^2)}\right)}{4e\sqrt{cx-1}\sqrt{cx+1}}-\frac{a+b\text{arccosh}(cx)}{4e\,(d+ex^2)^2}$$

input
```
Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcCosh[c*x])/(e*(d + e*x^2)^2) + (b*c*Sqrt[-1 + c^2*x^2]*(-1/
2*(e*x*Sqrt[-1 + c^2*x^2])/(d*(c^2*d + e)*(d + e*x^2)) + ((2*c^2*d + e)*Ar
cTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(3/2)*(c^2*d
 + e)^(3/2))))/(4*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

rule 6372
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(141) = 282$.

Time = 0.22 (sec) , antiderivative size = 1126, normalized size of antiderivative = 6.74

method	result	size
parts	Expression too large to display	1126
derivativedivides	Expression too large to display	1149
default	Expression too large to display	1149

input
```
int(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)-1/
16*c^4*e^2*(2*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1
/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^6*d^2*e*x^2+2*ln(2*((-(c^2*d+e)/e)^
(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2))
)*c^6*d^3-2*ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1
/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*c^6*x^2*d^2*e-2*ln(-2*(-(-(c^2*d+e)/e
)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2
)))*c^6*d^3+3*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1
/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^4*d*e^2*x^2+3*ln(2*((-(c^2*d+e)/e)^
(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2))
)*c^4*d^2*e-3*ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^
(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*c^4*x^2*d*e^2-3*ln(-2*(-(-(c^2*d+e)
/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1
/2)))*c^4*d^2*e+2*c^3*d*e*(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*(-c^2*d*e
)^(1/2)*x+ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*
c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*e^3*c^2*x^2+ln(2*((-(c^2*d+e)/e)^(1/2)*(c
^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*c^2*d*
e^2-ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+
e)/(e*c*x+(-c^2*d*e)^(1/2)))*c^2*x^2*e^3-ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2
*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*c^2*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(141) = 282$.

Time = 0.20 (sec) , antiderivative size = 1233, normalized size of antiderivative = 7.38

$$\int \frac{x(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(2*a + b)*c^4*d^4 + 2*(4*a + b)*c^2*d^3*e + 4*a*d^2*e^2 + 2*(b*c
^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 4*(b*c^4*d^3*e + b*c^2*d^2*e^2)*x^2 - (2*b
*c^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e +
b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*log(-(2*c^2*d^2 - (4*c^4*d^2 + 4*c^2*d
*e + e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)*((2*c^3*d + c*e)*x^2 - c*d) -
2*sqrt(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*c^2*d + e)*x + 2*(c^3*d^2 + c*
d*e)*x))/(e*x^2 + d)) - 4*((b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2
*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x + sqrt(c^2*x^2 - 1
)) - 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d
*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-
c*x + sqrt(c^2*x^2 - 1)) + 2*sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3
)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e
^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*
d^4*e^3 + d^3*e^4)*x^2), -1/8*((2*a + b)*c^4*d^4 + (4*a + b)*c^2*d^3*e + 2
*a*d^2*e^2 + (b*c^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 2*(b*c^4*d^3*e + b*c^2*d^
2*e^2)*x^2 - (2*b*c^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*
(2*b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*arctan((sqrt(-c^2*d^
2 - d*e)*sqrt(c^2*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c*e*x^2 + c*d))/(c^
2*d^2 + d*e)) - 2*((b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*
d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x + sqrt(c^2*x^2 - 1)) - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(a + b\text{arccosh}(cx))}{(d + ex^2)^3} dx = \int \frac{(b \operatorname{arcosh}(cx) + a)x}{(ex^2 + d)^3} dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/8*(c^4*log(e*x^2 + d)/(c^4*d^2*e + 2*c^2*d*e^2 + e^3) + 8*c*integrate(1
/4/(c^3*e^3*x^7 + (2*c^3*d*e^2 - c*e^3)*x^5 - c*d^2*e*x + (c^3*d^2*e - 2*c
*d*e^2)*x^3 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d^2*e + (c^2*d^2*e
- 2*d*e^2)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x) - (c^4*d^2 +
c^2*d*e + (c^4*d*e + c^2*e^2)*x^2 - 2*(c^4*d^2 + 2*c^2*d*e + e^2)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1)) + (c^4*e^2*x^4 + 2*c^4*d*e*x^2 + c^4*d^2)*l
og(c*x + 1) + (c^4*e^2*x^4 + 2*c^4*d*e*x^2 + c^4*d^2)*log(c*x - 1))/(c^4*d
^4*e + 2*c^2*d^3*e^2 + d^2*e^3 + (c^4*d^2*e^3 + 2*c^2*d*e^4 + e^5)*x^4 + 2
*(c^4*d^3*e^2 + 2*c^2*d^2*e^3 + d*e^4)*x^2))*b - 1/4*a/(e^3*x^4 + 2*d*e^2*
x^2 + d^2*e)
```

Giac [F]

$$\int \frac{x(a + b\text{arccosh}(cx))}{(d + ex^2)^3} dx = \int \frac{(b \operatorname{arcosh}(cx) + a)x}{(ex^2 + d)^3} dx$$

input
```
integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^3}\,dx = \int \frac{x\,(a + b\,\mathrm{acosh}(c\,x))}{(e\,x^2 + d)^3}\,dx$$

input
```
int((x*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a + b\operatorname{arccosh}(cx))}{(d + ex^2)^3}\,dx$$
$$= \frac{4\left(\int \frac{acosh(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e + 8\left(\int \frac{acosh(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)bd\,e^2x^2 + 4\left(\int \frac{acosh(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right.}{4e\,(e^2x^4 + 2de\,x^2 + d^2)}$$

input
```
int(x*(a+b*acosh(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((acosh(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x
)*b*d**2*e + 8*int((acosh(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 +
e**3*x**6),x)*b*d*e**2*x**2 + 4*int((acosh(c*x)*x)/(d**3 + 3*d**2*e*x**2 +
 3*d*e**2*x**4 + e**3*x**6),x)*b*e**3*x**4 - a)/(4*e*(d**2 + 2*d*e*x**2 +
e**2*x**4))
```

3.407 $\int \frac{a+b\text{arccosh}(cx)}{x(d+ex^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 745

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx = \frac{bcex\sqrt{-1+cx}\sqrt{1+cx}}{8d^2\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{a+b\operatorname{arccosh}(cx)}{4d\left(d+ex^2\right)^2}+\frac{a+b\operatorname{arccosh}(cx)}{2d^2\left(d+ex^2\right)}-\frac{bc\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc(2c^2d+e)\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{8d^{5/2}\left(c^2d+e\right)^{3/2}\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^3}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^3}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^3}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^3}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(1+e^{2\operatorname{arccosh}(cx)}\right)}{d^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^3}+\frac{b\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)}{2d^3}$$

output
```
1/8*b*c*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*d+e)/(e*x^2+d)+1/4*(a+b*a
rccosh(c*x))/d/(e*x^2+d)^2+1/2*(a+b*arccosh(c*x))/d^2/(e*x^2+d)-1/2*b*c*(c
^2*x^2-1)^(1/2)*arctanh((c^2*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(5/
2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*b*c*(2*c^2*d+e)*(c^2*x^
2-1)^(1/2)*arctanh((c^2*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(5/2)/(c
^2*d+e)^(3/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(a+b*arccosh(c*x))*ln(1-e^(1
/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3
-1/2*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*
(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3-1/2*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/2*(a+
b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/
2)+(-c^2*d-e)^(1/2)))/d^3+(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)/d^3-1/2*b*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3-1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3-1/2*b*polylog(
2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2
)))/d^3-1/2*b*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^
(1/2)+(-c^2*d-e)^(1/2)))/d^3+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 1147, normalized size of antiderivative = 1.54

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
((b*c*d*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] +
 Sqrt[e]*x)) + (b*c*d*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(
I*Sqrt[d] + Sqrt[e]*x)) + (4*a*d^2)/(d + e*x^2)^2 + (8*a*d)/(d + e*x^2) +
(b*d*ArcCosh[c*x])/(Sqrt[d] - I*Sqrt[e]*x)^2 + (5*b*Sqrt[d]*ArcCosh[c*x])/
(Sqrt[d] - I*Sqrt[e]*x) + (b*d*ArcCosh[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 + (
5*b*Sqrt[d]*ArcCosh[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + 16*b*ArcCosh[c*x]^2 +
16*b*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 8*b*ArcCosh[c*x]*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] - 8*b*ArcCosh
[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])] - 8*b*ArcCosh[c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqr
t[-(c^2*d) - e])] - 8*b*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c
*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 16*a*Log[x] - 8*a*Log[d + e*x^2] - ((5*I
)*b*c*Sqrt[d]*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*S
qrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x
))])/Sqrt[-(c^2*d) - e] + ((5*I)*b*c*Sqrt[d]*Log[(2*e*(-Sqrt[e] - I*c^2*Sq
rt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*
d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (b*c^3*d^(3/2)*Log
[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(c^2*d + e)^(3/
2) - (b*c^3*d^(3/2)*Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*...
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(-\frac{ex(a+b\operatorname{arccosh}(cx))}{d^3\left(d+ex^2\right)}+\frac{a+b\operatorname{arccosh}(cx)}{d^3x}-\frac{ex(a+b\operatorname{arccosh}(cx))}{d^2\left(d+ex^2\right)^2}-\frac{ex(a+b\operatorname{arccosh}(cx))}{d\left(d+ex^2\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^3}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^3}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d^3}+\frac{(a+b\operatorname{arccosh}(cx))^2}{bd^3}+\frac{\log\left(e^{-2\operatorname{arccosh}(cx)}+1\right)(a+b\operatorname{arccosh}(cx))}{d^3}+\frac{a+b\operatorname{arccosh}(cx)}{2d^2(d+ex^2)}+\frac{a+b\operatorname{arccosh}(cx)}{4d(d+ex^2)^2}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}(cx)}\right)}{2d^3}-\frac{bc\sqrt{c^2x^2-1}(2c^2d+e)\operatorname{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8d^{5/2}\sqrt{cx-1}\sqrt{cx+1}(c^2d+e)^{3/2}}-\frac{bc\sqrt{c^2x^2-1}\operatorname{arctanh}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2d^{5/2}\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}-\frac{bcex(1-c^2x^2)}{8d^2\sqrt{cx-1}\sqrt{cx+1}(c^2d+e)(d+ex^2)}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
-1/8*(b*c*e*x*(1 - c^2*x^2))/(d^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(d + e*x^2)) + (a + b*ArcCosh[c*x])/(4*d*(d + e*x^2)^2) + (a + b*ArcCosh[
c*x])/(2*d^2*(d + e*x^2)) + (a + b*ArcCosh[c*x])^2/(b*d^3) - (b*c*Sqrt[-1
+ c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d
^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(2*c^2*d + e)*
Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2]
)])/(8*d^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((a + b*A
rcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^3 - ((a + b*ArcCosh[c*x])*Log
[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3)
- ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*
x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(
2*d^3) - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d^3) - (b*PolyLog[2, -((S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^3) - (b*P
olyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*
d^3) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d
) - e]))])/(2*d^3) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 1225, normalized size of antiderivative = 1.64

method	result	size
parts	Expression too large to display	1225
derivativedivides	Expression too large to display	1278
default	Expression too large to display	1278

input
```
int((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/d^2/(e*x^2+d)+1/4*a/d/(e*x^2+d)^2-1/2*a/d^3*ln(e*x^2+d)+a/d^3*ln(x)+
b*(1/8*c^2*(6*c^4*d^2*arccosh(c*x)+4*arccosh(c*x)*c^4*d*e*x^2+c^3*d*e*x*(c
*x-1)^(1/2)*(c*x+1)^(1/2)+c^3*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c^4*d^2-
2*c^4*d*e*x^2-c^4*e^2*x^4+6*c^2*d*e*arccosh(c*x)+4*arccosh(c*x)*e^2*c^2*x^
2)/d^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)+5/8*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)
^2/d^3*e*arctanh(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)
/(c^4*d^2+c^2*d*e)^(1/2))+3/4*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*
arctanh(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2
+c^2*d*e)^(1/2))-1/4/(c^2*d+e)/d^3*e*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^
2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((
_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)
*_Z^2+e))+1/(c^2*d+e)/d^3*e*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))+1/(c^2*d+e)/d^3*e*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))+1/(c^2*d+e)/d^3*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/(
c^2*d+e)/d^3*e*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-1/4/(c^2*d+e)/
d^3*e^2*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R
1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/4/(c^2*d+e)/d^2*c^2*sum((_R
1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)...
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\,\text{acosh}(c\,x)}{x\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*acosh(c*x))/(x*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x\left(d+ex^2\right)^3}\,dx$$
$$=\frac{4\left(\int \frac{acosh(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right)b\,d^5+8\left(\int \frac{acosh(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right)b\,d^4e\,x^2+4\left(\int \frac{acosh(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}\right.}{}$$

input
```
int((a+b*acosh(c*x))/x/(e*x^2+d)^3,x)
```

output
```
(4*int(acosh(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**7),x)*
b*d**5 + 8*int(acosh(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x
**7),x)*b*d**4*e*x**2 + 4*int(acosh(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**
2*x**5 + e**3*x**7),x)*b*d**3*e**2*x**4 - 2*log(d + e*x**2)*a*d**2 - 4*log
(d + e*x**2)*a*d*e*x**2 - 2*log(d + e*x**2)*a*e**2*x**4 + 4*log(x)*a*d**2
+ 8*log(x)*a*d*e*x**2 + 4*log(x)*a*e**2*x**4 + 2*a*d**2 - a*e**2*x**4)/(4*
d**3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.408 $\int \frac{a+b\,\mathrm{arccosh}(cx)}{x^3\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 805

$$\begin{aligned}
\int \frac{a+b\operatorname{arccosh}(cx)}{x^3(d+ex^2)^3}\,dx &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} - \frac{bce^2x\sqrt{-1+cx}\sqrt{1+cx}}{8d^3(c^2d+e)(d+ex^2)} \\
&- \frac{a+b\operatorname{arccosh}(cx)}{2d^3x^2} - \frac{e(a+b\operatorname{arccosh}(cx))}{4d^2(d+ex^2)^2} \\
&- \frac{e(a+b\operatorname{arccosh}(cx))}{d^3(d+ex^2)} \\
&+ \frac{bce\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{d^{7/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{bce(2c^2d+e)\sqrt{-1+c^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{8d^{7/2}(c^2d+e)^{3/2}\sqrt{-1+cx}\sqrt{1+cx}} \\
&+ \frac{3e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^4} \\
&+ \frac{3e(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^4} \\
&+ \frac{3e(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^4} \\
&+ \frac{3e(a+b\operatorname{arccosh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^4} \\
&- \frac{3e(a+b\operatorname{arccosh}(cx))\log\left(1+e^{2\operatorname{arccosh}(cx)}\right)}{d^4} \\
&+ \frac{3be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^4} \\
&+ \frac{3be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^4} \\
&+ \frac{3be\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^4} \\
&+ \frac{3be\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2d^4} \\
&- \frac{3be\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(cx)}\right)}{2d^4}
\end{aligned}$$

output
```
1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/x-1/8*b*c*e^2*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^3/(c^2*d+e)/(e*x^2+d)-1/2*(a+b*arccosh(c*x))/d^3/x^2-1/4*e*(a+
b*arccosh(c*x))/d^2/(e*x^2+d)^2-e*(a+b*arccosh(c*x))/d^3/(e*x^2+d)+b*c*e*(
c^2*x^2-1)^(1/2)*arctanh((c^2*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(7
/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*e*(2*c^2*d+e)*(c^2
*x^2-1)^(1/2)*arctanh((c^2*d+e)^(1/2)*x/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(7/2)
/(c^2*d+e)^(3/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/2*e*(a+b*arccosh(c*x))*ln(1
-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2))
)/d^4+3/2*e*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^4+3/2*e*(a+b*arccosh(c*x))*ln(1-e^(
1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^
4+3/2*e*(a+b*arccosh(c*x))*ln(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/
(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^4-3*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2)/d^4+3/2*b*e*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,
e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))
/d^4+3/2*b*e*polylog(2,-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^
(1/2)+(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,e^(1/2)*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^4-3/2*b*e*polylog(2,-(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^4
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 1261, normalized size of antiderivative = 1.57

$$\int \frac{a+b\text{arccosh}(cx)}{x^3(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
```

output

```
-1/2*a/(d^3*x^2) - (a*e)/(4*d^2*(d + e*x^2)^2) - (a*e)/(d^3*(d + e*x^2)) -
 (3*a*e*Log[x])/d^4 + (3*a*e*Log[d + e*x^2])/(2*d^4) + b*((c*x*Sqrt[-1 + c
*x]*Sqrt[1 + c*x] - ArcCosh[c*x])/(2*d^3*x^2) + (((9*I)/16)*e*(ArcCosh[c*x
]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*
Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(S
qrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]))/d^(7/2) + (((9*I)/16)*e*(-(A
rcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[
d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d)
- e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]))/d^(7/2) - (e^(3/2)*((
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) -
 ArcCosh[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4
] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e
]*(c^2*d + e)^(3/2))))/(16*d^3) - (e^(3/2)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d
] + Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sq
rt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^
3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/(16*d^3) - (
3*e*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) - PolyLo
g[2, -E^(-2*ArcCosh[c*x])]))/(2*d^4) + (3*e*(ArcCosh[c*x]*(-ArcCosh[c*x...
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3 (d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{ 6374}$$

$$\int \left(\frac{3e^2x(a+b\text{arccosh}(cx))}{d^4 (d+ex^2)}-\frac{3e(a+b\text{arccosh}(cx))}{d^4x}+\frac{2e^2x(a+b\text{arccosh}(cx))}{d^3 (d+ex^2)^2}+\frac{a+b\text{arccosh}(cx)}{d^3x^3}+\frac{e^2x(a+b\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bcx(1-c^2x^2)\,e^2}{8d^3(dc^2+e)\sqrt{cx-1}\sqrt{cx+1}\,(ex^2+d)}-\frac{3(a+b\operatorname{arccosh}(cx))^2e}{bd^4}-\frac{(a+b\operatorname{arccosh}(cx))e}{d^3(ex^2+d)}-\frac{(a+b\operatorname{arccosh}(cx))e}{4d^2(ex^2+d)^2}+\frac{bc(2dc^2+e)\sqrt{c^2x^2-1}\operatorname{arctanh}\left(\frac{\sqrt{dc^2+ex}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)e}{8d^{7/2}(dc^2+e)^{3/2}\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{c^2x^2-1}\operatorname{arctanh}\left(\frac{\sqrt{dc^2+ex}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)e}{d^{7/2}\sqrt{dc^2+e}\sqrt{cx-1}\sqrt{cx+1}}-\frac{3(a+b\operatorname{arccosh}(cx))\log\left(1+e^{-2\operatorname{arccosh}(cx)}\right)e}{d^4}+\frac{3(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)e}{2d^4}+\frac{3(a+b\operatorname{arccosh}(cx))\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)e}{2d^4}+\frac{3(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)e}{2d^4}+\frac{3(a+b\operatorname{arccosh}(cx))\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{\sqrt{-d}c+\sqrt{-dc^2-e}}+1\right)e}{2d^4}+\frac{3b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}(cx)}\right)e}{2d^4}+\frac{3b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)e}{2d^4}+\frac{3b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)e}{2d^4}+\frac{3b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)e}{2d^4}+\frac{3b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)e}{2d^4}-\frac{a+b\operatorname{arccosh}(cx)}{2d^3x^2}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2d^3x}$$

input
```
Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
```

output
```
(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^3*x) + (b*c*e^2*x*(1 - c^2*x^2))/(
8*d^3*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2)) - (a + b*ArcCo
sh[c*x])/(2*d^3*x^2) - (e*(a + b*ArcCosh[c*x]))/(4*d^2*(d + e*x^2)^2) - (e
*(a + b*ArcCosh[c*x]))/(d^3*(d + e*x^2)) - (3*e*(a + b*ArcCosh[c*x])^2)/(b
*d^4) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqr
t[-1 + c^2*x^2])])/(d^(7/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*c*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqr
t[d]*Sqrt[-1 + c^2*x^2])])/(8*d^(7/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (3*e*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^4
+ (3*e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
 Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*e*(a +
b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyLog[2, -E^(
-2*ArcCosh[c*x])])/(2*d^4) + (3*b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/
(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^4) + (3*b*e*PolyLog[2, (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyL
og[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d
^4) + (3*b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.08 (sec) , antiderivative size = 1455, normalized size of antiderivative = 1.81

method	result	size
parts	Expression too large to display	1455
derivativedivides	Expression too large to display	1515
default	Expression too large to display	1515

input
```
int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-a*e/d^3/(e*x^2+d)-1/4*a*e/d^2/(e*x^2+d)^2+3/2*a*e/d^4*ln(e*x^2+d)-1/2*a/d
^3/x^2-3*a/d^4*e*ln(x)+b*c^2*(-1/8*(-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^7*d^3
*x-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^7*d^2*e*x^3-4*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*c^7*d*e^2*x^5+4*c^8*d^3*x^2+8*c^8*d^2*e*x^4+4*c^8*d*e^2*x^6+4*c^6*d^3*a
rccosh(c*x)+18*arccosh(c*x)*c^6*d^2*e*x^2+12*arccosh(c*x)*c^6*d*e^2*x^4-4*
c^5*d^2*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-7*c^5*d*e^2*x^3*(c*x-1)^(1/2)*(c*x
+1)^(1/2)-3*c^5*e^3*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)+3*c^6*d^2*e*x^2+6*c^6*
d*e^2*x^4+3*c^6*e^3*x^6+4*c^4*d^2*e*arccosh(c*x)+18*arccosh(c*x)*c^4*d*e^2
*x^2+12*arccosh(c*x)*e^3*c^4*x^4)/c^2/x^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2/d^
3-9/8*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^4/c^2*e^2*arctanh(1/4*(4*c^2*d
+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-5/4
*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arctanh(1/4*(4*c^2*d+2*e*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))+3/4/(c^2*d+e)/
d^4*e^2/c^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((
_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/(c^2*d+e)/d^4
*e^2/c^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3/(c^2*d+e
)/d^4*e^2/c^2*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3/(c^
2*d+e)/d^4*e^2/c^2*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3/(c^2*d+e
)/d^4*e^2/c^2*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+3/4/(c^2*d+e...
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x
^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arcosh}\left(cx\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x
^2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + b*integrate(log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3)
, x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^3(d+ex^2)^3}\,dx = \int \frac{b\operatorname{arcosh}(cx)+a}{(ex^2+d)^3x^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^3(d+ex^2)^3}\,dx = \int \frac{a+b\operatorname{acosh}(c\,x)}{x^3\,(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^3),x)
```

output
```
int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arccosh}(cx)}{x^3(d+ex^2)^3}\,dx$$
$$= \frac{4\left(\int \frac{acosh(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5+d^3x^3}dx\right)b\,d^6x^2+8\left(\int \frac{acosh(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5+d^3x^3}dx\right)b\,d^5e\,x^4+4\left(\int \frac{acosh(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5}\right.}{}$$

input
```
int((a+b*acosh(c*x))/x^3/(e*x^2+d)^3,x)
```

output
```
(4*int(acosh(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x**7 + e**3*x**9),
x)*b*d**6*x**2 + 8*int(acosh(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x*
*7 + e**3*x**9),x)*b*d**5*e*x**4 + 4*int(acosh(c*x)/(d**3*x**3 + 3*d**2*e*
x**5 + 3*d*e**2*x**7 + e**3*x**9),x)*b*d**4*e**2*x**6 + 6*log(d + e*x**2)*
a*d**2*e*x**2 + 12*log(d + e*x**2)*a*d*e**2*x**4 + 6*log(d + e*x**2)*a*e**
3*x**6 - 12*log(x)*a*d**2*e*x**2 - 24*log(x)*a*d*e**2*x**4 - 12*log(x)*a*e
**3*x**6 - 2*a*d**3 - 6*a*d**2*e*x**2 + 3*a*e**3*x**6)/(4*d**4*x**2*(d**2
+ 2*d*e*x**2 + e**2*x**4))
```

3.409 $\int \frac{x^4(a+b\,\mathbf{arccosh}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1224

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/16*b*c*(-d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/e^2/(c^2*d+e)/((-d)^(1/2)
-e^(1/2)*x)-1/16*b*c*(-d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/e^2/(c^2*d+e)/
((-d)^(1/2)+e^(1/2)*x)-1/16*(-d)^(1/2)*(a+b*arccosh(c*x))/e^(5/2)/((-d)^(1
/2)-e^(1/2)*x)^2+5/16*(a+b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)+1/
16*(-d)^(1/2)*(a+b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)+e^(1/2)*x)^2-5/16*(a+
b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)+e^(1/2)*x)-1/8*b*c^3*d*arctanh((c*(-d)
^(1/2)-e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(
1/2))/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)/e^(5/2)-5/
8*b*c*arctanh((c*(-d)^(1/2)-e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(
1/2))^(1/2)/(c*x-1)^(1/2))/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1
/2))^(1/2)/e^(5/2)+1/8*b*c^3*d*arctanh((c*(-d)^(1/2)+e^(1/2))^(1/2)*(c*x+1
)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2))/(c*(-d)^(1/2)-e^(1/2))
^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)/e^(5/2)+5/8*b*c*arctanh((c*(-d)^(1/2)+
e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2))/(
c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(5/2)+3/16*(a+b
*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2
)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a+b*arccosh(c*x))*ln(1+e^(1/
2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)
^(1/2)/e^(5/2)+3/16*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.12 (sec) , antiderivative size = 1185, normalized size of antiderivative = 0.97

$$\int \frac{x^4(a + b\text{arccosh}(cx))}{(d + ex^2)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
(a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((-5*(ArcCosh[c*x]/((-I)*Sqr
t[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*
d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*
Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + (5*(-(ArcCosh[c*x]/(I*Sq
rt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2
*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] +
 Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + ((I/16)*Sqrt[d]*((c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcC
osh[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + L
og[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^
2*d + e)^(3/2))))/e^2 - ((I/16)*Sqrt[d]*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/
((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] +
 Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[
e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(
d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/e^2 + (((3*I)/3
2)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c
*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sq
rt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/...
```

Rubi [A] (verified)

Time = 5.22 (sec) , antiderivative size = 1224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{d^2(a+b\text{arccosh}(cx))}{e^2(d+ex^2)^3}-\frac{2d(a+b\text{arccosh}(cx))}{e^2(d+ex^2)^2}+\frac{a+b\text{arccosh}(cx)}{e^2(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bd\text{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{5/2}}+\frac{bd\text{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{5/2}}-\frac{5b\text{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c}{8\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}+\frac{5b\text{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c}{8\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}-\frac{b\sqrt{-d}\sqrt{cx-1}\sqrt{cx+1}c}{16e^2\left(dc^2+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{b\sqrt{-d}\sqrt{cx-1}\sqrt{cx+1}c}{16e^2\left(dc^2+e\right)\left(\sqrt{e}x+\sqrt{-d}\right)}+\frac{5(a+b\text{arccosh}(cx))}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{5(a+b\text{arccosh}(cx))}{16e^{5/2}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{\sqrt{-d}(a+b\text{arccosh}(cx))}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2}+\frac{\sqrt{-d}(a+b\text{arccosh}(cx))}{16e^{5/2}\left(\sqrt{e}x+\sqrt{-d}\right)^2}+\frac{3(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3(a+b\text{arccosh}(cx))\log\left(\frac{e^{\text{arccosh}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)}{16\sqrt{-d}e^{5/2}}+\frac{3(a+b\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3(a+b\text{arccosh}(cx))\log\left(\frac{e^{\text{arccosh}(cx)}\sqrt{e}}{\sqrt{-d}c+\sqrt{-dc^2-e}}+1\right)}{16\sqrt{-d}e^{5/2}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16\sqrt{-d}e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*(b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(e^2*(c^2*d + e)*(Sqrt[-
d] - Sqrt[e]*x)) - (b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^
2*d + e)*(Sqrt[-d] + Sqrt[e]*x)) - (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(
5/2)*(Sqrt[-d] - Sqrt[e]*x)^2) + (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqr
t[-d] - Sqrt[e]*x)) + (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d
] + Sqrt[e]*x)^2) - (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqrt[
e]*x)) - (b*c^3*d*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt
[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(
c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(5/2)) - (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - S
qrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*Sq
rt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) + (b*c^3*d*Ar
cTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e
]]*Sqrt[-1 + c*x])])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e]
)^(3/2)*e^(5/2)) + (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x
])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(8*Sqrt[c*Sqrt[-d] - Sqrt
[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*Sqrt[-d]
*e^(5/2)) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] - Sqrt[-(c^2*d) - e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCosh[c
*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.97 (sec) , antiderivative size = 1752, normalized size of antiderivative = 1.43

method	result	size
parts	Expression too large to display	1752
derivativedivides	Expression too large to display	1754
default	Expression too large to display	1754

input
```
int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2)))+b/c^5*(-1/8*c^6*(c^4*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+c^4*d*e*x^
2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+3*arccosh(c*x)*d^2*c^5*x+5*arccosh(c*x)*d*c^
5*e*x^3+3*arccosh(c*x)*d*c^3*e*x+5*arccosh(c*x)*e^2*c^3*x^3)/e^2/(c^2*e*x^
2+c^2*d)^2/(c^2*d+e)-5/8*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(
-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/
2)*e)*c^6*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)^2+5/8*((2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/
2))*c^6/e^4/(c^2*d+e)-5/8*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*(2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1
/2)*e)*c^6*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d
*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)^2+5/8*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*
e)^(1/2))*c^6/e^4/(c^2*d+e)-3/16/e/(c^2*d+e)*c^6*sum(1/_R1/(_R1^2*e+2*c^2*
d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_
Z^2+e))+3/16/e/(c^2*d+e)*c^6*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*...
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccosh(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^4\,(a+b\,\text{acosh}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{acosh}{e^3x^6+3d\,e^2x}\right.}{8d}$$

input
```
int(x^4*(a+b*acosh(c*x))/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int((acosh(c*x)*x**4)/(d**3 + 3*d**2*e*
x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3 + 16*int((acosh(c*x)*x**4
)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**2*e**4*x**2 +
 8*int((acosh(c*x)*x**4)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6
),x)*b*d*e**5*x**4 - 3*a*d**2*e*x - 5*a*d*e**2*x**3)/(8*d*e**3*(d**2 + 2*d
*e*x**2 + e**2*x**4))
```

3.410 $\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1234

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(1/2)/e/(c^2*d+e)/((-d)^(1/2)-e
^(1/2)*x)-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(1/2)/e/(c^2*d+e)/((-d
)^(1/2)+e^(1/2)*x)-1/16*(a+b*arccosh(c*x))/(-d)^(1/2)/e^(3/2)/((-d)^(1/2)-
e^(1/2)*x)^2-1/16*(a+b*arccosh(c*x))/d/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)+1/16
*(a+b*arccosh(c*x))/(-d)^(1/2)/e^(3/2)/((-d)^(1/2)+e^(1/2)*x)^2+1/16*(a+b*
arccosh(c*x))/d/e^(3/2)/((-d)^(1/2)+e^(1/2)*x)+1/8*b*c^3*arctanh((c*(-d)^(
1/2)-e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/
2))/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)/e^(3/2)+1/8*
b*c*arctanh((c*(-d)^(1/2)-e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/
2))^(1/2)/(c*x-1)^(1/2))/d/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1
/2))^(1/2)/e^(3/2)-1/8*b*c^3*arctanh((c*(-d)^(1/2)+e^(1/2))^(1/2)*(c*x+1)^
(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2))/(c*(-d)^(1/2)-e^(1/2))^(
3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)/e^(3/2)-1/8*b*c*arctanh((c*(-d)^(1/2)+e^
(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2))/d/(
c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(3/2)-1/16*(a+b
*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2
)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccosh(c*x))*ln(1+e^(1/
2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)
^(3/2)/e^(3/2)-1/16*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 1143, normalized size of antiderivative = 0.93

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-8*a*Sqrt[e]*x)/(d + e*x^2)^2 + (4*a*Sqrt[e]*x)/(d^2 + d*e*x^2) + (4*a*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*b*(ArcCosh[c*x]/((-I)*Sqrt[d] + S
qrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*
x))])/Sqrt[-(c^2*d) - e]))/d + (2*b*(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)
+ (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[
-(c^2*d) - e]))/d - (2*I)*b*((c*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(Sqr
t[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) + ArcCosh[c*x]/(Sqrt[d]*(Sqrt
[d] + I*Sqrt[e]*x)^2) + (c^3*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e
] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d
 + I*Sqrt[d]*Sqrt[e]*x))]))/(c^2*d + e)^(3/2)) + (2*I)*b*((c*Sqrt[e]*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) + A
rcCosh[c*x]/(Sqrt[d]*(Sqrt[d] - I*Sqrt[e]*x)^2) - (c^3*(Log[4] + Log[(e*Sq
rt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(c^2*d + e)^(3/2)) +
 (I*b*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(
I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c
*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/
((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcC...
```

Rubi [A] (verified)

Time = 4.14 (sec) , antiderivative size = 1234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{6374}$$

$$\int \left(\frac{a+b\text{arccosh}(cx)}{e(d+ex^2)^2}-\frac{d(a+b\text{arccosh}(cx))}{e(d+ex^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{3/2}}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{3/2}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c}{8d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{3/2}}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c}{8d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{3/2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}c}{16\sqrt{-d}e\left(dc^2+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{b\sqrt{cx-1}\sqrt{cx+1}c}{16\sqrt{-d}e\left(dc^2+e\right)\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{a+b\operatorname{arccosh}(cx)}{16de^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\operatorname{arccosh}(cx)}{16de^{3/2}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{a+b\operatorname{arccosh}(cx)}{16\sqrt{-d}e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2}+\frac{a+b\operatorname{arccosh}(cx)}{16\sqrt{-d}e^{3/2}\left(\sqrt{e}x+\sqrt{-d}\right)^2}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-\frac{(a+b\operatorname{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{(a+b\operatorname{arccosh}(cx))\log\left(\frac{e^{\operatorname{arccosh}(cx)}\sqrt{e}}{\sqrt{-d}c+\sqrt{-dc^2-e}}+1\right)}{16(-d)^{3/2}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d]
 - Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d
+ e)*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(
Sqrt[-d] - Sqrt[e]*x)^2) - (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] -
Sqrt[e]*x)) + (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] + Sqrt[e
]*x)^2) + (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*
c^3*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] +
Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + S
qrt[e])^(3/2)*e^(3/2)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 +
 c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d*Sqrt[c*Sqrt[-d]
- Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) - (b*c^3*ArcTanh[(Sqrt[c*Sq
rt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x
])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(3/2))
 - (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d
] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt
[-d] + Sqrt[e]]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a
+ b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^
2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.79 (sec) , antiderivative size = 1222, normalized size of antiderivative = 0.99

method	result	size
parts	Expression too large to display	1222
derivativedivides	Expression too large to display	1237
default	Expression too large to display	1237

input
```
int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2)))+b/c^3*(1/8*c^4*(arccosh(c*x)*d*c^5*e*x^3-arccosh(c*x)*d^2*c^5*x+c^4*
d*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+c^4*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+ar
ccosh(c*x)*e^2*c^3*x^3-arccosh(c*x)*d*c^3*e*x)/e/(c^2*e*x^2+c^2*d)^2/d/(c^
2*d+e)+1/16/d/(c^2*d+e)*c^4*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((
_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/16/e/(c^2*d+e
)*c^6*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=
RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/8*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*
d*(c^2*d+e))^(1/2)*e)*c^4*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/d/e^3-1/8*((2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
+e)*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2))*c^4/d/(c^2*d+e)/e^3+1/8*(-(2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*(2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e+
(c^2*d*(c^2*d+e))^(1/2)*e)*c^4*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/d/e^3-1/8*(
-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*...
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccosh(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^2\,(a+b\,\text{acosh}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{acosh(cx}{e^3x^6+3d\,e^2x^4+}\right.}{8d^2e^2}$$

input
```
int(x^2*(a+b*acosh(c*x))/(e*x^2+d)^3,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 2*sqrt(e)*sqrt(d)*
atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*a*e**2*x**4 + 8*int((acosh(c*x)*x**2)/(d**3 + 3*d**2*e*x**2
 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2 + 16*int((acosh(c*x)*x**2)/(d
**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**2 + 8*i
nt((acosh(c*x)*x**2)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)
*b*d**2*e**4*x**4 - a*d**2*e*x + a*d*e**2*x**3)/(8*d**2*e**2*(d**2 + 2*d*e
*x**2 + e**2*x**4))
```

3.411 $\int \frac{a+b\,\mathrm{arccosh}(cx)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1234

$$\int \frac{a+b\,\mathrm{arccosh}(cx)}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)-e^(
1/2)*x)-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1
/2)+e^(1/2)*x)-1/16*(a+b*arccosh(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)-e^(1
/2)*x)^2-3/16*(a+b*arccosh(c*x))/d^2/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/16*(
a+b*arccosh(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)^2+3/16*(a+b*ar
ccosh(c*x))/d^2/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)-1/8*b*c^3*arctanh((c*(-d)^(
1/2)-e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/
2))/d/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)/e^(1/2)+3/
8*b*c*arctanh((c*(-d)^(1/2)-e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)+e^(
1/2))^(1/2)/(c*x-1)^(1/2))/d^2/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+
e^(1/2))^(1/2)/e^(1/2)+1/8*b*c^3*arctanh((c*(-d)^(1/2)+e^(1/2))^(1/2)*(c*x
+1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2))/d/(c*(-d)^(1/2)-e^(1
/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)/e^(1/2)-3/8*b*c*arctanh((c*(-d)^(1
/2)+e^(1/2))^(1/2)*(c*x+1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2
))/d^2/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)/e^(1/2)+3
/16*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccosh(c*x))*ln
(1+e^(1/2)*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2
)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccosh(c*x))*ln(1-e^(1/2)*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.24 (sec) , antiderivative size = 1161, normalized size of antiderivative = 0.94

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]
```

output
```
((8*a*d^(3/2)*x)/(d + e*x^2)^2 + (12*a*Sqrt[d]*x)/(d + e*x^2) + (12*a*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (6*b*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d
] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqr
t[e]*x))])/Sqrt[-(c^2*d) - e]))/Sqrt[e] - (6*b*Sqrt[d]*(-(ArcCosh[c*x]/(I*
Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c
^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d]
 + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]))/Sqrt[e] + (2*I)*b*d*((c*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/
(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqr
t[c^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^
(3/2))) - (2*I)*b*d*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt
[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^
3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x +
 Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e
]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))) + ((3*I)*b*(ArcCosh[c*x]*(-ArcCosh[c
*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e
])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]
)) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2...
```

Rubi [A] (verified)

Time = 2.85 (sec) , antiderivative size = 1234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6324, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arccosh}(cx)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{6324}$$

$$\int \left(-\frac{3e(a+b\operatorname{arccosh}(cx))}{8d^2(-de-e^2x^2)}-\frac{3e(a+b\operatorname{arccosh}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{3e(a+b\operatorname{arccosh}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e^{3/2}(a+b\operatorname{arccosh}(cx))}{8(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-ex\right)^3}-\frac{e}{8(\ldots}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{3b\,\text{arctanh}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c}{8d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{e}}-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c}{8d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{e}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{b\sqrt{cx-1}\sqrt{cx+1}c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{3(a+b\,\text{arccosh}(cx))}{16d^2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{3(a+b\,\text{arccosh}(cx))}{16d^2\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{a+b\,\text{arccosh}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)^2}+\frac{a+b\,\text{arccosh}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)^2}+\frac{3(a+b\,\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3(a+b\,\text{arccosh}(cx))\log\left(\frac{e^{\text{arccosh}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3(a+b\,\text{arccosh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3(a+b\,\text{arccosh}(cx))\log\left(\frac{e^{\text{arccosh}(cx)}\sqrt{e}}{\sqrt{-d}c+\sqrt{-dc^2-e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arccosh}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]
```

output
```
-1/16*(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]
 - Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d
+ e)*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]
*(Sqrt[-d] - Sqrt[e]*x)^2) - (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqr
t[-d] - Sqrt[e]*x)) + (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d
] + Sqrt[e]*x)^2) + (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] + S
qrt[e]*x)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sq
rt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/
2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*Sqrt[e]) + (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d]
 - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(
8*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (b*
c^3*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] -
Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] +
 Sqrt[e])^(3/2)*Sqrt[e]) - (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt
[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d^2*Sqrt[c*Sqr
t[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (3*(a + b*ArcCosh[c
*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqr...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6324
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.65 (sec) , antiderivative size = 1778, normalized size of antiderivative = 1.44

method	result	size
parts	Expression too large to display	1778
derivativedivides	Expression too large to display	1803
default	Expression too large to display	1803

input

```
int((a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))+b/c*(1/8*c^2*(5*arccosh(c*x)*d^2*c^5*x+3*arccosh(c*x)*d*c^
5*e*x^3-c^4*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c^4*d*e*x^2*(c*x-1)^(1/2)*(c*x
+1)^(1/2)+5*arccosh(c*x)*d*c^3*e*x+3*arccosh(c*x)*e^2*c^3*x^3)/d^2/(c^2*e*
x^2+c^2*d)^2/(c^2*d+e)+3/8*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(
1/2)*e)*c^2*arctan(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*
(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/d^2/e^2-3/8*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*c^2*arct
an(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)
+e)*e)^(1/2))/(c^2*d+e)/d^2/e^2+3/8*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e
)*e)^(1/2)*(2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e+(c^2*d*(c^
2*d+e))^(1/2)*e)*c^2*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/d^2/e^2-3/8*(-(2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)
+e)*c^2*arctanh(e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)/d^2/e^2-3/16/(c^2*d+e)/d^2*c^2*e*sum
(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(
e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+3/16/(c^2*d+e)/d^2*c^2*e*sum(_R1/(_R1^2*e...
```

Fricas [F]

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^3}\,dx = \int \frac{b\,\text{arcosh}\,(cx) + a}{(ex^2 + d)^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(cx)}{(d + ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\,\text{arcosh}\,(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\,\text{acosh}(c\,x)}{(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*acosh(c*x))/(d + e*x^2)^3,x)
```

output
```
int((a + b*acosh(c*x))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(cx)}{(d+ex^2)^3}\,dx$$
$$= \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{acos}{e^3x^6+3d\,e^2x}\right.}{8d}$$

input
```
int((a+b*acosh(c*x))/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int(acosh(c*x)/(d**3 + 3*d**2*e*x**2 +
3*d*e**2*x**4 + e**3*x**6),x)*b*d**5*e + 16*int(acosh(c*x)/(d**3 + 3*d**2*
e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2*x**2 + 8*int(acosh(c*x)
/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**4 +
5*a*d**2*e*x + 3*a*d*e**2*x**3)/(8*d**3*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.412 $\int (fx)^m \left(d+ex^2\right)^3 (a+b\,\mathbf{arccosh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 518

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\operatorname{arccosh}(cx))\,dx =$$

$$-\frac{be\Big(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+342m+119m^2+18m^3+m\right.}{c^5f^2(3+m)^2(5+m)^2(7+m)^2}$$

$$-\frac{be^2\left(3c^2d(7+m)^2+e\left(30+11m+m^2\right)\right)(fx)^{4+m}\sqrt{-1+cx}\sqrt{1+cx}}{c^3f^4(5+m)^2(7+m)^2}$$

$$-\frac{be^3(fx)^{6+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^6(7+m)^2}+\frac{d^3(fx)^{1+m}(a+b\operatorname{arccosh}(cx))}{f(1+m)}$$

$$+\frac{3d^2e(fx)^{3+m}(a+b\operatorname{arccosh}(cx))}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}(a+b\operatorname{arccosh}(cx))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\operatorname{arccosh}(cx))}{f^7(7+m)}$$

$$-\frac{b\left(\frac{c^6d^3(3+m)(5+m)(7+m)}{1+m}+\frac{e(2+m)\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+342m+119m^2+18m^3+m^4\right)\right)}{(3+m)(5+m)(7+m)}\right)}{c^5f^2(2+m)(3+m)(5+m)(7+m)\sqrt{-1+c}}$$

output
```
-b*e*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18
*m^3+119*m^2+342*m+360))*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/f^2/(
3+m)^2/(5+m)^2/(7+m)^2-b*e^2*(3*c^2*d*(7+m)^2+e*(m^2+11*m+30))*(f*x)^(4+m)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/f^4/(5+m)^2/(7+m)^2-b*e^3*(f*x)^(6+m)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/f^6/(7+m)^2+d^3*(f*x)^(1+m)*(a+b*arccosh(c*x))/
f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccosh(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+
m)*(a+b*arccosh(c*x))/f^5/(5+m)+e^3*(f*x)^(7+m)*(a+b*arccosh(c*x))/f^7/(7+
m)-b*(c^6*d^3*(3+m)*(5+m)*(7+m)/(1+m)+e*(2+m)*(3*c^2*d*e*(7+m)^2*(m^2+7*m+
12)+3*c^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18*m^3+119*m^2+342*m+360))/(3+m)/(5
+m)/(7+m))*(f*x)^(2+m)*(-c*x+1)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c
^2*x^2)/c^5/f^2/(2+m)/(3+m)/(5+m)/(7+m)/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.77

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx$$
$$= x(fx)^m \left(\frac{d^3(a+b\text{arccosh}(cx))}{1+m}+\frac{3d^2ex^2(a+b\text{arccosh}(cx))}{3+m}+\frac{3de^2x^4(a+b\text{arccosh}(cx))}{5+m}+\frac{e^3x^6(a+b\text{arccosh}(cx))}{7+m}-\frac{bce^3x^7\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},4+\frac{m}{2},5+\frac{m}{2},c^2x^2\right)}{(7+m)(8+m)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^3x\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3bcd^2ex^3\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},c^2x^2\right)}{(12+7m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3bcde^2x^5\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6+m}{2},\frac{8+m}{2},c^2x^2\right)}{(5+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
x*(f*x)^m*((d^3*(a + b*ArcCosh[c*x]))/(1 + m) + (3*d^2*e*x^2*(a + b*ArcCos
h[c*x]))/(3 + m) + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/(5 + m) + (e^3*x^6*(
a + b*ArcCosh[c*x]))/(7 + m) - (b*c*e^3*x^7*Sqrt[1 - c^2*x^2]*Hypergeometr
ic2F1[1/2, 4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (b*c*d^3*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/
2, (4 + m)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
3*b*c*d^2*e*x^3*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m
)/2, c^2*x^2])/((12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*d*
e^2*x^5*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2
*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6373, 27, 2113, 2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (fx)^m (a+b\text{arccosh}(cx)) \, dx$$

$$\Big\downarrow \text{6373}$$

$$-bc\int \frac{(fx)^{m+1}\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{(fx)^{m+1}\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}$$

$$\Big\downarrow \; 2113$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}$$

$$\Big\downarrow \; 2340$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int \frac{(fx)^{m+1}\left(\frac{e^2\left(3c^2d(m+7)^2+e\left(m^2+11m+30\right)\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+7)x^2}{m+3}+\frac{c^2d^3(m+7)}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+7)}+\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+6}}{c^2f^5(m+7)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}$$

$$\Big\downarrow \; 1590$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\int \frac{(fx)^{m+1}\left(\frac{d^3(m+5)(m+7)c^4}{m+1}+\frac{e\left(3d^2\left(m^2+12m+35\right)^2c^4+3de(m+7)^2\left(m^2+7m+12\right)c^2+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{c^2x^2-1}}d}{c^2(m+5)}}{c^2(m+7)}\right.}{f\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}$$

$$\Big\downarrow \; 363$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\left(\frac{c^4d^3(m+5)(m+7)}{m+1}+\frac{e(m+2)\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de(m+7)^2\left(m^2+7m+12\right)+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{c^2(m+3)^2(m+5)(m+7)}\right)\int\frac{(fx)^m}{\sqrt{c^2x^2}}}{c^2(m+5)}\right.}{}$$

$$\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}$$

$\downarrow$ 279

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\sqrt{1-c^2x^2}\left(\frac{c^4d^3(m+5)(m+7)}{m+1}+\frac{e(m+2)\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de(m+7)^2\left(m^2+7m+12\right)+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{c^2(m+3)^2(m+5)(m+7)}\right.}{\sqrt{c^2x^2-1}}}{c^2(m}\right.}{}$$

$$\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}$$

$\downarrow$ 278

$$\frac{d^3(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\text{arccosh}(cx))}{f^7(m+7)}-$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+6}}{c^2f^5(m+7)^2}+\frac{\frac{e^2\sqrt{c^2x^2-1}(fx)^{m+4}\left(3c^2d(m+7)^2+e\left(m^2+11m+30\right)\right)}{c^2f^3(m+5)^2(m+7)}+\frac{e\sqrt{c^2x^2-1}(fx)^{m+2}\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de\right.}{c^2f(m}}{}\right.}{}$$

input
```
Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
 m)*(a + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*Ar
cCosh[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7
*(7 + m)) - (b*c*Sqrt[-1 + c^2*x^2]*((e^3*(f*x)^(6 + m)*Sqrt[-1 + c^2*x^2]
)/(c^2*f^5*(7 + m)^2) + ((e^2*(3*c^2*d*(7 + m)^2 + e*(30 + 11*m + m^2))*(f
*x)^(4 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f^3*(5 + m)^2*(7 + m)) + ((e*(3*c^2*d
*e*(7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 +
 342*m + 119*m^2 + 18*m^3 + m^4))*(f*x)^(2 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f
*(3 + m)^2*(5 + m)*(7 + m)) + (((c^4*d^3*(5 + m)*(7 + m))/(1 + m) + (e*(2
+ m)*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2
 + e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4)))/(c^2*(3 + m)^2*(5 + m)*(7
+ m)))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (
4 + m)/2, c^2*x^2])/(f*(2 + m)*Sqrt[-1 + c^2*x^2]))/(c^2*(5 + m)))/(c^2*(7
 + m))))/(f*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2113

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2340

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6373

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b \operatorname{arccosh}(cx))\, dx$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\, dx = \int \left(ex^2+d\right)^3 (b \operatorname{arcosh}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\mathrm{arccosh}(cx))\, dx = \int \left(ex^2+d\right)^3 (b\,\mathrm{arcosh}\,(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + ((m^3 + 9*m^2 + 23*m +
 15)*b*e^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5 + 3*(m^3
 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3
*f^m*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^4 + 16*m^3 + 86*m^2
+ 176*m + 105) + integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c*e^3*f^m*x^7 + 3*
(m^3 + 11*m^2 + 31*m + 21)*b*c*d*e^2*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35
)*b*c*d^2*e*f^m*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*c*d^3*f^m*x)*x^m/((m^4
 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 - (m^4 + 16*m^3 + 86*m^2 + 176*m
 + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^
3 - 86*m^2 - 176*m - 105)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate(((m
^3 + 9*m^2 + 23*m + 15)*b*c^2*e^3*f^m*x^8 + 3*(m^3 + 11*m^2 + 31*m + 21)*b
*c^2*d*e^2*f^m*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^2*e*f^m*x^4 + (m
^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2 +
 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105), x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\mathrm{arccosh}(cx))\, dx = \int \left(ex^2+d\right)^3 (b\,\mathrm{arcosh}\,(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \int (a+b\,\text{acosh}(c\,x))\,(f\,x)^m\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^3,x)
```

output
```
int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^3, x)
```

Reduce [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\text{arccosh}(cx))\,dx = \text{Too large to display}$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*acosh(c*x)),x)
```

output
```
(f**m*(x**m*a*d**3*m**3*x + 15*x**m*a*d**3*m**2*x + 71*x**m*a*d**3*m*x + 1
05*x**m*a*d**3*x + 3*x**m*a*d**2*e*m**3*x**3 + 39*x**m*a*d**2*e*m**2*x**3
+ 141*x**m*a*d**2*e*m*x**3 + 105*x**m*a*d**2*e*x**3 + 3*x**m*a*d*e**2*m**3
*x**5 + 33*x**m*a*d*e**2*m**2*x**5 + 93*x**m*a*d*e**2*m*x**5 + 63*x**m*a*d
*e**2*x**5 + x**m*a*e**3*m**3*x**7 + 9*x**m*a*e**3*m**2*x**7 + 23*x**m*a*e
**3*m*x**7 + 15*x**m*a*e**3*x**7 + int(x**m*acosh(c*x)*x**6,x)*b*e**3*m**4
 + 16*int(x**m*acosh(c*x)*x**6,x)*b*e**3*m**3 + 86*int(x**m*acosh(c*x)*x**
6,x)*b*e**3*m**2 + 176*int(x**m*acosh(c*x)*x**6,x)*b*e**3*m + 105*int(x**m
*acosh(c*x)*x**6,x)*b*e**3 + 3*int(x**m*acosh(c*x)*x**4,x)*b*d*e**2*m**4 +
 48*int(x**m*acosh(c*x)*x**4,x)*b*d*e**2*m**3 + 258*int(x**m*acosh(c*x)*x*
*4,x)*b*d*e**2*m**2 + 528*int(x**m*acosh(c*x)*x**4,x)*b*d*e**2*m + 315*int
(x**m*acosh(c*x)*x**4,x)*b*d*e**2 + 3*int(x**m*acosh(c*x)*x**2,x)*b*d**2*e
*m**4 + 48*int(x**m*acosh(c*x)*x**2,x)*b*d**2*e*m**3 + 258*int(x**m*acosh(
c*x)*x**2,x)*b*d**2*e*m**2 + 528*int(x**m*acosh(c*x)*x**2,x)*b*d**2*e*m +
315*int(x**m*acosh(c*x)*x**2,x)*b*d**2*e + int(x**m*acosh(c*x),x)*b*d**3*m
**4 + 16*int(x**m*acosh(c*x),x)*b*d**3*m**3 + 86*int(x**m*acosh(c*x),x)*b*
d**3*m**2 + 176*int(x**m*acosh(c*x),x)*b*d**3*m + 105*int(x**m*acosh(c*x),
x)*b*d**3))/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

3.413 $\int (fx)^m \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx)) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 322

$$\begin{aligned}
&\int (fx)^m \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx)) \, dx \\
&= -\frac{be(2c^2d(5+m)^2+e(12+7m+m^2))\,(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{c^3f^2(3+m)^2(5+m)^2} \\
&\quad -\frac{be^2(fx)^{4+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^4(5+m)^2}+\frac{d^2(fx)^{1+m}(a+b\text{arccosh}(cx))}{f(1+m)} \\
&\quad +\frac{2de(fx)^{3+m}(a+b\text{arccosh}(cx))}{f^3(3+m)}+\frac{e^2(fx)^{5+m}(a+b\text{arccosh}(cx))}{f^5(5+m)} \\
&\quad -\frac{b\left(\frac{c^4d^2(3+m)(5+m)}{1+m}+\frac{e(2+m)(2c^2d(5+m)^2+e(12+7m+m^2))}{(3+m)(5+m)}\right)(fx)^{2+m}\sqrt{1-cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2}\right.}{c^3f^2(2+m)(3+m)(5+m)\sqrt{-1+cx}}
\end{aligned}$$

output

```
-b*e*(2*c^2*d*(5+m)^2+e*(m^2+7*m+12))*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/c^3/f^2/(3+m)^2/(5+m)^2-b*e^2*(f*x)^(4+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c/f^4/(5+m)^2+d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)
*(a+b*arccosh(c*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)
-b*(c^4*d^2*(3+m)*(5+m)/(1+m)+e*(2+m)*(2*c^2*d*(5+m)^2+e*(m^2+7*m+12))/(3+
m)/(5+m))*(f*x)^(2+m)*(-c*x+1)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^
2*x^2)/c^3/f^2/(2+m)/(3+m)/(5+m)/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.91

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\operatorname{arccosh}(cx))\,dx$$
$$=x(fx)^m\left(\frac{d^2(a+b\operatorname{arccosh}(cx))}{1+m}+\frac{2dex^2(a+b\operatorname{arccosh}(cx))}{3+m}+\frac{e^2x^4(a+b\operatorname{arccosh}(cx))}{5+m}-\frac{bcd^2x\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcdex^3\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},c^2x^2\right)}{(12+7m+m^2)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bce^2x^5\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6+m}{2},\frac{8+m}{2},c^2x^2\right)}{(5+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
x*(f*x)^m*((d^2*(a + b*ArcCosh[c*x]))/(1 + m) + (2*d*e*x^2*(a + b*ArcCosh[
c*x]))/(3 + m) + (e^2*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (b*c*d^2*x*Sqrt[
1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 +
3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*e*x^3*Sqrt[1 - c^2*x^2
]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7*m + m^2)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*e^2*x^5*Sqrt[1 - c^2*x^2]*Hypergeome
tric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6373, 27, 1905, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (fx)^m (a+b\text{arccosh}(cx))\,dx$$

$$\Big\downarrow \ 6373$$

$$-bc\int \frac{(fx)^{m+1}\left(\frac{e^2x^4}{m+5}+\frac{2dex^2}{m+3}+\frac{d^2}{m+1}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}$$

$$\Big\downarrow \ 27$$

$$-\frac{bc\int \frac{(fx)^{m+1}\left(\frac{e^2x^4}{m+5}+\frac{2dex^2}{m+3}+\frac{d^2}{m+1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}$$

$$\Big\downarrow \ 1905$$

$$-\frac{bc\sqrt{c^2x^2-1}\int \frac{(fx)^{m+1}\left(\frac{e^2x^4}{m+5}+\frac{2dex^2}{m+3}+\frac{d^2}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}$$

$$\Big\downarrow \ 1590$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\int \frac{(fx)^{m+1}\left(\frac{c^2(m+5)d^2}{m+1}+\frac{e\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)x^2}{(m+3)(m+5)}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+5)}+\frac{e^2\sqrt{c^2x^2-1}(fx)^{m+4}}{c^2f^3(m+5)^2}\right)}{f\sqrt{cx-1}\sqrt{cx+1}}+\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}$$

$$\Big\downarrow \ 363$$

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\left(\frac{c^2d^2(m+5)}{m+1}+\frac{e(m+2)\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{c^2(m+3)^2(m+5)}\right)\int\frac{(fx)^{m+1}}{\sqrt{c^2x^2-1}}dx+\frac{e\sqrt{c^2x^2-1}(fx)^{m+2}\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{c^2f(m+3)^2(m+5)}}{c^2(m+5)}+\cdots}{f\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}$$

$\Big\downarrow$ 279

$$-\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\sqrt{1-c^2x^2}\left(\frac{c^2d^2(m+5)}{m+1}+\frac{e(m+2)\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{c^2(m+3)^2(m+5)}\right)\int\frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\frac{e\sqrt{c^2x^2-1}(fx)^{m+2}\left(2c^2d(m+5)^2+e\left(m^2+7m+1\cdots\right)\right)}{c^2f(m+3)^2(m+5)}}{c^2(m+5)}\cdots\right)}{f\sqrt{cx-1}\sqrt{cx+1}}$$

$$\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}$$

$\Big\downarrow$ 278

$$\frac{d^2(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\text{arccosh}(cx))}{f^5(m+5)}-$$

$$\frac{bc\sqrt{c^2x^2-1}\left(\frac{\frac{\sqrt{1-c^2x^2}(fx)^{m+2}\left(\frac{c^2d^2(m+5)}{m+1}+\frac{e(m+2)\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{c^2(m+3)^2(m+5)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f(m+2)\sqrt{c^2x^2-1}}+\frac{e\sqrt{c^2x^2-1}(fx)\cdots}{\cdots}}{c^2(m+5)}\cdots\right)}{f\sqrt{cx-1}\sqrt{cx+1}}$$

input
```
Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m
)*(a + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCosh[
c*x]))/(f^5*(5 + m)) - (b*c*Sqrt[-1 + c^2*x^2]*((e^2*(f*x)^(4 + m)*Sqrt[-1
 + c^2*x^2])/(c^2*f^3*(5 + m)^2) + ((e*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m +
m^2))*(f*x)^(2 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f*(3 + m)^2*(5 + m)) + (((c^2
*d^2*(5 + m))/(1 + m) + (e*(2 + m)*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2)
))/(c^2*(3 + m)^2*(5 + m)))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric
2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f*(2 + m)*Sqrt[-1 + c^2*x^2]))/(
c^2*(5 + m))))/(f*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 (a+b\,\mathrm{arccosh}\,(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\text{arccosh}(cx)) \, dx = \int (ex^2+d)^2 (b \operatorname{arcosh}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2)^2 (a+b\text{arccosh}(cx)) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\text{arccosh}(cx)) \, dx = \int (ex^2+d)^2 (b \operatorname{arcosh}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) + ((m^2 + 4*m + 3)*b*e^2*f^m*x^5 + 2*(m^2 + 6*m + 5)*b*d*e
*f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(((m^2 + 4*m + 3)*b*c*e^2*f
^m*x^5 + 2*(m^2 + 6*m + 5)*b*c*d*e*f^m*x^3 + (m^2 + 8*m + 15)*b*c*d^2*f^m*
x)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^3*x^3 - (m^3 + 9*m^2 + 23*m + 15)*c*x
+ ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 15)*sqrt(c*x +
 1)*sqrt(c*x - 1)), x) - integrate(((m^2 + 4*m + 3)*b*c^2*e^2*f^m*x^6 + 2*
(m^2 + 6*m + 5)*b*c^2*d*e*f^m*x^4 + (m^2 + 8*m + 15)*b*c^2*d^2*f^m*x^2)*x^
m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 15), x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx=\int (ex^2+d)^2(b\,\text{arcosh}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\text{arccosh}(cx))\,dx=\int (a+b\,\text{acosh}(c\,x))\,(f\,x)^m\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^2,x)
```

output
```
int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\operatorname{arccosh}(cx))\,dx$$
$$= \frac{f^m \left(x^m a\, d^2 m^2 x + 8x^m a\, d^2 m x + 15x^m a\, d^2 x + 2x^m a d e\, m^2 x^3 + 12x^m a d e m\, x^3 + 10x^m a d e\, x^3 + x^m a\, e^2 m^2 x\right.}{}$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*acosh(c*x)),x)
```

output
```
(f**m*(x**m*a*d**2*m**2*x + 8*x**m*a*d**2*m*x + 15*x**m*a*d**2*x + 2*x**m*
a*d*e*m**2*x**3 + 12*x**m*a*d*e*m*x**3 + 10*x**m*a*d*e*x**3 + x**m*a*e**2*
m**2*x**5 + 4*x**m*a*e**2*m*x**5 + 3*x**m*a*e**2*x**5 + int(x**m*acosh(c*x
)*x**4,x)*b*e**2*m**3 + 9*int(x**m*acosh(c*x)*x**4,x)*b*e**2*m**2 + 23*int
(x**m*acosh(c*x)*x**4,x)*b*e**2*m + 15*int(x**m*acosh(c*x)*x**4,x)*b*e**2
+ 2*int(x**m*acosh(c*x)*x**2,x)*b*d*e*m**3 + 18*int(x**m*acosh(c*x)*x**2,x
)*b*d*e*m**2 + 46*int(x**m*acosh(c*x)*x**2,x)*b*d*e*m + 30*int(x**m*acosh(
c*x)*x**2,x)*b*d*e + int(x**m*acosh(c*x),x)*b*d**2*m**3 + 9*int(x**m*acosh
(c*x),x)*b*d**2*m**2 + 23*int(x**m*acosh(c*x),x)*b*d**2*m + 15*int(x**m*ac
osh(c*x),x)*b*d**2))/(m**3 + 9*m**2 + 23*m + 15)
```

3.414 $\int (fx)^m (d+ex^2) (a+b\text{arccosh}(cx)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\int (fx)^m \left(d+ex^2\right) (a+b\text{arccosh}(cx)) \, dx = -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}(a+b\text{arccosh}(cx))}{f(1+m)} + \frac{e(fx)^{3+m}(a+b\text{arccosh}(cx))}{f^3(3+m)} - \frac{b(e(1+m)(2+m)+c^2d(3+m)^2)\,(fx)^{2+m}\sqrt{1-cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{cf^2(1+m)(2+m)(3+m)^2\sqrt{-1+cx}}$$

output
```
-b*e*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/f^2/(3+m)^2+d*(f*x)^(1+m)*(
a+b*arccosh(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccosh(c*x))/f^3/(3+m)-b*(e*
(1+m)*(2+m)+c^2*d*(3+m)^2)*(f*x)^(2+m)*(-c*x+1)^(1/2)*hypergeom([1/2, 1+1/
2*m],[2+1/2*m],c^2*x^2)/c/f^2/(1+m)/(2+m)/(3+m)^2/(c*x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.01

$$\int (fx)^m \left(d+ex^2\right) (a+b\text{arccosh}(cx)) \, dx$$
$$= x(fx)^m \left(-\frac{bcdx\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\frac{(d(3+m)+e(1+m)x^2)(a+b\text{arccosh}(cx))}{1+m} - \frac{bcex^3\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},c^2x^2\right)}{(4+m)\sqrt{-1+cx}\sqrt{1+cx}}}{3+m}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
x*(f*x)^m*(-((b*c*d*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2,
(4 + m)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (((
d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcCosh[c*x]))/(1 + m) - (b*c*e*x^3*Sqrt
[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((4 +
 m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(3 + m))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6371, 960, 136, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right) (fx)^m (a+b\text{arccosh}(cx)) \, dx$$
$$\Big\downarrow \; 6371$$
$$-\frac{bc\int \frac{(fx)^{m+1}\left(e(m+1)x^2+d(m+3)\right)}{\sqrt{cx-1}\sqrt{cx+1}}dx}{f\left(m^2+4m+3\right)} + \frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)} + \frac{e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}$$

$$\Big\downarrow \; 960$$

$$-\frac{bc\left(\left(\frac{e(m+1)(m+2)}{c^2(m+3)}+d(m+3)\right)\int \frac{(fx)^{m+1}}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{e(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{c^2f(m+3)}\right)}{f\left(m^2+4m+3\right)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}$$

$$\Big\downarrow \; 136$$

$$-\frac{bc\left(\frac{\sqrt{c^2x^2-1}\left(\frac{e(m+1)(m+2)}{c^2(m+3)}+d(m+3)\right)\int \frac{(fx)^{m+1}}{\sqrt{c^2x^2-1}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{e(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{c^2f(m+3)}\right)}{f\left(m^2+4m+3\right)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}$$

$$\Big\downarrow \; 279$$

$$-\frac{bc\left(\frac{\sqrt{1-c^2x^2}\left(\frac{e(m+1)(m+2)}{c^2(m+3)}+d(m+3)\right)\int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{e(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{c^2f(m+3)}\right)}{f\left(m^2+4m+3\right)}+\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}$$

$$\Big\downarrow \; 278$$

$$\frac{d(fx)^{m+1}(a+b\text{arccosh}(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\text{arccosh}(cx))}{f^3(m+3)}-\frac{bc\left(\frac{\sqrt{1-c^2x^2}(fx)^{m+2}\left(\frac{e(m+1)(m+2)}{c^2(m+3)}+d(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{e(m+1)\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}}{c^2f(m+3)}\right)}{f\left(m^2+4m+3\right)}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a +
 b*ArcCosh[c*x]))/(f^3*(3 + m)) - (b*c*((e*(1 + m)*(f*x)^(2 + m)*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(c^2*f*(3 + m)) + (((e*(1 + m)*(2 + m))/(c^2*(3 + m))
 + d*(3 + m))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, c^2*x^2])/(f*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(f*(
3 + 4*m + m^2))
```

Defintions of rubi rules used

rule 136
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^Fr
acPart[m])   Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b, c, d, f,
 m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6371
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (Simp[e*(f*x)^(m + 3)*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x] - Sim
p[b*(c/(f*(m + 1)*(m + 3)))   Int[(f*x)^(m + 1)*((d*(m + 3) + e*(m + 1)*x^2
)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Maple [F]

$$\int (fx)^m \left(e\,x^2 + d\right) \left(a + b \operatorname{arccosh}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d + ex^2\right) \left(a + b\operatorname{arccosh}(cx)\right) dx = \int \left(ex^2 + d\right)\left(b \operatorname{arcosh}\left(cx\right) + a\right)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d + ex^2\right) \left(a + b\operatorname{arccosh}(cx)\right) dx = \int (fx)^m \left(a + b \operatorname{acosh}\left(cx\right)\right) \left(d + ex^2\right) \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (fx)^m (d+ex^2) (a+b\text{arccosh}(cx)) \, dx = \int (ex^2+d)(b\,\text{arcosh}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

output
```
a*e*f^m*x^3*x^m/(m + 3) + (b*e*f^m*(m + 1)*x^3 + b*d*f^m*(m + 3)*x)*x^m*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^2 + 4*m + 3) + (f*x)^(m + 1)*a*d/(
f*(m + 1)) + integrate((b*c*e*f^m*(m + 1)*x^3 + b*c*d*f^m*(m + 3)*x)*x^m/(
(m^2 + 4*m + 3)*c^3*x^3 - (m^2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^2*x^2 -
 m^2 - 4*m - 3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate((b*c^2*e*f^m*
(m + 1)*x^4 + b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2*x^2 - m^2
- 4*m - 3), x)
```

Giac [F]

$$\int (fx)^m (d+ex^2) (a+b\text{arccosh}(cx)) \, dx = \int (ex^2+d)(b\,\text{arcosh}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2) (a+b\text{arccosh}(cx)) \, dx = \int (a+b\,\text{acosh}(c\,x))\, (f\,x)^m \, (e\,x^2+d) \, dx$$

input
```
int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2),x)
```

output
```
int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2), x)
```

Reduce [F]

$$\int (fx)^m (d+ex^2) (a+b\text{arccosh}(cx)) \, dx$$
$$= \frac{f^m \left(x^m a d m x + 3x^m a d x + x^m a e m \, x^3 + x^m a e \, x^3 + \left(\int x^m acosh(cx) \, x^2 dx\right) b e \, m^2 + 4\left(\int x^m acosh(cx) \, x^2 dx\right.\right.}{}$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*acosh(c*x)),x)
```

output
```
(f**m*(x**m*a*d*m*x + 3*x**m*a*d*x + x**m*a*e*m*x**3 + x**m*a*e*x**3 + int
(x**m*acosh(c*x)*x**2,x)*b*e*m**2 + 4*int(x**m*acosh(c*x)*x**2,x)*b*e*m +
3*int(x**m*acosh(c*x)*x**2,x)*b*e + int(x**m*acosh(c*x),x)*b*d*m**2 + 4*in
t(x**m*acosh(c*x),x)*b*d*m + 3*int(x**m*acosh(c*x),x)*b*d))/(m**2 + 4*m +
3)
```

3.415 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2} \, dx$$

$$\Big\downarrow \text{ 6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\ \text{arccosh}\,(cx))}{e\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 18.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(fx)^m\,(a+b\operatorname{acosh}(cx))}{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d),x)
```

output
```
Integral((f*x)**m*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{d+ex^2}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{d + ex^2} \, dx = \int \frac{(b\,\text{arcosh}\,(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{d + ex^2} \, dx = \int \frac{(a + b\,\text{acosh}(c\,x))\,(f\,x)^m}{e\,x^2 + d} \, dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2),x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{(fx)^m(a+b\operatorname{arccosh}(cx))}{d+ex^2}\,dx = f^m\left(\left(\int \frac{x^m}{e\,x^2+d}dx\right)a+\left(\int \frac{x^m\,acosh(cx)}{e\,x^2+d}dx\right)b\right)$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(e*x^2+d),x)
```

output
```
f**m*(int(x**m/(d + e*x**2),x)*a + int((x**m*acosh(c*x))/(d + e*x**2),x)*b
)
```

3.416 $\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2} dx = \text{Int}\left(\frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}, x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2} dx = \int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2} dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2} \, dx$$

$$\Big\downarrow \; 6375$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\,\text{arccosh}\,(cx))}{(e\,x^2+d)^2}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(a+b\operatorname{acosh}(c\,x))\,(f\,x)^m}{(e\,x^2+d)^2}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^2,x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.83

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^2}\,dx=f^m\left(\left(\int \frac{x^m}{e^2x^4+2de\,x^2+d^2}dx\right)a+\left(\int \frac{x^m\,acosh(cx)}{e^2x^4+2de\,x^2+d^2}dx\right)b\right)$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(e*x^2+d)^2,x)
```

output
```
f**m*(int(x**m/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*a + int((x**m*acosh(c*x)
)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b)
```

3.417 $\int \frac{(fx)^m(a+b\,\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m(a+b\,\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \text{Int}\left(\frac{(fx)^m(a+b\,\text{arccosh}(cx))}{(d+ex^2)^3},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\,\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(fx)^m(a+b\,\text{arccosh}(cx))}{(d+ex^2)^3}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{6375}$$

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^3}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\,\text{arccosh}\,(cx))}{(e\,x^2+d)^3}dx$$

input
```
int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

output
```
int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d + ex^2)^3}\,dx = \int \frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{(ex^2 + d)^3}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccosh(c*x) + a)*(f*x)^m/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d + ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a + b\text{arccosh}(cx))}{(d + ex^2)^3}\,dx = \int \frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{(ex^2 + d)^3}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arcosh}\,(cx)+a)(fx)^m}{(ex^2+d)^3}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(a+b\,\text{acosh}(c\,x))\,(f\,x)^m}{(e\,x^2+d)^3}\,dx$$

input
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^3,x)
```

output
```
int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^3, x)
```

Reduce [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int \frac{(fx)^m(a+b\text{arccosh}(cx))}{(d+ex^2)^3}\,dx = f^m\left(\left(\int \frac{x^m}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)a + \left(\int \frac{x^m\,acosh(cx)}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\right)$$

input
```
int((f*x)^m*(a+b*acosh(c*x))/(e*x^2+d)^3,x)
```

output
```
f**m*(int(x**m/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*a + i
nt((x**m*acosh(c*x))/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)
*b)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file